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# Differential Cross Sections of the Neutral Pion Photoproduction from Hydrogen in the Energy Range between 390 and 975 MeV 

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# DIFFERENTIAL CROSS SECTIONS OF THE NEUTRAL PION PHOTOPRODUCTION FROM HYDROGEN IN THE ENERGY RANGE BETWEEN 390 AND 975 MeV 

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#### Abstract

The differential cross sections of the reaction $\gamma \mathrm{p} \rightarrow \pi^{\circ} \mathrm{p}$ have been measured at the photon energies between 390 MeV and 975 MeV with the energy steps of 20 MeV or 25 MeV . The angular range covered between $15^{\circ}$ and $130^{\circ}$ in the center of mass system. Recoil protons were detected with a magnetic spectrometer and a photon decayed from a $\pi^{\circ}$-meson was also detected in coincidence with a photon detector at the angles from $50^{\circ}$ to $130^{\circ}$. Two photons from a $\pi^{\circ}$-meson were detected by a pair of photon detectors at the angles of $15^{\circ}, 35^{\circ}$ and $50^{\circ}$. The experimental results are compared with the recent partial wave analyses.


## Chapter I. Introduction

Over the last twenty years many measurements of the differential cross sections $\sigma(\theta)$ on the single pion photoproduction processes have been performed in the resonance region to investigate the electromagnetic properties of the nucleon resonances and the behaviour of the non-resonant terms. As is well known, there are four invariant complex amplitudes in the single pion photoproduction process. Therefore, we need at least seven different experimental measurements to determine the photoproduction amplitudes at one kinematical point (incident photon energy $[K]$ and pion angle in the center of mass system $\left[\theta_{\pi^{c m}}^{\mathrm{cm}}\right]$ ). Many experimental efforts have been devoted to measure the spin dependent parameters such as a polarization of the recoil nucleon $\mathrm{P}(\theta)$, a polarized photon asymmetry $\Sigma(\theta)$ and a polarized target asymmetry $T(\theta)$, in the last ten years. Recently, double spin correlation parameters were measured using the polarized photon and the polarized target ${ }^{1)}$.

The gross features of the above experimental results ${ }^{2)}$ are well explained by the recent phenomenological analyses. Metcalf and Walker (MW) ${ }^{3), 4)}$ decomposed the photoproduction amplitudes into three parts; an electric Born term, a resonant term described with a Breit-Winger formula and an additional "background" term in the lower partial waves. They used a considerable amount of data and determined the electromagnetic couplings of the nucleon resonance below 2 GeV . Feller et al. (NAGOYA) ${ }^{5}$ fitted the data at a photon energy below 1.2 GeV specially emphasizing their polarization measurement of $T(\theta)^{6}$. Their model was essentially the same as

[^1]the one by MW. Knies-Moorhause-Oberlack (KMO) ${ }^{7}$, Moorhause-OberlackRosenfeld (MOR) ${ }^{8)}$ and Devenish-Lyth-Rankin (DLR) ${ }^{9}$ ) used the fixed-t dispersion relations in their analyses. They calculated the real part of amplitudes from the imaginary part of amplitudes assuming that the imaginary part is dominated at the resonance. On the other hand, there are some theoretical predictions on the photon couplings basing on a simple quark model ${ }^{10) \sim 12)}$. These predictions show a fairly good agreement with above analyses only on the dominant resonances such as $\mathrm{P}_{33}(1236), \mathrm{D}_{13}(1513)$ and $\mathrm{F}_{15}(1688)$.

The number of data on single pion photoproduction is not so abandant in comparison with those by nucleon scattering experiments and there are still some discrepancies among the experimental results of the photoproduction from different laboratories. Fig. 1 shows the summary of the existing experimental data on the differential cross section for the reaction $\gamma \mathrm{p} \rightarrow \pi^{\circ} \mathrm{p}$. The aim of the present work is to offer accurate data of differential cross sections for the reaction $\gamma \mathrm{p} \rightarrow \pi^{\circ} \mathrm{p}$ over a wide angular range in the incident photon energy range between 390 and 975 MeV .


Fig. 1. Summary of the existing experimental data for the reaction $\gamma \mathrm{p} \rightarrow \pi^{\circ}$ p below 1.1 GeV .
In this energy region, there are some interferences between the tail of the first resonance $\left[P_{33}(1236)\right]$ and the second resonances dominated by $D_{13}(1520)$ and other small resonances such as $P_{11}(1470)$ and $S_{11}(1535)$. It is promising to study the reaction $\gamma \mathrm{p} \rightarrow \pi^{\circ} \mathrm{p}$ for the investigation on the small resonant amplitudes because of the relatively small contribution of the Born term comparing with the reaction $\gamma \mathrm{p} \rightarrow \pi^{+} \mathrm{n}$.

There are few systematic experiments on the cross sections for the reaction $\gamma \mathrm{p} \rightarrow \pi^{\circ} \mathrm{p}$ in the energy range between 450 and 600 MeV covering a wide angular range with the exception of Dougan et al. ${ }^{13), 14)}$. In their experiment, only recoil protons
were detected with a range telescope. In the present work a recoil proton was detected with a magnetic spectrometer having a large momentum acceptance and one of the photons decayed from a $\pi^{\circ}$-meson was also detected with the photon detector in coincidence in the angular range from $50^{\circ}$ to $130^{\circ}$. Moreover, the differential cross sections of this process at forward angles $\left(\theta_{\pi^{\mathrm{cm}}}^{\mathrm{cm}} \rightarrow 50^{\circ}\right)$ have been measured simultaneously with a $\pi^{\circ}$-detector consisting of a pair of photon detectors. Experimental equipments and procedure are described in Chapter II. The reduction of differential cross section is stated in Chapter III. Experimental results and discussions are given in Chapter IV and V, respectively.

## Chapter II. Experimental Equipments and Procedure

## II-1. General description

The experimental layout is shown in Fig. 2. A photon beam from the 1.3 GeV electron synchrotron at Institute for Nuclear Study, University of Tokyo, was incidented on a liquid hydrogen target. Recoil protons were detected with the magnetic spectrometer (proton arm) and a photon decayed from a $\pi^{\circ}$-meson was detected with the photon detector (gamma arm) in coincidence. These detection systems were mounted on rotatable platforms. The $\pi^{\circ}$-detector for the measurement at the forward angle was also mounted on an another platform. A pair spectrometer was set at the upstream of the beam line and was used to calibrate the photon detector. All informations were stored in a computer system and analysed in a real time.


Fig. 2. Schematic view of the experimental arrangement of the present experiment.

## II-2. Photon beam and the liquid hydrogen target

Electrons in the synchrotron were striked onto a platinum radiator of $50 \mu \mathrm{~m}$ thick to produce the bremsstrahlung after accelerated near the maximum energy.

The spill of the bremsstrahlung was kept to be about 4 m sec to reduce the rate of a chance coincidence throughout the experiment. The resulting bremsstrahlung was defined by two lead collimators to give a beam spot of 30 mm in diameter at the target position. A sweeping magnet was placed behind each collimator to reject charged particles in the photon beam. The intensity of the photon beam was monitored with a thick walled ionization chamber which was calibrated with a Faraday cup using extracted electron beam ${ }^{15)}$. The typical value of the beam intencity was $1.5 \times 10^{9}$ equivalent quanta per second. A thin walled ionization chamber was also used as a supplementary monitor for the beam intensity. Scintillation counter telescopes in the pair spectrometer was used to monitor the beam condition.

The container of liquid hydrogen was a cylindrical shape of 106.5 mm length and 40.6 mm diameter made of Mylar and placed with its axis along the bremsstrahlung. The thickness of the cylindrical wall was $125 \mu \mathrm{~m}$ and that of the walls of both ends across the bremsstrahlung beam was $75 \mu \mathrm{~m}$. The container was put in a vacuum vessel whose window was made of Mylar of $250 \mu \mathrm{~m}$ thick.

## II-3. Magnetic spectrometer (proton arm)

Recoil protons were detected with the magnetic spectrometer ${ }^{16)}$ which consisted of four planes of a multiwire proportional chamber (MWPC \#1~\#4) ${ }^{17}$, six planes of a multiwire spark chamber (WSC $\# 1 \sim \# 6)^{18}$ ), an analysing magnet (Spark Chamber Magnet) and scintillation counters (T1, T2, T3 and ANTI), as shown in Fig. 3. MWPC and WSC determined the trajectory of the charged particle before and behind the magnet, respectively. The momentum of the particle was calculated from the trajectories and the magnetic field of the analysing magnet. The recoil proton was identified by the method as described afterward (II-5). The momentum of the


Fig. 3. Magnetic spectrometer for the measurement of the recoil protons.
recoil proton in the present experiment ranged from 330 to $920 \mathrm{MeV} / \mathrm{c}$. The momentum resolution calculated by the Monte Carlo method was $1.3 \%$ above $700 \mathrm{MeV} / \mathrm{c}$ and $6.0 \sim 1.3 \%$ between 330 and $700 \mathrm{MeV} / \mathrm{c}$. The momentum acceptance ranged from 100 to $400 \mathrm{MeV} / \mathrm{c}$ and the typical angular acceptance was 3.7 msr .
(a) MWPC

The construction of MWPC is shown in Fig. 4. The basic design parameters were similar to those of Charpak et al. ${ }^{19}$. The dimension of the useful area was $280 \times 280 \mathrm{~mm}^{2}$. Two cathode planes were installed both sides of the anode plane. The gap between the anode and cathode plane was 6 mm . The cathode plane consisted of stainless steel wire of $100 \mu \mathrm{~m}$ in diameter with 1 mm spacing. The anode plane was consisted of gold plated tungsten wire of $29 \mu \mathrm{~m}$ in diameter with 2 mm spacing. Windows of each chamber were made of Mylar of $125 \mu \mathrm{~m}$ thick. Frames of each chamber were made of epoxy fiber glass. The anode plane was connected to the ground potential and a constant high voltage of 5.05 kV was supplied to the cathode plane. Anode wires were connected to the amplifier card. The mixing gas of $66 \%$ Argon, $33 \%$ isobutane and $1 \%$ Freon was continuously flowed to each chamber. The signal from the anode wire was amplified and encoded, then transferred to a small computer of TOSBAC- 40 through CAMAC modules. The gate width to read out the data was chosen to be 100 n sec to achieve a high detection efficiency and a small background.

MWPC \#1, \#3 and \#4 were used to read the vertical coordinate and \#2 was used for the horizontal coordinate.


Fig. 4. Construction of the multiwire proportional chamber.
(b) WSC

The construction and dimension of WSC are shown in Fig. 5. A gap of the chamber was 8 mm and the useful area was $370 \times 650 \mathrm{~mm}^{2}(\# 1 \sim \# 3)$ and $660 \times 1040$ $\mathrm{mm}^{2}$ (\#4~6\#). Both plates of chambers were made of stainless steel wire of $100 \mu \mathrm{~m}$
in diameter with 1 mm spacing. Windows of chambers were made of Mylar of $125 \mu \mathrm{~m}$ thick and frames were made of the same material as MWPC. Nelium gas were continuously flowed after mixing of Helium. The mixing gas was $67 \%$ Helium and $33 \%$ Neon. A clearing field of 80 Volt was applied to make the memory time short. High voltage pulse of $5 \sim 6 \mathrm{kV}$ and $1 \mu \mathrm{sec}$ was generated by a Thyratron ${ }^{20)}$ which was triggered by a Krytron pulser. The total delay time of high voltage pulses after the passage of particles through the chamber was $1 \mu \mathrm{sec}$.

The spark position was determined by the magnetostrictive delay line read out system which consisted of a magnetostrictive delay line, front and back fiducial wires, pick up coils, amplifiers, zero-crossing discriminators, a clock generator ( 20 MHz ), interval scalers and a WSC controller.

|  | EFFECTIVEAREA |  | $\begin{aligned} & \mathrm{C} \\ & \mathrm{~cm} \end{aligned}$ | $10$ | $\begin{aligned} & \text { GAP } \\ & \mathrm{cm} \end{aligned}$ | $\left[\begin{array}{cc} x & (\mathrm{em}) \\ f \cdot F i D & -\mathrm{B} \cdot \mathrm{FD} \end{array}\right]$ | $\begin{array}{lr} \mathrm{y} & (\mathrm{~cm}) \\ \mathrm{F} \cdot \mathrm{FiD}-\mathrm{Bf} 10 \end{array}$ | Wire angle |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A cm | B cm |  |  |  |  |  |  |
| \#1 | 65 | 37 | 100 | 62 | 0.8 | 95 | 58 | $10^{\circ}$ |
| \# 2 | 65 | 37 | 100 | 62 | 0.8 | 95 | 58 | $-10^{\circ}$ |
| 苇3 | 65 | 37 | 100 | 62 | 0.8 | 95 | 58 | $0^{\circ}$ |
| \#4 | 104 | 66 | 160 | 100 | 0.8 | 148 | 90 | $-10^{\circ}$ |
| \$ | 104 | 66 | 160 | 100 | 0.8 | 148 | 90 | $10^{\circ}$ |
| \#6 | 104 | 66 | 160 | 100 | 0.8 | 148 | 90 | $0^{\circ}$ |
| FFIO : FRONT FIDUCIAL |  |  |  |  |  |  |  |  |
| B FID : BACK FIDUCIAL |  |  |  |  |  |  |  |  |



Fig. 5. Dimension and construction of the multiwire spark chamber.
(c) Analysing magnet

An analysing magnet with a maximum field strength of 7.03 kG were used to measure the momentum of protons. The magnetic field was measured by the NMR method before and after the experiment. The field distribution in the useful volume of the magnet was also measured by the rotating coil method. The aperture of the magnet was 250 mm high and 50 mm wide, and the effective field length was 1016 mm .
(d) Scintillation counter

Four sets of scintillation counters were used to trigger the proton arm. T1 and ANTI were placed in front of the analysing magnet and T2 and T3 were placed behind the magnet. T1 consisted of four scintillation counters. The dimension of each scintillator was $30 \times 35 \times 3 \mathrm{~mm}^{3}$ (width $\times$ height $\times$ thickness). ANTI consisting of
two scintillation counters ( $200 \times 250 \times 10 \mathrm{~mm}^{3}$ ) were placed in front of the magnet to restrict the acceptance. T2 consisted of two scintillation counters ( $280 \times 280$ $\times 3 \mathrm{~mm}^{3}$ ) and T3 consisted of four scintillation counters ( $170 \times 440 \times 6 \mathrm{~mm}^{3}$ ). Scintillators of T1, T2 and ANTI were viewed by photomultipliers from one side and those of T3 were viewed by photomultipliers from both sides. A light emission diode was connected to each photomultiplier to monitor the performance of the scintillation counter.

## II-4. Photon detector (gamma arm)

## (a) Construction

A construction of the photon detector ${ }^{21)}$ is shown in Fig. 6. The photons from the $\pi^{\circ}$ mesons were converted into electrons and positrons with a lead plate of 5.8 mm thick after passing through a lead collimator, a Lithium hydrate ( LiH ) absorber and veto counters. The hodoscope array of $20 \times 20$ placed right behind of the lead converter determined the conversion position of the photons. Then, the lead glass Čerenkov counter absorbed the total energy of converted electrons and positrons.

The Čerenkov material was lead glass SF-2 and the dimension was 300 mm cube. The density, radiation length, a reflective index for the D -line and critical energy are $3.85 \mathrm{~g} / \mathrm{cm}^{3}, 10.9 \mathrm{~g} / \mathrm{cm}^{2}, 1.648$ and 16.9 MeV , respectively. The Čerenkov light was collected on nine photomultipliers and their signals were added by a linear adder circuit. Output pulses from nine photomultipliers were adjusted in height and in time. The glass and photomultipliers were mounted in an iron box of 6 mm


Fig. 6. The whole assembly of the photon detector for the measurement of a photon from the $\pi^{\circ}$-meson.
thick. A light emission diode was mounted on the lead glass to monitor the gain of photomultipliers.

The scintillator size of the hodoscope array was $3 \times 16 \times 302 \mathrm{~mm}^{3}$. The veto counter was separated into two scintillation counters. Each scintillator was $153 \times$ $302 \times 6 \mathrm{~mm}^{3}$ and overlapped 1 mm each other.

The whole assembly was shielded with lead blocks of 50 mm thick and mounted in an iron box of 19 mm thick. A LiH absorber was placed to reduce the electromagnetic background in front of the veto counters. The thickness of LiH was 100 mm and the vessel was made of acrylite of 5 mm thick.
(b) Calibration

The photon detector was calibrated up to 550 MeV by using mono-energetic electron beams which were analysed by the pair spectrometer. The momentum spread of the electron beam was $1 \%$. The block diagram of the electronics is shown in Fig. 7. Signals from the Čerenkov counter were splitted into two parts. One was used for the fast coincidence with the pair counters (PAIR 1, PAIR 2 and PAIR 3) and triggered the linear gate circuit. Another was fed to a small computer (OKITAC4300 ) through the linear gate and analogue to digital converter (ADC) of CAMAC standard. The pulse height distribution was displayed on a storage scope and


Fig. 7. Block diagram of the electronics for the calibration of the photon detector.
punched out on a paper tape for an off-line analysis. As is shown in Fig. 8, the energy resolution of the Čerenkov counter can be expressed as

$$
\Delta E / E=4.3 \times(E-30)^{-1 / 2} \quad(E \text { in } \mathrm{MeV})
$$

when a lead plate of one radiation length was placed ${ }^{22) \sim 24)}$. Detection efficiencies of the Čerenkov counter for electrons were also measured with different sets of the threshold level of the discriminator circuit as is shown in Fig. 9.


Fig. 8. Energy resolution of the Cerenkov counter. The lead plate converter of 5.8 mm thick was placed.


Fig. 9. Efficiency of the photon detector. The threshold level for each set was 70,90 and 122 MeV , respectively.

## II-5. Electronics and the on-line system

(a) Electronics

A block diagram and timing chart of the fast electronics were shown in Fig. 10 and Fig. 11, respectively.


Fig. 10. Block diagram of the electronics for the coincidence measurement of the reaction $\gamma \mathrm{p} \rightarrow \pi^{\circ} \cdot \mathrm{p}$. All accidental channel was not described.


Fig. 11. Timing chart of the fast electronics. The two signals presented in this chart show the timing of the signal due to the particle with minimum momentum for maximum path and with maximum momentum for minimum path.

## Logic of the gamma arm

Output signals of the Čerenkov counter were served as the Čerenkov signal $(\check{C})$. They were splitted into three parts and were used as a logic signal, as an analogue signal for pulse height analysis and as a stop signal for the measurement of the flight time difference between the gamma arm and the proton arm. Anyone of signals from the hodoscopes for the vertical directions called $X$ and those for the horizontal direction called $Y$ were fed into coincidence circuit ( $X Y$ ). Anyone of signals from the veto counters was called $V$. Finally the coincidence signal ( $X Y \cdot C \cdot \bar{V}$ ) was adopted as the $\gamma$ signal.
Logic of the proton arm
Signals from T1, T2, T3 and ANTI were coincided (T1•T2•T3• $\overline{A N T I}$ ) and adopted as the proton signal $(P)$. The coincidence width was chosen to be 80 ns to detect the proton with the momentum ranged from 200 to $1000 \mathrm{MeV} / \mathrm{c}$.

Signals from T1, T2 and T3 were fed to the strobed coincidence circuit to read the bit informations of the each counter. Signals from T1 and T3 counters were used as the stop and start pulses for the measurement of the flight time of the particle. Pulse heights of T3 (DE/DX) was used to identify the proton. Double arm coincidence

A coincidence between gamma arm $(\gamma)$ and proton arm $(P)$ was made to identify the reaction $\gamma \mathrm{p} \rightarrow \pi^{\circ} \mathrm{p}$ and used as the MASTER signal. It triggered the high voltage pulsing system of WSC, linear gate circuits, ADC, time to digital converter (TDC), strobed coincidence circuits, MWPC and WSC read out systems. When MASTER signal was generated, the regeneration was inhibited for 20 m sec to provide enough time to charge up the high voltage power supply of WSC.
(b) On-Line system

The block diagram of the on-line system is shown in Fig. 12. The following informations were stored in a small computer (TOSBAC-40C); pulse heights of the Čerenkov counter and T3 ( $D E / D X$ ), flight time difference between the gamma arm and T3 (TOF-1), flight time between T1 and T3 (TOF-2), bit patterns of the hodoscope of the gamma arm, T1, T2 and T3, and the firing positions of MWPC and WSC. All informations for each event required 968 bits. The firing positions of MWPC and WSC were displayed on an oscilloscope of TOSBAC-40 event by event. These informations were sent to a central computer (TOSBAC-3400) by on-line and analysed in a real time. Each devices was monitored by the following distributions analysed by TOSBAC-3400; firing positions of MWPC, WSC, gamma hodoscope, T1, T2 and T3 and the pulse height of the Cerenkov counter. The distributions of TOF-1, TOF-2, DE/DX and the kinematical parameters such as $K$ and $\theta_{\pi^{\mathrm{m}}}^{\mathrm{m}}$ calculated by TOSBAC- 3400 were sent back to TOSBAC- 40 at the end of each run and printed out on a line printer. Finally, raw data were recorded on a magnetic tape for off-line analyses.


Fig. 12. Block diagram of the on-line system.

## II-6. Data Taking

The set up parameters of the detection system were summarized in Table 1. The maximum energy of the bremsstrahlung was varied with the step of 100 MeV with the exception of the RUN (870-90). The magnetic field was adjusted to have a proper momentum acceptance according to each RUN. Usually the photon detector was set at 1132 mm apart from the target to get a high geometrical efficiency. When the counting rate of the veto counters become high, the photon detector was set at 1573 mm to reduce the accidental coincidence. The bias level of the Čerenkov counter was varied according to the counting rate and the momentum of a photon from a $\pi^{\circ}$-meson. Each device was maintained to operate with a good condition by the informations described in the previous section.

Table 1. Set up parameters for each run. $\theta_{p}$; set up angle of the magnetic spectrometer, Mag.; field strength of the analyzing magnet, $\theta_{\tau}$; set up angles of the photon detector, $L_{r}$; distances between the photon detector and the target, Bias; threshold level of the photon detector.

| RUN $\left(K-\theta_{\mathrm{cm}}\right)$ <br> $(\mathrm{MeV}-$ deg. $)$ | $K^{\max }$ <br> $(\mathrm{MeV})$ | $\theta_{p}$ <br> $($ deg. $)$ | Mag. <br> $(\mathrm{kG})$ | $\theta_{\tau}$ <br> $($ deg. $)$ | $L_{7}$ <br> $(\mathrm{~cm})$ | Bias <br> $(\mathrm{MeV})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $700-50$ | 750 | 60.9 | 3.56 | 35.6 | 157.3 | 90 |
| $750-60$ | 850 | 57.4 | 4.65 | 43.1 | 113.2 | 90 |
| $700-60$ | 750 | 56.0 | 4.16 | 42.9 | 157.3 | 90 |
| $600-60$ | 650 | 56.0 | 3.95 | 44.3 | 157.3 | 90 |
| $500-60$ | 550 | 55.6 | 3.40 | 46.1 | 157.3 | 90 |
| $700-70$ | 750 | 51.1 | 5.15 | 53.0 | 157.3 | 90 |
| $600-70$ | 650 | 51.2 | 4.75 | 53.0 | 157.3 | 90 |
| $500-70$ | 550 | 51.0 | 3.97 | 54.0 | 157.3 | 70 |
| $650-80$ | 750 | 46.2 | 5.75 | 58.2 | 157.3 | 90 |
| $600-80$ | 650 | 46.3 | 5.13 | 60.4 | 113.2 | 90 |
| $500-80$ | 550 | 46.3 | 4.57 | 62.6 | 113.2 | 70 |
| $870-90$ | 1000 | 41.3 | 6.86 | 62.2 | 113.2 | 122 |
| $750-90$ | 850 | 42.1 | 4.82 | 66.3 | 113.2 | 90 |
| $700-90$ | 750 | 41.6 | 6.51 | 67.1 | 113.2 | 90 |
| $550-90$ | 650 | 41.6 | 5.75 | 69.2 | 113.2 | 90 |
| $450-90$ | 550 | 46.3 | 4.58 | 62.6 | 113.2 | 70 |
| $700-100$ | 750 | 36.6 | 7.03 | 76.2 | 113.2 | 90 |
| $600-100$ | 650 | 36.6 | 5.95 | 78.5 | 113.2 | 70 |
| $500-100$ | 550 | 37.6 | 5.16 | 81.1 | 113.2 | 70 |
| $700-110$ | 750 | 32.0 | 7.03 | 86.1 | 113.2 | 90 |
| $600-110$ | 650 | 32.1 | 6.70 | 88.5 | 113.2 | 70 |
| $500-110$ | 550 | 32.3 | 5.55 | 91.2 | 113.2 | 70 |
| $800-120$ | 850 | 27.7 | 7.00 | 98.2 | 113.2 | 90 |
| $700-120$ | 750 | 27.2 | 7.03 | 97.3 | 113.2 | 70 |
| $600-120$ | 650 | 27.6 | 7.03 | 99.7 | 113.2 | 70 |
| $500-120$ | 550 | 27.7 | 5.95 | 102.2 | 113.2 | 70 |
| $500-130$ | 550 | 24.8 | 6.34 | 109.0 | 113.2 | 70 |
|  |  |  |  |  |  |  |

## Chapter III. Data Reduction

## III-1. General description

The differential cross sections were reduced from the event informations recorded on the magnetic tape. The flow chart of the data processing program is shown in Fig. 13. The four momentum of the recoil proton $\left(\vec{P}_{P}\right)$ was calculated from the firing positions of MWPC, WSC and the magnetic field of the analysing magnet. Kinematical parameters ( $K, \theta_{\pi^{\mathrm{c}}}^{\mathrm{m}}$ etc.) for the reaction $\gamma \mathrm{p} \rightarrow \pi^{\circ} \mathrm{p}$ were reconstructed from the $\vec{P}_{P}$. The detection efficiencies of the whole detection system were evaluated by the Monte Carlo method.

## III-2. Reduction of the proton momentum

(a) MWPC fitting

The trajectory of protons from the target to the analysing magnet was determined by the linear fitting method using the data of MWPC event by event. Firstly, the event with multiple firings in anyone of MWPC was rejected except the case where neighbouring two or three wires were fired. In that case the center of them was


Fig. 13. Flow chart of the data processing program.
adopted as the firing position. Secondly, the event in which the deviation between the fired and the fitted position was greater than 5 mm this event was rejected. The full width at half maximum of the distribution of this deviation for good events was about 1.5 mm . The angle between the incident photon axis and the recoil proton $\left(\theta_{p}\right)$, and the injection angle to the analysing magnet $(\alpha)$ were determined by this fitted line.
(b) WSC fitting

The trajectory behind the analysing magnet was also determined by the linear fitting method using the WSC data. For each WSC, the data which gave the following characteristics were rejected from the fitting process; the multi-track, the fluctuation of the back fiducial count being greater than $0.1 \%$, the difference between the fired and fitted position being greater than 3 mm . Finally, the event in which more than three of the WSC data were rejected by the reason above mentioned, were rejected. The full width at half maximum of the distribution of the deviation between the fired and fitted position was about 0.3 mm . The azimuthal angle $\left(\phi_{p}\right)$ and the ejection angle of the proton from the analysing magnet $(\beta)$ was determined by this fitted line.
(c) Calculation of the proton momentum

The momentum of the proton $\left(P_{P}^{r}\right)$ was calculated by the effective edge approximation and was represented in the following relation

$$
P_{P}^{r}=\frac{0.3 \times B \times L}{\sin (\alpha)+\sin (\beta)}
$$

where $B$ is the strength of the magnetic field, $L$ is the effective field length. The momentum of the recoil proton at the interacting point was given with

$$
P_{P}=P_{P}^{r}+\delta P
$$

where $\delta P$ is the momentum loss of the proton in the liquid hydrogen target, air and scintillators and other materials between the target and the magnet. It was calculated from the range energy relation ${ }^{25)}$. $K$ and $\theta_{\pi^{\circ}}^{\text {cm }}$ were reconstructed from $P_{P}, \theta_{P}$ and $\phi_{p}$.

## III-3. Rejection of background events

Although the reaction $\gamma \mathrm{p} \rightarrow \pi^{\circ} \mathrm{p}$ was identified by the present coincidence measurement between the gamma arm and the proton arm, the distribution of TOF-1,TOF-2 and $D E / D X$ were used to reduce the backgrounds due to other processes. However, among the multi-pion production, the reaction $\gamma \mathrm{p} \rightarrow \pi^{\circ} \pi^{\circ} \mathrm{p}$ can contributes to the background of the present experiment. This background was reduced to be less than $3 \%$ by choosing the events whose reconstructed energy of the initial photons laid between ( $K^{\max }-250 \mathrm{MeV}$ ) and $K^{\max }$, where the $K^{\max }$ is the maximum energy of the bremsstrahlung. The background due to the reaction $\gamma \mathrm{p} \rightarrow \pi^{+} \mathrm{n}$ was negligibly small because of the coincidence measurement.

The reaction $\gamma \mathrm{p} \rightarrow \gamma^{\prime} \mathrm{p}$ (proton Compton scattering) is very similar kinematically to the reaction $\gamma \mathrm{p} \rightarrow \pi^{\circ} \mathrm{p}$. Moreover, the detection efficiency of the gamma arm for the former reaction is higher than that of the latter reaction. In the present experiment, the fraction of the proton Compton events was determined from the distribution of the difference between the predicted and observed positions of a photon on the hodoscope of the gamma $\mathrm{arm}^{26)}$. Assuming that the measured photons were produced by the proton Compton process, the predicted position of a photon on the hodoscope of the gamma arm was calculated using the momentum of a recoil proton measured with the proton arm. The distributions of the difference between the predicted and measured positions are shown in Fig. 14. The distributions of the difference thus obtained was simulated by the Monte Carlo calculation for the reaction


Fig. 14. Distribution of the difference between predicted and experimental position of a photon on the gamma hodoscope for the vertical direction ( $\Delta Y$ ). The peak around $\Delta Y=0$ are due to the events of the proton Compton process.
$\gamma \mathrm{p} \rightarrow \pi^{\circ} \mathrm{p}$ and is also shown in Fig. 14. The background due to the proton Compton process was estimated from this distributions and amounted from $3 \%$ to $30 \%$.

## III-4. Monte Carlo simulation

The detection efficiency and resolution of the whole detection system for the reaction $\gamma \mathrm{p} \rightarrow \pi^{\circ} \mathrm{p}$ was simulated by the Monte Carlo calculation. The block diagram of this calculation is shown in Fig. 15.


Fig. 15. Flow chart of the Monte Carlo program.
(a) Random numbers on $K_{\gamma}, \vec{X}_{t}, \theta_{p}^{\mathrm{cm}}$ and $\phi_{p}^{\mathrm{cm}}$

The energy spectrum of the incident photon was approximated by the Schiff's formula. The distribution of the reaction point $\vec{X}_{t}$ was assumed to be Gaussian for the radial and uniform along the beam axis. The standard deviation of the radial distribution was determined from the X-ray film exposed to the photon beam.

The distribution of $\cos \theta_{p}^{\mathrm{cm}}$ and $\phi_{p}^{\mathrm{cm}}$ were assumed to be uniform in this calculation, where $\theta_{p}^{\mathrm{cm}}$ and $\phi_{p}^{\mathrm{cm}}$ are the angle of the recoil proton in the center of mass system. The other kinematical parameters were calculated from $K_{\gamma}, \theta_{p}^{\mathrm{cm}}$ and $\phi_{p}^{\mathrm{cm}}$.
(b) Proton arm

The energy loss and multiple Coulomb scattering of the proton in the following materials were taken into account to reproduce the experimental condition; the liquid hydrogen target, Mylar of the target, MWPC and WSC, air, scintillators and wires.
(c) Gamma arm

A $\pi^{\circ}$-meson decays isotropically into two photons in the $\pi^{\circ}$ rest system. The
distribution of the random number on $\cos \theta_{\gamma}^{* *}$ and $\phi_{\gamma}^{* *}$ were assumed to be uniform, where $\theta_{\gamma}^{* *}$ and $\phi_{\gamma}^{* *}$ were the decay angle of the $\pi^{\circ}$-meson in the $\pi^{\circ}$ rest system.

The following factors were calculated in the program; geometrical efficiency, conversion efficiency of a photon into electron-position pairs in the lead plate placed in front of the hodoscopes, attenuation factor of photons in the target and $\mathrm{LiH} \mathrm{ab}-$ sorber and counting losses due to discrimination of the electronics.

The conversion efficiency and attenuation factor were approximated by the following formula;

$$
\eta_{c}=1.0-\exp (-\sigma \cdot \rho \cdot N \cdot x / A)
$$

where $\rho$ is the density of the material, $N$ is the Avogadro number, $A$ is the atomic number, $x$ is the thickness of the material and $\sigma$ is the cross section of the pair production. $\sigma$ was calculated by the formula given by Sorensen ${ }^{27}$, which was consistent with experimental results ${ }^{28}$ ) within $1 \%$. Fig. 16 shows energy dependence of the conversion efficiency and the attenuation factor.


Fig. 16. Conversion efficiency of a photon in the lead plate of 5.8 mm thick and attenuation factor in the liquid hydrogen target and LiH absorber.

The energy distribution of photons in the lead glass was assumecd to be Gaussian, whose standard deviation was obtained from the result of the calibration run described in II-4. If the energy of photon was lower than the threshold level of the electronics, the event was rejected.
(d) Kinematical reconstruction

Kinematical parameters were reconstructed using observed values only on the event which could be detected. The procedure for the reconstruction of events was similar to the one for the experimental data. The difference between initial (generated) and simulated observables were also calculated. The distribution of these difference show the resolution of the detection system.

## III-5. Reduction of the differential cross section

(a) Experimental yield

The experimental yield can be represented in the following relation;

$$
Y_{e}=\frac{\mathrm{d} \sigma}{\mathrm{~d} \Omega^{*}} \mathrm{~d} \Omega^{*} \cdot Q \cdot N_{t} \cdot \eta(\gamma, \mathrm{p})
$$

with $\quad Y_{e}$; coincidence rate of the photon from $\pi^{\circ}$-meson and recoil proton of the reaction $\gamma \mathrm{p} \rightarrow \pi^{\circ} \mathrm{p}$,
$\frac{\mathrm{d} \sigma}{\mathrm{d} \Omega^{*}}$; differential cross section in the center of mass system,
$Q$; number of incident photons,
$N_{t}$; number of target protons, $\eta(\gamma, \mathrm{p})$; detection efficiency for the photon and proton,
$\eta(\gamma, \mathrm{p})$ is given with

$$
\eta(\gamma, \mathrm{p})=\eta(\text { LiH }) \cdot \eta(\text { conv. }) \cdot \eta(\text { veto }) \cdot \eta(\text { bias }) \cdot \eta(\text { WSC }) \cdot \eta(\mathrm{p})
$$

$\eta(\mathrm{LiH})$ and $\eta($ conv.) are the factors due to the attenuation of photons in the LiH absorber and the conversion of photons in the lead plate. $\eta($ bias $)$ is the loss of $\gamma$ signals due to the discrimination of photons by electronics of the Čerenkov counter. The counting rate of the veto counter of the gamma arm was very high ( $\sim 5 \times 10^{9}$ $\mathrm{sec}^{-1}$ ) when it was placed near the beam line and target. Then the accidental coincidence between the veto counter and the other counter was recorded on a scaler circuit. The loss of $\gamma$ signal due to this reason [ $\eta$ (veto $)$ ] was summarized in Table 2. The loss of the MASTER signals [ $\eta(\mathrm{WSC})$ ] during the inhibited time was measured and is also listed in Table 2. $\eta(\mathrm{p})$ is the loss of the recoil protons in the proton arm due to energy loss, multiple scattering and the nuclear absorption. The loss due to the nuclear absorption in the scintillation counter was estimated to be less than $2 \%$. The loss due to the energy loss and multiple scattering were taken into account in the Monte Carlo calculation.

The accidental coincidence between the gamma and proton arm was amounted to $0.6 \sim 5.7 \%$ for the almost all experimental set up as listed in Table 2. On the other hand, the time resolution of MWPC and WSC were not so good in comparison with the scintillation counter system. Therefore, the trajectories of multi particles were recorded in MWPC and WSC accidentally. The ratio of the events which satisfied the criteria of reconstructions in III-2, to the total events ranged from $63 \%$ to $89 \%$. The correction factor $\left[\eta_{r}\right]$ due to this reason was given by the following relation,

$$
\eta_{r}=\frac{N_{r}}{N(1-r)}
$$

where $N_{r}$ is the number of events satisfied the criteria, $N$ is the number of total events and $r$ is the ratio of the accidental coincidence to the total events. The results were also listed in Tabl 2.

The counting rate with the empty target was less than $0.5 \%$ of the one with the full target and we neglect this correction.
$Q$ is given with

$$
Q=\frac{1}{K^{\max }} \int K \frac{B(K)}{K} \mathrm{~d} K=\frac{U}{K^{\max }}
$$

where $B(K)$ is the spectral function of bremsstrahlung and $U$ is the total energy of incident photons measured with the thick walled ionization chamber. Fig. 17(a)~ (d) show the distributions of the experimental yield normalized by the equivalent quanta.
(b) Monte Carlo yield and cross sections

The Monte Carlo yield can be expressed in the following formula,

Table 2. Normalization corrections and the rate of the accidental coincidence. $\pi$ (Veto) is the inefficiency due to the accidental coincidence between the veto counter and the other counters, $\pi$ (WSC) is the inefficiency due to the inhibited time of the MASTER signal and $\eta_{r}$ is the reconstruction efficiency.

| RUN $\left(K-\theta_{\pi}^{\text {mm }}\right)$ | $\pi($ Veto $)$ | $\pi($ WSC $)$ | Accidental <br> Coincidence <br> $($ MeV-deg. $)$ | $(\%)$ |
| :---: | :---: | :---: | :---: | :---: |

$$
Y_{m}=\int \mathrm{d} \Omega^{*} \cdot Q_{m} \cdot \eta_{m}(\gamma, \mathrm{p}) \cdot \alpha_{m}^{-1}
$$

with $\quad Y_{m}$; Monte Carlo yields,
$Q_{m}$; number of photons generated in the simulation,
$\eta_{m}(\gamma, \mathrm{p})$; detection efficiency taken into account in the simulation described in III-4 with

$$
\eta_{m}(\gamma, \mathrm{p})=\eta_{m}(L i H) \cdot \eta_{m}(c o n v .) \cdot \eta_{m}(\text { bias }) \cdot \eta_{m}(\mathrm{p}),
$$

$\alpha_{m}$; correction factor due to the restriction of the angle $\theta_{p}^{\mathrm{cm}}$ and $\theta_{p}^{\mathrm{cm}}$ to increase the Monte Carlo efficiency.
$Q_{m}$ is given with

$$
Q_{m}=\frac{C}{K^{\max }} \int K \frac{B(K)}{K} \mathrm{~d} K
$$

where $C$ is the normalization constant and is expressed with

$$
C=\frac{M_{t}}{\int_{K^{1}}^{K^{2}} \frac{B(K)}{K} \mathrm{~d} K}
$$



Fig. 17. Experimental yield as a function of the incident photon energy and $\theta_{\pi \circ}^{\text {cm }}$, where $E_{\max }$ is the maximum energy of the bremsstrahlung.
(a) $E_{\max }=550 \mathrm{MeV}$,
(b) $E_{\max }=650 \mathrm{MeV}$,
(c) $E_{\max }=750 \mathrm{MeV}$ and
(d) $E_{\max }=$ 850 MeV .
with $M_{t}$ is the total trial number is the simulation, $K^{1}$ and $K^{2}$ is the minimum and maximum energies of the incident photon generated in the simulation, respectively.

Finally, the differential cross section can be expressed with

$$
\frac{\mathrm{d} \sigma}{\mathrm{~d} \Omega^{*}}=\frac{Y_{e}}{Q}\left(N_{t} \cdot \frac{Y_{m}}{M_{t}} \cdot \alpha_{m} \frac{\int_{K^{1}}^{K^{2}} \frac{(B K)}{K} \mathrm{~d} K}{\frac{1}{K^{\max }} \int K \frac{B(K)}{K} \mathrm{~d} K}\right)^{-1}
$$

## Chapter IV. Experimental Results

The present results $\left(\mathrm{d} \sigma / \mathrm{d} \Omega^{*}\right)$ on the differential cross sections are summarized
in Table 3. The resolutions for the incident photon energy $\Delta K$ and for the $\pi^{\circ}$-c.m. angle $\Delta \theta_{\pi^{c}}^{\mathrm{cm}}$ are expressed in the following relations;

$$
\begin{aligned}
& \Delta K=\left(\left(\frac{\partial K}{\partial P} \Delta P\right)^{2}+\left(\frac{\partial K}{\partial \theta_{p}} \Delta \theta_{p}\right)^{2}\right)^{1 / 2} \\
& \Delta \theta_{\pi^{\mathrm{om}}}^{\mathrm{cm}}=\left(\left(\frac{\partial \theta_{\pi^{\mathrm{o}}}^{\mathrm{cm}}}{\theta_{p}} \Delta P\right)^{2}+\left(\frac{\partial \theta_{\pi^{\mathrm{om}}}^{\mathrm{om}}}{\partial \theta_{p}} \theta_{p}\right)^{2}\right)^{1 / 2}
\end{aligned}
$$

where $\Delta P$ and $\Delta \theta_{p}$ are the momentum and the angular resolution of the magnetic spectrometer calculated by the Monte Carlo method as described in III-4. These resolutions are summarized in Table 4 for each data point. The energy step between the data points was not chosen to be equal to the energy resolution. The energy resolution is about the same as the energy step at angles greater than $90^{\circ}$. However at the forward angles, the energy resolution becomes rather wider than the energy step. The angular acceptance of each data point was ranged from $3^{\circ}$ to $4^{\circ}$ (FWHM). The angular resolution of the detection system is almost better than the angular acceptance.

Table 3. Differential cross sections for the reaction $\gamma \mathrm{p} \rightarrow \pi^{\circ} \mathrm{p}$, obtained by the coincidence measurement with the magnetic spectrometer and the photon detector. The presented errors include only statistical errors.

| $\begin{gathered} K \\ (\mathrm{MeV}) \end{gathered}$ | $\begin{gathered} \theta_{5}^{\mathrm{cm}} \\ (\mathrm{deg} . \end{gathered}$ | $\begin{aligned} & \mathrm{d} \sigma / \mathrm{d} \Omega^{*} \\ & (\mu \mathrm{~b} / \mathrm{sr}) \end{aligned}$ |
| :---: | :---: | :---: |
|  | $50^{\circ}$ |  |
| 610 | 53.5 | $1.90 \pm 0.12$ |
| 630 | 53.0 | $1.89 \pm 0.12$ |
| 650 | 53.0 | $1.96 \pm 0.12$ |
| 670 | 53.0 | $1.88 \pm 0.11$ |
| 690 | 53.0 | $1.84 \pm 0.11$ |
|  | $60^{\circ}$ |  |
| 490 | 64.0 | $5.29 \pm 0.32$ |
| 510 | 63.5 | $4.25 \pm 0.26$ |
| 530 | 63.5 | $3.49 \pm 0.22$ |
| 550 | 63.5 | $3.05 \pm 0.20$ |
| 570 | 63.5 | $2.61 \pm 0.18$ |
| 590 | 63.0 | $2.33 \pm 0.16$ |
| 610 | 61.0 | $1.98 \pm 0.05$ |
| 630 | 61.0 | $1.98 \pm 0.05$ |
| 650 | 61.0 | $2.11 \pm 0.05$ |
| 670 | 60.5 | $2.31 \pm 0.05$ |
| 690 | 60.5 | $2.66 \pm 0.05$ |
| 710 | 60.5 | $2.96 \pm 0.05$ |
| 730 | 60.5 | $3.02 \pm 0.05$ |
| 750 | 60.0 | $3.00 \pm 0.05$ |
|  | $70^{\circ}$ |  |
| 450 | 73.0 | $8.17 \pm 0.34$ |
| 470 | 73.0 | $6.70 \pm 0.28$ |
| 490 | 73.0 | $5.64 \pm 0.24$ |
| 510 | 72.0 | $4.16 \pm 0.20$ |

Table 4. Resolutions of the present experiment at each data point. $\Delta K$ is the energy resolution of the incident photon and $\Delta \theta_{\pi}^{\mathrm{cm}}$ is the angular resolutions of the $\pi^{\circ}$-angle in the center of mass system.

| $\begin{gathered} K \\ (\mathrm{MeV}) \end{gathered}$ | $\begin{gathered} \Delta K(\text { FWHM }) \\ (\mathrm{MeV}) \end{gathered}$ | $\begin{gathered} \Delta 0_{\mathrm{s} \mathrm{\circ}(\mathrm{~cm}(\mathrm{FWHM})} \\ \text { (deg.) } \end{gathered}$ |
| :---: | :---: | :---: |
| $\theta_{\pi}^{\mathrm{cm}}=50^{\circ}$ |  |  |
| 610 | 84 | 4.1 |
| 630 | 83 | 4.0 |
| 650 | 85 | 3.9 |
| 670 | 87 | 3.8 |
| 690 | 87 | 3.7 |


| $\begin{gathered} K \\ (\mathrm{MeV}) \end{gathered}$ | $\begin{gathered} \theta_{\overline{\operatorname{sem}}} \\ \text { (deg.) } \end{gathered}$ | $\begin{aligned} & \mathrm{d} \sigma / \mathrm{d} \Omega^{*} \\ & (\mu \mathrm{~b} / \mathrm{sr}) \end{aligned}$ | $\begin{gathered} K \\ (\mathrm{MeV}) \end{gathered}$ | $\begin{gathered} \Delta K(\text { FWHM }) \\ (\mathrm{MeV}) \end{gathered}$ | $\begin{gathered} \Delta \theta_{\pi}^{\mathrm{cm}(\mathrm{~m}(\mathrm{FWHM})} \\ (\mathrm{deg} .) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 530 | 73.5 | $3.66 \pm 0.31$ | 530 | 44 | 3.0 |
| 550 | 73.5 | $3.28 \pm 0.28$ | 550 | 44 | 2.9 |
| 570 | 73.5 | $2.56 \pm 0.24$ | 570 | 45 | 2.7 |
| 590 | 73.5 | $2.56 \pm 0.24$ | 590 | 46 | 2.6 |
| 610 | 73.0 | $2.73 \pm 0.19$ | 610 | 46 | 2.5 |
| 630 | 73.0 | $2.81 \pm 0.19$ | 630 | 47 | 2.4 |
| 650 | 73.0 | $2.84 \pm 0.19$ | 650 | 47 | 2.3 |
| 670 | 73.0 | $2.79 \pm 0.18$ | 670 | 47 | 2.2 |
| 690 | 73.0 | $2.71 \pm 0.18$ | 690 | 47 | 2.2 |
|  | $80^{\circ}$ |  | $\theta_{\stackrel{\circ}{\text { cma }}}^{\text {cm }}=80^{\circ}$ |  |  |
| 410 | 83.0 | $12.9 \pm 0.42$ | 410 | 33 | 3.6 |
| 430 | 83.0 | $10.7 \pm 0.35$ | 430 | 33 | 3.3 |
| 450 | 83.0 | $8.60 \pm 0.29$ | 450 | 34 | 3.1 |
| 470 | 83.0 | $7.03 \pm 0.25$ | 470 | 34 | 2.9 |
| 490 | 83.0 | $5.81 \pm 0.21$ | 490 | 35 | 2.7 |
| 510 | 83.0 | $4.83 \pm 0.18$ | 510 | 36 | 2.6 |
| 530 | 83.0 | $3.94 \pm 0.17$ | 530 | 36 | 2.5 |
| 550 | 83.0 | $3.88 \pm 0.16$ | 550 | 36 | 2.4 |
| 570 | 83.0 | $3.38 \pm 0.14$ | 570 | 37 | 2.3 |
| 590 | 83.0 | $2.70 \pm 0.12$ | 590 | 37 | 2.2 |
| 610 | 83.0 | $2.98 \pm 0.18$ | 610 | 37 | 2.1 |
| 630 | 83.0 | $2.77 \pm 0.17$ | 630 | 37 | 2.0 |
| 650 | 83.0 | $3.00 \pm 0.17$ | 650 | 38 | 1.9 |
| 670 | 83.0 | $2.95 \pm 0.16$ | 670 | 38 | 1.9 |
|  | $90^{\circ}$ |  | $\theta_{\pi}^{\mathrm{em}}=90^{\circ}$ |  |  |
| 390 | 92.0 | $15.3 \pm 0.50$ | 390 | 28 | 3.3 |
| 410 | 92.0 | $11.9 \pm 0.39$ | 410 | 28 | 3.1 |
| 430 | 92.0 | $10.0 \pm 0.33$ | 430 | 28 | 2.8 |
| 450 | 91.5 | $7.63 \pm 0.27$ | 450 | 29 | 2.6 |
| 470 | 91.5 | $6.78 \pm 0.24$ | 470 | 29 | 2.5 |
| 490 | 91.5 | $5.63 \pm 0.22$ | 490 | 28 | 2.3 |
| 510 | 91.5 | $5.54 \pm 0.22$ | 510 | 29 | 2.2 |
| 530 | 93.5 | $4.89 \pm 0.31$ | 530 | 29 | 2.1 |
| 550 | 93.0 | $4.05 \pm 0.27$ | 550 | 30 | 2.0 |
| 570 | 93.0 | $3.16 \pm 0.23$ | 570 | 30 | 1.9 |
| 590 | 93.0 | $3.15 \pm 0.22$ | 590 | 31 | 1.8 |
| 610 | 93.0 | $2.79 \pm 0.18$ | 610 | 31 | 1.8 |
| 630 | 93.0 | $2.77 \pm 0.17$ | 630 | 28 | 1.7 |
| 650 | 90.5 | $3.26 \pm 0.09$ | 650 | 27 | 1.7 |
| 670 | 90.5 | $3.40 \pm 0.09$ | 670 | 31 | 1.6 |
| 690 | 90.0 | $3.82 \pm 0.09$ | 690 | 32 | 1.6 |
| 710 | 90.0 | $4.29 \pm 0.09$ | 710 | 32 | 1.5 |
| 730 | 90.0 | $4.35 \pm 0.09$ | 730 | 32 | 1.5 |
| 750 | 90.0 | $4.55 \pm 0.10$ | 750 | 32 | 1.4 |
| 770 | 90.0 | $4.17 \pm 0.10$ | 770 | 32 | 1.4 |
| 790 | 91.5 | $3.99 \pm 0.12$ | 790 | 33 | 1.4 |
| 810 | 91.0 | $3.56 \pm 0.11$ | 810 | 33 | 1.3 |
| 830 | 91.0 | $3.09 \pm 0.10$ | 830 | 33 | 1.3 |
| 850 | 91.0 | $2.67 \pm 0.09$ | 850 | 33 | 1.3 |
| 870 | 91.0 | $2.19 \pm 0.08$ | 870 | 34 | 1.2 |
|  | $100^{\circ}$ |  | $\theta_{\frac{\mathrm{c}}{\mathrm{cm}} \mathrm{O}}^{\mathrm{c}}=100^{\circ}$ |  |  |
| 390 | 102.5 | $16.6 \pm 0.63$ | 390 | 24 | 2.9 |

Table 3. (Continued)

| $\begin{gathered} K \\ (\mathrm{MeV}) \end{gathered}$ | $\left.\begin{array}{c} \theta_{\mathrm{c} \mathrm{~cm}}^{\mathrm{a}} \\ (\mathrm{deg} . \end{array}\right)$ | $\begin{gathered} \mathrm{d} / \sigma \mathrm{d} \Omega^{*} \\ (\mu \mathrm{~b} / \mathrm{sr}) \end{gathered}$ |
| :---: | :---: | :---: |
| 410 | 102.0 | $13.2 \pm 0.51$ |
| 430 | 102.0 | $10.7 \pm 0.44$ |
| 450 | 102.0 | $8.69 \pm 0.37$ |
| 470 | 101.0 | $7.69 \pm 0.34$ |
| 490 | 101.0 | $5.55 \pm 0.28$ |
| 510 | 103.0 | $4.71 \pm 0.26$ |
| 530 | 103.0 | $4.74 \pm 0.27$ |
| 550 | 102.0 | $4.09+10.24$ |
| 570 | 102.0 | $3.76 \pm 0.23$ |
| 590 | 103.2 | $3.77 \pm 0.22$ |
| 610 | 102.0 | $3.02 \pm 0.19$ |
| 630 | 103.5 | $3.65 \pm 0.21$ |
| 650 | 103.0 | $3.08 \pm 0.19$ |
| 670 | 102.0 | $3.61 \pm 0.20$ |
| 690 | 102.0 | $3.62 \pm 0.20$ |
|  | $110^{\circ}$ |  |
| 390 | 113.5 | $14.5 \pm 0.49$ |
| 410 | 113.0 | $11.9 \pm 0.42$ |
| 430 | 113.0 | $9.76 \pm 0.35$ |
| 450 | 112.5 | $7.18 \pm 0.28$ |
| 470 | 112.0 | $5.27 \pm 0.24$ |
| 490 | 112.0 | $4.71 \pm 0.22$ |
| 510 | 111.5 | $4.53 \pm 0.21$ |
| 530 | 113.0 | $4.17 \pm 0.21$ |
| 550 | 112.0 | $3.54 \pm 0.18$ |
| 570 | 112.0 | $3.54 \pm 0.17$ |
| 590 | 112.0 | $2.90 \pm 0.16$ |
| 610 | 112.0 | $2.53 \pm 0.15$ |
| 630 | 112.0 | $2.50 \pm 0.14$ |
| 650 | 111.5 | $2.21 \pm 0.13$ |
| 670 | 111.5 | $2.44 \pm 0.14$ |
| 690 | 111.5 | $2.35 \pm 0.13$ |
|  | $120^{\circ}$ |  |
| 390 | 123.5 | $12.6 \pm 0.50$ |
| 410 | 123.0 | $9.69 \pm 0.40$ |
| 430 | 123.0 | $7.56 \pm 0.34$ |
| 450 | 122.5 | $6.38 \pm 0.30$ |
| 470 | 122.0 | $5.42 \pm 0.26$ |
| 490 | 123.5 | $4.77 \pm 0.24$ |
| 510 | 123.0 | $4.48 \pm 0.18$ |
| 530 | 123.0 | $3.78 \pm 0.19$ |
| 550 | 122.5 | $3.24 \pm 0.16$ |
| 570 | 122.0 | $3.10 \pm 0.16$ |
| 590 | 121.5 | $2.46 \pm 0.14$ |
| 610 | 121.5 | $2.26 \pm 0.13$ |
| 630 | 122.0 | $2.61 \pm 0.15$ |
| 650 | 120.0 | $2.79 \pm 0.07$ |
| 670 | 120.0 | $2.79 \pm 0.07$ |
| 690 | 120.0 | $2.86 \pm 0.07$ |
| 710 | 120.0 | $3.25 \pm 0.07$ |
| 730 | 120.0 | $3.22 \pm 0.07$ |
| 750 | 119.5 | $3.15 \pm 0.07$ |

Table 4. (Continued)

| $K$ <br> $(\mathrm{MeV})$ | $\Delta K(\mathrm{FWHM})$ <br> $(\mathrm{MeV})$ | $\Delta \theta_{8}^{\text {cm }}(\mathrm{FWHHM})$ <br> $(\mathrm{deg})$. |
| :---: | :---: | :---: |
| 410 | 25 | 2.6 |
| 430 | 25 | 2.5 |
| 450 | 25 | 2.3 |
| 470 | 25 | 2.2 |
| 490 | 26 | 2.1 |
| 510 | 26 | 1.9 |
| 530 | 26 | 1.9 |
| 550 | 26 | 1.8 |
| 570 | 26 | 1.7 |
| 590 | 27 | 1.6 |
| 610 | 27 | 1.6 |
| 630 | 27 | 1.5 |
| 650 | 27 | 1.5 |
| 670 | 27 | 1.4 |
| 690 | 27 | 1.4 |


| $\theta_{\mathrm{r}}^{\mathrm{cm} \mathrm{O}}=110^{\circ}$ |  |  |
| :---: | :---: | :---: |
|  | 390 |  |
| 410 | 22 | 2.6 |
| 430 | 22 | 2.4 |
| 450 | 23 | 2.2 |
| 470 | 23 | 2.1 |
| 490 | 23 | 1.9 |
| 510 | 23 | 1.8 |
| 530 | 20 | 1.7 |
| 550 | 23 | 1.7 |
| 570 | 23 | 2.3 |
| 590 | 23 | 1.5 |
| 610 | 23 | 1.5 |
| 630 | 23 | 1.4 |
| 650 | 24 | 1.4 |
| 670 | 24 | 1.3 |
| 690 | 24 | 1.3 |
|  |  | 1.2 |


| $\theta_{\pi}^{\mathrm{cm}}=120^{\circ}$ |  |  |
| :---: | :---: | :---: |
|  |  |  |
| 390 | 20 | 2.3 |
| 410 | 20 | 2.2 |
| 430 | 20 | 2.0 |
| 450 | 20 | 1.9 |
| 470 | 21 | 1.8 |
| 490 | 21 | 1.7 |
| 510 | 20 | 1.6 |
| 530 | 21 | 1.5 |
| 550 | 21 | 1.5 |
| 570 | 21 | 1.4 |
| 590 | 21 | 3.1 |
| 610 | 21 | 1.3 |
| 630 | 21 | 1.2 |
| 650 | 21 | 1.2 |
| 670 | 22 | 1.2 |
| 690 | 22 | 1.1 |
| 710 | 23 | 1.1 |
| 730 | 23 | 1.0 |
| 750 | 24 | 1.0 |


| $K$ <br> $(\mathrm{MeV})$ | $\theta_{\pi}^{\mathrm{cma}}$ <br> $(\mathrm{deg})$. | $\mathrm{d} \sigma / \mathrm{d} \Omega^{*}$ <br> $(\mu \mathrm{~b} / \mathrm{sr})$ |
| :---: | :---: | :---: |
| 770 | 119.5 | $3.03 \pm 0.07$ |
| 790 | 119.5 | $2.80 \pm 0.07$ |
|  | $130^{\circ}$ |  |
| 410 | 130.0 | $9.57 \pm 0.48$ |
| 430 | 129.0 | $7.61 \pm 0.40$ |
| 450 | 128.5 | $6.00 \pm 0.34$ |
| 470 | 128.5 | $5.40 \pm 0.31$ |
| 490 | 128.5 | $4.66 \pm 0.27$ |
| 510 | 128.0 | $3.60 \pm 0.23$ |


| $K$ <br> $(\mathrm{MeV})$ | $\Delta K(\mathrm{FWHM})$ <br> $(\mathrm{MeV})$ | $\Delta \theta_{\mathrm{c}}^{\mathrm{cm}}(\mathrm{FWHM})$ <br> $(\mathrm{deg})$. |
| :---: | :---: | :---: |
| 770 | 25 | 1.0 |
| 790 | 26 | 1.0 |
| $\theta_{\mathrm{f}}^{\mathrm{cm}=130^{\circ}}$ |  |  |
| 410 | 19 | 2.0 |
| 430 | 19 | 1.9 |
| 450 | 19 | 1.8 |
| 470 | 19 | 1.7 |
| 490 | 19 | 1.6 |
| 510 | 19 | 1.5 |

The errors listed in Table 3 include only the statistical errors. However there are the following origins of the systematic error.

1) The uncertainties due to the maximum energy of the bremsstrablung ( $\left.\Delta K^{\max }\right)$ $K^{\text {max }}$ ) were about $0.5 \%$, because the beam spill was kept to be about 4 m sec during the experiment.
2) The uncertainty of the number of target protons due to evaporating of the liquid hydrogen was estimated to be less than $1.0 \%$.
3) The total energy of the incident photon beam was measured by the thick walled ionization chamber. The total energy $U$ is represented with

$$
U=\alpha_{i} \cdot \alpha_{c},
$$

where $\alpha_{i}$ is the calibration constant of the current integrator and $\alpha_{c}$ is the calibration constant of the thick walled ionization chamber, whose error were $1.3 \%$ and $2.0 \%$, respectively.
4) The uncertainties of the efficiency of the gamma arm was considered as follows. The cross section of pair production in the material was calculated as described in III-4(c). It is consistent with the experimental value within $1.0 \%$. The threshold level of the Čerenkov counter was measured frequently during the experiment and their fluctuation was less than $0.3 \%$. The geometrical efficiency of the gamma arm was defined by the hodoscope. The pulse height of the Čerenkov counter for the photon entered in the edge of the counter become smaller than that for the photon entered in the central region because of the leakage of the electromagnetic shower from the lead glass. The error due to this origin was estimated to be negligibly small.
5) The ambiguity caused by the Monte Carlo calculation of the momentum acceptance and the solid angle of the proton arm was the most heavy source of the systematic error. The detection efficiencies at the border of the acceptance were fall steeply against the proton momentum or angles. Therefore, we adopt only the central region of the acceptance for each experimental set up, where the efficiencies varied smoothly. The error due to this origin was estimated to be less than $5.0 \%$.
6) The error in estimating the fraction of proton Compton events was almost less than $1.0 \%$, but it was mounted to $2.0 \%$ at $60^{\circ}$ and $5.0 \%$ at $50^{\circ}$.
7) The other factors of the uncertainties such as $\eta(v e t o), \eta(W S C)$ and $\eta_{r}$ were less than $1.0 \%$.

The quadrature sum of these systematic errors amount to be $5.9 \%$ at angles between $70^{\circ}$ and $130^{\circ}$, and $6.2 \%$ at $60^{\circ}$ and $7.7 \%$ at $50^{\circ}$.

## Chapter V. Discussions



Fig. 18. The energy dependence of the differential cross sections for the reaction $\gamma \mathrm{p} \rightarrow \pi^{\circ} \mathrm{p}$. The solid and dashed curves are from partial wave analyses by NAGOYA and MOR, respectively. $\phi$; represents the present results obtained by the coincidence measurement with the magnetic spectrometer and the photon detector, $\phi$; the present results obtained by the $\pi^{\circ}$-detector, $* ;$ Y. Hemmi et al., $\dagger$; J. S. Barton et al.

The energy dependence of the cross sections between $15^{\circ}$ and $130^{\circ}$ are plotted in Fig. 18 and Fig. 20 (a) $\sim(\mathrm{k})$ together with the results obtained with the $\pi^{\circ}$ detector at forward angles. The recent results obtained at other laboratories ${ }^{13), 14), 29) \sim 36)}$ are also plotted in these figures. The data noted by the open circles were the results


Fig. 19. The angular dependence of the differential cross sections for the reaction $\gamma \mathrm{p} \rightarrow \pi^{\circ} \mathrm{p}$. represents the results by P. Dougan et al. The other notations are the same in Fig. 18.
obtained with the $\pi^{\circ}$ detector. Fig. 19 shows the angular distributions at the energies between 450 and 750 MeV . The results obtained by the two different detection systems were overlapped at the angle of $50^{\circ}$ between 600 and 750 MeV . They show a fairly good agreement with each other. Systematic errors caused by the beam properties, the liquid hydrogen target and the beam monitor were the same in these results because of the simultaneous measurements.

In the energy region below 490 MeV , present results were overlapped with the combined data of experiments carried out at the 500 MeV electron synchrotron at Bonn University ${ }^{31,}{ }^{32}$ ), ${ }^{34}$ ). They measured the differential cross sections by detecting only the recoil protons using the magnetic spectrometer or the range telescope. The consistency of their results were very well. The agreement of the present results with theirs is seen to be almost good. Our experimental region overlaps widely with the one by Dougan et al. ${ }^{13), 14)}$. They also detected only the recoil protons by the range telescope. Our results were consistent with them at the angle of $50^{\circ}, 80^{\circ}, 90^{\circ}, 100^{\circ}$ and $120^{\circ}$, but there are some discrepancies at the angle of $60^{\circ}$ and $110^{\circ}$. In the energy region above the second resonances, present results were overlapped with the results by Feller et al. ${ }^{33)}$, Barton et al. ${ }^{30)}$ and Booth et al. ${ }^{35)}$ at the angle of $60^{\circ}, 90^{\circ}$ and $120^{\circ}$. Feller et al. detected only the recoil proton with the magnetic spectrometer. In these single arm experiments, the backgrounds due to proton Compton processes can not be rejected in the experiments. Barton et al. detected the recoil protons with the range telescope in coincidence with the one of the photons from the $\pi^{\circ}$-meson. However, the Compton backgrounds were not eliminated in the measurement. Therefore, they made a correction using the Compton differential cross sections which is not well established by the experiments. Booth et al. also detected the recoil proton and the photon and subtructed the Compton processes by the method nearly the same as the present experiment. In these coincidence experiment, the contamination due to the reaction $\gamma \mathrm{p} \rightarrow \pi^{+} \pi^{-} \mathrm{p}$ and $\gamma \mathrm{p} \rightarrow \pi^{+} \mathrm{n}$ is negligibly small as described in III-4. The present results at the angle of $60^{\circ}$ are somewhat smaller than the other ones.

There are few experiments at the forward angles $\left(\theta_{\pi^{\mathrm{c}}}^{\mathrm{cm}} \leq 50^{\circ}\right)$ with the exception of Hemmi et al. ${ }^{29)}$ in this energy range. They are well agree between the first and second resonance region, but there are discrepancies at the energy of 950 MeV .

The solid and dashed curves in Fig. 18 and Fig. 19 represent the predicted value by MOR and NAGOYA, respectively. The solid curves in Fig. 20 are the predicted value by MW. The results by MW and NAGOYA have the same feature concerned with the differential cross section. In comparing these predicted value with the present results, following discussions can be made. In the angular range from $60^{\circ}$ to $110^{\circ}$, the fit by MW is better than the fit by MOR. The predicted value by MOR



Fig. 20(a)~(i). The energy dependence of the differential cross sections for the reaction $\gamma \mathrm{p} \rightarrow \pi^{\circ} \mathrm{p}$. $\diamond ;$ G. Fischer et al., $\diamond$; W. Braunschweig et al. $\diamond$ P. Feller et al., $\Delta$; H. Genzel, $\oplus$; P.S. L. Booth et al. ${ }^{35)}$ and $\odot$; P.S.L. Booth et al. ${ }^{36)}$. The solid curves are from partial wave analysis by MW. The dashed line are the results of modified partial wave analysis of MW at the present work.
are too small around the second resonances near $90^{\circ}$. The present results are higher than both analyses at the forward angles and backward angles ( $15^{\circ}, 35^{\circ}, 120^{\circ}$ and $130^{\circ}$ ) below 600 MeV .

Using the s-channel helicity amplitudes in the notation of Walker ${ }^{3)}$, the differential cross section in single pion photoproduction of nucleons can be given by

$$
\frac{\mathrm{d} \sigma}{\mathrm{~d} \Omega}=\frac{1}{2} \frac{q}{k} \sum_{i=1}^{4}\left|H_{i}\right|^{2}
$$

with

$$
\begin{aligned}
& H_{1}=\frac{1}{\sqrt{2}} e^{i \phi} \sin \theta \cos \frac{1}{2} \theta \sum_{n=1}^{\infty}\left(B_{n+}-B_{(n+1)-}\right)\left(P_{n}^{\prime \prime}-P_{n+1}^{\prime \prime}\right) \\
& H_{2}=\sqrt{2} \cos \frac{1}{2} \theta \sum_{n=0}^{\infty}\left(A_{n+}-A_{(n+1)-}\right)\left(P_{n}^{\prime}-P_{n+1}^{\prime}\right) \\
& \left.\left.H_{3}=\frac{1}{\sqrt{2}} e^{2 i \phi} \sin \theta \sin \frac{1}{2} \theta \sum_{n=1}^{\infty}\left(B_{n+}+B_{(n+1)}\right)\right) P_{n}^{\prime \prime}+P_{n+1}^{\prime \prime}\right) \\
& H_{4}=\sqrt{2} e^{i \phi} \sin \frac{1}{2} \theta \sum_{n=0}^{\infty}\left(A_{n+}+A_{(n+1)-}\right)\left(P_{n}^{\prime}+P_{n+1}^{\prime}\right)
\end{aligned}
$$

where $k$ and $q$ are the momenta of photons and nucleons in the c.m. system, respectively, $P_{l}^{\prime}$ and $P_{l}^{\prime \prime}$ are derivatives of Legendre polynominals. $H_{1}$ and $H_{3}$ are the initial-helicity- $3 / 2$ amplitudes and $H_{2}$ and $H_{4}$ are the initial-helicity- $1 / 2$ amplitudes. $A_{l \pm}$ and $B_{l \pm}$ are the helicity element corresponding to the state with pion orbital angular momentum $l$ and total angular momentum $j=l \pm 1 / 2 . \quad H_{2}$ and $H_{4}$ are the amplitudes which do not vanish at the forward and backward angles, respectively. In order to improve the fit at forward and backward angles by modifying the MW analysis, the helicity elements $A_{l \pm}$ must be changed. The largest amplitude of $A_{l \pm}$ below 600 MeV is the resonant amplitude of $\mathrm{P}_{33}(1236)$. By modifying the resonance parameter of $\mathrm{P}_{33}(1236)$, a better fit can be obtained in our energy region. However, at the energy around the first resonance, the fit gives very large $\chi^{2}$. And then, we modified only the background amplitude to explain these characteristics of the present results. The input value of the $\operatorname{Re}\left(A_{1+}\right)$ of the reaction $\gamma \mathrm{p} \rightarrow \pi^{\circ} \mathrm{p}$ was changed at the

Table 5. (a) The input value of the background partial wave $\operatorname{Re}\left(A_{1}{ }^{+}\right)$of the reaction $\gamma \mathrm{p} \rightarrow \pi^{\circ} \mathrm{p}$. The units are $\mu \mathrm{b}^{1 / 2}$.

|  | $K(\mathrm{MeV})$ |  | 525 | 700 |
| :---: | :--- | :---: | :---: | :---: |
| $\operatorname{Re}\left(A_{1+}\right)$ <br> $\left(\gamma \mathrm{p} \rightarrow \pi^{\circ} \mathrm{p}\right)$ | Original | Metcalf-Walker | -0.144 | -0.114 |
|  | Modified | -0.25 | -0.20 | -0.100 |

(b) $\chi^{2} /($ number of data).

| $\theta_{\mathrm{K} \mathrm{c}^{\circ}}$ | $\chi^{2} /$ (no. of data) |  |
| :---: | :---: | :---: |
|  | Metcalf-Walker | Modified |
| $15^{\circ}$ | 3.59 | 1.86 |
| $35^{\circ}$ | 3.61 | 2.29 |
| $120^{\circ}$ | 3.36 | 2.07 |
| $130^{\circ}$ | 4.40 | 3.20 |
| all data | 2.47 | 2.19 |

energy of 525,700 and 850 MeV as listed in Table 5(a). Other helicity elements were not changed.

In this modification, the major constraint of MW's model, that the background amplitude should vary smooth with energy, is ensured. Other observables such as $\mathrm{P}(\theta), \mathrm{T}(\theta)$ and $\Sigma(\theta)$ are scarcely changed by this modification. The fitted curves are shown in Fig. 20 by the dashed line. $\chi^{2} /($ number of data) are listed in Table 5(b). It is clear that our modification can reproduce our results at the forward and backwark angles and give smaller values for $\chi^{2}$. Our results indicate that the MW's resonance amplitudes can reproduce the experimental results on the reaction $\gamma \mathrm{p} \rightarrow \pi^{\circ} \mathrm{p}$ below 1 GeV and are seemed to be established in this energy region. However, it is needed to understand the background amplitudes by the experimental and theoretical efforts such as double polarization measurements and analysis basing on the dispersion relation.

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