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LONG-TERM PROBABILITY DISTRIBUTION OF FIXED OFFSHORE STRUCTURALRESPONSE USING ANIMPROVED VERSION OF FINITE MEMORY NONLINEAR SYSTEM PROCEDURE

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Abstract. Offshore structures are exposed to random wave loading in the ocean environment and hence the probability distribution of the extreme values of their response to wave loading is required for their safe and economical design. Due to nonlinearity of the drag component of Morison's wave loading and also due to intermittency of wave loading on members in the splash zone, the response is often non-Gaussian [1-2]; therefore, simple techniques for derivation of the probability distribution of extreme responses are not available. However, it has recently been shown that the short-term response of an offshore structure exposed to Morison wave loading can be approximated by the response of an equivalent finite-memory nonlinear system (FMNS) [3]. Previous investigation shows that the developed FMNS models reduce the computational effort but the predictions are not very good for low intensity sea states. Therefore, to overcome this deficiency, a modified version of FMNS models is referred to as MFMNS models is used to determine the extreme response values which improves the accuracy but is computationally less efficient than FMNS models. In this paper, the 100-year responses derived from the long-term probability distribution of the extreme responses from MFMNS and FMNS models are compared with corresponding distributions from the CTS method is investigated with the effect of current to establish their level of accuracy. The methodology for derivation of the long-term distribution of extreme responses (and the evaluation of 100-year responses) is discussed. The accuracy of the predictions of the 100-year responses from MFMNS and FMNS models will then be investigated.

1 INTRODUCTION

For offshore structural design, the load due to wind-generated random waves is usually the most important source of loading. Whilst these structures can be designed by exposing them to extreme regular waves (100-year design wave), it is much more satisfactory to use a probabilistic approach to account for the inherent randomness of the wave loading [1]. This approach allows the statistical properties of the loads and structural responses to be determined, which is essential for risk-based assessment of these structures. The major obstacle in achieving this objective is the nonlinearity of the wave load mechanism resulting in non-Gaussian response distributions [2], due to the drag component of Morison wave loading. Furthermore, dynamic effects, the presence of current and load intermittency in the splash zone, all have an effect on the statistical properties of structural response [3, 4], increasing the complexity of the problem.

Many different techniques [5] have been introduced for evaluation of statistical properties of offshore structural response. Examples of time domain techniques include (standard or conventional) Monte Carlo time simulation, Finite-memory nonlinear system and NewWave theory. Also, in the frequency domain, the Volterra series has been used to calculate the skewness and kurtosis of structural response from its higher order spectra (bi-spectra and tri-spectra). However, in the most part, these methods have major shortcomings such as their inability to account for current or load intermittency, and/or are limited to quasi-static responses or to very simple structures with a few nodal loads.

In reality, Monte Carlo time simulation is the most reliable technique in that it could readily account for all forms of nonlinearities [3]. In this technique, a random wave record is first simulated from a given frequency spectrum (representing a particular sea state), and a corresponding response record such as a base shear record is then simulated. The extreme value of this record will be a sample extreme response. The process is then repeated many times to have a large sample of extreme responses, which will then be fitted to a suitable extreme value probability distribution model. However, very large samples (tens of thousands) are required to prevent excessive sampling variability [6]. Naturally, this results in an unacceptably high computational cost, especially as structures become more complex and increasingly economical designs are required.

A new technique has recently been introduced [1] by using the mean upcrossing rate function in predicting extreme response values. The results of the simulation were optimized with linear extrapolation for more accurate prediction. Najafian [3] applied the same technique but proposed a finite-memory nonlinear system (FMNS) model to increase efficiency. Based on the validation test, the model was able to improve efficiency, but it was less accurate as compared to Monte Carlo time simulation. Hence, Najafian and Mohd Zaki [7] improvised the model that is able to improve both efficiency and accuracy. However, the model is only relevant for structures under high significant wave height (H_s) values.

With further investigation, Mohd Zaki *et al.* [8] introduced a method to increase the accuracy on low H_s values by dividing the structure into two parts. Each leg of structure is modelled separately using the FMNS model and the total extreme response will be the summation of the extreme responses in each part. This is known as a modified finite-memory nonlinear system (MFMNS). However, the method is still limited only to certain sea state conditions, such as the presence of currents along wave propagations.

2 CONVENTIONAL TIME SIMULATION (CTS) PROCEDURE FOR SIMULATION OF RESPONSES

Conventional time simulation (CTS) technique has been used in this study for the following two reasons: a) to identify system B of the FMNS model (refer to Figure 1), and b) to establish the accuracy of the FMNS models by comparing the probabilistic properties of the response from FMNS models with those from the CTS method. To identify system B, a long record (about 4.5 hours in this study) of both nonlinear and its associated linear quasi-static responses were simulated; hence in this section the simulation of both nonlinear and linear quasi-static responses are discussed.

The procedure of simulating a sample of response record is summarized as follows:

1. Identify the appropriate frequency wave spectrum (i.e. Pierson-Moskowitz spectrum) based on the location of the offshore structure, provided the significant wave height, $H_s$value and its corresponding T_z value.

2. Generate surface elevation based on the appropriate frequency wave spectrum at an arbitrary reference point for a given period of time.The surface elevation at point, x and time, $t$can then be expressed as:

$$\eta(x,t) = \sum_{i=1}^{NW} A_i cos(2\pi f_i t - k_i x - \varphi_i) \tag{1}$$

   where NW is the total number of wavelets used in the simulation, f are a set of equally-spaced discrete wave frequencies, k are their associated wave numbers, φ are random phase angles distributed uniformly in the range $0 < \varphi < 2\pi$, and finally A are wave amplitudes

3. Compute the component of water particle kinematics (velocities and accelerations) according to linear random wave theory at each node elevation using the appropriate transfer function and account for the intermittency load at the member of the splash zone as defined below.

$$\Gamma_u(f_i) = \omega_i \frac{\cosh(k_i z)}{\sinh(k_i d)} e^{-jk_i(x-x_0)} \tag{2}$$

$$\Gamma_{\dot{u}}(f_i) = j\omega_i \Gamma_u(f_i), \qquad j = \sqrt{-1} \tag{3}$$

   where Γ_u and $\Gamma_{\dot{u}}$ are the transfer functions for water particles kinematics; velocity and acceleration, respectively at a particular nodal point, in which x is the horizontal coordinate of the node and $x_0$is the coordinate of arbitrary reference point where the surface elevation has been simulated. ω stands for angular frequency where $\omega_i = 2\pi f_i$, k is their corresponding wave numbers, d is the water depth that indicates vertical distance between mean water level (MWL) and the seabed and z refer to elevation above the seabed. The vertical stretching approach will be applied for the condition above the mean water level.

4. Compute the Morison load at different nodes corresponding to its water particle kinematics. That is,

$$F = F_d + F_i = k_d u|u| + k_i \dot{u} \tag{4}$$

$$k_d = C_d \rho \frac{D}{2} \quad ; \quad k_i = C_m \rho \frac{\pi D^2}{4}$$

where F_d and F_i are the drag and inertial components of fluid loading; C_d and C_m are empirical drag and inertia coefficients; D is the cylinder diameter; ρ is the water density; u and $\dot{u}$ are the horizontal component of water particle velocity and acceleration at the centre of the cylinder. For a more complete review of Morison's equation refer to Sarpkaya and Isaacson [9].

5. Calculate the drag-induced, $\tilde{R}_d$ and inertia-induced, $\tilde{R}_i$ components of the response (quasi-static base shear and overturning moment in this study). As the structural system is assumed to be linear, the total response would then be equal to the sum of the foregoing two components.

3 FINITE-MEMORY NONLINEAR SYSTEMS

Finite-memory nonlinear systems, as depicted in Figure 1, are extensively used in system identification techniques to establish a relationship between the output and input of some nonlinear systems [3]. In the case of a jacket structure exposed to random waves, the input is the surface elevation at a reference point and the output is the dynamic response. The following is meant to briefly explain how a finite-memory nonlinear system can be used to establish a simple relationship between the output and input of a jacket structure. For a more complete description refer to [3, 7].

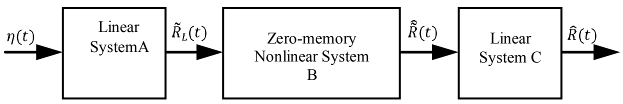

Figure 1: The structure of Finite-memory nonlinear system.

1. The water particle kinematics at different nodes of the structure are calculated based on linear random wave theory by applying appropriate transfer functions to the surface elevation at a given point, $\eta(t)$. On the other hand, the linear quasi-static response (denoted by $\tilde{R}_L(t)$, where R stands for response; the symbol ∽ on top of R indicates quasi-static; and finally, the subscript L stands for linear) is equal to a linear combination of linearised Morison loads acting at different nodes. Since the linearised Morison loads are

themselves linear combinations of their corresponding water particle kinematics, it can be concluded that the linear quasi-static response is linearly related to the surface elevation, via a transfer function. This linear process is represented by the first linear system (system A) in Figure 1. In other words, system A converts the surface elevation at a reference point to the linear quasi-static response.

2. The next stage is the calculation of approximate values of the (nonlinear) quasi-static response (i.e. $\hat{\hat{R}}$ where the symbol $\hat{}$ on top of R stands for approximate value) from their corresponding linear response values through a memoryless nonlinear transformation. That is, instantaneous values of quasi-static response are assumed to depend on the values of the linear quasi-static response at the same instant. This process is represented by the nonlinear system B in Figure 1. System B accounts for nonlinearities due to both drag component of Morison loading and also nonlinearities due to load intermittency on members in the splash zone.
3. The final stage is the calculation of approximate (dynamic) response, from its corresponding (approximate) quasi-static response through a suitable transfer function. This process is represented by the linear system C in Figure 1. This stage is quite straightforward as the appropriate transfer functions (frequency response functions) are determined from established procedures in modal analysis of linear structures.

3.1 Modified Finite-Memory Nonlinear System (MFMNS)

Current finding shows that the modified finite-memory nonlinear system (MFMNS) technique able to determine the short-term probability distribution of response extreme values efficiently [8]. This modified version of FMNS performs better in terms of accuracy. The poorer performance of standard FMNS, particularly for low H_s values, can be improved by dividing the structure into two parts (parts 1 and 2) in order to minimize the horizontal distance between nodes in each part compared to the wavelengths.

By applying systems A and B (refer to Figure 1), the quasi-static response is modelled separately according to the part. Then, the quasi-static responses at each part need to be combined to obtain the total quasi-static response. In this paper, the waves are assumed to propagate in the global Y direction. Therefore, the first two columns in a plane perpendicular to the wave direction (XZ plane) constitute the first part, with the remaining two columns belong to the second part (refer to Figure 1). Note that the standard FMNS was developed analogous to a single column. In effect, long waves perceive the structure as a single column.

4 LONG-TERM PROBABILITY DISTRIBUTION OF EXTREME RESPONSES

In this section, the long-term probability distribution of the extreme responses from MFMNS models are compared with corresponding distributions from CTS method to investigate the level of accuracy of the MFMNS models in predicting the probability distribution of the extreme responses. The long-term distribution of extreme responses can be derived by convoluting the short-term distribution of extreme values by the long-term distribution of sea states. That is,

$$P_{LT,r_{max}}(q) = \int_{H_s=0}^{\infty}\int_{T_z=0}^{\infty} P_{r_{max}}(q|H_s,T_z)dH_s dT_z \tag{5}$$

$$= \int_{H_s=0}^{\infty}\int_{T_z=0}^{\infty} P_{r_{max}}(q|H_s,T_z)p(H_s,T_z)dH_s dT_z$$

where $p(H_s, T_z)$ is the joint probability density function of significant waveheight and mean-zero upcrossing period. Using the extended scatter diagram as an approximation for the joint probability density function, the above equation can be written as [10],

$$P_{LT,r_{max}}(q) = \sum_i \sum_j P_{r_{max}}(q|H_{si},T_{Zj}) * \frac{W_{ij}}{W} \tag{6}$$

where W_{ij} is the number of occurrences of the sea states represented by H_{si} and T_{Zj} in the scatter diagram and $W = \sum_i \sum_j W_{ij}$ is the total number of sea states. In other words, W_{ij}/W is the probability of occurrence of the sea state characterised by H_{si} and T_{Zj}. Meanwhile, $P_{r_{max}}(q|H_{si},T_{Zj})$ is the short-term probability distribution of response extreme values for the sea state characterised by H_{si} and T_{Zj}.

In this study, for each sea state, the distribution of extreme values is calculated based on 1000 simulated records, each of 128sec duration. In order to calculate $P_{LT,r_{max}}(q)$ from Eq. (6), the values of $P_{r_{max}}(q|H_{si},T_{Zj})$ must be known for all simulated q values belonging to all sea states. $P_{r_{max}}(q_n|H_{si},T_{Zj})$ is calculated in the following way. Rank all the simulated extreme values for a given sea state from smallest to largest. Then assuming that the number of simulated extreme values for each sea state is N (N = 1000 in this study) and assuming that q_n is the nth smallest simulated extreme value, use the following equations to calculate the short-term distributions

$$P_{r_{max}}(q_n|H_{si},T_{Zj}) = \frac{n-0.44}{N+0.12} \tag{7}$$

and

$$P_{r_{max}}(q < q_1|H_{si},T_{Zj}) = 0$$
$$P_{r_{max}}(q > q_N|H_{si},T_{Zj}) = 1$$
$$P_{r_{max}}(q_1 < q < q_N\ |H_{si},T_{Zj}) = \text{ determine by the interpolation from Eq. (7)}$$

Furthermore, the method of moments was used to fit a Gumbel distribution to the simulated extreme values for each sea state. The long-term distributions of extreme values are then calculated by convoluting the short-term Gumbel distributions of extreme values with the probability of occurrence of each sea states (Eq. (7)). The foregoing long-term distributions are then used to calculate the 100-year responses for CTS, FMNS and MFMNS methods.

4.1 Derivation of the 100-Year Extreme Response from the Scatter Diagram

The 100-year extreme response is defined as the extreme response which, on average, is equalled or exceeded once every 100 years. Now, the number of 128-sec intervals in 100 years is *N*= 100*365*24*3600/128. Therefore, on average, only in one 128-sec interval out of *N*intervals, the extreme response would be higher than R_{100}. (In other words, the extreme response in (*N*–1) intervals must be less than R_{100}). Then, the probability of the extreme response in a 128-sec interval being larger than R_{100}would be 1/*N*. That is,

$$\text{Prob}\{extreme\ response > R_{100}\} = \frac{1}{N} \tag{8}$$

$$= \frac{1}{100 * 365 * 24 * 3600/128}$$

$$= 4.0589 * 10^{-8}$$

Knowing the probability of exceedence, its corresponding 100-year response can be calculated from the long-term distribution of extreme responses (based on Gumbel distributions fitted to simulated extreme responses for each sea state).

5 TEST STRUCTURE AND RESPONSES

The preceding sections demonstrated the procedure of evaluating offshore structural responses based on the appropriate wave theory. The evaluation was applied to the test structure that is similar to a fixed jack-up platform by applying the drag and inertia coefficient according to API RP2A-WSD standard code (2007) of C_D = 1.05 and C_m = 1.20. As shown in Figure 2, the test structure is a quasi-static fixed platform consisting of four vertical legs, each with a 1.5m diameter and wall thickness of 40mm, located in a water depth of 110m. The legs support a 35m *x* 38m platform deck and the hydrodynamic load is distributed along 30 points on each leg. The foregoing test structures were subjected to various uni-directional sea-states simulated from Pierson–Moskowitz (P–M) frequency spectrum. The responses chosen for investigation were the base shear and overturning moment of the test structure.

Adjustment of the Young's Modulus of the bracing elements of the structure (not shown in the figure) allows control of the overall stiffness and hence the natural frequency of the structure (the internal bracings were assumed to receive no fluid loading to reduce the computational effort). JCP2, JCP5 and JCP8 are used to refer to three FE models with first mode natural periods of 2.53, 5.21 and 8.12 seconds, respectively. For a more complete description of the test structure refer to [5].

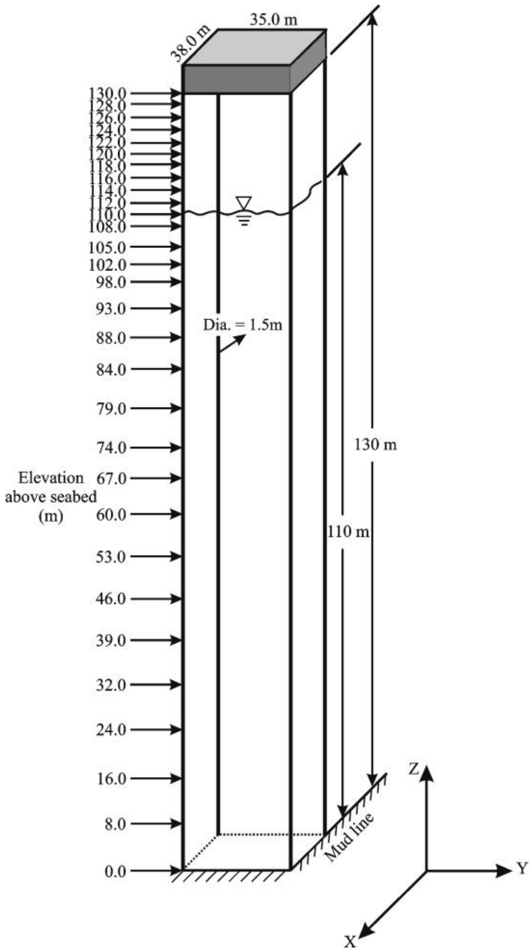

Figure 2: Schematic diagram of the test structure.

6 COMPARISON OF 100-YEAR RESPONSES FROM MFMNS AND CTS METHODS

The long-term probability distributions of the maxima for the case of (total) quasi-static base shear with zero current are shown in Figure 3. As observed, there is good agreement between the distributions from CTS and MFMNS methods (both simulated and Gumbel-based distributions). The long-term probability distributions of the maxima for JCP5 base shear with zero current, and for JCP2 base shear with negative current, are shown in Figures 4 and 5, respectively, and similar conclusions can be made. It is observed that the 100-year extreme response for all cases are predicted with very good accuracy with maximum inaccuracies of 6% for negative current case.

It is also observed that the 100-year maxima are very significantly underestimated by linear modelling of the response. The worst case is that of the JCP5 overturning moment with zero current with a ratio of 0.40 between linear and nonlinear 100-year maxima. The 100-year minima are also significantly underestimated by the linear response model in most cases. The only exception is that of the positive current, where due to extreme asymmetry, the linear 100-year minima are sometimes higher than their corresponding nonlinear values.

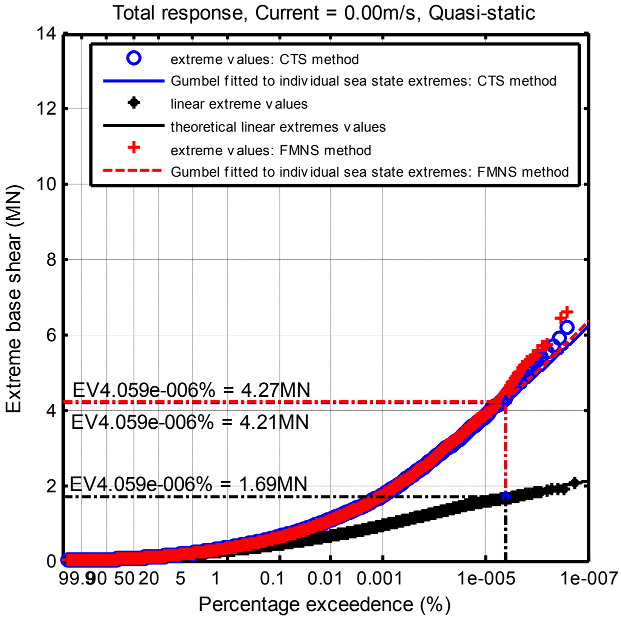

Figure 3: Comparison of 100-year (maxima) from CTS and MFMNS methods. 1000 sample records for each sea state, T = 128sec.Total base shear, quasi-static, current = 0.00m/sec.

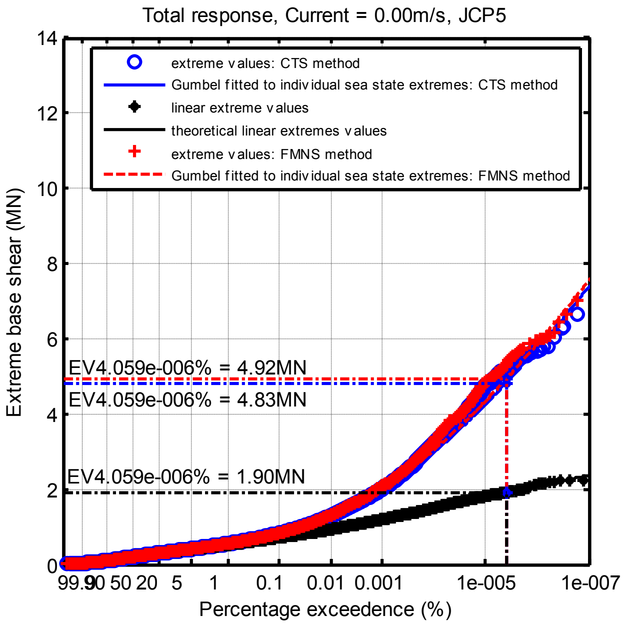

Figure 4: Comparison of 100-year (maxima) from CTS and MFMNS methods. 1000 sample records for each sea state, T = 128sec.Total base shear, JCP5, current = 0.00m/sec.

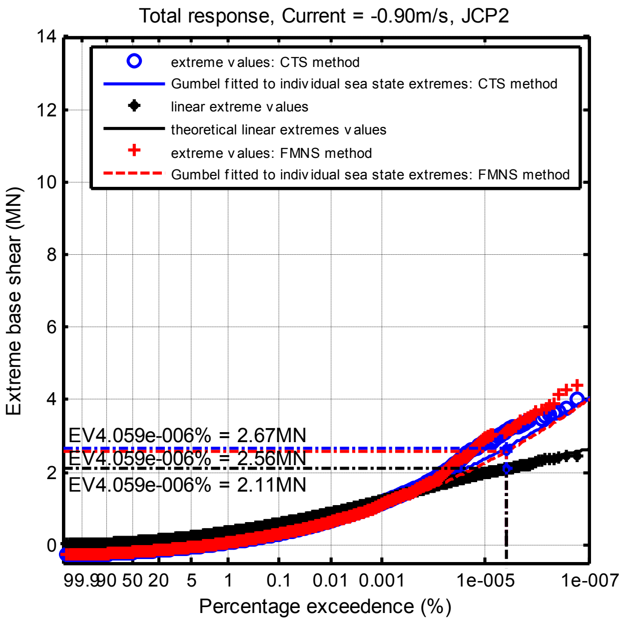

Figure 5: Comparison of 100-year (maxima) from CTS and MFMNS methods. 1000 sample records for each sea state, T = 128sec.Total base shear, JCP2, current = -0.90m/sec.

7 CONCLUSIONS

- Two different versions of finite-memory nonlinear systems for modelling offshore structural response due to Morison's wave loading have been reviewed: FMNS and MFMNS. It was concluded that the accuracy of FMNS, is poor for low H_s values (H_s = 5m), and that MFMNS is more appropriate.
- The level of accuracy of MFMNS/FMNS models is established by calculating the ratios between the 100-year responses from the MFMNS/FMNS methods and corresponding values from the conventional time simulation method (CTS). The maximum inaccuracies in predicting the 100-year responses from MFMNS and FMNS methods are 3% and 7%, respectively. So while MFMNS is more accurate, the accuracy of the FMNS is also good.
- Finally, it has also been confirmed that the 100-year maxima are very significantly underestimated by linear modelling of the response with ratios as low as 0.40 between linear and nonlinear 100-year maxima. Therefore, linearization can lead to significant under design of the structure with the very high risk of structural failure when the structure is exposed to a severe sea state.

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