

# HULL-FORM OPTIMIZATION OF A LUXURY YACHT UNDER DETERMINISTIC AND STOCHASTIC OPERATING CONDITIONS VIA GLOBAL DERIVATIVE-FREE ALGORITHMS

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**Abstract.** Simulation-based design optimization (SBDO) techniques are used in the shape design of complex engineering systems. SBDO methods integrate an optimization algorithm, a tool for the design modification, and analysis tools. In the context of ship/ocean applications the objective function is often noisy, its derivatives are not directly provided and local minima cannot be excluded, therefore global derivative-free algorithms are widely used. The objective of this work is to investigate the efficiency of three global deterministic derivative-free optimization algorithms for the deterministic and stochastic hull-form optimization of a luxury yacht. Particle Swarm Optimization (DPSO), Dolphin Pod Optimization (DPO), and Dividing RECTangles (DIRECT) are applied to reduce the total resistance over a variety of conditions. The approach includes a comparison of the performances of the optimization algorithms, based on deterministic results for two separate operating conditions. DPSO is identified as the most promising optimization algorithm and is used for the robust design optimization (RDO) performed considering a stochastic variation of the cruise speed with uniform distribution within a speed range from 8 to 16 kn. The resistance curve of deterministic and robust solutions is finally presented.

## 1 INTRODUCTION

Simulation-based design analysis and optimization (SBDO) techniques represent an actual paradigm to assist designers in assessing and designing complex engineering systems, where both objectives and constraints often involve several concurrent disciplines. SBDO for shape design has been widely used for diverse engineering applications, such as aerospace, automotive, and naval, where the vehicle performances (e.g., aerodynamic and aeroelasticity [1, 2], hydrodynamics and hydro-structural interaction [3, 4, 5], structures [6], etc.) strongly depend on the shape design. In this context, automatic SBDO approaches need to efficiently integrate three elements:

an optimization algorithm, a geometry modification and automatic meshing tools, and simulation solvers.

Real life applications are affected by several sources of uncertainty, such as environmental and/or operating conditions. Specifically, uncertainty quantification (UQ) methods are needed to evaluate the effects of stochastic input parameters on the relevant outputs, which can be represented in terms of expected value, standard deviation, and probability distribution [7, 8, 9]. Integrating UQ within the design optimization process leads to robust design optimization (RDO) formulations. SBD/RDO still represents a significant theoretical, algorithmic, and computational challenge, especially when high-fidelity and/or multi-disciplinary solvers are employed [10, 11].

The objective of the present work is to investigate the efficiency of three global deterministic derivative-free optimization algorithms in the hull-form optimization of a luxury yacht under deterministic and stochastic operating conditions. The interest in investigating and using deterministic global derivative free-algorithms for deterministic and or robust simulation-based shape design optimization for ship/ocean applications depends on several concurrent issues. Objective and constraints are often solved using black-box tools through systems of partial differential equations, they might be noisy and derivatives are not directly provided. Furthermore, the presence of local minima in the search space cannot be excluded a priori.

Particle Swarm Optimization (DPSO) [12], Dolphin Pod Optimization (DPO) [13], and Dividing RECTangles (DIRECT) [14] are applied to reduce the total resistance of a 35m length overall yacht. The hull form is modified using two design variables, controlling global modification functions. Design constraints include fixed displacement, fixed length between perpendiculars, fixed beam and limited (5%) variation of the draft. Two deterministic operating conditions (namely, cruise speed equal to 10 and 14 kn) are considered separately, along with a stochastic variation of the cruise speed (uniformly distributed from 8 to 16 kn) for the RDO. A stationary 2DOF problem with free surface is studied by potential flow simulations [15]. Specifically, the ship advances in calm water and is free to heave and pitch. A quasi Monte Carlo method coupled with a deterministic variant of the Latin Hypercube Sampling (LHS, [10, 11]) is used for UQ within RDO.

For the sake of confidentiality, the original geometry of the yacht hull and its optimized versions are not shown in the current paper.

## 2 DETERMINISTIC VERSUS ROBUST OPTIMIZATION: PROBLEM FORMULATION

The general deterministic single-objective design optimization problem may be formulated, within the SBD context, as

$$\text{maximize or minimize } f(\mathbf{x}), \quad \mathbf{x} \in X \subseteq \mathbb{R}^{N_{\text{DV}}} \quad (1)$$

where  $f$ , provided by simulations, is the deterministic objective function and the vector  $\mathbf{x}$  collects the  $N_{\text{DV}}$  deterministic design variables. Equality and inequality functional constraints may be applied to the problem formulated in Eq. 1, if required. Such a formulation implies that all the inputs of the simulation tools are deterministic and the design optimization is performed for specific design, operating, and environmental conditions.

When dealing with real life applications, the problem presented in Eq. 1 is affected by several sources of uncertainties, such as those related to operating and/or environmental conditions, therefore UQ methods need to be embedded within the optimization process. In RDO the effects of the input uncertainties may be evaluated using diverse formulations of the original objective function. The expected value, the standard deviation, the cumulative distribution function or the weighted sum of expected value and standard deviation of the deterministic objective function might be used to assess the RDO problem in its general form. The current work focuses on the expected value only, correspondingly the problem in Eq. 1 is reformulated as

$$\text{maximize or minimize } \mu(f) = \int_Y f(\mathbf{x}, \mathbf{y})p(\mathbf{y})d\mathbf{y}, \quad \mathbf{x} \in X \subseteq \mathbb{R}^{N_{DV}} \quad (2)$$

where  $\mu$  is the expected value of the original deterministic objective function  $f$ ,  $\mathbf{x}$  collects the  $N_{DV}$  deterministic design variables, and  $p(\mathbf{y})$  is the probability density function of the stochastic operating and environmental conditions, collected in  $\mathbf{y} \in Y$ . Functional constraints may be applied and included in the formulation, if required.

### 3 SBD OPTIMIZATION PROBLEM AND TOOLBOX AT A GLANCE

The objective function for the problem in Eq. 1 is the total resistance of the yacht in calm water at 10 and 14 kn, whereas the objective function for the problem in Eq. 2 is the expected value of the total resistance in calm water, evaluated over a stochastic speed  $y$ , with  $y \in [8;16]$  kn following a uniform probability density function.

The SBD optimization toolbox includes three essential and interconnected elements: (a) the optimization algorithm, (b) a tool for the design modification, and (c) the analysis tools. For the current application three deterministic, single-objective, derivative-free, and global optimization algorithms are used, the shape modifications are performed using orthogonal basis functions, the hydrodynamic performances are evaluated using a potential flow solver, and for the RDO problem, the UQ is performed using a quasi MC method coupled with deterministic LHS. An overview of the tools implemented is presented in the following.

#### 3.1 Global Derivative-Free Optimization Algorithms

The numerical optimization is performed using DPSO algorithm, DPO, and DIRECT, which are briefly recalled in the following.

##### 3.1.1 Deterministic Particle Swarm Optimization [DPSO]

PSO, originally introduced by [16], is based on the social-behavior metaphor of a flock of birds or a swarm of bees searching for food. PSO belongs to the class of heuristic algorithms for single-objective evolutionary derivative-free global optimization. In order to make PSO more efficient for SBDO, a deterministic version of the algorithm, namely DPSO, was formulated by [17] and further investigated in [12]. Accordingly, the location  $\mathbf{x}_j^i$  of the  $j$ -th particle at the  $i$ -th iteration is evaluated as

$$\begin{cases} \mathbf{v}_j^{i+1} = \chi[\mathbf{v}_j^i + c_1(\mathbf{x}_{j,pb} - \mathbf{x}_j^i) + c_2(\mathbf{x}_{gb} - \mathbf{x}_j^i)] \\ \mathbf{x}_j^{i+1} = \mathbf{x}_j^i + \mathbf{v}_j^{i+1} \end{cases} \quad (3)$$

where  $\mathbf{x}_{j,pb}$  is the *personal optimum* or *cognitive term* (the best position ever visited by the  $j$ -th particle),  $\mathbf{x}_{gb}$  is the *global or social optimum* (the overall best position ever visited by all the particles),  $\chi$  is the constriction factor and  $c_1$  and  $c_2$  are respectively the cognitive and the social learning rate coefficients.

DPSO performances depend on the number of swarm particles interacting during the optimization, on the initialization of the particles in terms of position and speed in the search space, and on the set of coefficients defining the personal or global behaviour of the swarm dynamics. Following [12], the swarm dimension is set to  $4 \cdot N_{DV}$ , the swarm is initialized using Hammersley sequence sampling (HSS) [18] over the design variables domain and its boundary with non-null velocity, and the coefficients are defined as  $\chi = 0.721$ ,  $c_1 = c_2 = 1.655$  [19]. For the current application the number of function evaluations is set to  $N_{feval} = 128 \cdot N_{DV}$ .

### 3.1.2 Dolphin Pod Optimization [DPO]

DPO algorithm was introduced by [13], based on the metaphor of a pod of dolphins. The  $j$ -th dolphin position  $\mathbf{x}_j^i$  and velocity vector  $\mathbf{v}_j^i$  at the  $i$ -th iteration are respectively defined as

$$\begin{cases} \mathbf{v}_j^{i+1} = (1 - \xi \Delta t) \mathbf{v}_j^i + \Delta t (-k \boldsymbol{\delta}_j + h \boldsymbol{\varphi}_j) \\ \mathbf{x}_j^{i+1} = \mathbf{x}_j^i + \mathbf{v}_j^{i+1} \Delta t \end{cases} \quad (4)$$

The coefficient of Eq. 4 are defined as

$$k = h = \frac{q}{N_d}, \quad \Delta t = \frac{\Delta t_{\max}}{p}, \quad \xi \Delta t < 1 \quad (5)$$

where  $N_d$  is the number of dolphins,  $q$  defines the weights for the attraction forces  $\boldsymbol{\delta}_j$  (congregation) and  $\boldsymbol{\varphi}_j$  (self-awareness, communication and memory), whereas  $p$  defines the integration time step  $\Delta t$ .

DPO performances depend on the number of dolphins interacting during the optimization, on its initialization in terms of position and speed in the design space, and on the set of coefficients that control its dynamic. Following the guideline suggested in [13] the pod dimension is set to  $N_d = 4 \cdot N_{DV}$ , the pod is initialised using HSS on the domain, and the coefficients are defined as  $\xi = 1.00$ ,  $q = 1.00$ ,  $p = 8.00$ ,  $\alpha = 0.50$ . For the current application the number of function evaluations is set to  $N_{feval} = 128 \cdot N_{DV}$ .

### 3.1.3 DIviding RECTangles [DIRECT]

DIRECT is a sampling deterministic global derivative-free optimization algorithm and a modification of the Lipschitzian optimization method [14, 20, 21]. The optimization process starts transforming the domain  $\mathcal{D}$  of the problem into the unit hyper-cube  $D$ . At the first step  $f(x)$  is evaluated at the center ( $c$ ) of the search domain; the hyper-cube is then partitioned into a set of smaller hyper-rectangles and  $f(x)$  is evaluated at their centers. The partition of  $D$  at iteration  $k$  is defined as

$$\mathcal{H}_k = \{D_i : i \in \mathcal{I}_k\}, \quad \text{with } D_i = \{x \in \mathbb{R}^{N_{DV}} : \ell_j^{(i)} \leq x_j \leq u_j^{(i)}, j = 1, \dots, N_{DV}, \forall i \in \mathcal{I}_k\} \quad (6)$$

where  $\ell_j^{(i)}$  and  $u_j^{(i)} \in [0, 1]$ , with  $i \in \mathcal{I}_k$ , are the lower and upper bounds defining the hyper-rectangle  $D_i$  and  $\mathcal{I}_k$  is the set of indices identifying the subsets defining the current partition. At a generic  $k$ -th iteration, starting from the current partition  $\mathcal{H}_k$  of  $D$ , a new partition,  $\mathcal{H}_{k+1}$ , is built subdividing a set of promising hyper-rectangles of the previous one. The identification of *potentially optimal* hyper-rectangles is based on the measure of the hyper-rectangle itself and on the value of  $f(x)$  at its center  $c^i$ . The refinement of the partition continues until a prescribed number of function evaluations is performed, or another stopping criterion is satisfied. The minimum of  $f(x)$  over all the centers of the final partition, and the corresponding centers, provide an approximate solution to the problem. For the current application the number of function evaluations is again set to  $N_{\text{feval}} = 128 \cdot N_{\text{DV}}$ .

### 3.2 Shape Modification Method

Shape modifications are represented in terms of orthogonal basis functions  $\psi_j$  ( $j = 1, \dots, N_{\text{DV}}$ ), defined over surface-body patches as [20, 21, 22]

$$\psi_j(\xi, \eta) := \alpha_j \sin\left(\frac{p_j \pi \xi}{A_j} + \phi_j\right) \sin\left(\frac{q_j \pi \eta}{B_j} + \chi_j\right) \mathbf{e}_{k(j)} \quad (\xi, \eta) \in [0; A] \times [0; B] \quad (7)$$

where  $\alpha_j$  is the  $j$ -th (dimensional) design variable;  $p_j$  and  $q_j$  define the order of the basis function in  $\xi$  and  $\eta$  direction respectively;  $\phi_j$  and  $\chi_j$  are the corresponding spatial phases;  $A_j$  and  $B_j$  are the patch extension in  $\xi$  and  $\eta$  respectively, and  $\mathbf{e}_{k(j)}$  is a unit vector. Modifications may be applied in  $x$ ,  $y$  or  $z$  direction ( $k(j) = 1, 2, 3$  respectively).

In this work two (normalized) design variables,  $x_1 = \alpha_1/2$  and  $x_2 = \alpha_2$ , are used. The shape modifications are obtained as per Eq. 7 with  $j = 1, 2$  and  $k = 3$ . The associated parameters are  $p_1 = 2.0$ ,  $\phi_1 = 0.0$ ,  $q_1 = 1.0$ ,  $\chi_1 = 0.0$ ,  $\alpha_1 \in [-2.0; 2.0]$  and  $x_1 \in [-1.0; 1.0]$ ;  $p_2 = 1.0$ ,  $\phi_2 = 0.0$ ,  $q_2 = 2.0$ ,  $\chi_2 = 0.0$ ,  $\alpha_2 \in [-1.0; 1.0]$  and  $x_2 \in [-1.0; 1.0]$ . Geometric constraints include fixed length between perpendiculars ( $L_{\text{BP}}$ ), fixed displacement, and fixed beam, whereas a 5% change in draft is allowed.

### 3.3 Hydrodynamic analysis

The hydrodynamics is solved using the code WARP (WAVE Resistance Program), developed at CNR-INSEAN. For details of equations, numerical implementation and validation the reader is addressed to [15]. Specifically, the wave resistance computations are based on the double-model linear potential flow (PF) theory [23], whereas the frictional resistance is estimated using a flat-plate approximation, based on the local Reynolds number [24].

The following assumptions are used for the numerical investigations. The numerical model, free to sink and trim (2DOFs problem), advances in calm water with  $v \in [8; 16]$  kn. Its reference length  $L_{\text{BP}}$  equals to 34.5 m ( $0.224 \leq \text{Fr} \leq 0.447$ ) and the displacement  $\Delta$  equals to 240 tons. The fluid conditions are  $\rho = 1000 \text{ kg/m}^3$ ,  $\nu = 1.1\text{E-}06 \text{ m}^2/\text{s}$  and  $g = 9.81 \text{ m/s}^2$ .

PF calculations are performed using a 150x50 panel grid for the hull surface. The computational domain for the free surface is defined within 1 hull length upstream, 3 lengths downstream and 1.5 lengths aside, discretized with 30x44, 90x44 and 30x44 surface panels, respectively. Grid convergence analysis is provided in [25].

### 3.4 UQ method

The integral in Eq. 2 is approximated using a quasi MC method as

$$\mu(f) = \frac{1}{N_{\text{UQ}}} \sum_{i=1}^{N_{\text{UQ}}} f(\mathbf{x}, \mathbf{y}_i) \quad (8)$$

A deterministic variant of the LHS method is applied, dividing the uncertain parameter domain (1D) with  $N_{\text{UQ}} = 2^h + 1$  ( $h \in \mathbb{N}^+$ ) evenly spaced items [10, 11].

## 4 NUMERICAL RESULTS

Numerical results are organized as follows. Subsection 4.1 shows the results of the deterministic design optimization aimed at minimizing the total resistance,  $R_T$ , in two operating conditions (*i.e.*  $v=10$  and  $v=14$ kn, respectively indicated as Problem I and II in the following). Moreover the most promising optimization algorithm is identified, based on deterministic design optimization results. The RDO results are shown in subsection 4.2 and include a parametric performance analysis of the RDO solution and the best performing deterministic configurations.

### 4.1 Deterministic optimization

For Problem I, DPSO, DPO, and DIRECT have a similar objective function reduction (differences are below 0.5%) even if DPSO and DPO show a faster convergence to the global minimum, as depicted in Fig. 1a. The final configurations found by the three algorithms are very close in terms of design variables, as shown in Fig. 2a. The deterministic optimization results, summarized in Tab. 1, show that the three algorithms achieved approximately the same total resistance reduction equal about to 11.60%. Figure 3a shows a parametric performance analysis on the three optimized configurations compared to the original one, versus the cruise speed. As desired, the optimized shapes outperform the original for the design speed ( $v = 10$  kn). For higher speeds, the optimized hulls show a total resistance larger than the original.

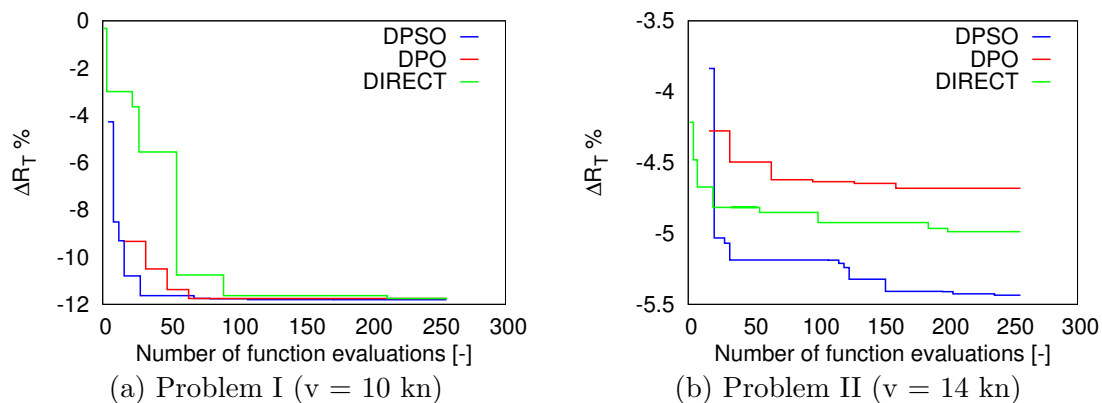


Figure 1: Deterministic optimization, algorithm convergence

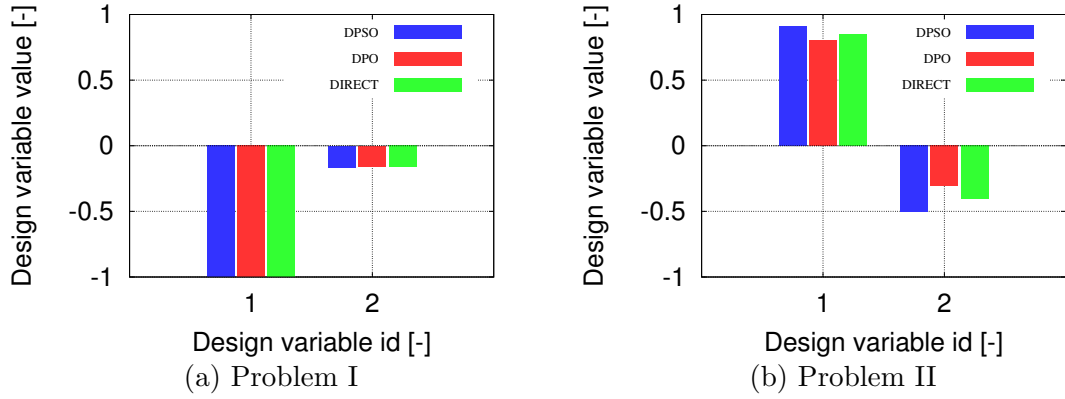


Figure 2: Deterministic optimization, optimal design variables

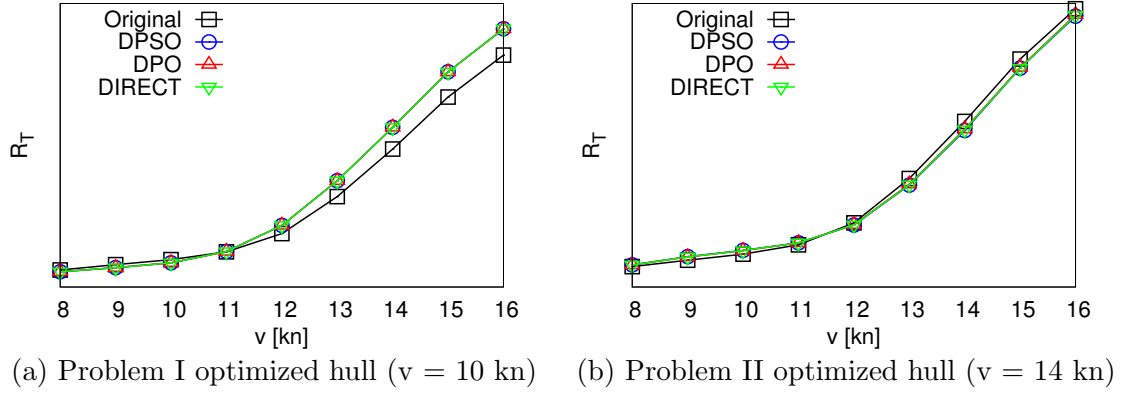


Figure 3: Resistance curve of optimal configurations

Table 1: Deterministic Problem I results ( $v = 10$  kn)

ID	$x_1$	$x_2$	$\Delta R_T\%$
Original	0.000	0.000	-
DPSO	-1.000	-0.159	-11.65
DPO	-1.000	-0.167	-11.60
DIRECT	-1.000	-0.162	-11.62

For Problem II, DPSO, DPO, and DIRECT achieve a comparable objective function reduction (the differences are smaller than 1%) even if DPSO shows a faster convergence to the global minimum and achieves a better solution than DPO and DIRECT, as depicted in Fig. 1b. The final configurations found by DPSO and DIRECT are close in terms of design variables (see Fig. 2b), whereas DPO design variables values are slightly different. The deterministic optimization

Table 2: Deterministic Problem II results ( $v = 14$  kn)

ID	$x_1$	$x_2$	$\Delta R_T\%$
Original	0.000	0.000	-
DPSO	0.911	-0.503	-5.45
DPO	0.806	-0.308	-4.66
DIRECT	0.850	-0.404	-4.98

results are summarized in Tab. 2, showing a total resistance reduction of 5.45%, achieved by DPSO. Figure 3b shows a parametric performance analysis of the three optimized configurations compared to the original one, versus the cruise speed. As desired, the three optimized shapes outperform the original for the design speed ( $v = 14$  kn). For lower speeds, their total resistance is found larger than the original.

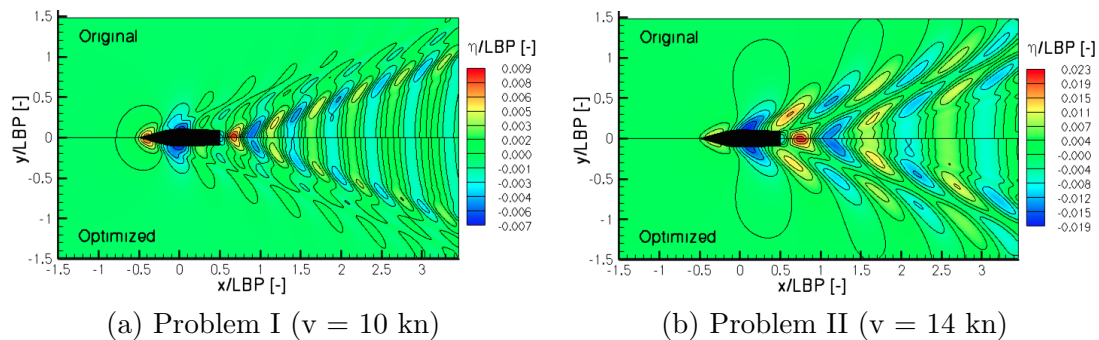


Figure 4: Wave elevation of original and DPSO optimized hulls

Finally, Fig. 4 shows the comparison between the wave elevation of the original and the DPSO optimized configurations for Problem I ( $v = 10$ ) and II ( $v = 14$  kn), respectively. It may be noted how, for Problem I, the optimized hull reduces bow and stern diverging Kelvin waves and stern transverse wave. For Problem II, the optimized hull only reduces the bow wave while slightly increases stern diverging and transverse waves, resulting in a smaller performance improvement than Problem I.

## 4.2 Robust optimization

The UQ convergence is studied versus the number of samples  $N_{UQ} = 2^h + 1$ ,  $h \in \mathbb{N}$ , for the design variables at centre domain value ( $x_1 = x_2 = 0$ ). Figure 5a shows the solution change of the expected value of the total resistance ( $\Delta\mu/\mu$ ) versus  $N_{UQ}$ . A benchmark tolerance for solution changes is set to  $10^{-3}$ , correspondingly a number of UQ samples  $N_{UQ} = 33$  is identified.

Figure 5b represents the DPSO convergence history. DPSO achieves an objective function reduction of 7.6% showing a quite fast convergence to the global minimum. The RDO is performed using a number of DPSO function evaluations set to  $128 \cdot N_{DV}$ , the same as Problem I and II. However, it should be noted that the overall number of function evaluation for the RDO procedure is  $128 \cdot N_{DV} \cdot N_{UQ}$ , since each evaluation of  $\mu$  requires 33 evaluations of the total



resistance.

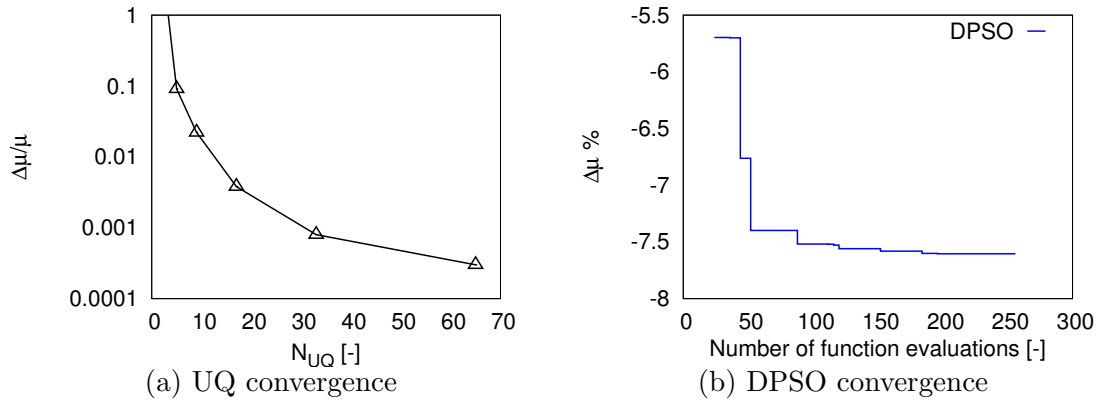


Figure 5: RDO, UQ and DPSO convergence

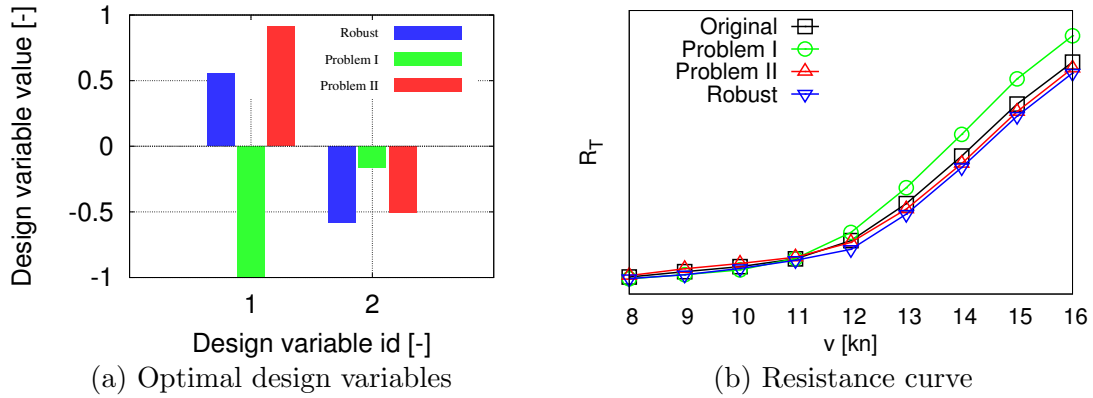


Figure 6: Comparison of deterministic optimization versus RDO, optimal design variables and resistance curve

Figure 6a shows the design variables values obtained performing the RDO compared to the deterministic DPSO values of Problem I and II. The RDO optimum falls nearby the optimum of Problem II.

Table 3 summarizes the results of the RDO optimization and, furthermore, shows the values of the expect value of the total resistance evaluated *ex-post* for the optimal configurations of Problem I and II. Finally, a parametric analysis of the total resistance versus the cruise speed is presented in Fig. 6b, comparing the performances of the optimal RDO configuration and of the optimal configurations of Problem I and II to the original ones. A close agreement is found between Problem II solution and the RDO solution, which however present the best overall improvement in the speed range (Tab. 3). Furthermore, although the RDO solution presents

the overall best performance, Problem I solution is slightly better than the others for  $v = 10$  kn. It should be also noted that the RDO solution outperforms Problem II solution for  $v = 14$  kn, revealing that probably none of the three algorithms had completely explored the search space while solving Problem II.

Table 3: Comparison of deterministic optimization versus RDO, summary of results

ID	$x_1$	$x_2$	$\Delta\mu\%$
Original	0.000	0.000	-
DPSO Problem I	-1.000	-0.159	11.14
DPSO Problem II	0.911	-0.503	-2.39
DPSO RDO	0.554	-0.580	-7.62

## 5 CONCLUDING REMARKS AND FUTURE WORK

An exploratory comparative study was presented on the performances of three global derivative-free optimization algorithms for simulation-based shape design optimization of hydrodynamic applications. Specifically, the ability of DPSO, DPO and DIRECT was studied for the minimization of the total resistance of a luxury yacht. Two separate deterministic cruise conditions (Problem I and II) were addressed. A stochastic variation of the cruise speed (uniformly distributed within the speed range) was also considered for a RDO problem. The RDO was performed using the best performing algorithm identified from Problem I and II. Two design variables controlled global shape modifications, an in-house potential solver was used to evaluate the hydrodynamic performances of the yacht, and a quasi MC method with a deterministic version of LHS was used to solve the UQ within RDO.

The three algorithms showed overall similar performances, achieving for both deterministic problems comparable reductions of the objective function. For Problem I, DPSO and DPO converged to the minimum faster than DIRECT, whereas for Problem II DPSO showed a faster convergence than the other algorithms. Overall, DPSO was identified as the best performing algorithm and used for the RDO problem.

The deterministic and robust optimization results were compared in terms of total resistance and its expected value. A parametric analysis over the speed range revealed that the RDO solution presents the overall best performance. Moreover, Problem I optimal configuration shows the best performance for its optimization speed ( $v = 10$ kn) whereas its performances decrease in the high speed range. Differently, RDO solution is found slightly outperforming Problem II solution at Problem II optimization speed ( $v = 14$  kn), thus revealing that probably during the deterministic optimization the design space was not completely explored. Furthermore, the *ex-post* evaluation of the expected value of the total resistance of the deterministic solutions showed that Problem I optimal solution has the worst behavior, whereas the deterministic optimal solution of Problem II presents a similar trend to the RDO solution. Future work will extend this study addressing higher-dimensional problems using higher-fidelity hydrodynamic solvers, taking into account diverse probability distributions of cruise speed.

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