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Hydrodynamical Gravity

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Abstract

The hydrodynamical reaction of an incommpressible, inviscid, and irrotational fluid of gravitons emerging from massive particles gives the Newton's inverse-square force acting on the particles. Due to this mass loss, any self-gravitating systems disintegrate in a time scale of mass loss. The magnitude of the gravitational acceralation is the graviton velocity divided by the mass-loss time scale. The mass-loss time scale in the solar system must be much longer than a Hubble time, so that the graviton has speeds greater than c.

1 Introduction

В.Р.Келер (1960) in his book "НА ПОРОГЕ НЕВЕДОНОГО" presents a hypothesis that gravitons isotropically emitted from a massive body cause gravity. If only one body exists, the reactive force of emitted particles balances and the body remains at rest. However if another body exists near it, the space between the two bodies is filled with gravitons and the number of gravitons emitted there decreases gradually, so that the reaction of gravitons emitted to outer regions attracts the two bodies as shown in Figure 1.

In this paper we will show that the hydrodynamical reaction of mass loss from massive particles gives the Newton's inverse-square force acting on the particles.

2 Basic Assumptions

The basic assumptions made in this paper are

(1) Each particle is a source of gravitons, which are carrying away a part of mass from it at a rate proportional to its present mass M:

$$\frac{\mathrm{d}M}{\mathrm{d}t} = -\alpha M, \tag{1}$$

or

$$M = M_0 \mathrm{e}^{-\alpha t}, \tag{2}$$

where α^{-1} is the mass-loss time scale. Let be the density in the graviton flow ρ and the source strength $4\pi m$. Then, we have

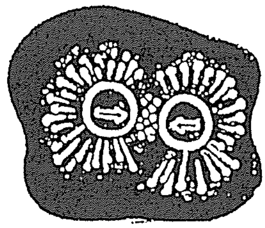

Fig. 1. Reactive force due to gravitons isotropically emitted from two bodies (Келер 1960).

$$\alpha M = 4\pi \rho m. \tag{3}$$

This "human" law of particle decay is essential to rule a self-gravitating system of different masses in good order (mentioned below).

(2) The graviton flow is assumed to be incompressible, inviscid and irrotational. The density in the flow ρ is kept constant.

(3) A graviton does not emit secondary particles, so that a graviton itself can not be a cause of gravity.

(4) The graviton velocity is much higher than the particle velocity, so that the particle seems to move with a constant velocity as seen on the graviton flow.

3. Inverse-square law

As well-known(Lagally 1922; Yih 1969; Imai 1973), an isotropic source at rest in a steady, incompressible, inviscid and irrotaional uniform flow gets a thrust in the upstream direction. This can be explained by a simple argument:

We consider an isotropic source of strength $4\pi m$, moving with velocity $\mathbf{v}$ in the uniform flow of velocity $\mathbf{U}$ and of density ρ. The emerging flow from the source also has the same density.

Following Jeans(1928), the equation of motion for a body losing its mass isotropically in the rest frame of the body is

$$\frac{\mathrm{d}}{\mathrm{d}t}(M\mathbf{v}) = \frac{\mathrm{d}M}{\mathrm{d}t}\mathbf{v} + \mathbf{F}, \tag{4}$$

where $\mathbf{F}$ is the force exerted on the source. Equation (4) assures that, if $\mathbf{F}=0$, the velocity $\mathbf{v}$ does not change.

The structure of the cometary flow emerging from a source in a uniform flow is shown in Figure 2. The velocity of the uniform flow remains unchanged in the upstream and downstream flow since the velocity potential is given by

$$\Phi = \mathbf{U}.\,\mathbf{x} - \frac{\mathrm{m}}{r}. \tag{5}$$

In fact, by integrating the pressure force exerted on the cometary surface by the surrounding fluid, it can be shown that, in Figure 2, the inward momentum flux into the hatched region of the cometary flow ($\theta \geqq 2\sin^{-1}\sqrt{2/3} = 110°$) balances the outward momentum flux from the blank region (see Appendix I and also Batchelor 1967). The surrounding fluid therefore exerts no force on the cometary flow and vice versa.

The downstream flow emerging from the source finally gains the momentum flux $4\pi\rho\, m\mathbf{U}$. This holds whether the source is moving or not. Hence, from the momentum-conservation law, the rate of change in the momentum of the source becomes

$$\frac{\mathrm{d}}{\mathrm{d}t}(M\mathbf{v}) = -4\pi\,\rho m\mathbf{U}. \tag{6}$$

Thus, using equations (1) and (3), we obtain the equation of motion for the source

$$M\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \mathbf{F} = -4\pi\,\rho m(\mathrm{U}-\mathbf{v}) = -\,\alpha M(\mathbf{U}-\mathbf{v}). \tag{7}$$

The same rocket reaction produced by the sublimating gases of a comet nucleus causes the non-gravitational force acting on the comet, as first pointed owt by Whipple (1950).

Following Imai(1973) and Yih(1969), the force acting upon a source of strength $4\pi m_\mu(t)$ moving with velocity $\mathbf{v}_\mu(t)$ at $\mathbf{r}=\mathbf{r}_\mu(t)$ can be also calculated by integrating the momentum flux density tensor over a small spherical surface surrounding the source.

The equation of motion for the graviton fluid in the translating frame moving with the source m_μ is given by

$$\frac{\partial}{\partial t'}(\rho\mathbf{v}') = -\mathrm{grad}p - \frac{\partial}{\partial x_k'}(\rho\mathbf{v}'v_k') + \sum_\nu 4\pi\,\rho m_\nu(t)\,\mathbf{v}'\delta\,(\mathbf{r}-\mathbf{r}_\nu(t)), \tag{8}$$

where $\mathbf{v}'=\mathbf{v}-\mathbf{v}_\mu$(see Appendix II equation (B7)). In the following, the apostrophe(') above the time in the translating frame are omitted except for the partial time derivative.

The momentum flux density tensor is given by

$$G_{\mathrm{ik}} = \rho v_i' v_k' + p\delta_{\mathrm{ik}}. \tag{9}$$

The total force acting on the source is equal to the integral of G_{ik} taken over a

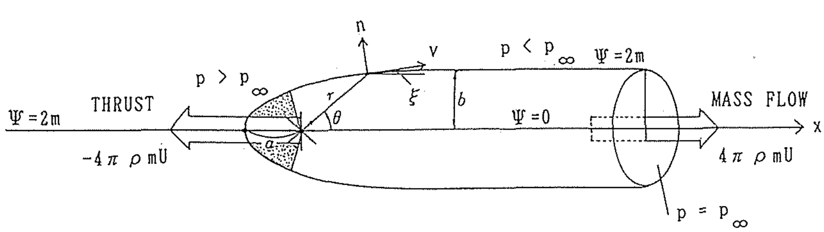

Fig. 2. A semi-infinite cometary flow emerging from a source in a uniform flow.

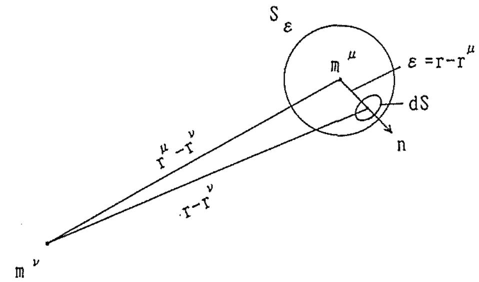

Fig. 3. A spherical surface surrounding the relevant source.

spherical surface moving with the source m_μ of radius ε, S_ε (t)(see Figure 3).

$$F_i = -\int\int_{S\varepsilon} G_{ik} n_k dS. \tag{10}$$

where n_k is the kth component of the outward normal to a surface element dS. The velocty in the flow for a distribution of moving sources $m_\nu(t)$ at $\mathbf{r}=\mathbf{r}_\nu(t)$(ν =1,..,N) is determined from

$$\text{div } \mathbf{v}' = 4\pi \sum_\nu m_\nu\ (t)\ \delta\ (\mathbf{r}-\mathbf{r}_\nu\ (t)) \quad \text{amd} \quad \text{rot } \mathbf{v}' = 0, \tag{11}$$

(see Appendix II (B9)).
Accordingly, we can introduce the velocity potential $\boldsymbol{\Phi}'$ defined by

$$\mathbf{v}' = \text{grad } \Phi'. \tag{12}$$

The velocity potential satisfies the Poisson equation

$$\Delta\Phi' = \sum_\nu 4\pi\, m_\nu\ (t)\ \delta\ (\mathbf{r}-\mathbf{r}_\nu\ (t)). \tag{13}$$

The solution which tends to $-\mathbf{v}_\mu$ at infinity is given by

$$\Phi' = \Phi - \mathbf{v}_\mu.\mathbf{x}, \tag{14}$$

where

$$\Phi = -\sum_\nu \frac{m_\nu\ (t)}{|\mathbf{r}-\mathbf{r}_\nu(t)|}. \tag{15}$$

The velocity is given by

$$\mathbf{v}' = \sum_\nu \frac{m_\nu\ (t)\,(\mathbf{r}-\mathbf{r}_\nu(\mathrm{t}))}{|\mathbf{r}-\mathbf{r}_\nu(t)|^3} - \mathbf{v}_\mu. \tag{16}$$

The force exerted on the moving source $m_\mu(t)$ with velocity $\mathbf{v}_\mu(t)$ at $\mathbf{r}=\mathbf{r}_\mu(t)$ by the other sources is our aim. Putting $\mathbf{r}-\mathbf{r}_\mu(t)=\varepsilon(t)$, the summation is split into two terms:

$$\mathbf{v}' = \sum_{\nu\neq\mu} \frac{m_\nu(t)\,(\mathbf{r}-\mathbf{r}_\nu(t))}{|\mathbf{r}-\mathbf{r}_\nu(t)|^3} + \frac{m_\mu(t)\,\varepsilon(t)}{\varepsilon(t)^3} - \mathbf{v}_\mu. \tag{17}$$

Taylor expansion about $\mathbf{r}=\mathbf{r}_\mu(t)$ yields

$$\mathbf{v}' = \mathrm{U}' + \sum_{\nu\neq\mu} \frac{m_\nu(t)\,(\varepsilon(t) - 3\varepsilon_{\mu\nu}(t))}{|\mathbf{r}_\mu(t) - \mathbf{r}_\nu(t)|^3} + \frac{m_\mu(t)\ \varepsilon(t)}{\varepsilon(t)^3}, \tag{18}$$

where

$$\mathbf{U}'\ (t) = \mathbf{U}\ (t) - \mathbf{v}_\mu, \tag{19}$$

$$\mathbf{U}\ (t) = \sum_{\nu\neq\mu} \frac{m_\nu(t)\,(\mathbf{r}_\mu(t) - \mathbf{r}_\nu(t))}{|\mathbf{r}_\mu(t) - \mathbf{r}_\nu(t)|^3}, \tag{20}$$

$$\varepsilon_{\mu\nu\mathrm{i}}(t) = \frac{(x_{\mu\mathrm{i}}(t) - x_{\nu\mathrm{i}}(t))^2}{|\mathbf{r}_\mu(t) - \mathbf{r}_\nu(t)|^2}\,\varepsilon_\mathrm{i}(t). \tag{21}$$

The second term in the right hand side of equation (18) becomes negligible compared to the third term as ε tends to zero. Thus, the velocity field near the source $m_\mu(t)$ with moving with velocity $\mathbf{v}_\mu(t)$ is a combination of a uniform flow and the source flow,

$$\mathbf{v}' = \mathbf{U}'(t) + \frac{m_\mu(t)\ \varepsilon(t)}{\varepsilon(t)^3}. \tag{22}$$

Bernoulli's theorem in an incompressible, inviscid and irrotational flow with sources $m_\nu(t)$ at $\mathbf{r}=\mathbf{r}_\nu(t)$ ($\nu=1,\ldots,\mathrm{N}$) leads to the same form, even when sources are present,

$$p = \rho f(t) - \rho\frac{\partial\Phi'}{\partial t'} - \frac{1}{2}\rho v'^2 \tag{23}$$

(see Appendix II equation (B17)). From equations (9), (22) and (23), using $n_\mathrm{k} = \varepsilon_\mathrm{k}/\varepsilon$, we have

$$\begin{aligned}
G_{\mathrm{ik}}n_\mathrm{k} &= G_{\mathrm{ik}}\frac{\varepsilon_\mathrm{k}}{\varepsilon}\\
&= \rho(U_\mathrm{i}' + \frac{m_\mu\,\varepsilon_\mathrm{i}}{\varepsilon^3})\ (U_\mathrm{k}' + \frac{m_\mu\,\varepsilon_\mathrm{k}}{\varepsilon^3})\ \frac{\varepsilon_\mathrm{k}}{\varepsilon}\\
&\quad + \delta_{\mathrm{ik}}\,\{\rho f(t) - \rho\frac{\partial\Phi'}{\partial t'} - \frac{1}{2}\rho\,[(U_1' + \frac{m_\mu\,\varepsilon_1}{\varepsilon^3})^2 + (U_2' + \frac{m_\mu\,\varepsilon_2}{\varepsilon^3})^2\\
&\quad + (U_3' + \frac{m_\mu\,\varepsilon_3}{\varepsilon^3})^2]\}\ \frac{\varepsilon_\mathrm{k}}{\varepsilon}\\
&= \rho U_\mathrm{i}'U_\mathrm{k}'\,\frac{\varepsilon_\mathrm{k}}{\varepsilon} + \rho m_\mu(U_\mathrm{i}'\,\varepsilon_\mathrm{k} + U_\mathrm{k}'\,\varepsilon_\mathrm{i})\,\frac{\varepsilon_\mathrm{k}}{\varepsilon^4} + \rho m_\mu^{\ 2}\,\frac{\varepsilon_\mathrm{i}}{\varepsilon^5}\\
&\quad + [\rho f(t) - \frac{\partial\Phi'}{\partial t'} - \frac{1}{2}\rho U'^2]\frac{\varepsilon_\mathrm{i}}{\varepsilon} - \rho m_\mu U_\mathrm{k}'\,\frac{\varepsilon_\mathrm{k}\varepsilon_\mathrm{i}}{\varepsilon^4} - \rho m_\mu^{\ 2}\,\frac{\varepsilon_\mathrm{i}}{2\varepsilon^5}\\
&= \rho m_\mu\ (U_\mathrm{i}'\varepsilon^2 + U_\mathrm{k}'\varepsilon_\mathrm{i}\varepsilon_\mathrm{k} - U_\mathrm{k}'\varepsilon_\mathrm{k}\varepsilon_\mathrm{i})/\varepsilon^4 + (\text{terms with a single } \varepsilon_\mathrm{i} \text{ or } \varepsilon_\mathrm{k})\\
&= \rho m_\mu U_\mathrm{i}'/\varepsilon^2 + (\text{terms with a single } \varepsilon_\mathrm{i} \text{ or } \varepsilon_\mathrm{k}).
\end{aligned} \tag{24}$$

Here, we will make use of the formula $\int\!\!\int_{S\varepsilon} \varepsilon_\mathrm{i}\mathrm{d}S=0$. From equations (10) and (24), we find the thrust acting on the source m_μ

$$\begin{aligned}
F_\mathrm{i} &= -\frac{\rho m_\mu U_\mathrm{i}'}{\varepsilon^2}\int\!\!\int_{S\varepsilon} \mathrm{d}S\\
&= -4\pi\,\rho m_\mu U_\mathrm{i}'\\
&= -4\pi\,\rho m_\mu(U_\mathrm{i} - v_{\mu\mathrm{i}}) \qquad (25)\\
&= -\alpha M_\mu(U_\mathrm{i} - v_{\mu\mathrm{i}}). \qquad (26)
\end{aligned}$$

Thus, we arrive at equation (7).

Putting equation (20) into equation (26) and using equation (3), we find

$$\mathbf{F} = -\sum_{\nu\neq\mu} \frac{\alpha^2 M_\mu M_\nu}{4\pi\rho} \frac{(\mathbf{r}_\mu - \mathbf{r}_\nu)}{|\mathbf{r}_\mu - \mathbf{r}_\nu|^3} + \alpha M_\mu \mathbf{v}_\mu. \tag{27}$$

Putting the inverse of the mass-loss time scale

$$\alpha = (4\pi G\rho)^{\frac{1}{2}}, \tag{28}$$

we obtain the Newton's inverse-square law

$$F = -\sum_{\nu\neq\mu} \frac{GM_\mu M_\nu(\mathbf{r}_\mu - \mathbf{r}_\nu)}{|\mathbf{r}_\mu - \mathbf{r}_\nu|^3} + \alpha M_\mu \mathbf{v}_\mu. \tag{29}$$

4 A binary problem

As a simple illustration, we consider a binary of the primary star M_1 with velocity $\mathbf{v}_1$ at $\mathbf{r}=\mathbf{r}_1$ and the secondary star M_2 with velocity $\mathbf{v}_2$ at $\mathbf{r}=\mathbf{r}_2$. Using equation (29), we have the force exerted on M_1

$$\mathbf{F} = -\frac{GM_1M_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3}(\mathbf{r}_1 - \mathbf{r}_2) + \alpha M_1\mathbf{v}_1. \tag{30}$$

The first term is the gravitational force exerted on a source at rest by the other source. The second term is the mass-loss force acting on a moving source even when the fluid is at rest.

Thus, the equation of motion for M_1 is

$$M_1\frac{d\mathbf{v}_1}{dt} = -\frac{GM_1M_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3}(\mathbf{r}_1 - \mathbf{r}_2) + \alpha M_1\mathbf{v}_1. \tag{31}$$

The equation of motion for M_2 is

$$M_2\frac{d\mathbf{v}_2}{dt} = \frac{GM_1M_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3}(\mathbf{r}_1 - \mathbf{r}_2) + \alpha M_2\mathbf{v}_2. \tag{32}$$

Equations (31) and (32) can be rewritten as

$$\frac{d\mathbf{v}_1}{dt} = -\frac{GM_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3}(\mathbf{r}_1 - \mathbf{r}_2) + \alpha\mathbf{v}_1, \tag{33}$$

$$\frac{d\mathbf{v}_2}{dt} = \frac{GM_1}{|\mathbf{r}_1 - \mathbf{r}_2|^3}(\mathbf{r}_1 - \mathbf{r}_2) + \alpha\mathbf{v}_2. \tag{34}$$

Putting $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$, $\mathbf{v}=\mathbf{v}_1-\mathbf{v}_2$ and $M = M_1 + M_2$ and subtracting equation (34) from equation (33), we find

$$\frac{d\mathbf{v}}{dt} = -\frac{GM}{r^3}\mathbf{r} + \alpha\mathbf{v}. \tag{35}$$

In polar coordinates, this equation leads to

$$\ddot{\mathrm{r}} - \mathrm{r}\omega^2 = \frac{GM_1}{\mathrm{r}^2} + \alpha\,\dot{\mathrm{r}}\,, \tag{36}$$

$$\frac{1}{r}\frac{d}{dt}(\mathrm{r}^2\omega) = \alpha r\omega, \tag{37}$$

where ω is the angular velocity.

From equation (37), we find the time evolution of specific angular momentum

$$r^2\omega = J_0 e^{\alpha t}, \tag{38}$$

where J_0 is the initial specific angular momentum. The tangential component of the relative velocity of M_1 and M_2 $r\omega$ brings about a tangential thrust. Its effects increases the specific angular momentum exponentially in a time scale of mass loss.

The radial motion can be solved approximately. From equations (36) and (38), we obtain

$$\ddot{r} = \left(\frac{J_0^2}{r^3}\,e^{2\alpha t} - \frac{GM_{10}}{r^2}\,e^{-\alpha t}\right) + \alpha\,\dot{r}\,. \tag{39}$$

We look for an approximate solution satisfying the initial conditions

$$r = r_0, \quad \ddot{r} = \dot{r} = 0 \quad \text{and} \quad \frac{GM_{10}}{r_0^2} = \frac{J_0^2}{r_0^3}. \tag{40}$$

Such a solution is found to be

$$r = r_0 + \frac{J_0^2}{r_0^3}\left(\frac{\alpha}{2}\,t^3 + \frac{\alpha^2}{4}\,t^4\right). \tag{41}$$

Accordingly, the secondary M_2 gradually leaves the circular orbit $\mathbf{r}=\mathbf{r}_0$. As time goes on, the solution tends to

$$r \propto e^{\alpha t}, \qquad \dot{r} = \alpha r \quad \text{and} \quad \omega \propto e^{-\alpha t}. \tag{42}$$

Hence, the binary system will disintegrate gradually in a time scale of mass loss α^{-1}. The time variation of the total energy of the binary system can be readly deduced from equations (33) and (34) as

$$\frac{d}{dt}\left[\tfrac{1}{2}M_1 v_1^2 + \tfrac{1}{2}M_2 v_2^2 - \frac{GM_1M_2}{|\mathbf{r}_1-\mathbf{r}_2|}\right]$$

$$= \frac{dM_1}{dt}\left[\tfrac{1}{2}v_1^2 - \frac{GM_2}{|\mathbf{r}_1-\mathbf{r}_2|} - v_1^2\right] + \frac{dM_2}{dt}\left[\tfrac{1}{2}v_2^2 - \frac{GM_1}{|\mathbf{r}_1-\mathbf{r}_2|} - v_2^2\right] \tag{43}$$

$$= \alpha L_1 + \alpha L_2. \tag{44}$$

The last terms in the parentheses are lacking in Jeans'(1928) energy equation. Equation (43) surely shows that the total energy increases all the time due to the mass-loss force acting on a moving source. This is also a general conclusion that holds for any self-gravitating system because the equation of motion (35) is quite general in nature. The two terms of equation (44) are the Lagrangians of M_1 and M_2. It means that *the principle of least action minimizes the increase in the total energy.*

5 Discussions

Recent investigations (Pierce et al. 1994; Freedman et al. 1994) show that

$$H = 80\text{km sec}^{-1}\text{Mpc}^{-1} = 2.6\ 10^{-18}\ \text{sec}^{-1},$$

$$H^{-1} = 3.9\ 10^{17}\ \mathrm{sec} = 1.2\ 10^{10}\ \mathrm{y}, \tag{45}$$

$$\rho_c = \frac{3H^2}{8\pi G} = 1.2\ 10^{-29}\ \mathrm{g\ cm^{-3}}.$$

The total energy density of the relic radiation is $\varepsilon_\gamma = 4.2\ 10^{-13}$ erg cm^{-3} (Mather et al. 1990; Gush et al. 1991), which corresponds to mass density

$$\rho_\gamma = 4.7\ 10^{-34}\ \mathrm{g\ cm^{-3}}. \tag{46}$$

The solar system keeps its stability for 4.5 10^9 y, one third of a Hubble time H^{-1}. Hence, the disintegration time scale should be much longer than a Hubble time. Thus, we have

$$\alpha^{-1} \gg H^{-1} = 1.2\ 10^{10}\ \mathrm{y}. \tag{47}$$

That is,

$$\alpha \ll H = 2.6\ 10^{-18}\ \mathrm{sec^{-1}}. \tag{48}$$

From equations (28) and (45), we find that the density in the graviton flow should satisfy

$$\frac{\rho}{\rho_c} = \frac{2\alpha^2}{3H^2} \ll 1. \tag{49}$$

If $\rho \approx \rho_\gamma$, we have

$$\alpha \approx 2.0\ 10^{-20}\ \mathrm{sec^{-1}}, \tag{50}$$

$$\alpha^{-1} \approx 1.6\ 10^{12}\ \mathrm{y}. \tag{51}$$

From equation (C5) in Appendix Ⅲ, the velocity in the graviton flow $\mathbf{u}$ is related to the gravitational acceleration $\mathbf{a}$ as $\mathbf{u} = \mathbf{a}/\alpha$.
The lower limit of gravitational accelerations in the Universe a_0 is equal to 2 10^{-8} cm sec^{-2} (Milgrom 1983 and 1995; Ishizawa 1987). A new estimate in spiral galaxies is 2.4 10^{-9} cm sec^{-2} (Ishizawa 1995). If we adopt the above estimate of α (50), this gives the velocity in the graviton flow u

$$\frac{u}{c} = \frac{a_0}{\alpha c} \approx 4. \tag{52}$$

The velocity of the graviton flow at the earth is

$$\frac{u_\oplus}{c} = \frac{GM_\odot}{\alpha c(1\mathrm{AU})^2} = 1.0\ 10^9. \tag{53}$$

Therefore, the speed of the graviton flow is greater than c. This is the most serious problem in this work, which is inconsistent with the classical relativistic field theory. However the classical field theory also has some fatal defects:

1. The classical fields do work but not tire. While our mass devotes himself to attract other masses.
2. The light can not escape from a black hole but the gravitational field comes oozing out of it.

The stagnation point of the earth for the sun $\mathbf{r} = \mathbf{r}_s$ is determined from the condition that $\frac{\partial \Phi}{\partial r} = 0$ (Imai 1973). This gives

$$\frac{GM_{\odot}}{(1_{\mathrm{AU}}-r_{\mathrm{s}})^2}=\frac{Gm_{\oplus}}{r_{\mathrm{s}}{}^2}. \tag{54}$$

Thus, we have

$$\begin{aligned} r_{\mathrm{s}} &\approx \left(\frac{m_{\oplus}}{M_{\odot}}\right)^{\frac{1}{2}} \mathrm{AU} \\ &= 1.7\ 10^{-3}\,\mathrm{AU} \\ &= 40\ R_{\oplus} \\ &= \tfrac{2}{3} r_{☽}. \end{aligned} \tag{55}$$

The stagnation point is at two thirds of a distance to the moon.

Appendix I

A combination of a source of strength $4\pi m$ and a uniform flow of velocity U is described by the stream function

$$\Psi = \tfrac{1}{2}\,Ur^2\sin^2\theta + m(1-\cos\theta) \tag{A1}$$

(Imai 1973).
The surface of the cometary flow is expressed by the condition that $\Psi=2m$, that is,

$$r=\left(\frac{m}{U}\right)^{\frac{1}{2}}\frac{1}{\sin(\theta/2)}. \tag{A2}$$

The diatance of the stagnation point from the source a and the radius of the cometary tail b are

$$a=\sqrt{m/U} \quad \text{amd} \quad b=2\sqrt{m/U}. \tag{A3}$$

The velocity potential is given by equation (5)

$$\Phi = \mathbf{U}.\mathbf{x} - \frac{m}{r}. \tag{A4}$$

The velocity components are

$$v_{\mathrm{x}} = U + \frac{m\cos\theta}{r^2}, \qquad v_{\mathrm{y}} = \frac{m\sin\theta}{r^2}, \tag{A5}$$

$$v = U[1+2\sin^2(\theta/2) - 3\sin^4(\theta/2)]^{\frac{1}{2}}. \tag{A6}$$

The angle between the velocity v and the direction of the positive x-axis ξ is given by

$$\tan\xi = \frac{v_{\mathrm{y}}}{v_{\mathrm{x}}} = \frac{\sin^2(\theta/2)\,\sin\theta}{1+\sin^2(\theta/2)\,\cos\theta}, \tag{A7}$$

and

$$\sin\xi = \left[\frac{\tan^2\xi}{1+\tan^2\xi}\right]^{\frac{1}{2}} = \left[\frac{4\sin^6(\theta/2)}{1+3\sin^2(\theta/2)}\right]^{\frac{1}{2}}. \tag{A8}$$

The x-component of the normal to the surface of the cometary flow is

$$n_{\mathrm{x}} = -\sin\xi. \tag{A9}$$

From Bernoulli's theorem, we have

$$p = p_\infty(t) - \tfrac{1}{2}\rho(v^2 - U^2) \tag{A10}$$

$$= p_\infty(t) - \tfrac{1}{2}\rho U^2[2\sin^2(\theta/2) - 3\sin^4(\theta/2)]. \tag{A11}$$

The surface element $\mathrm{d}S$ is

$$\begin{aligned} \mathrm{d}s &= 2\pi\, r\sin\theta\ \mathrm{d}s \\ &= 2\pi\, r\sin\theta\,(\mathrm{d}r^2 + r^2\mathrm{d}\theta^2)^{\frac{1}{2}} \\ &= \frac{2\pi\, m\sin\theta}{U\sin^2(\theta/2)}\left[\frac{1+3\sin^2(\theta/2)}{4\sin^2(\theta/2)}\right]^{\frac{1}{2}}\mathrm{d}\theta. \end{aligned} \tag{A12}$$

The pressure force exerted on the cometary surface by the surrounding fluid is, using equations (A2), (A8), (A9), (A11) and (A12),

$$-\int p n_x \mathrm{d}S = \frac{4\pi m}{U}\int\{p_\infty - \tfrac{1}{2}\rho U^2[2\sin^2(\theta/2) - 3\sin^4(\theta/2)]\}$$

$$\times\sin(\theta/2)\cos(\theta/2)\mathrm{d}\theta - p_\infty\pi b^2 = \frac{4\pi\, m p_\infty}{U} - \frac{4\pi\, m p_\infty}{U} = 0. \tag{A13}$$

Thus, the total pressure force exerted on the cometary flow by the surrounding fluid vanishes. It is found from equations (A6) and (A10) that, for $\theta \geqq 2\sin^{-1}\sqrt{\frac{2}{3}} = 110^\circ$, $v \leqq U$ and $p \geqq p_\infty$ and, for $0^\circ \leqq \theta < 2\sin^{-1}\sqrt{\frac{2}{3}}$, they are the opposites. The contribution of $\theta \geqq 110^\circ$ to the pressure integral cancels out that of $0^\circ \leqq \theta < 110^\circ$ exactly.

Appendix II

The equation of motion of a source m_μ is derived from equations (1), (3) and (6) as

$$\frac{\mathrm{d}}{\mathrm{d}t}(M_\mu \mathbf{v}_\mu) = \frac{\mathrm{d}M_\mu}{\mathrm{d}t}\mathbf{v}, \quad \mu = 1,\ldots,\mathrm{N}, \tag{B1}$$

or

$$M_\mu\frac{\mathrm{d}\mathbf{v}_\mu}{\mathrm{d}t} = \frac{\mathrm{d}M_\mu}{\mathrm{d}t}(\mathbf{v} - \mathbf{v}_\mu), \quad \mu = 1,\ldots,\mathrm{N}, \tag{B2}$$

where v is the velocity of the graviton fluid and

$$\frac{\mathrm{d}M_\mu}{\mathrm{d}t} = -4\pi\,\rho m_\mu = -\alpha M_\mu. \tag{B3}$$

Under the law of particle decay (B3), the orbits of particles in a cluster are independent of their masses. This is similar to the human law in our soceity, under which heavy and light, rich and poor, old and young, man and woman, able and unable equally have right and responsibility. Such a law would rule the system in good order.

The equation of continuity and the equation of motion for the graviton fluid are given in the presence of sources with variable strength $m_\nu(t)$ and velocity $\mathbf{v}_\nu(t)$ at $\mathbf{r} = \mathbf{r}_\nu(t)$ ($\nu = 1,\ldots,\mathrm{N}$). To find the pressure about the source $m_\mu(t)$ moving with a constant velocity $\mathbf{v}_\mu$ (see Assumption (4)), we choose the translating frame fixed in the source. Following Goldstein (1960), the velocity and acceralation in the translating frame can be related to those in the rest frame fixed to the space as

$$\begin{aligned} \mathbf{r}' &= \mathbf{r}-\mathbf{v}_\mu t, \\ t' &= t \\ \mathbf{v}' &= \mathbf{v}-\mathbf{v}_\mu, \\ \frac{\partial}{\partial t} &= \frac{\partial}{\partial t'} - \mathbf{v}_\mu \frac{\partial}{\partial \mathbf{r}'}, \\ \frac{\partial}{\partial \mathbf{r}} &= \frac{\partial}{\partial \mathbf{r}'}, \end{aligned} \tag{B4}$$

and

$$\frac{\mathrm{D}\mathbf{v}}{\mathrm{D}t} = \frac{\mathrm{D}\mathbf{v}'}{\mathrm{D}t'}. \tag{B5}$$

The derivative D**v**/Dt denotes the rate of change of the velocity of a given fluid particle as it moves about in space(Landau and Lifshitz 1966). Relative motion of frames does not affect any scalar property of a point mass. The equation of continuity does not change in the translating frame:

$$\frac{\partial \rho}{\partial t'} + \mathrm{div}\rho\mathbf{v}' = \sum_\nu 4\pi\rho m_\nu(t)\delta(\mathbf{r}-\mathbf{r}_\nu(t)). \tag{B6}$$

The equation of motion in the translating frame is, using eqnations (B4) and (B6),

$$\frac{\partial}{\partial t'}(\rho\mathbf{v}') = -\mathrm{gradp} - \frac{\partial}{\partial x_{\mathrm{k}}'}(\rho\mathbf{v}'v_{\mathrm{k}}') + \sum_\nu 4\pi\rho m_\nu(t)\mathbf{v}'\,\delta(\mathbf{r}-\mathbf{r}_\nu(t)). \tag{B7}$$

The last term presents the momentum flux of the fluid supplied to the graviton flow by a distant source m_ν moving with velocity $\mathbf{v}_\nu(t)$ at $\mathbf{r}=\mathbf{r}_\nu(t)$. The velocity of the supplied fluid is taken equal to $\mathbf{v}' = \mathbf{v} - \mathbf{v}_\mu$ by considering equation (B1). For the relevant source, we must use $\mathbf{v}'=\mathbf{v}-\mathbf{v}_\mu=0$ because the mean velocity at the isotropic source is equal to $\mathbf{v}_\mu$.

Using equation (B6) and the formula $(\mathbf{v}.\mathrm{grad})\mathbf{v}=\mathrm{grad}(v^2/2)-\mathbf{v}\mathrm{x}\mathrm{rot}\,\mathbf{v}$, the equation of motion (B7) can be rewritten as

$$\frac{\partial \mathbf{v}'}{\partial t'} = -\frac{1}{\rho}\,\mathrm{grad}\,p - \mathrm{grad}(\tfrac{1}{2}v'^2). \tag{B8}$$

Thus, the equation of motion of the graviton flow remains unchanged even in the presence of sources because the final velocity of graviton fluid flowing out from a source coincides with that of the nearby flow. We consider an incompressible, inviscid and irrotational fluid. From equation (B6), the equation of continuity leads to

$$\mathrm{div}\,\mathbf{v}' = \sum_\nu 4\pi m_\nu(t)\delta(\mathbf{r}-\mathbf{r}_\nu(t)) \quad \text{and} \quad \mathrm{rot}\,\mathbf{v}' = 0. \tag{B9}$$

The velocity potential Φ' defined by

$$\mathbf{v}' = \mathrm{grad}\,\Phi' \tag{B10}$$

satisfies the Poisson's equation

$$\Delta\Phi' = \sum_\nu 4\pi\, m_\nu(t)\delta(\mathbf{r}-\mathbf{r}_\nu(t)). \tag{B11}$$

Thus, we find

$$\Phi' = \Phi - \mathbf{v}_\mu . \mathbf{x}, \tag{B12}$$

where

$$\Phi = -\sum_\nu \frac{m_\nu(t)}{|\mathbf{r}-\mathbf{r}_\nu(t)|}. \tag{B13}$$

Putting equation (B12) into equation (B10), we have

$$\mathbf{v}' = \sum_\nu \frac{m_\nu(t)\,(\mathbf{r}-\mathbf{r}_\nu(t))}{|\mathbf{r}-\mathbf{r}_\nu(t)|^3} - \mathbf{v}_\mu. \tag{B14}$$

Putting equation (B10) into equation (B8), we find

$$\mathrm{grad}\left(\frac{\partial \Phi'}{\partial t'} + \tfrac{1}{2}v'^2 + \frac{p}{\rho}\right) = 0. \tag{B15}$$

Thus, we find Bernoulli's theorem

$$\frac{\partial \Phi'}{\partial t'} + \tfrac{1}{2}v'^2 + \frac{p}{\rho} = f(t'). \tag{B16}$$

The pressure in the graviton flow is given by

$$p = \rho f(t') - \rho \frac{\partial \Phi'}{\partial t'} - \tfrac{1}{2}\rho v'^2. \tag{B17}$$

Using equations (B4) and (B12), the pressure in the rest frame fixed to the space is

$$p = \rho f(t) - \rho \frac{\partial \Phi}{\partial t} - \tfrac{1}{2}\rho v^2 + \tfrac{1}{2}\rho v_\mu^2,$$

$$= p_\Lambda(t) - \rho \frac{\partial \Phi}{\partial t} - \tfrac{1}{2}\rho v^2. \tag{B18}$$

where $p_\Lambda(t)$ is the pressure at infinity where the velocity potential Φ and the velocity v vanish.

Appendix III

Bateman's variational principle uses the pressure as the Lagrangian density (Bateman 1929 and 1944; Seliger and Whitham 1968). This is of particular important for potential flows.

Let us start with a combination of the Lagrangian for a system of particles and the pressure integral.

$$I = \int\int \Sigma_\mu \left[\tfrac{1}{2} M_\mu r_\mu(t)^2 + \frac{\mathrm{d}M_\mu}{\mathrm{d}t}\Phi\right] \delta(\mathbf{r}-\mathbf{r}_\mu(t))\,\mathrm{d}V\,\mathrm{d}t + \int\int p\,\mathrm{d}V\,\mathrm{d}t. \tag{C1}$$

where the velocity potential Φ given by equation (B13) is split to the μ th term and the other terms as

$$\Phi = -\sum_{\nu\neq\mu} \frac{m_\nu}{|\mathbf{r}-\mathbf{r}_\nu(t)|} - \frac{m_\mu}{|\mathbf{r}-\mathbf{r}_\mu(t)|},$$

$$= \Phi_1(|\mathbf{r}-\mathbf{r}_\nu|) + \Phi_2(|\mathbf{r}-\mathbf{r}_\mu|). \tag{C2}$$

The pressure is given by equation (B18)

$$p = p_\Lambda(t) - \rho \frac{\partial \Phi}{\partial t} - \tfrac{1}{2}\rho(\nabla \Phi)^2. \tag{C3}$$

Putting equation (C3) into equation (C1), we have

$$I = \int \int \sum_{\mu} [\tfrac{1}{2} M_\mu \dot{r}_\mu{}^2 + \frac{\mathrm{d}M_\mu}{\mathrm{d}t} \Phi]\, \delta(\mathbf{r}-\mathbf{r}_\mu) \mathrm{d}V \mathrm{d}t$$
$$+ \int \int [p_\Lambda - \rho \frac{\partial \Phi}{\partial t} - \tfrac{1}{2} \rho (\nabla \Phi)^2]\, \mathrm{d}V \mathrm{d}t. \qquad \text{(C4)}$$

If $\mathrm{d}M/\mathrm{d}t = -\alpha M$ and if the gravitational potential is defined by

$$\phi = \alpha \Phi, \qquad \text{(C5)}$$

this action integral coincides with Landau and Lifshits' (1979) total action integral for the field plus masses except for the p_Λ term and $\partial \Phi / \partial t$ term. The action with the p_Λ term of the cosmological constant is adopted in the study of the gravitational channel by Ishizawa(1987).

We consider the variations $\delta r_\mu(t)$ and $\delta \Phi$. The boundary conditions are

(1) $\delta r_\mu(t=\pm\infty) = 0$.
(2) $\delta \Phi(t=\pm\infty) = 0$.
(3) $(\nabla \Phi) n = 0$ or $\delta \Phi = 0$ on the boundary surface.
(4) The velocity potential is finite on the boundary. (C6)

The variation of the action integral (C4) with respect to $\mathbf{r}_\mu(t)$ gives

$$\delta I = \int \int \sum_{\mu} [M_\mu \dot{\mathbf{r}}_\mu \delta \dot{\mathbf{r}}_\mu + \frac{\mathrm{d}M_\mu}{\mathrm{d}t} \{\frac{\partial}{\mathrm{d}r_\mu} (\Phi_1(|\mathbf{r}-\mathbf{r}_\nu|)\, \delta(\mathbf{r}-\mathbf{r}_\mu))$$
$$+ \frac{\partial}{\partial \mathbf{r}_\mu} (\Phi_2(|\mathbf{r}-\mathbf{r}_\mu|)\, \delta(\mathbf{r}-\mathbf{r}_\mu))\}\, \delta \mathbf{r}_\mu]\, \mathrm{d}V \mathrm{d}t \qquad \text{(C7)}$$

The Φ_1 term becomes gradΦ_1 by use of the formula $\int f(x)\, \delta'(x) \mathrm{d}x = -\int f'(x)\, \delta(x) \mathrm{d}x$ if f is finite at the boundary.

The Φ_2 term vanishes because $\frac{\partial}{\partial x_\mu} [\Phi_2(|\mathbf{r}-\mathbf{r}_\mu|)\, \delta(\mathbf{r}-\mathbf{r}_\mu)]$ is a odd function of $x - x_\mu$ etc. Thus, we have

$$\delta \mathrm{I} = \int \Sigma_\mu M_\mu \dot{\mathbf{r}}_\mu \delta(\mathbf{r}-\mathbf{r}_\mu)\, \delta r_\mu |_{t=-\infty}^{t=\infty} \mathrm{d}V$$
$$- \int \int \Sigma_\mu [\frac{\mathrm{d}}{\mathrm{d}t} (M_\mu \dot{\mathbf{r}}_\mu) - \frac{\mathrm{d}M_\mu}{\mathrm{d}t} \mathrm{grad}\, \Phi_1]\, \delta(\mathbf{r}-\mathbf{r}_\mu)\, \delta \mathbf{r}_\mu \mathrm{d}V \mathrm{d}t. \qquad \text{(C8)}$$

Hence, we have

$$\frac{\mathrm{d}}{\mathrm{d}t} (M_\mu \dot{\mathbf{r}}_\mu) = \frac{\mathrm{d}M_\mu}{\mathrm{d}t} \mathrm{grad}\, \Phi_1. \qquad \text{(C9)}$$

This is the same as equation (B1). Using eqnations (1) and (C5), this leads to the equation of motion for the source m_μ

$$\frac{\mathrm{d}\mathbf{v}_\mu}{dt} = -\, \mathrm{grad}\, \Phi_1 + \alpha \mathbf{v}_\mu, \qquad \text{(C10).}$$

where

$$\phi_1 = \alpha \Phi_1. \qquad \text{(C11).}$$

The variation with respect to Φ is

$$\delta I = -\int \rho \delta \Phi |_{t=-\infty}^{t=\infty} \mathrm{d}V - \int \int \rho\, (\nabla \Phi)_n \delta \Phi \mathrm{d}S \mathrm{d}t$$

$$+ \int \int \left[\rho \Delta \Phi + \sum_{\mu} \frac{\mathrm{d}M_\mu}{\mathrm{d}t} \delta\,(\mathbf{r}-\mathbf{r}_\mu)\right] \delta \Phi \mathrm{d}V \mathrm{d}t. \tag{C12}$$

Thus, we have

$$\rho \Delta \Phi = -\sum_{\mu} \frac{\mathrm{d}M_\mu}{\mathrm{d}t} \delta\,(\mathbf{r}-\mathbf{r}_\mu). \tag{C13}$$

Using eqnations (1), (28) and (C5), we obtain the Poisson's equation.

$$\Delta \phi = \sum_{\mu} 4\pi G M_\mu \delta(\mathbf{r} - \mathbf{r}_\mu). \tag{C14}$$

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References

Batchelor, G.K. 1967, *An Introduction to Fluid Dynamics*(Cambridge University Press: Cambridge), p. 462.

Bateman, H. 1929, *Proc. Roy. Soc.* A**125**, 598.

Bateman, H. 1944, *Partial differential equations* (Cambridge University Press: Cambridge)

Bekenstein, J., and Milgrom, M. 1984, *Astrophys. J.* **286**, 7.

Freedman, W.L., Madore, B.F., Mould, J.R., Hill, R., Ferrarese, L., Kennicutt Jr, R.C., Saha, A., Stetson, P.B., Graham, J.A., Ford, H., Hoessel, J.G., Huchra, J., Hughes, S.M., and Illingworth, G.D., 1994, *Nature,* **371**, 757.

Goldstein, S. 1960, *Lectures on Fluid Mechanics* (Interscience Publishers: London), p. 11.

Gush, H.P., Halpern, M., and Wisshnow, E.H. 1991, *Phys. Rev. Lett.* **65**, 537.

Imai, H. 1973, *Fluid Dynamics* (Syokabo: Tokyo), pp. 88, 158 and 381.

Ishizawa, T. 1987, in IAU Symposium No. 117, *Dark Matter in the Universe* ed. J. Kormendy, and G.R. Knapp (Dordrecht: Reidel), p. 334.

Ishizawa, T. 1995, in preparation.

Jeans, J.H. 1928, *Astronomy and Cosmogony* (Cambridge University Press: Cambridge), pp. 290 and 292.

Ке л ер, В.Р. 1960, НА ПОЛОГЕ НЕВЕДОМОГО(цк влксм "молодаяа гвардия": москва), p.191.

Lagally, M. 1922, *Z. Angew. Math. Mech.*, **2**, 407.

Landau, L.D. and Lifshitz, E.M. 1966, *Fluid Mechanics* (Pergamon Press: Oxford), p. 3.

Landau, L.D. and Lifshitz, E.M. 1979, *The Classical Theory of Fields* (Pergamon Press: Oxford), p. 341.

Mather, J.C., Cheng, E.S., Eplee Jr., R.E., Isaacman, R.B., Meyer, S.S., Shafer, R.A., Weiss, R., Wright, E.L., Bennet, C.L., Boggess, N.W., Dwek, E., Gulkis, S., Hauser, M.G., Janssen, M., Kelsall, T., Lubin, P.M., Moseley Jr., S.H., Murdock, T.L., Silverberg, R.F., Smoot, G.F., and Wilkinson, D.T. 1990, *Astrophys. J. (Letters),* **345**, L37.

Milgrom, M. 1983, *Astrophys. J.* **270**, 365, 371, and 384.
Milgrom, M. 1995, a preprint of Weizmann Institute of Science, Israel.
Pierce, M.J., Welch, D.L., McClure, R.D., van den Bergh, S., Racine, R., and Stetson, P.B., 1994, *Nature,* **371**, 385.
Seliger, R.L. and Whitham, G.B. 1968, *Proc. Roy. Soc.* A**305**, 1.
Yih, C-S. 1969, *Fluid Mechanics* (MaGraw-Hill: New York), p. 111.
Whipple, F. L. 1950, Astrophys. J. **111**, 375.