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NUMERICAL MODELLING OF GRANULAR CARGO ON BULK CARRIERS IN SEAWAY

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Abstract. This paper outlines the development of a numerical model for granular cargo on bulk carriers. In order to study the vessel behaviour including the motion of the load, a monolithic approach is chosen to model the fully coupled problem. The formulation of the granular material therefore has to be fully Eulerian. A nonlinear elastic solid phase is implemented in the Finite Volume solver FreSCo^{+} following the approach of Richter et. al [19] and Sugyiama et. al [22]. The method is then verified with the help of different Fluid-Structure interaction test cases.

1 INTRODUCTION

In the last decade a significant number of incidents and accidents occurred while bulk carriers were carrying nickel or iron ore. Sixty-six casualties were reported alone between October 2010 and December 2011 with further accidents and incidents arising since 2011, the latest accident on January 2nd 2015. The reason for these losses is not well understood, since data describing the behaviour of the cargo on board is rare. No data for cargo state exists for vessels which were actually lost and therefore all accident investigations are nowadays heavily based on the accounts of eyewitnesses. The general explanation for these sinkings is believed to be the liquefaction of the bulk cargo due to cyclic excitation in seaways. After the cargo has liquefied a free surface flow establishes in the holds of the bulk carrier which can lead to an abrupt capsize of the vessel. The correct geotechnical term for this failure mechanism (liquefaction) is cyclic mobility and depends on many physical characteristics as well as the saturation level of the cargo. The phenomena is studied intensively in the field of geotechnology and is well described in [10]. A possible numerical model is given by Oka et. al in [17]. Nevertheless it is not proven that cyclic mobility is the only possible failure mechanism and it is suggested that also structural failure mechanisms (e.g. sliding) of the cargo could cause the loss of the vessels.

In order to study the behaviour of the bulk cargo on board, a general elasto-plastic model which is able to depict cyclic mobility as well as structural failures has to be derived. Since the goal is to be able to perform a fully-coupled simulation of a bulk carrier in seaway carrying its cargo, the elasto-plastic model has to be implemented in an existing code which is able to perform seaway studies. The Finite-Volume code FreSCo$^+$ includes all features (6DoF solver, overset grids, regular and irregular seaway, three volume-of-fluid phases) needed for the full simulation of the bulk carrier in seaway (see [1, 14]). In order to circumvent data transfer problems, a monolithic approach is taken. Therefore an additional elasto-plastic phase is introduced to the Finite-Volume fluid solver which is based upon the VoF methodology.

Due to the Eulerian framework of the numerical procedure an Eulerian formulation of the elasto-plastic model has to be used. In order to build a sound numerical elasto-plastic model, a fully Eulerian nonlinear elastic model is derived. This model is described in Section 2 and verified in Section 3 on some fully Eulerian Fluid-Structure interaction test cases. Monolithic Fluid-Structure interaction in Eulerian coordinates using an interface tracking technique has already been explored by some authors. Richter et. al ([19] and [3]) introduced this technique using Finite Elements and the level-set method. Sugyiama et al. [22] modified the approach and discretised it with the Finite Difference method and used the volume-of-fluid method.

In the PhD-dissertation of El Feghali [4] the approach of Sugyiama is translated to the Finite Element methode and the level-set method is used. Further work on Eulerian Fluid-Structure interaction can be found in [2, 6, 8, 9, 11, 13, 15, 18, 20] and [21] among others. The list does not claim to be complete but the most important research, on which the method applied in this study is built, is referenced. The present nonlinear elastic numerical model is based on a combination of the approaches of Richter et. al [19] and Sugiyama et. al [22] and is discretised with the Finite-Volume method. A pressure correction scheme (SIMPLE), typical for implicit Finite-Volume codes applied to incompressible or weakly compressible media, is used to satisfy the continuity equation.

2 MATHEMATICAL FORMULATION AND NUMERICAL ALGORITHM

In this section the governing equations for the fluid domain as well as the solid domain are introduced. The combination of the governing equations via the volume-of-fluid method is described and the implemented solution algorithm is stated.

2.1 Governing Equations

The incompressibe Navier-Stokes equations describe the fluid behaviour. If the momentum equation is formulated generally, it holds true in the fluid domain as well as the solid domain

$$\rho \frac{\partial v_i}{\partial t} + \rho \frac{\partial v_i v_j}{\partial x_j} = \rho g_i + \frac{\partial \sigma_{ij}}{\partial x_j} \ , \tag{1}$$

where v_i denotes the cartesian coordinates of the velocity vector. The difference between the fluid and solid phase lies in the value of the Cauchy tensor σ_{ij} stemming from different constitutive equations for the materials, as well as the different material properties. All fluid properties are denoted by a capital F where all solid properties are denoted by a capital S. The well known volume-of-fluid method introduced by Hirt et. al [7] is used to separate the fluid from the solid domain for an assumed unique kinematic field. Accordingly, all material properties, e.g. the density ρ or the viscosity μ can be expressed by

$$\begin{aligned} \rho &= c_S \rho_S + (1 - c_S)\rho_F \\ \mu &= c_S \mu_S + (1 - c_S)\mu_F \end{aligned} \tag{2}$$

with c_S being the volumetric solid concentration ($c_S = 1$ inside the solid domain and $c_S = 0$ inside the fluid domain). Equation 2 can also be reformulated by substituting c_S with c_S to the power of a constant. The constant can be chosen arbitrarily, leading to a nonlinear dependency of the density and viscosity on the solid concentration. The solid concentration is transported by a simple convective transport equation as common in the volume-of-fluid method for immiscible phases

$$\frac{\partial c_S}{\partial t} + \frac{\partial v_j c_S}{\partial x_j} = 0 \; . \tag{3}$$

By using a monolithic approach with only one velocity field calculated from the momentum equation kinematic matching conditions are automatically fulfilled at the solid/fluid-interface.

The Cauchy stress for an exemplary incompressible laminar fluid $\sigma_{F,ij}$ can be expressed as

$$\sigma_{F,ij} = -p\delta_{ij} + \mu \frac{\partial v_i}{\partial x_j} \; . \tag{4}$$

An incompressible Neo-Hooke material is chosen as solid material for the validation test cases. The general formulation nevertheless can be applied with arbitrary constitutive equations if the volumetric and deviatoric parts are separated. For Neo-Hookean materials the Cauchy stress tensor $\sigma_{S,ij}$ yields

$$\sigma_{S,ij} = -p\delta_{ij} + \eta_S (F_{ij}^{-1} F_{ij}^{-T} - \delta_{ij}) \tag{5}$$

where F_{ij} is the Eulerian deformation gradient and η_S is the Lamé constant. The stress tensors can be combined using the solid concentration and the viscosity, which allows for a visco-elastic behaviour in the solid phase

$$\sigma_{ij} = -p\delta_{ij} + \mu \frac{\partial^2 v_i}{\partial x_j^2} + c_S \eta_S (F_{ij}^{-1} F_{ij}^{-T} - \delta_{ij}) \; . \tag{6}$$

The Eulerian deformation gradient F_{ij} depends on the displacement in cartesian Eulerian coordinates u_i and can be expressed by

$$F_{ij} = \delta_{ij} - \frac{\partial u_i}{\partial x_j} \ . \tag{7}$$

Since the Eulerian displacement is not naturally known in a fluid code, an equation for the displacement has to be introduced. The total derivative of the Eulerian displacement is equal to the velocity leading to following convective transport equation

$$\frac{\partial u_i}{\partial t} + v_j \frac{\partial u_i}{\partial x_j} = v_i \ . \tag{8}$$

The displacement is only needed for the calculation of the stress tensor $\sigma_{S,ij}$ inside the Neo-Hookean material. Unfortunately, the use of velocity v_i to convect the displacement leads to large deformations in the fluid phase resulting in numerical problems. Therefore the velocity v_i is substituted by the velocity of the Neo-Hookean material $v_{S,i}$

$$\frac{\partial u_i}{\partial t} + v_{S,j} \frac{\partial u_i}{\partial x_j} = v_{S,i} \ . \tag{9}$$

Richter et. al [19] suggested a harmonic continuation of the velocity of the Neo-Hookean material to the computational domain, based upon an elliptic reaxation approach. The velocity of the Neo-Hookean material $v_{S,i}$ is calculated by applying this technique as follows

$$c_S v_{S,i} - a_w \frac{\partial^2 v_{S,i}}{\partial x_j^2} = c_S v_i \tag{10}$$

where a_w is a very small positive constant ($a_w = 0.001$ is found to be the best estimate for all validation test cases).

2.2 Numerical Algorithm

The in-house code FreSCo$^+$ is a Finite-Volume RANS solver especially developed for incompressible free surface flows but also able to solve more complex problems as e.g. cavitation, compressible flows and six-degrees-of freedom motion. FreSCo$^+$ uses a sequential approach to solve the Navier-Stokes equations. A pressure correction scheme is used to calculate the pressure and ensure a divergence free velocity. The scheme of choice is the SIMPLE algorithm described in Ferziger et. al [5] which is derived from the continuity equation. As can be seen in equation 6 the model is formulated with only one monolithic pressure field which takes the value of the fluid pressure in the fluid domain and the solid pressure in the solid domain. Time discretisation follows from a simple implicit Euler scheme in the present study, although other options are available. The elastic part of the Cauchy stress as well as the pressure are added explicitly in the momentum equation, whereas the deviatoric part of the fluid Cauchy stress is treated implicitly in the momentum equation. A CICSAM (introduced by Ubbink [23]) or HRIC (introduced by Muzaferija et. al [16]) scheme is used for the discretisation of the convective term in the volume-of-fluid transport equation. For all other equations the convective term is discretised with a QUICK scheme introduced by Leonard [12].

3 NUMERICAL RESULTS

Three test cases are described in this section. First the movement of a soft Neo-Hookean elastic disk is simulated in a two-dimensional lid driven cavity. Subsequently, the behaviour of an elastic bottom of a two-dimensional lid driven cavity is examined and finally extended to three dimensions. The latter two cases are important for assessing the prospects of the model's application to cargo in holds.

3.1 Elastic disk in two dimensional lid driven cavity

An elastic disk in a two dimensional lid driven cavity is simulated with the method described above. The test case was introduced by Zhao et. al [24] and also used for validation purposes by Sugiyama et. al [22]. The cavity has dimensions 1m $\times$1m and consists of three solid walls at rest and a solid lid at $y = 1$m which moves with $v_x = 1$m/s and drives the flow. Initially the whole system is at rest and the lid wall starts moving at $t = 0$s. The initial position of the elastic disk is illustrated in figure 1 (center of the disk is at $x_C = 0.6$m and $y_C = 0.5$m, radius of disk is $r = 0.2$m). The density of the fluid and solid phase are equal ($\rho_F = \rho_S = 1$kg/m^3), hence no buoyancy forces occur at rest. The elastic disk complies to the incompressible Neo-Hookean constitutive equation and its stiffness is determined through its Lamé constant being equal to $\eta_S = 0.1$N/m^2. The viscosity of the fluid reads $\mu_F = 0.01$Pa s. A purely elastic disk is simulated and therefore the viscosity of the solid phase is assigned to $\mu_S = 0$Pa s. The acceleration due to body forces, e.g. gravity, is neglected. The CICSAM scheme is applied for the discretisation of the convective term in the free surface equation.

Results obtained with FreSCo$^+$ are compared to the literature values in Figure 2. Overall a fair agreement with the reference values is seen. For later times the shape of the elastic disk is influenced by numerical diffusion of the solid concentration. The mesh used for the simulations with FreSCo$^+$ has dimensions 120×120 and is coarse in comparison to the mesh used by Sugiyama et. al [22] (1024×1024). Identical results were obtained on two finer meshes (150×150 and 200×200). A Courant number of 0.06 is applied and the results indicated no sensitivitiy to a decrease of the time step.

3.2 Two dimensional lid driven cavity with elastic bottom

A further two-dimensional test case is simulated in order to validate the implementation of the Dirichlet boundary conditions for the flow velocity, the displacement and the solid velocity. The test case was introduced by Richter et. al [19] and is a lid driven cavity with an elastic bottom. The size of the domain is again 1m$\times$1m and the boundary between

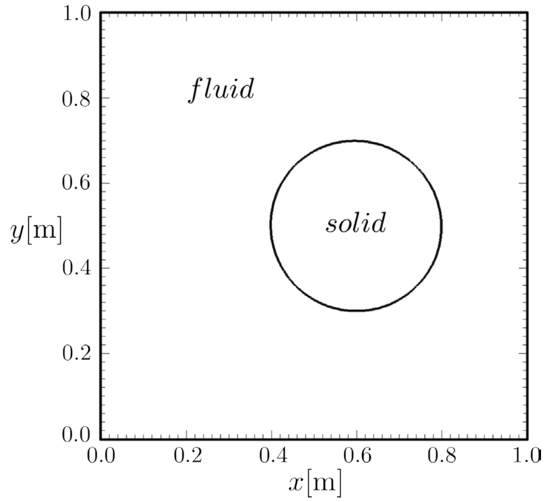

Figure 1: Initial position of the elastic disk in the lid driven cavity.

the fluid and solid part is initially at $y = 0.25$m (see figure 3). The lid is moving with the velocity $v_x = 4x(1-x)$m/s. All other walls are rigid walls at which a Dirichlet boundary condition is applied for the flow velocity, displacement and solid velocity. An incompressible Neo-Hookean material is chosen for the elastic bottom and three different stiffnesses are simulated ($\eta_S = 0.05\text{N/m}^2$, $\eta_S = 0.1\text{N/m}^2$ and $\eta_S = 1.0\text{N/m}^2$). The densities and viscosities are identical to the first example. As in the test case above body forces are neglected. Richter et. al [19] simulated this case in a steady framework. The present approach is inherently unsteady, thus displayed solutions show results that were iterated to steady state. The HRIC scheme is used for the discretisation of the convective term in the free surface equation and a Courant number of 0.2 is applied. As in Richter et. al [19] a convergence study is conducted. The numerical results are listed in tables 1, 2 and 3. J_1 is the x-component of the displacement at point $x = 0.25$m, $y = 0.25$m and J_2 is the y-component of the displacement at point $x = 0.25$m, $y = 0.25$m. It can be

Table 1: Numerical results for the lid driven cavity with elastic bottom ($\eta_S = 0.05\text{N/m}^2$).

	Numerical Results		Richter et. al [19]	
Δh[m]	J_1[m]	J_2[m]	J_1[m]	J_2[m]
2^{-6}	$-8.4591 \cdot 10^{-2}$	$2.2402 \cdot 10^{-2}$	$-2.7917 \cdot 10^{-2}$	$1.9036 \cdot 10^{-2}$
2^{-7}	$-5.8475 \cdot 10^{-2}$	$2.0593 \cdot 10^{-2}$	$-2.8423 \cdot 10^{-2}$	$2.0207 \cdot 10^{-2}$
2^{-8}	$-3.9430 \cdot 10^{-2}$	$1.6869 \cdot 10^{-2}$	$-2.7802 \cdot 10^{-2}$	$2.0177 \cdot 10^{-2}$

seen that for the coarsest mesh both values J_1 and J_2 are overestimated by this method for all three η_S. Since the VoF method is applied, the boundary between the fluid and solid domain is not sharp for coarse meshes and therefore the elastic bottom behaves too

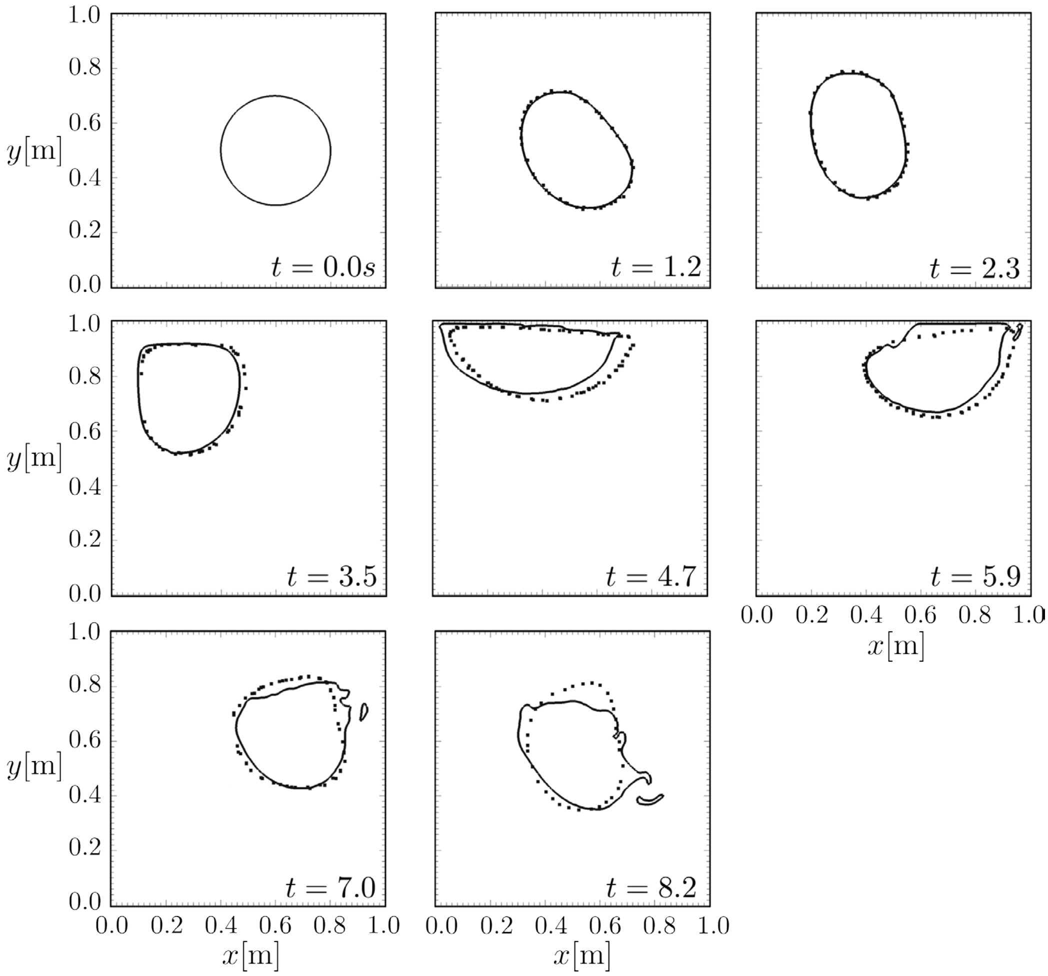

Figure 2: Comparison of the deformation of the elastic disk in the lid driven cavity with literature values from Sugiyama et. al [22]. The dots refer to the reference solution for 1024×1024 nodes and the solid outline represents results of the present study obtained on a mesh of 120×120 control volumes.

Table 2: Numerical results for the lid driven cavity with elastic bottom ($\eta_S = 0.1\text{N/m}^2$).

	Numerical Results		Richter et. al [19]	
Δh[m]	J_1[m]	J_2[m]	J_1[m]	J_2[m]
2^{-6}	$-3.7445 \cdot 10^{-2}$	$1.0664 \cdot 10^{-2}$	$-1.5377 \cdot 10^{-2}$	$1.0197 \cdot 10^{-2}$
2^{-7}	$-3.6467 \cdot 10^{-2}$	$1.0865 \cdot 10^{-2}$	$-1.5383 \cdot 10^{-2}$	$1.0191 \cdot 10^{-2}$
2^{-8}	$-2.5148 \cdot 10^{-2}$	$9.8359 \cdot 10^{-3}$	$-1.4716 \cdot 10^{-2}$	$1.0092 \cdot 10^{-2}$

soft. The value J_2 is underestimated for the finest mesh. It should be reminded that the locations, at which J_1 and J_2 are evaluated, lie directly on the interface between the fluid

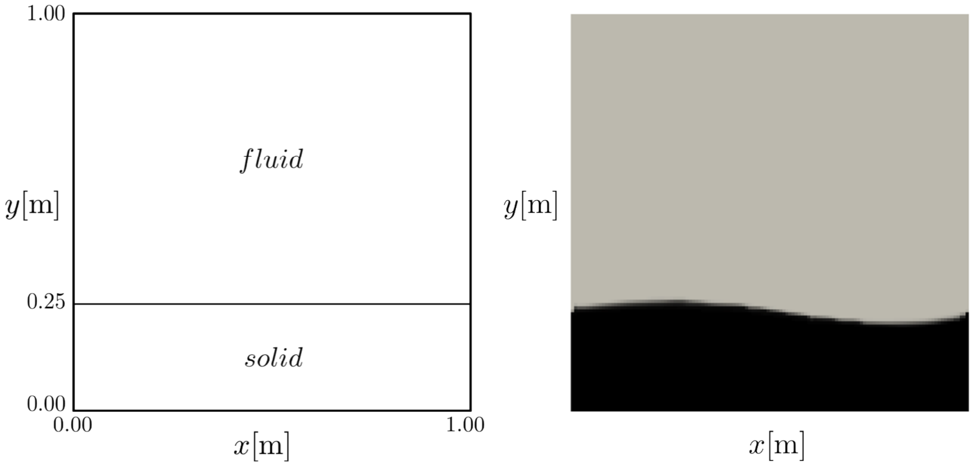

Figure 3: Left: Initial position of elastic bottom. Right: Converged position of elastic bottom with $\eta_S = 0.05\text{N/m}^2$ for a mesh of 256×256 control volumes.

Table 3: Numerical results for the lid driven cavity with elastic bottom ($\eta_S = 1.0\text{N/m}^2$).

	Numerical Results		Richter et. al [19]	
Δh[m]	J_1[m]	J_2[m]	J_1[m]	J_2[m]
2^{-6}	$-5.7629 \cdot 10^{-3}$	$1.0635 \cdot 10^{-3}$	$-1.7548 \cdot 10^{-3}$	$1.0296 \cdot 10^{-3}$
2^{-7}	$-4.1352 \cdot 10^{-3}$	$1.2179 \cdot 10^{-3}$	$-1.6103 \cdot 10^{-3}$	$1.0028 \cdot 10^{-3}$
2^{-8}	$-3.0079 \cdot 10^{-3}$	$9.5730 \cdot 10^{-4}$	$-1.5405 \cdot 10^{-3}$	$9.9413 \cdot 10^{-4}$

and solid phase. The interface region is most challenging for an interface capturing VoF scheme. A nonlinear dependency of the viscosity on the solid concentration in equation 2 might lead to better results. Generally the results for J_2 are close to the reference results, whereas the results for J_1 could still be improved. A finer mesh is needed to obtain a converged result and to reach the reference result for J_1.

3.3 Three dimensional lid driven cavity with elastic bottom

The test case described in section 3.2 is extended to become a three dimensional test case. A three dimensional cavity with dimensions $1m \times 1m \times 1m$ inlcuding an elastic bottom is simulated. The solid domain reaches from $y = 0$m to $y = 0.25$m and the fluid domain is above $y = 0.25$m. The lid at $y = 1$m is moving with a velocity of $v_x = 4x(1-x)$m/s while the velocity components in x- and y-direction are equal to zero. Dirichlet boundary conditions for the flow velocity, the displacement and the solid velocity are set at all other walls of the cavity. Therefore the displacement of the elastic bottom will be zero at the walls. The fluid and solid properties are as described in section 3.1, where a Lamé constant of $\eta_S = 0.05\text{N/m}^2$ is chosen for this case. The mesh is

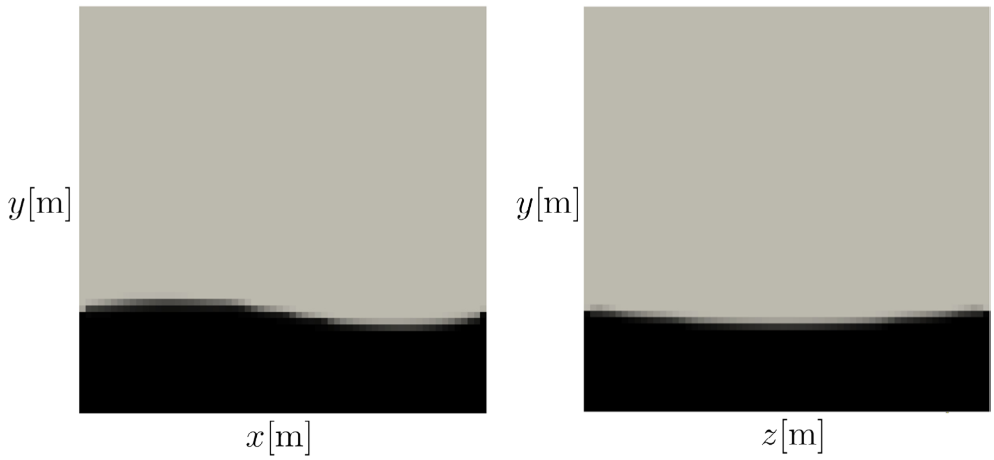

Figure 4: Plane views of three dimensional lid driven cavity with elastic bottom at $t = 40$s. The anchor for the x-y plane is set at $x = 0.5$m, $y = 0.5$m and $z = 0.5$m. The anchor for the z-y plane is set at $x = 0.75$m, $y = 0.5$m and $z = 0.5$m.

coarse ($64 \times 64 \times 64$) and a Courant number of 0.64 is applied. The deformation of the

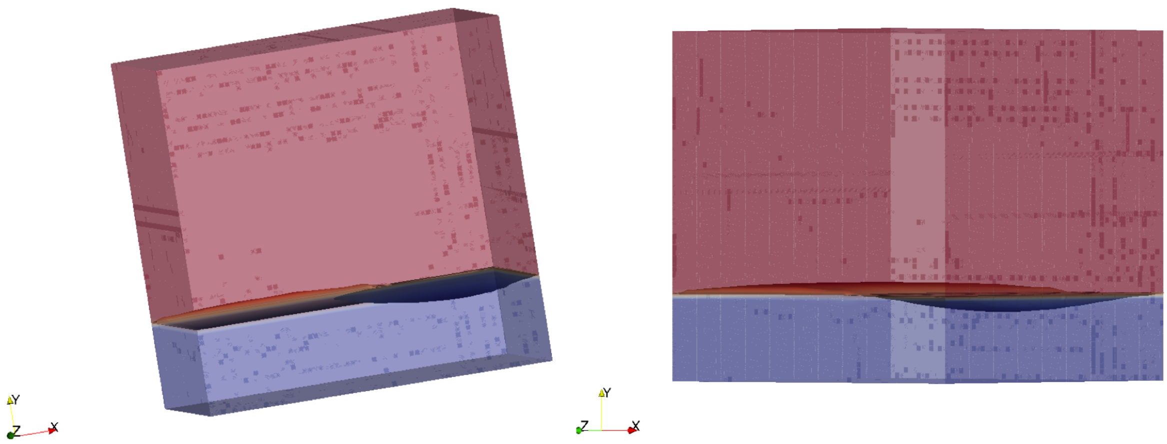

Figure 5: Two views of the three dimensional lid driven cavity with elastic bottom. The contour of the surface of the elastic bottom and its y coordinate is displayed.

elastic bottom at $t = 40$s is displayed in figures 4 and 5. A similarity of the shape of the elastic bottom displayed in the x-y plane with the shape of the elastic bottom in the two-dimensional test case can be observed. With this case it is demonstrated that the elastic model introduced in FreSCo$^+$ is able to simulate three-dimensional cases.

4 CONCLUSIONS

A fully Eulerian Fluid-Structure interaction method is introduced in FreSCo$^+$. The behaviour of elastic cargo in holds of bulk carriers can be investigated with this model. The method is validated on two two-dimensional test cases and also successfully applied to a three-dimensional case. A convergence study is conducted which shows that the grid size has an influence on the results directly on the boundary between solid and fluid domain. This flaw of the model is explained by the usage of the VoF method and suggested to be solved by a nonlinear dependency of the viscosity on the solid concentration.

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