

# NUMERICAL STUDY OF WAVE TRANSFORMATION USING THE FREE SURFACE RECONSTRUCTION METHOD

ANKIT AGGARWAL\*, MAYILVAHANAN ALAGAN CHELLA\* , HANS  
BIHS\*, CSABA PÁKZODI†, ØIVIND ASGEIR ARNTSEN\*

\*Department of Civil and Environmental Engineering  
Norwegian University of Science and Technology  
Web page: <http://www.reef3d.com/>

†SINTEF Ocean  
Trondheim, Norway

**Key words:** Reconstruction Method, Irregular Waves, CFD, Free Surface Elevation

**Abstract.** The study of irregular wave field is complex due to its random hydrodynamic characteristics. Many experimental studies have been performed in the past to study irregular waves. However, numerical investigations are less time consuming and expensive as compared to the experimental studies. For a good validation of the numerical model, it is essential to reproduce the laboratory waves numerically. The reconstruction of the numerical irregular free surface elevation is necessary because the paddle signal for the wave-maker in experiments is unknown in most of the cases. It is quite challenging to reconstruct the time history of free surface elevation of irregular waves because of the random wave phases and wave periods. In the present work, a numerical investigation is performed using the open-source computational fluid dynamics (CFD) model REEF3D to test and validate the reconstruction of free surface profiles for irregular wave propagation. Two-dimensional irregular waves are generated by super-positioning of the regular wave components. In the current reconstruction approach, the free surface is reconstructed by representing the irregular free surface elevation as a summation of its Fourier components. First, the free surface reconstruction method is tested for irregular waves in a two-dimensional wave tank with constant water depth. The reconstructed free surface elevations shows a good match with the theoretical wave profiles. Further, the method is used to reconstruct the wave transformation over an impermeable fully submerged bar where the complex phenomena such as shoaling and wave breaking occur. The reconstructed numerical free surface elevations along the wave tank are compared with the experimental free surface elevations. The complex phenomena such as shoaling and breaking are represented with reasonable accuracy in the numerical model.

## 1 INTRODUCTION

Offshore structures like offshore wind turbines are exposed to irregular breaking wave loads. Offshore engineering applications require a detailed study of the wave hydrodynamics for a safer

design. Thus, the study of irregular breaking waves is important for the associated loads. The complexity involved in the different stages of breaking makes it difficult to study and investigate. Many experimental investigations have been performed to study the transformations of irregular waves in shallow and deep water. Vincet et al. [1] investigated the transformation of monochromatic and directional irregular waves passing over a submerged elliptical mound in a controlled laboratory experiment. Beji et al. [2] investigated the irregular wave propagation over a submerged bar. They examined the wave transformation processes such as wave shoaling, breaking, post-breaking and deshoaling under irregular waves. Ting et al. [3] studied the wave and turbulence characteristics in broad-banded irregular waves over a slope. They also investigated the probability distributions for the wave height and peak velocities for the irregular breaking waves.

Computational fluid dynamics (CFD) can be used as an effective tool to investigate the phenomenon like wave breaking in detail. Many researchers in the past used CFD based two phase flow models for simulating breaking waves [4][5][6][7][8]. The complex dynamics evolve during the breaking process such as the formation of overturning wave crests, enclosed air-packet and the splash-up phenomena. Their results revealed important flow structures in breaking waves. Bredmose et al. [9] investigated breaking wave impacts on monopiles with the focused wave groups using CFD. They compared the numerical and theoretical free surface in time-domain by using the linear reconstruction of waves. However, their numerical model under predicts the irregular wave surface. Also, the phases were not correctly represented by the numerical results. The goal of the present work is to study and validate the reconstruction method for free surface with non-breaking and breaking irregular waves. The investigation is performed is for irregular waves propagating over a submerged bar using REEF3D [10]. The numerical free surface elevations are compared with experimental measurements for different wave gauge locations over the bar. The free surface elevation is reconstructed using the Fast Fourier Transformation (FFT) [11]. The numerical model is able to simulate the wave transformation process including shoaling during the different stages of breaking.

## 2 NUMERICAL MODEL

The present numerical model is based on the governing equations of fluid dynamics: continuity equation and the Reynolds Averaged Navier-Stokes equations (RANS) with the assumption of an incompressible fluid given as:

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (1)$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ (\nu + \nu_t) \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] + g_i \quad (2)$$

where,  $u$  is the velocity averaged over time  $t$ ,  $\rho$  is the fluid density,  $p$  is the pressure,  $\nu$  is the kinematic viscosity,  $\nu_t$  is the eddy viscosity,  $i$  and  $j$  denote the indices in  $x$  and  $y$  direction, respectively and  $g_i$  is the acceleration due to gravity.

The numerical model uses a fifth-order finite difference Weighted Essentially Non-Oscillatory (WENO) scheme in multi-space dimensions for the spatial discretization [12]. The third order TVD Runge Kutta scheme is used for the time discretization [13]. An adaptive time stepping

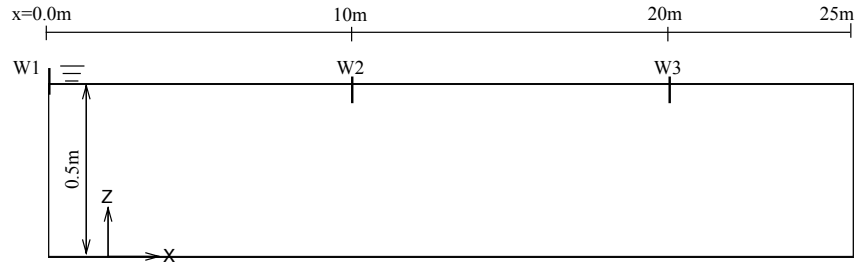


Figure 1: Setup of the numerical wave tank (side view)

scheme is used in the numerical model [14]. The present study uses the  $k - \omega$  model [15] along with the Reynolds Averaged Navier Stokes (RANS) equation. The level set method is used to capture the free surface [16]. Detailed information about the numerical model can be obtained in Bihs et al. [10]. REEF3D has been used in the past for a wide range of marine applications, such as wave-structure interaction [17], breaking wave forces [18], floating body dynamics [19] and sediment transport [20]. For the reconstruction of irregular waves, the Fast Fourier Transformation (FFT) algorithm is used. The wave amplitudes ( $A_k$ ), angular frequencies ( $\omega_k$ ) and the phase angles ( $\epsilon_k$ ) are computed for the target irregular wave train at the wave generation using FFT. A time series of free surface elevations can be written as a summation of the Fourier components [11]:

$$\eta(t) = \sum_{k=1}^N C_k e^{ik\omega t} \quad (3)$$

where,  $C_k$  denotes the Fourier coefficients.

The computed wave amplitudes ( $A_k$ ), angular frequencies ( $\omega_k$ ) and the phase angles ( $\epsilon_k$ ) are given as an input to the numerical model. The first-order irregular waves are generated by the super-positioning of the linear regular waves components [21][22].

### 3 VALIDATION WITH THEORY

#### 3.1 Computational setup

The numerical model is tested and validated by comparing the numerically reconstructed wave free surface elevation and the theoretical free surface elevation. The numerical simulations are performed in a two-dimensional (2D) numerical wave tank (NWT) without any structures as shown in Fig. 1. The NWT which is 25 m long and 1 m high with a water depth of 0.5 m is used in the simulations. Three wave gauges are placed along the NWT to study the numerical wave propagation and changes in the wave surface elevation and the numerical results are compared with the theoretical results.

The numerical tests are conducted for three different grid sizes  $dx = 0.10$  m, 0.05 m and 0.025 m for the grid refinement study. The irregular waves are generated with a significant wave height  $H_s = 0.054$  m and a peak period  $T_p = 2.5$  s. Fig. 2 presents a comparison of the numerical

and theoretical irregular wave free surface elevation for three different grid sizes  $dx = 0.10$  m,  $0.05$  m and  $0.025$  m at different wave gauge locations. It is observed that for the wave gauge located next to the wavemaker (W1), phase differences are observed between the numerical and theoretical results with  $dx = 0.10$  m (Fig. 2(a)). These phase differences are slightly reduced for the grid size  $dx = 0.05$  m and they are reduced to almost zero for the grid size  $dx = 0.025$  m. The numerical wave crests and troughs are in a good agreement with the theoretical results for all grid sizes.

The wave propagation over constant water depth is studied with two more wave gauges along the NWT. The numerical results show some phase differences with the theory for the wave gauge located at  $x = 10$  m (W2) and  $20$  m (W3) for  $dx = 0.10$  m and  $0.05$  m. With the grid size  $dx = 0.025$  m, the theoretical waves are accurately captured with the correct phases in the numerical simulations (Figs. 2(c) and 2(e)). Figs. 2(b), (d) and (f) present the irregular wave surface elevation for a longer time series at three different wave gauge locations with the grid size  $dx = 0.025$  m. It is noticed that the numerical model is able to accurately reproduce the theoretical irregular wave train (Figs. 2(b)(W1), 2(d)(W2) and (f)(W3)). The wave generation, wave propagation and the numerical wave crests and wave troughs are computed with good accuracy. Thus, it can be concluded that the numerical model is able to accurately reconstruct the irregular waves. Fig. 3 presents the simulated free surface changes with velocity magnitude (m/s) variation during the propagation of the irregular wave train. A scaled up view of the wave free surface elevation shows the irregularity in the wave train.

## 4 IRREGULAR BREAKING WAVES OVER A SUBMERGED BAR

### 4.1 Setup of the numerical wave tank

In this section, the numerical simulations are performed in a 2D NWT to study the wave propagation and transformation of irregular breaking waves over a submerged bar. The numerical results for the free surface elevation are compared with the experimental results in time-domain. [2]. The numerical wave tank is  $22$  m long and  $0.8$  m high with a water depth of  $0.4$  m. A submerged trapezoidal bar with a weather side slope of  $1:20$  and  $2$  m horizontal crest followed by a  $1:10$  lee side slope is placed at a distance of  $6$  m from the wave generation. The JONSWAP spectrum is used for irregular wave generation in the experiments. The numerical setup is shown in Fig. 4. In the numerical setup, three different wave gauges are placed along the numerical wave tank (NWT) at  $x = 6.0$  m,  $11$  m and  $13$  m. The numerical tests are performed with input of significant wave height  $H_s = 0.054$  m and peak time period  $T_p = 2.5$  s.

### 4.2 Results

Fig. 5 presents the comparison of the numerical and experimental wave free surface elevation over time at different wave gauge locations. Numerical irregular wave train is in a good agreement with the experimental data measured at the wave gauge located at the toe of weather side slope (W1) (Fig. 5(a)). When the waves propagate over the bar with decreasing water depth, it leads to shoaling. The wave crests and wave troughs show a good match with the experimental results. This complex shoaling process over a submerged bar is well captured in the numerical model as shown in Fig. 5(b) (W2). When the waves propagate over the flat part of the bar, the

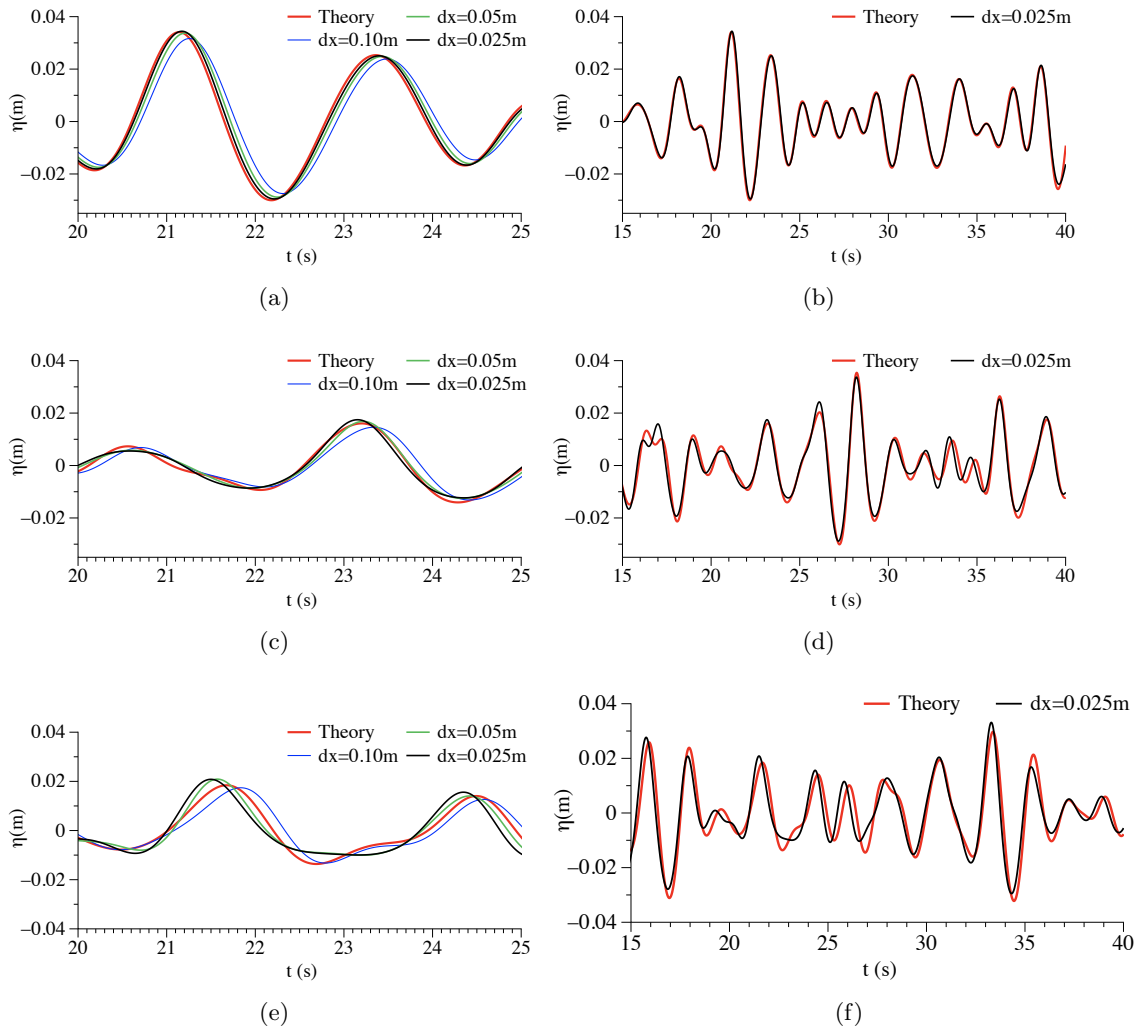


Figure 2: Comparison of numerical and experimental wave free surface elevation ( $m$ ) over time (sec) at a) W1 (shorter time series with different  $dx$ ) b) W1 (longer time series with  $dx = 0.025$  m) c) W2 (shorter time series with different  $dx$ ) d) W2 (longer time series with  $dx = 0.025$  m) e) W3 (shorter time series with different  $dx$ ) f) W3 (longer time series with  $dx = 0.025$  m)

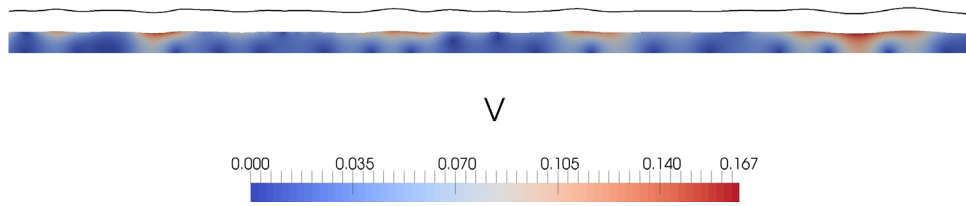


Figure 3: Simulated free surface changes with velocity magnitude (m/s) variation during the propagation of the irregular wave train

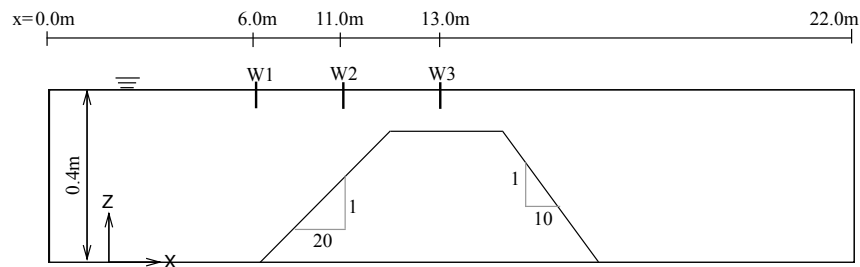


Figure 4: Setup of the numerical wave tank (side view)

water depth is significantly reduced. The wave crests become sharp and eventually the waves break (Fig. 5(c)). The numerical results show a good comparison with the experimental results for the wave gauge located at  $x = 13$  m (W3) over the flat bed.

Thus, the numerical results of present study show that the numerical model is able to reconstruct the breaking irregular waves propagating over the submerged bar accurately.

## 5 CONCLUSIONS

The numerical model REEF3D is used to validate the wave reconstruction method for irregular waves. The waves are reconstructed using the Fast Fourier Transformation (FFT). First, a grid refinement study is conducted to study the effects of grid size on numerical results. The numerical simulations are carried out in a two-dimensional (2D) wave tank without any structures. The grid refinement study evinces that the results with the grid size ( $dx = 0.025$  m) show a good agreement with the theory. The wave amplitudes and phases are correctly reproduced in the numerical model. Next, a complex case of irregular breaking waves over a submerged bar is investigated. The free surface elevations computed at three different locations along the bar are compared with the experimental results. A good match between the experimental and numerical results shows that the correct inlet wave is reproduced in the numerical model. The numerical wave amplitudes and wave phases during the wave transformation processes such as shoaling and breaking are represented in the numerical model with reasonable accuracy.

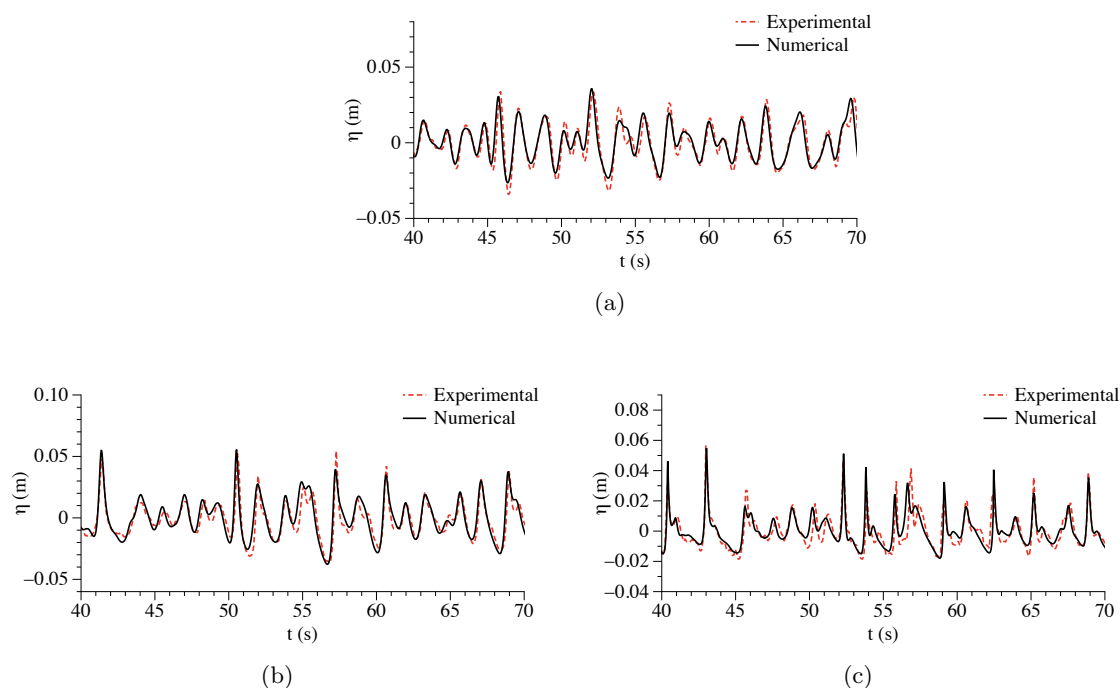


Figure 5: Comparison of numerical and experimental wave free surface elevation ( $m$ ) over time (sec) at a) W1 b) W2 c) W3

## ACKNOWLEDGEMENTS

The research work has been funded by the Research Council of Norway through the project "Hydrodynamic Loads on Offshore Wind Turbine Substructures" (project number: 246810). The authors gratefully acknowledge the computing time granted by NOTUR (project number: NN2620k).

## REFERENCES

- [1] Vincent, C. L., and Briggs, M. J., 1989. "Refraction-diffraction of irregular waves over a mound". *J. Waterway Port Coastal Ocean Engng*, **115**, pp. 269–285.
- [2] Beji, S., and Battjes, J., 1993. "Experimental investigation of wave propagation over a bar". *Coastal Engineering*, **19**, pp. 151–162.
- [3] Ting, F., 2001. "Laboratory study of wave and turbulence velocities in a broad-banded irregular wave surf zone". *Coastal Eng.*, **43**, pp. 183–208.
- [4] Lubin, P., Vincent, S., Abadie, S., and Caltagrione, J., 2006. "Three-dimensional LES simulation of air entrainment under plunging breaking waves". *Coast. Eng.*, **53**, pp. 631–655.
- [5] Christensen, E. D., 2006. "LES simulation of spilling and plunging breakers". *Coast. Eng.*, **53**, pp. 463–485.

- [6] Jacobsen, N. G., Fuhrman, D., and Fredsøe, J., 2012. “A wave generation toolbox for the open-source cfd library: Openfoam”. *International Journal for Numerical Methods in Fluids*, **70**, pp. 1073–1088.
- [7] Alagan Chella, M., Bihs, H., and Myrhaug, D., 2015. “Characteristics and profile asymmetry properties of waves breaking over an impermeable submerged reef”. *Coastal engg.*, **100**, pp. 26–36.
- [8] Alagan Chella, M., Bihs, H., Myrhaug, D., and Michael, M., 2016. “Hydrodynamic characteristics and geometric properties of plunging and spilling breakers over impermeable slopes”. *Ocean Modelling*, **103**, pp. 53–72.
- [9] Bredmose, H., and Jacobsen, G. N., 2010. “Breaking wave impacts on offshore wind turbine foundations: focused wave groups and CFD”. *Proceedings of OMAE, June 6-11*.
- [10] Bihs, H., Kamath, A., Alagan Chella, M., Aggarwal, A., and Arntsen, A., O., 2016. “A new level set numerical wave tank with improved density interpolation for complex wave hydrodynamics”. *Computers and fluids*, **140**, pp. 191–208.
- [11] Hoffmann, J. “Matlab und simulink”. *Addison Wesley Longman, Inc.*
- [12] Jiang, G. S., and Peng, D., 2000. “Weighted ENO schemes for Hamilton Jacobi equations”. *SIAM Journal of Scientific Computing*, **21**, pp. 2126–2143.
- [13] Harten, A., Engquist, B., Osher, S., and Chakravarthy, S., 1987. “Uniformly high-order accurate essentially non-oscillatory schemes iii”. *Journal of Computational Physics*, **71**, pp. 231–303.
- [14] Griebel, M., Dornseifer, T., and Neunhoffer, T., 1998. *Numerical Simulations in Fluid Dynamics*. SIAM.
- [15] Wilcox, D., 1994. “Turbulence modeling for CFD”. *DCW Industries Inc. La Canada, California*.
- [16] Osher, S., and Sethian, J. A., 1988. “Fronts propagating with curvature- dependent speed: Algorithms based on hamilton-jacobi formulations”. *Journal of Computational Physics*, **79**, pp. 12–49.
- [17] Kamath, A., Alagan Chella, M., Bihs, H., and Muskulus, M., 2015. “CFD investigations of wave interaction with a pair of large tandem cylinders”. *Ocean Engg.*, **108**, pp. 734–748.
- [18] Bihs, H., Kamath, A., Alagan Chella, M., and Arntsen, A., O., 2016. “Breaking-wave interaction with tandem cylinders under different impact scenarios”. *Journal of Waterway, Port, Coastal, and Ocean Engineering*.
- [19] Bihs, H., and Kamath, A., 2017. “A combined level set/ghost cell immersed boundary representation for simulations of floating bodies.”. *International Journal for Numerical Methods in Fluids*.



- [20] Afzal, M., Bihs, H., Kamath, A., and Arntsen. A., O., 2015. “Three dimensional numerical modeling of pier scour under current and waves using level set method.”. *Journal of Offshore Mechanics and Arctic Engineering-Transactions of The Asme*.
- [21] Aggarwal, A., Alagan Chella, M., Kamath, A., Bihs, H., and Arntsen. A., O., 2016. “Irregular wave forces on a large vertical circular cylinder”. *Energy Procedia*, **94**, pp. 504–516.
- [22] Aggarwal, A., Alagan Chella, M., Kamath, A., Bihs, H., and Arntsen. A., O., 2016. “Numerical simulation of irregular wave forces on a horizontal cylinder”. *ASME 2016 35th International Conference on Ocean, Offshore and Arctic Engineering, Volume 2: CFD and VIV*.