Non-linear MHD simulations of magnetically confined plasma using OpenFOAM

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I. Introduction

Investigation of alternative energy resources for future is utmost important topic. One of the possibilities is nuclear fusion which powers the sun, being this very attractive since it is clean, safe and virtually unlimited energy. The achievement of controlled nuclear fusion will require a wide variety of fields such as plasma physics, materials physics, electrical engineering, heat transfer, etc. The computational fluid dynamics (CFD) contribute to understand the behaviour of confined plasma and in guiding experiments. This project aims to assess the feasibility of open source software, OpenFOAM to study the physics of magnetically confined plasma, and to expect realistic modelling of fusion plasma as a long-term project objective. The OpenFOAM magnetohydrodynamics (MHD) solver has been applied to solve plasma dynamics.

II. OpenFOAM.

OpenFOAM [1] is a free and open source CFD software released and developed under the GPL license. Being free and open source implies that one can find a large experienced community and a very collaborative environment. This, together with an apparently user-friendly program, has been sufficient claim to verify the viability of this code as a potential high-performance solver for simulations close to fusion plasmas. OpenFOAM has been applied to the study of liquid metal dynamics of breeding blankets [2]. In this work, OpenFOAM is applied to magnetically confined plasma.

A. The OpenFOAM MHD solver.

The simulations of plasma dynamics which is represented by MHD equations have been performed using mhdFoam, the OpenFOAM's MHD solver based on the PISO algorithm. This code provides numerical solutions for the single-fluid non-linear visco-resistive MHD equations for the pressure-constant density-constant (also magnetic diffusivity and kinematic viscosity are constant) incompressible approach [3]:

$$\frac{\partial \mathbf{v}}{\partial t} = \mathbf{j} \wedge \mathbf{b} + \nu \nabla^2 \mathbf{v} - (\mathbf{v} \cdot \nabla)\, \mathbf{v}, \tag{1}$$

$$\frac{\partial \mathbf{b}}{\partial t} = (\mathbf{b} \cdot \nabla)\, \mathbf{v} - (\mathbf{v} \cdot \nabla)\, \mathbf{b} + \lambda \nabla^2 \mathbf{b}, \tag{2}$$

$$\nabla \cdot \mathbf{b} = \nabla \cdot \mathbf{v} = 0; \tag{3}$$

where $\mathbf{j} = \nabla \wedge \mathbf{b}$ is the charge current density. Here, all variables are normalized to Alfvén units. The dimensionless kinematic viscosity, ν, and magnetic diffusivity, λ, are the inverse of Reynolds and magnetic Reynolds number, respectively.

The mhdFoam solver has been extended to include an external magnetic field by presenting two part; a fixed part which is time-independent and a perturbation part which evolves by time,

$$\mathbf{b}\,(t, \mathbf{r}) = \mathbf{b}_{\text{Fixed}}\,(\mathbf{r}) + \mathbf{b}_{\text{Vble}}\,(t, \mathbf{r})\,. \tag{4}$$

The equations on which resistiveMhdFoam (the modified mhdFoam) works are the result of replacing (4) in (1)-(3).

B. Benchmark with Orszag-Tang test.

The Orszag-Tang test [4] has been performed to assess the robustness of the OpenFOAM code for magnetohydrodynamics, mhdFoam, to benchmark with [5]. This is a simple and classic test to check the validity of MHD codes based on simple, non-random and periodic initial conditions (that is, at $t = 0$). The time evolution of the magnetic and the velocity fields are calculated in a cube (whose size is $2\pi \times 2\pi \times 2\pi$) with periodic boundary conditions and without external sources. The MHD simulations have been studied for three cases; $\nu = \lambda = 0.01$ with $N^3 = 64^3$ cells (using 2 CPUs, $\sim$ 2 hours of wall-time), $\nu = \lambda = 0.005$ with $N^3 = 128^3$ cells (2 CPUs, $\sim$ 2 days of wall-time) and $\nu = \lambda = 0.001$ with $N^3 = 256^3$ cells (8 CPUs, $\sim$ 1 month of wall-time). First and second order discretization's were used for the temporal and spatial derivatives, respectively.

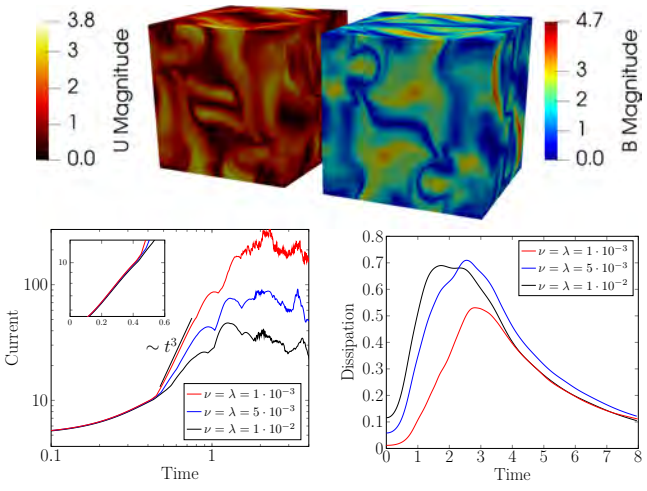

Fig. 1: Top panel, velocity and magnetic fields profiles for $\nu = \lambda = 0.01$ in the cube domain at $t = 2$. Bottom panel, time evolution of the maximum charge current density (left) and energy dissipation (right) has been benchmarked with [5].

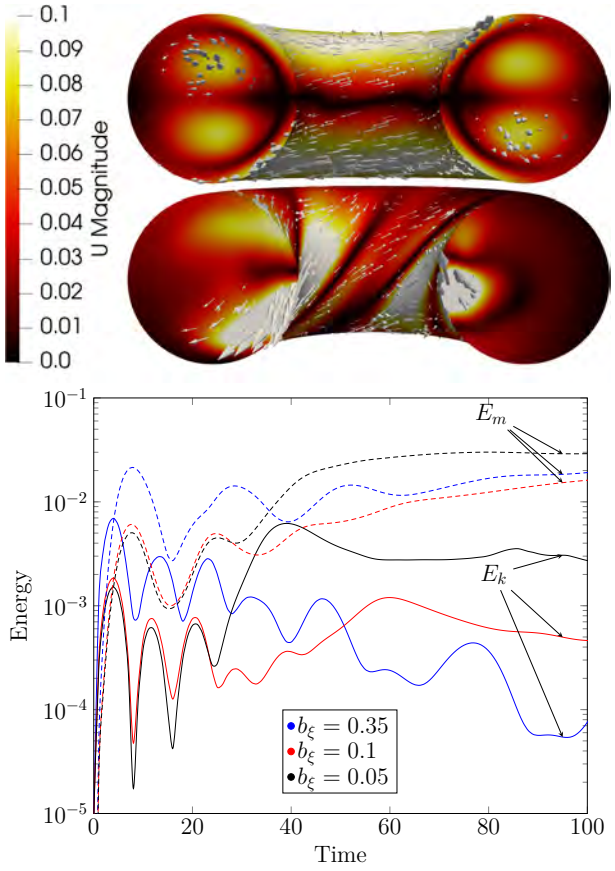

Fig. 2: Top panel, contour plot of velocity profiles for $b_\theta = 0.35$, $\nu = \lambda = 0.002$ and $b_\xi = 0.35$ at $t = 30$, $b_\xi = 0.1$ at $t = 70$. Bottom panel, time evolution of kinetic (solid) and magnetic (dashed) energies $b_\xi = 0.35$ (blue), $b_\xi = 0.1$ (red), $b_\xi = 0.05$ (black), keeping $b_\theta = 0.35$ for all cases.

Figure 1 shows the contour plots of the magnetic end velocity fields profiles during the MHD decay process and the results for maximum charge current density, $\max \|\mathbf{j}\| = \max \sqrt{j_x^2 + j_y^2 + j_z^2}$, and the energy dissipation, $\epsilon = \nu\langle\boldsymbol{\omega}^2\rangle + \lambda\langle\mathbf{j}^2\rangle$, where $\boldsymbol{\omega} = \boldsymbol{\nabla}\wedge\mathbf{v}$ represents the vorticity and $\langle ...\rangle$, the volumetric average. The magnetic and the velocity fields interact through non-linear MHD dynamics process, forming the filamentary structures correlating each other. The benchmark has been carried out with the time evolution of $\max \|\mathbf{j}\|$. The $\max \|\mathbf{j}\|$ evolution in an early phase turns out to be independent from the ν and λ chosen. The case with $\nu = \lambda = 0.001$ shows a dependence of $\sim t^3$ from $t \approx 0.5$ to $t \approx 1$. A saturation of $\max \|\mathbf{j}\|$ is observed beyond $t \approx 1$. It is inverse proportion against the transport parameters ν and λ. The $\max \|\mathbf{j}\|$ plot is consistent with the observations of [5]. The plot of the time evolution of ϵ which corresponds to small vortex structures of the field is different from [5]. The possible reason could be due to the numerical methods used in the work.

III. MHD SIMULATIONS IN TOROIDAL GEOMETRY.

The MHD simulations have been applied to a toroidal geometry ($\sim$ 700k cells, 3 to 8 CPUs, $\sim$ 1-3 days of wall-time). The geometry consists of a toroid of circular cross section of a major radius $R_0 = 0.55\pi$ and a minor radius $r_0 = 0.3\pi$. Two magnetic fields, poloidal and toroidal, are imposed by an external source assuming the external coils. Since the interspacing between the coils is smaller inside than outside, the toroidal field decreases with the inverse of the distance as one moves away from the axis of symmetry, $\mathbf{b}_\xi = b_\xi R_0/R\ \hat{\boldsymbol{\xi}}$. The poloidal field, $\mathbf{b}_\theta = b_\theta r/r_0\ \hat{\boldsymbol{\theta}}$, is produced by the circulation of a toroidal current. Here, $b_\theta = b_\theta|_{(r=r_0)}$ is the value of the poloidal field at the boundary and $b_\xi = b_\xi|_{(R=R_0)}$ is the value of the toroidal magnetic field at centre of the cross section. The toroidal magnetic field has been scanned, $b_\xi = 0.35$, 0.1 and 0.05, keeping the poloidal magnetic field fixed to $b_\theta = 0.35$. The simulations were performed with $\nu = \lambda = 0.002$. An initial velocity field is given by random perturbation of $E_k \approx 1.24\times10^{-5}$. Figure 2 shows the contour plots of velocity field of $b_\xi = 0.35$ at $t = 30$ and $b_\xi = 0.1$ at $t = 70$ keeping $b_\theta = 0.35$ for both cases. They show four counter rotating vortices in the case of $b_\xi = 0.35$ and a helical structure formed in the case of $b_\xi = 0.1$. The ratio of b_θ and b_ξ characterizes the MHD stability. The energy evolutions show oscillations before $t = 100$. In the case of low b_ξ, the MHD dynamo is induced forming the helical structures. These results are consistent with the observations of the work which has been carried out by other code [6].

IV. CONCLUSIONS AND PERSPECTIVES

In this work, the viability of OpenFOAM as a potential code for high performance simulations oriented to fusion plasmas has been assessed. The results of Orszag-Tang test are comparable to the studies of [5]. In the toroidal domain, the energy plots turn out to be in good agreement with [6]. The non-linear MHD simulations via OpenFOAM gives plausible results although there is a room to improve, for example, the MHD modelling and physics parameters.

The generation of simulation mesh has been applied to other geometries. The capability of generating arbitrary mesh is potential interest for numerical studies of fusion research. The study of those geometries will be carried out in the future.

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David Garrido González is a Physics bachelor student at Universidade de Santiago de Compostela (USC). He is at his last year and is currently carrying out his bachelor thesis about non-linear MHD simulations of magnetically confined plasma using OpenFOAM in close collaboration with Universitat Politècnica de Catalunya (UPC), Spain.