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HOMOGENIZED AND NON-CLASSICAL BEAM THEORIES IN SHIP STRUCTURAL DESIGN – CHALLENGES AND OPPORTUNITIES

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Abstract. The paper gives an overview of the recent developments on the application of homogenized, non-classical beam theories used to predict the micro- and macrostructural stresses in the design of marine structures. These theories become important when ultra-lightweight marine structures are developed and one needs to explore the regions where the length scales of beam openings are in the range of the characteristic lengths of the beams or when lattice/frame-type beams are used to reduce the weight of ship structures. The homogenized beam models are based on non-classical continuum mechanics that allow local bending inside the beams. This added feature allows the treatment of size effects with great accuracy. The resulting analytical and finite element models have special features in terms of shape functions and iterative solutions in non-linear problems. Non-classical beam models enable localization processes that recover the microstructural effects from homogenized solutions accurately and the models are able to handle limit states of serviceability and ultimate strength. The non-classical models are validated by experiments and 3D FE simulations of periodic beams and plates. The non-classical beam theories converge to the physically correct solutions for wider range of beam parameters than the classical beam theories do.

1 INTRODUCTION

Thin-walled structures form the basis of transportation of people and goods as they are widely used in different length-scales in vehicles, containers and tanks, in packaging and they are used even in the microelectronics to control vehicles' operation; see Figure 1. The thin-walled structures contribute significantly to the overall weight of a vehicle itself and to the payload. Structural performance is measured by the ratio between the payload and the structural weight. We aim to build increasingly lighter vehicles, to diminish the environmental impact of transportation. However, the limits to which we can push the structural performance ratio are defined by safety and further by the probability of failure of the transportation system, of the vehicles, and of their structures all the way to the materials.

The probability of failure is defined by the relation between the demand (load) and the capacity (strength) of the structure. Demand results from operations in random environments, whereas capacity stems from structural design, manufacturing, materials and operations. Since both of them have a statistical nature, the definition of uncertainty and the acceptable levels for the probability of failure are to a great extent affected by computational modelling. Due to global warming, the random environments are proven more aggressive which increases the loads and further the probability of failure. Simultaneously, new ultra-lightweight materials and lattice structures are introduced to small length-scales via advanced manufacturing methods such as additive manufacturing and laser-welding, see Figure 1 and Ref. [1].

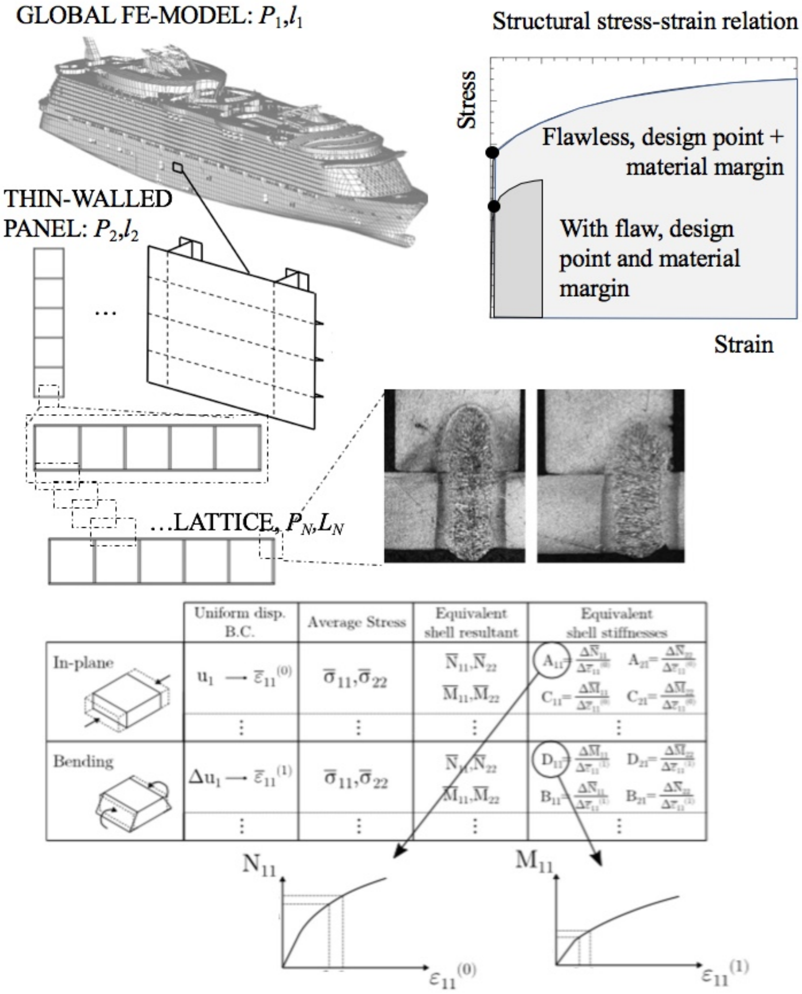

Figure 1: Lightweight design of marine structures through hierarchical structures with finite scale separation based multi-scale modelling. Classical length-scales range from ship hull girder (primary response) to bulkheads (secondary response) and further to stiffened panels (tertiary response). Modern manufacturing and materials technology is enabling additional length-scales with geometrical and material non-linearity.

The ultra-lightweight features are obtained by applying to ever smaller scales the ideas of civil engineers (e.g. Eiffel tower), bridge-builders (e.g. Vierendeel frames), off-shore engineers and naval architects (e.g. 3D-truss-structures, side shells with balcony openings), and aeronautical engineers (e.g. wing-spars) who have over long time found these feasible solutions to very complex problems where different length-scales interact. The ultra-lightweight materials and structures have relative densities, ρ_{mat}/ρ_{bulk}, often as low as 0.1-0.2, but stiffness-to-strength values are not reduced as much. However, the material reserve in terms of plastic capacity with respect to the elastic limit (design point) is significantly reduced especially when the material contains flaws or perforations, see Figure 1. This forces designers to impose larger safety factors for these ultra-lightweight builds. Thus, even though we gain better performance in the material scale, we tend to suffer larger overall uncertainty in structural scale as the integration into structural systems contains increasing complexity and layers of advanced structures. This uncertainty originates partly from production challenges, such as the positioning of laser-welds, initial imperfections and residual stresses, but more importantly from the limitations of the current strength-assessment methods which need to evaluate response and strength at all relevant length-scales simultaneously.

This paper gives an overview of the recent developments on the application of homogenized beam theories based on non-classical continuum mechanics used to predict the micro- and macrostructural stresses in the design of marine structures. The paper is organized so that we first present the main assumptions introduced by homogenization and localization and discuss their relevancy in terms of marine structures. Then we move to the resulting differential equations and discuss their analytical and numerical solutions in linear elastic problems, eigenvalue vibration and buckling problems and in a problem where geometrical non-linearity is faced in micro and macroscales. In order to demonstrate the gains and remaining challenges we present case studies where comparison to experiments and high-fidelity finite element simulations are performed.

2 HOMOGENIZATION AND LOCALIZATION

In the assessment of structural performance, the response is needed to ensure that the load-carrying mechanism is correct within a complex structure and strength is needed to ensure capacity. In terms of solid mechanics, the response is effectively determined by using homogenized continuum models, while strength requires localization and understanding of how damage spreads in the structure in a realistic manner. The challenge between the two approaches is the scale-transition so that the energy, stresses and strains are all kept in balance between the continuum model and the sub-model where localization and strength are defined. This difference between the two approaches is presented here by an example of periodic, linear elastic web-core sandwich beam for which the accurate solution can be derived by assuming that the faces and webs bend locally as Euler-Bernoulli beams; we consider this here as *Assumption 0* for the reason that more advanced local deformation models can be developed easily by more advanced beam models or superelements. The beam deflection by using discontinuity functions [2] is given as:

$$w^{face}(x) = w^{l,bend}(x) + w^{g,bend} \tag{1a}$$

$$w^{l,bend}(x) = w_0^l + \theta_0^l x + \frac{1}{D_i}\left[\sum_{i=1}^{I}\frac{H(x-a_i^M)M_i(x-a_i^M)^2}{2!} + \sum_{j=1}^{J}\frac{H(x-a_j^F)F_j(x-a_j^F)^3}{3!} + \sum_{k=1}^{K}\frac{H(x-a_k^q)q_k(x-a_k^q)^3}{4!}\right] \tag{1b}$$

$$w^{g,bend}(x) = w_0^g + \theta_0^g x - \frac{1}{D_g}\left[\sum_{l=1}^{L} \frac{\left(M_l^t + M_l^b\right) H(x - a_l^{Mi})(x - a_l^{Mi})^2}{2!}\right] \tag{1c}$$

where $w^{l,bend}$ is the local deflection of the face due to point forces and moments and distributed external and internal loads applied directly to the face and $w^{g,bend}$ is the global deflection caused by elongation of the faces to opposite directions. w_0 and θ_0 are the local and global transverse displacement and rotation boundary conditions, respectively. Symbols a indicate the locations of the point moments (superscript M), forces (superscript F) and uniform distributed loads (superscript q). H is the Heaviside function with first and second derivative being Dirac's delta and unit doublet functions respectively.

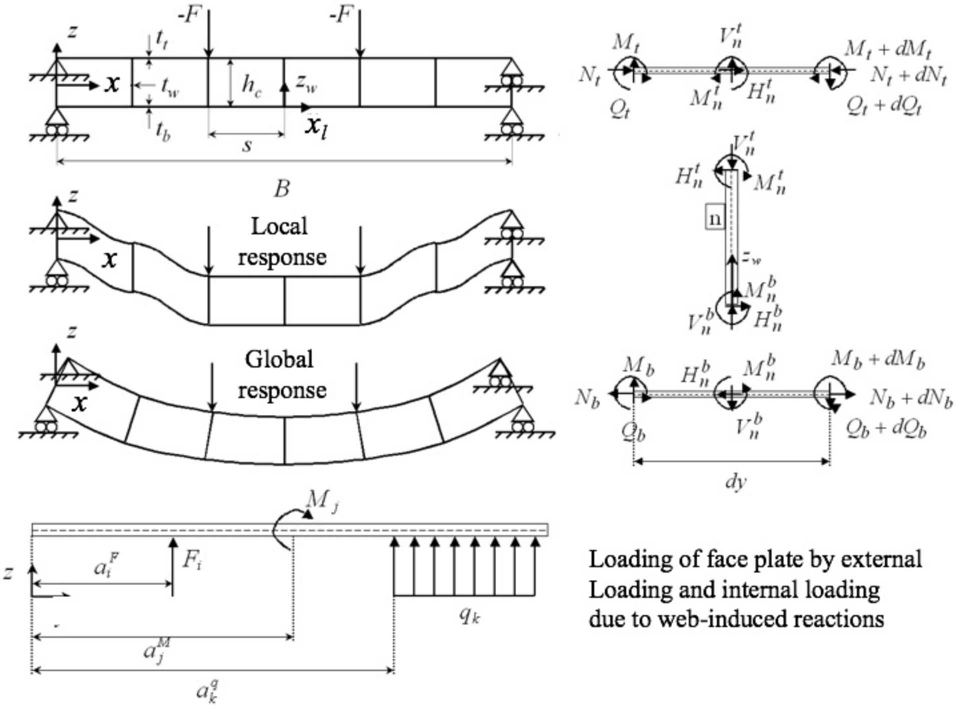

Figure 2: Two-scale modeling of web-core beam with discontinuity functions and an assumption of microstructural Euler-Bernoulli beam behavior.

This solution is accurate, but needs to be solved for each location of flexible core members (including joint rigidity) along x-axis resulting in $3L$+4 equations, where L is the number of web-plates in the beam [1]. The solution can be simplified towards a homogenized solution by neglecting distributed loads and moving all loads to the locations of webs; this is *Assumption 1* that currently limits the use of continuum models to the cases of finite length-scale ratio. The result is:

$$w^{face}(x) = w_0^l + \theta_0^l x + w_0^g + \theta_0^g x \tag{2}$$

$$+\sum_{l=1}^{L} \frac{H(x - a_l\ )(x - a_l\ )^2}{2!}\left[\frac{1}{D_i}\left(M_l + \frac{F_l(x - a_l\ )}{3}\right) - \frac{1}{D_g}\left(M_l^t + M_l^b\right)\right]$$

When in **localization** the peak response at the location of high gradients must be assessed, we can use Taylor-series expansion around point a as

$$w(x) = w^0(a) + \frac{w^1(a)}{1!}(x-a) + \frac{w^2(a)}{2!}(x-a)^2 + \cdots \tag{3}$$

The comparison of Eqs. (1)-(3) reveals that the $(x\text{-}a)^n$-terms are maintained to various degrees and the accuracy around the point of interest can be maintained by including enough terms in the Taylor series to compensate for the neglection of the Heaviside operator. In the case of an entire beam, we end up with the same number of terms as in the accurate solution given by Eq. (1). Thus, this expansion is useful, but should be only performed locally. In order to approximate the response in smooth, periodic, fields of deformation away from high gradients, the **homogenization** theory can be used, where the two length-scale asymptotic expansion is used instead to give:

$$w^k(x) = w^0(x,y) + k^1 w^1(x,y) + k^2 w^2(x,y) + \cdots \tag{4}$$

$$k = \frac{l_{micro}}{l_{macro}} \ll 1 \tag{5}$$

$$w^k(x,y) = w^k(x, y + l_{micro}) \tag{6}$$

where the microlevel coordinate $y=x/k$ is the local coordinate which is assumed to be infinitely small in comparison to x. In Eq. (4), the $(x\text{-}a)^n$-terms are approximated with x^n-terms multiplied by powers of the length-scale ratio, k, Eq. (5). The microscale responses are assumed to be fully periodic, Eq. (6) and these should be recoverable from the macroscale responses. Thus, the actual positioning of the unit cell along the beam is not considered and the continuity conditions on deflection and its derivatives at the edges of unit cell are used to secure that microscale responses do not transfer through unit cell edges, except through macro-scale phenomena; this is *Assumption 2* that limits the use of continuum theories.

The *Assumption 3* that the length-scale ratio is infinitely small is never true. Especially when applied to ultra-lightweight structures, the geometrical and physical characteristic lengths of the vehicles themselves (e.g. hull girder of the ship, l_1~300m), the secondary structures (e.g. bulkheads and decks, l_2~30m) and the tertiary structures (e.g. stiffened or corrugated sandwich panels, l_3~3m) can be close. The approximation can be improved when more terms are included in the asymptotic expansion especially close to the locations of high gradients. These additional terms, however, increase the computational costs. These different approaches have very similar form. While the asymptotic expansion with increasing terms aims to model with continuum the discrete structure, the Taylor series aims to do this from the discrete structure towards continuum. When and how these two approaches meet for finite length scale ratio is a grand-challenge. Analytical solutions exist for the simplest of cases. However, these solutions have very little practical relevance. An alternative is to use the fine mesh finite element method, but this is not a sustainable solution in the long run, since we expect new emerging length scales in future structures developed with novel manufacturing technologies and materials, see Figure 1. In these FEM models, the element size is defined by the smallest detail that affects strength. For the limit state of fatigue, the size of such detail can be 10^{-6} m, which calls for a mesh size at least 5-10 times smaller. This scale difference of 10^9 m in ship structures results in enormous computational models beyond current and near future computational capabilities. Therefore, the natural choice of computational approach is the extended non-classical continuum description. Here we focus on two sub-classes of namely those based on strain-gradient/couple stress and micropolar models; extensive review of different models is given by Srinivasa and Reddy [3].

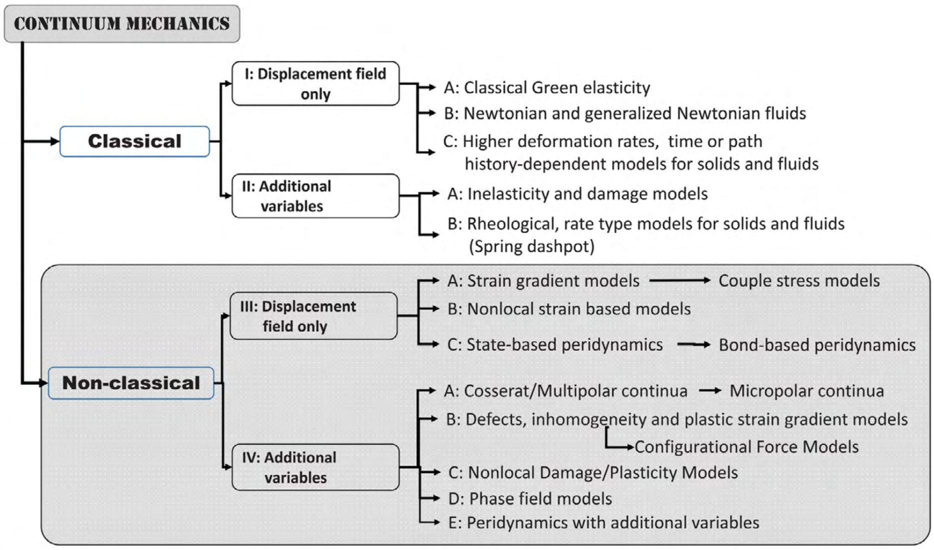

Figure 3: Taxonomy of continuum mechanics as described by Srinivasa and Reddy [3].

3 DIFFERENTIAL EQUATIONS AND THEIR SOLUTIONS

3.1 Beam formulations based on micropolar and couple-stress models

The displacements for a micropolar Timoshenko beam can be written as [4-7]

$$u(x,y) = u_x(x) + y\theta(x), w(x,y) = w(x), \Psi(x,y) = \varphi(x) \tag{7}$$

where the classical displacements u and w and rotation θ are complemented by a non-classical microrotation φ. In the micropolar formulation the microrotation enters the beam through strain formulation as [4-5]

$$\varepsilon_x = \frac{\partial u}{\partial x}, \varepsilon_{xy} = \frac{\partial w}{\partial x} - \Psi, \varepsilon_{yx} = \frac{\partial u}{\partial y} + \Psi, \kappa_{xz} = \frac{\partial \Psi}{\partial x} \tag{7}$$

which ultimately means that the shear strain will consist of symmetric and antisymmetric parts and there is an additional curvature κ_{xz} that can be used to describe local bending. In a couple stress based approach the strains read [6-7]

$$\varepsilon_x = \frac{\partial u}{\partial x}, \gamma = \theta - \frac{\partial w}{\partial x}, \chi_{xy} = \frac{1}{4}\left(\frac{\partial \theta}{\partial x} - \frac{\partial^2 w}{\partial x^2}\right) \tag{7}$$

Thus, instead of formulating shear into antisymmetric and symmetric parts, average shear strain is used and non-classical, local bending features are introduced through curvature χ_{xy}. By employing a constitutive model and variational principles, one obtains the governing equations for the micropolar and couple stress beam models.

3.2 Analytical and Finite Element Solutions

The fact that the beam strains include microrotations or gradients of local rotations increases the total differential order of the governing beam differential equations. This means that the analytical solutions will include higher-order terms that appear as exponential or hyperbolic functions. Such terms are significant in the vicinity of strain gradients and in structures where internal stiffness of the microstructure is significant in comparison to macroscale strain gradients. In analytical solutions this effect is of course directly included to the solution and does not require special attention. However, in finite element approximations it creates a need to include higher-order polynomials to shape function approximations. Karttunen et al. [8] derived also exact shape functions which do not suffer from convergence issues due to numerical procedures. However, these elements have been tested only for linear elastic cases.

3.3 Scale Interaction

The scale transition is needed between micro- and macroscale analyses, see Figure 4 (couple stress approach). Up-scaling results in homogenized stiffness properties for which in addition to classical in-plane, bending, membrane-bending and shear stiffness, the local unit cell stiffness in terms of strain gradients are needed. This process can be linear or non-linear in terms of geometry. The inverse-process of homogenization, i.e. down-scaling/localization instead results periodic stresses that can be used to assess the stress peaks inside microstructure. This improves to a great extent the strength predictions essential in marine structures.

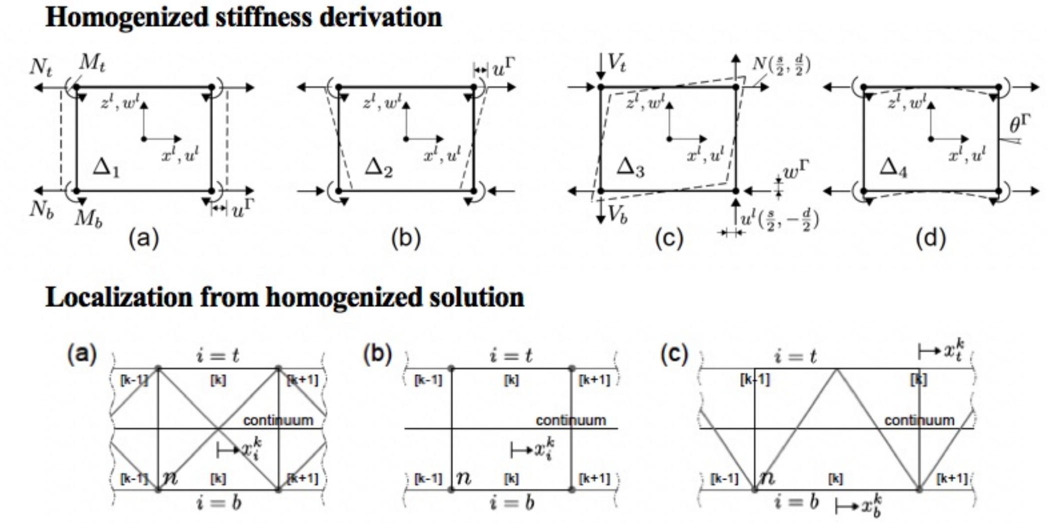

Figure 4: Top. Derivation of homogenized stiffness properties for in-plane, bending, shear and couple stresses. Localization of microstresses from homogenized solution. [6].

4 LIMIT STATES

4.1 General

In the following examples we demonstrate the different aspects of non-local beam theories in terms of gains in accuracy and remaining challenges. These have been collected from various sources and presented here to give insight to the current state of the art. The limit states selected are those categorized as by International Ships and Offshore Structures Congress (ISSC); serviceability, ultimate, fatigue and accidental.

4.2 Serviceability

The functionality of a ship structure is crucial in terms fulfilling the mission needs. This means that the deformation/stress levels, stiffness and vibratory (e.g. comfort) need to be within the design limits. Figure 5 shows examples from Refs. [4, 9], that demonstrate clearly the main benefits of the non-classical formulations and the challenges faced due to homogenization for the linear-elastic, static case. Figure 6 presents this for the eigenvalue vibratory responses. In this example the homogenous core sandwich panels with thick-faces and the discrete web-core sandwich panels are considered both with couple stress and micropolar frameworks respectively.

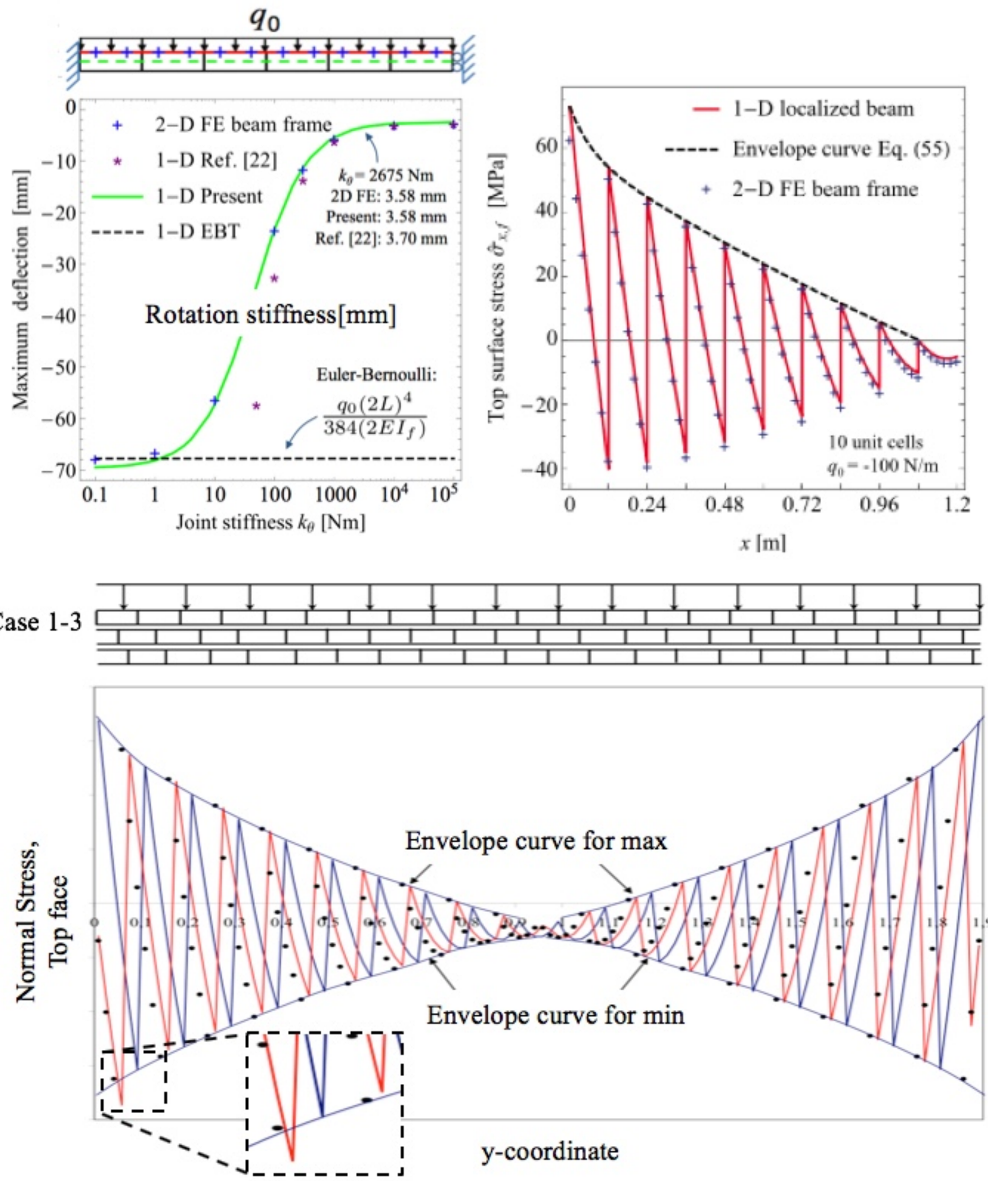

Figure 5: Top. Comparison between developed micropolar method [4] and high-fidelity FEA with the envelope curves for maximum stress from localization. Bottom. Effect of different unit cell positioning on observed discrete and envelope curve stress distributions [9].

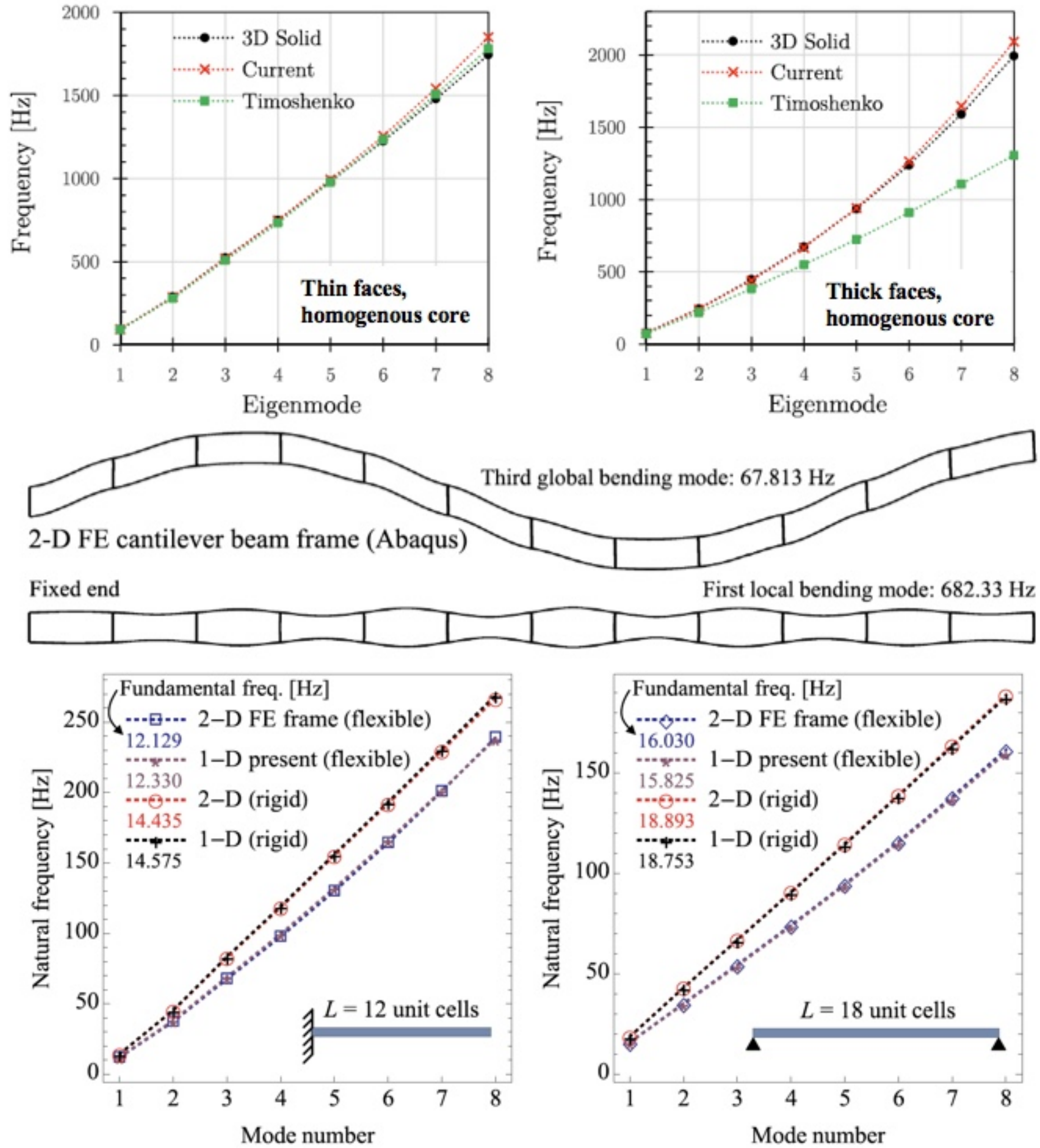

Figure 6: Comparison of beam solutions for eigenfrequency analysis. Top: 1-D Couple stress, Middle: 2-D FE frame example Bottom: 1-D Micropolar. [6,7].

Figure 5 shows that for beams with length-scale aspect ratio of l_{micro}/l_{macro}=1/12 the deflection can be obtained very accurately with the micropolar model. It is shown that the agreement between the high-fidelity Finite Element Analysis is perfect for various joint rotation stiffness values between the face and web-plates. In the case of classical Timoshenko theory, the results are in agreement with FEA in the cases of high rotation stiffness, but start to deviate as the rotation stiffness approaches zero. This means that micropolar model is able to converge to physically correct behavior for the entire rotation stiffness range as long as the microstructure is periodically regular over beam span. The accuracy of the method is further highlighted in the top surface normal stress comparison. The surface stress is very important when for example fatigue strength is assessed. It is also shown that the strain/stress localization can be approached with envelope curve capturing the maximum stresses from each unit cell. This results in a continuous curve far above the floating unit cell average, indicating that in these structures with high length-scale aspect ratio, stress localization is more important design criterion that stiffness. It is also seen that the unit cell positioning with respect to beam boundaries has significant effect on single stress peaks near the boundaries. In these cases, the application of the Taylor series would become important to consider in localization process, i.e. Assumption 1 should be relaxed.

Figure 6 shows that the influence of non-classical continuum mechanics become important when the faces of sandwich panels have significant local bending stiffness in relation to bending and shear deformations. The fact that these non-classical solutions are considering the finite curvature of microstructure stiffens the structure and it can be seen that the agreement with 3D-FEA is excellent. As the formulation is continuum-based, the local vibrations at the unit cell level cannot be properly assessed. In terms of beam theories based on classical continuum corrections have been proposed for this issue recently, thus there is possibilities to correct this also in non-classical setting.

4.3 Ultimate, Accidental and Fatigue Strength

Ultimate, accidental and fatigue strength are important strength criteria for design of marine structures as they define the maximum stresses that the structure can tolerate. Figure 7 presents results of ultimate strength assessment based on classical one-scale and non-classical two-scale geometrical nonlinearity and the influence of rotation stiffness to the linear elastic response used to assess the fatigue strength.

Figure 7 shows clearly that conducting a two-scale geometric non-linear analysis coupled with a non-classical model is very important in predicting the non-linear buckling load of sandwich beams regardless of the core type. The microscale effect is larger in stretch-dominated cores, where a rapid decrease of the beam stiffness properties occurs after local buckling and the macroscale load carrying mechanism changes. In case of bending-dominated cores such as web-core panels, the geometric non-linearity is only relevant at the macroscale and described through the *von Kármán* term. Differences between the classical and non-classical solutions are due to the incorporation of an additional scale in stretch-dominated cores, whereas in bending-dominated cores the effect is similar to the one observed in the linear case. Figure 7 also shows that as rotation stiffness of the joint connecting the core and face plates changes, the dominating deformation modes change rapidly. The rotation stiffness can change for example through plasticity induced by high-level monotonic loading or by propagating fatigue cracks trough the laser-welds. Here it should be recognized that the beam theories based on classical continuum mechanics fail to predict the response accurately on the low rotation stiffness values due to lack of inclusion of the finite curvature condition to the beam formulation. When the couple stress or micropolar formulations are used, the responses are accurately captured. The reason is better illustrated through spectral analysis of the deformation wave-lengths which reveals that the deformation amplitude is dominated by the response at the level of characteristic length of the beam, while the second most important average length corresponds to that of the unit cell. The spectral analysis also reveals that there is significant characteristic length amplitude between these two wave lengths which is associated with the strain gradient. The amplitudes of these different characteristic wave lengths are related to the T-joint rotation stiffness. This means that in principle it should be possible to extract damage from the panels by analysis the deformation mode changes in the structure over the lifetime. However, in this case it should be recognized that the continuum assumption cannot handle the variation of beam stiffness properties along beam length. This influence is especially important that the closeness of boundaries where for example shear response can be very different between consecutive unit cells due to variation of laser-weld positioning.

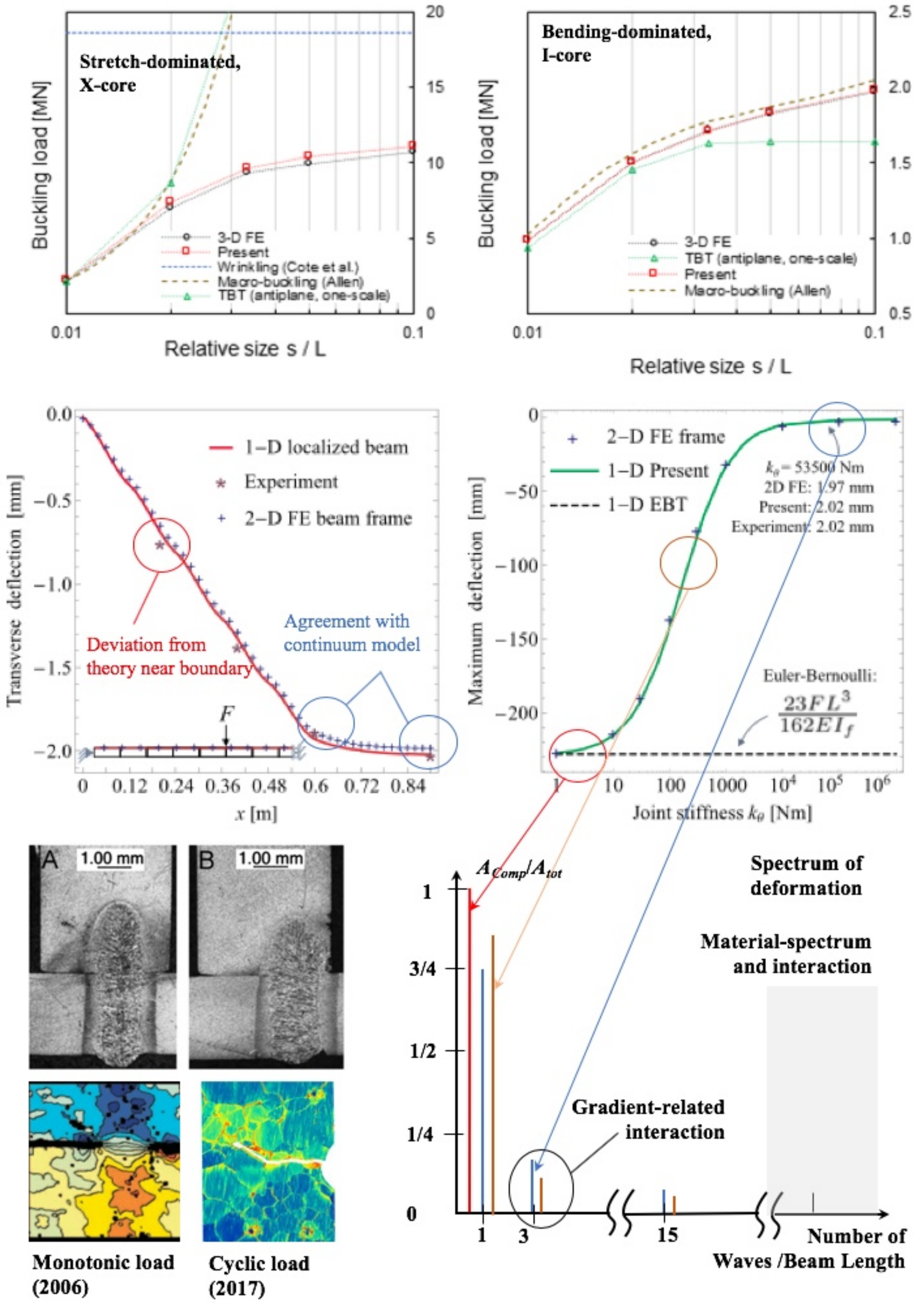

Figure 7: Beam solutions ultimate strength.

5 CONCLUSIONS

The paper gave an overview of the recent developments on the application of homogenized, non-classical beam theories used to predict the micro- and macrostructural stresses in the design of marine structures. These theories are important when ultra-lightweight marine structures are developed and one needs to explore the regions where the length scales of beam openings are in the range of the characteristic lengths of the beams or when lattice/frame-type beams are used to reduce the weight of ship structures. The homogenized beam models are based on non-classical continuum mechanics that allow local bending inside the beams. This added feature allows the treatment of size effects with great accuracy. The resulting analytical and finite element models have special features in terms of shape functions and iterative solutions in non-linear problems.

The non-classical beam theories converge to the physically correct solutions for wider range of beam parameters than the classical beam theories do. The investigations show that the localization of stresses is accurate for cases where the periodic continuum type of boundary conditions for the unit cells ae valid. The theories should be developed that allow random positioning in order to satisfy the needs of maritime solutions. For the vibratory response the eigenfrequencies and -modes are predicted with much better accuracy than for classical models. However, the coupling between local and global modes should be developed and analyses should be extended to allow assessment of forced vibrations. In ultimate strength analysis 2-way coupling accounting geometrical non-linearity have been developed and it shows excellent agreement with 3D high fidelity FEA. These successes should be extended to cover material non-linearity and formulations for plates and shells.

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