Subdividing linear and curved meshes preserving sharp features of a model

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***Abstract*—To provide straight-edged and curved piece-wise polynomial meshes that target a unique smooth geometry while preserving the sharp features and smooth regions of the model, we propose a new fast curving method based on hierarchical subdivision and blending. There is no need for underlying target geometry. It is only required a straight-edged mesh with boundary entities marked to characterize the geometry features, and a list of features to recast through a unique sharp-to-smooth modeling capability. The examples show that the method is well-suited to curve large quadratic and quartic meshes in low-memory configurations.**

Keywords*—*mesh curving, surrogate geometry, geometry modeling, subdivision, blending

I. Extended Abstract

A. Introduction

In flow simulations for wind energy, transport of pollutants, and bio-engineering the boundary of the computational domain is usually represented by a straight-edged mesh obtained by sampling real data. This straight-edged mesh approximates the geometry, at different scales, corresponding to the viscous surfaces to analyze such as topography, urban areas and human organs. The mesh also presents a series of sharp features, vertices, polylines bounded by vertices, that the method should preserve, and that bound the smooth regions of the computational model.

The resolution to approximate the geometry could be insufficient for the required flow analysis, and thus, additional refinement of the boundary mesh would be required. However, a standard refinement approach, when no target geometry is available, could be inadequate for flow simulation in a twofold way. First, the refined mesh might reproduce precisely the geometry of the first straight-edged mesh and thus, introduce artificial flow artifacts close to initially non-smooth features that should be smooth. Second, the refined mesh might target a smooth surface geometry, implicitly determined by the initial straight-edged mesh, but without adequately respecting after successive refinement the sharp features, curves, and vertices, of the computational model. Ideally, vertices should remain fixed, and polylines should target a smooth limit curve.

Solving these issues is essential for those flow analyses that start from a mesh obtained by sampling real data where the computational model presents smooth regions bounded by sharp features. Even they can be useful in aeronautical applications where only legacy data, in a format of vertices, and polyline and surface meshes, is available. In some applications, practitioners might also need, a non-standard but flexible *sharp-to-smooth modeling* capability, to remove some sharp features ensuring that surrounding regions become smooth along with the removed feature.

Intending to provide piece-wise linear meshes or curved piece-wise polynomial meshes that target a unique smooth geometry while preserving the sharp features of the model, our contribution is to propose a new fast curving method based on hierarchical subdivision and blending with sharp-to-smooth modeling capabilities. Our approach only needs an initial straight-edged mesh with boundary triangles marked with surface identifiers, and a list of features to recast. There is no need for underlying target geometry. The goal of the method is to obtain a volume mesh of the flow domain that under successive refinement leads to smooth regions bounded by the sharp features determined after recasting. The recasting operation is devised to implement a sharp-to-smooth modeling capability. We favored a fast and explicit curving method, based on subdivision and blending, to an implicit approach formulation that features validity guarantees or untangling capabilities, based on boundary curving and optimization, but slower and more memory demanding. This favoring is so since the appearance of invalid elements is small compared with the scale of the generated meshes, and fast local untangling can repair those invalid elements. This work details our mesh curving methodology and illustrates its application with the included numerical examples.

B. Example

The curved high-order mesh generation procedure proposed in this work is composed of four main steps:

1) **Sharp-to-smooth modeling**. We recast some of the feature entities present in the original model, and thus provide a new model improving the smoothness of the surrogate geometry. Each feature vertex (node of the mesh), curve (set of edges of the mesh) and surface (set of triangles of the mesh) is associated with a unique identifier. Therefore, to recast a feature, it is enough to know its identifier. The initial model, Fig. 1(a), contains some vertex and curve features, such as the leading edge or the curve along the fuselage, that have been recast in the final model, Fig. 1(b), to obtain a smoother virtual surface.
We highlight that the presence of a non-desired geometry feature has an impact on the smoothness of the generated mesh. We illustrate the continuity of the normal vectors using a zebra mapping. For the initial model, the normals are discontinuous, see Fig. 1(c), and therefore, the mesh is $\mathcal{C}^0$-continuous;

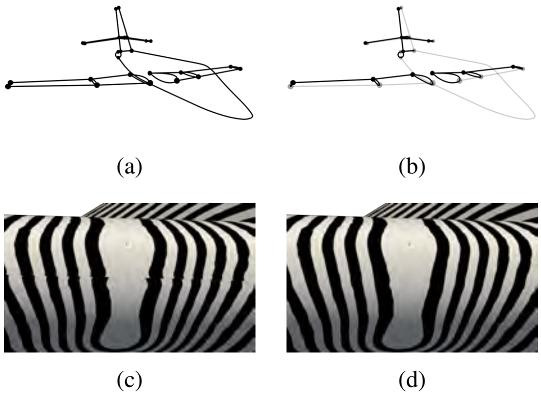

Fig. 1: (a) Marks on boundary entities of the initial model: curve and vertex features. (b) Marks of the final model: curve and vertex features recast (gray) and preserved (black). Zebra mapping showing isophotes along the leading edge for the (c) initial model, and (d) the final model.

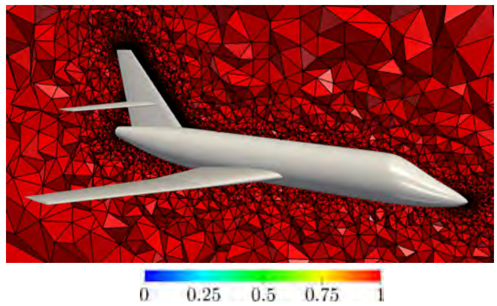

Fig. 2: Generated valid curved tetrahedral mesh of polynomial degree $p = 4$ composed of $1.8{\cdot}10^7$ nodes and $1.7{\cdot}10^6$ elements.

while in the final model the normals are continuous, see Fig. 1(d), so the mesh is $\mathcal{C}^1$-continuous.

2) **Approximate a surrogate boundary**. Given a linear tetrahedral mesh, we extract its boundary. The boundary is a linear triangular mesh with its entities marked, and by means of the hierarchical subdivision of its elements we generate a curved high-order triangular surface mesh. The curved surface mesh approximates a surrogate boundary composed of feature surfaces with an interior that is $\mathcal{C}^1$-continuous and $\mathcal{C}^2$-continuous almost everywhere [1], [2].
This surrogate is determined by the subdivision of the curves and surfaces, and preserves the sharp features and smooth regions marked on the boundary of the initial volume mesh.
3) **Accommodate the curvature of the boundary**. We accommodate the curvature of the curved surface mesh to the boundary volume elements using an explicit hierarchical blending [3]. In this example, the blending reduces from 358 to 24 the number of invalid elements, that is, in 237 seconds a 93% of the invalid elements have been untangled.
4) **Local untangling**. If necessary, we optimize the inverted elements locally following the approach detailed in [4]. Since the mesh after the previous step is close to be optimal, it is a good initial condition for the implicit optimization and in 60 seconds the mesh becomes valid achieving a minimum quality of 0.7. The final quartic mesh is shown in Fig. 2.

C. Conclusion

The obtained results show that we can generate, from an initial straight-edged mesh, successively refined piecewise linear, quadratic and quartic meshes, that target smooth curves and surfaces, while preserving the initially marked sharp features and smooth regions. The interior of the obtained limit curves is of class $\mathcal{C}^2$, and the interior of the surfaces is at least $\mathcal{C}^1$-continuous, being of class $\mathcal{C}^2$ when the surface mesh is structured. For manifolds with boundaries, only a straight-edged mesh with boundary triangles marked with surface identifiers is required. Then, the method automatically computes the boundary curves and vertices from the triangle marks. A unique sharp-to-smooth modeling capability - not fully available in standard CAD packages - allows removing sharp features, *i.e.* vertices and curves, and smoothly merge the incident entities, *i.e.* curves and surfaces. The resulting surrogate geometry features $\mathcal{C}^1$-continuity along the merging region. The proportion of invalid elements is small compared to the size of the meshes, and thus, we showed that it can be fixed using local untangling and curving without the need for a global solver.

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References

[1] J. M. Lane and R. F. Riesenfeld, "A theoretical development for the computer generation and display of piecewise polynomial surfaces," *IEEE TPAMI*, no. 1, pp. 35–46, 1980.

[2] C. Loop, "Smooth subdivision surfaces based on triangles," Master's thesis, The University of Utah, Masters Thesis, 1987.

[3] A. Perronnet, "Interpolation transfinie sur le triangle, le tétraèdre et le pentaèdre. application à la création de maillages et à la condition de dirichlet," *CR ACAD SCI I-MATH*, vol. 326, no. 1, pp. 117–122, 1998.

[4] A. Gargallo-Peiró, X. Roca, J. Peraire, and J. Sarrate, "Optimization of a regularized distortion measure to generate curved high-order unstructured tetrahedral meshes," *IJNME*, vol. 103, no. 5, pp. 342–363, 2015.

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