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EFFECT OF THE SPIN OF AN INTERPLANETARY DUST ON ITS MOTION

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ABSTRACT

Effect of the spin of an interplanetary dust on its motion was studied. The spin effect is larger for larger dust and for smaller heliocentric distance. The equation of heat conduction of a rotating sphere under the irradiation of sun light was solved. The numerical result was also given. The spin effect is as follows; (1) the change of the magnitude of an orbital angular momentum L, (2) the change of inclination of the orbital plane and (3) the precession of L. The approximate solution that the orbit radius of dust approaches a constant value was obtained. If any mechanism maintaining the spin exists, the fallin of dust into the sun is prevented.

Key words: Interplanetary dust; Spin effect.

1. Introduction

It has been considered that the main effect perturbing the motion of an interplanetary dust comes from the drag force due to the absorption and the re-emission of solar radiation (Poynting-Robertson effect). This effect works to decrease an orbital angular momentum of the dust and consequently the dust particle spirals around and finally into the sun. The lifetime of the dust is rather short. For example, a dust of 1 μ radius falls into the sun from the heliocentric distance of 1 AU for 700 ρ years (ρ is the density of dust in g/cm^{-3}). Therefore the dust materials must be always supplied into the interplanetary space from any source. The comets, the moon, the asteroid and the interstellar medium would be the candidates of the sources.

There are some effects other than the Poynting-Robertson effect. Among these, the drag force due to the solar wind and the friction due to the neutral gas decelerate the dust, and the above mentioned situation would not change. Thus the mechanisms accelerating dust particles are very interesting problems.

The effect of a spin of the dust and the effect of the Lorentz force by the interplanetary magnetic field acting on a charged dust do not always decelerate it. As these forces have a tangential component along the orbit of dust, they can accelerate the dust under certain condition. Then the dust will survive for a long

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time. The present paper deals with the spin effect, and the Lorentz force effect will be discussed elsewhere.

The precise consideration shows that the surface temperature of dust is higher on the dayside than on the nightside. The momentum of emitted radiation in the rest system of dust is larger on the dayside, so the reaction pushes the dust outward from the sun.

When the dust has a spin, a part of the surface of dust reaches the maximum temperature not at just noon but at certain time in the afternoon. Then the reactive force has a tangential component besides the outward component, (see figure 1). The tangential component may accelerate the dust when both the orbital rotation and the spin of dust are of the same sense. This effect was pointed out previously by YARKOVSKY and a few subsequent papers dealt with the problem, but the detailed analysis and the numerical results have not been shown yet (ÖPIK 1951; JACCIA 1963).

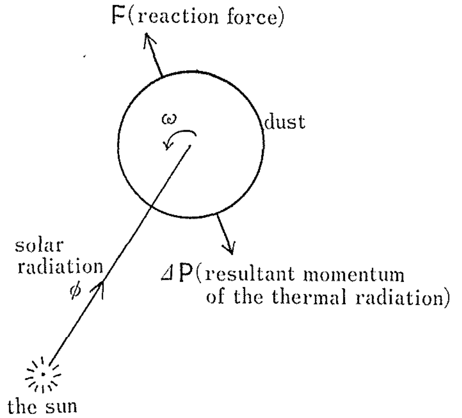

Fig. 1. The effect of a spin of dust.

2. Order of Magnitude of the Spin Effect

The forces acting on an interplanetary dust are classified into the conservative and non-conservative forces. The gravitational force acting on the dust of mass m at a heliocentric distance r is $GM_{\odot}/mr^2$, where G is the gravitational constant and $M_{\odot}$ is the solar mass. The radiation pressure of the sun is ϕ/c, where ϕ is the energy of the sun light absorbed by the dust per unit time and c is the light velocity. The Poynting-Robertson drag force, which is non-conservative, is $(\phi/c)(v/c)$, where v is the dust velocity. The spin effect concerns with the nonuniformity of the temperature ΔT and is given by $(\phi/c)(\Delta T/T)$, where T is the average temperature over the dust. Both the conservative and the non-conservative forces are of the same order of magnitude.

The non-uniformity of the dust temperature ΔT is very small because of the smallness of the dust size. For a thin layer of dust materials of thickness s irradiated normally by the solar radiation at 1 AU, the temperature difference ΔT between the both sides of the layer is shown in table 1. As the dust materials, silicate glass and iron are chosen. The s and r dependence of ΔT is sr^{-2}. Since that of T is $r^{-0.5}$, the s and r dependence of $(\Delta T/T)$ is $sr^{-1.5}$. The factor (v/c) appearing in the Poynting-Robertson drag force is 10^{-4} for a circular orbit of 1 AU. The r dependence of this factor is $r^{-0.5}$. At 1 AU, the factor $(\Delta T/T)$ is 10^{-4} for a silicate glass particle of 100 μ radius, if one takes the temperature T as 300 K. Thus, the spin effect dominates the Poynting-Robertson effect when the radius of a dust is larger than the critical value of 100 μ. The critical value is proportional to the heliocentric distance r.

Table 1. The temperature difference at 1 AU and the physical constants of the dust materials.

Material	Density ρ [g/cm³]	Specific heat C [cal/gK]	Thermal conductivity [cal/cm·s·K]	ΔT[K] at 1 AU $s=1\mu$	100μ	1 cm	$\left(\frac{\rho C}{2\kappa}\right)^{1/2}$ [s$^{1/2}$/cm]	$\left(\frac{\kappa}{C}\right)$ [erg·s/cm³]
Silicate glass	2.1	0.20	3.4×10^{-3}	5×10^{-4}	5×10^{-2}	5	7.86	1.7×10^{-2}
Iron	7.9	0.105	0.18	9×10^{-6}	9×10^{-4}	9×10^{-2}	2.15	1.7

3. Calculations

The stationary state is assumed. A sphere of radius s spinning with a constant angular velocity ω is irradiated by an omnidirectional light, absorbs all the incident light and emits the thermal radiation. At each part of the surface the emitted radiation energy is proportional to the fourth power of the surface temperature. The spin axis is choson as z axis and the light is assumed to come in the xz-plane with an angle ψ with z axis.

In the polar corrdinate system, the temperature T of the sphere is determined by the equation,

$$\frac{\omega\rho C}{\kappa}\frac{\partial T}{\partial\varphi}=\Delta T\,, \tag{1}$$

where C is the specific heat of the material and Δ means the Laplacian. The boundary condition is

$$\kappa\frac{\partial T}{\partial r}+\sigma T^4=\begin{cases}(\phi/\pi s^2)\,(\boldsymbol{n}\boldsymbol{e}_0) & \text{for}\quad(\boldsymbol{n}\boldsymbol{e}_0)>0\\ 0 & \text{for}\quad(\boldsymbol{n}\boldsymbol{e}_0)<0\end{cases} \tag{2}$$

on the surface of sphere. The unit vectors $\boldsymbol{n}$ and $\boldsymbol{e}_0$ direct the outward normal on the surface and towards the light source respectively and σ is the Stefan-Boltzmann constant.

In order to solve equation (1), we expand the temperature T in terms of the normalized spherical harmonics $Y_{lm}(\theta,\varphi)$ as follows,

$$T(r,\theta,\varphi)=T_0\{1+\sum_{l=1}^{\infty}\sum_{m=-l}^{l}Y_{lm}(\theta,\varphi)\cdot R_{lm}(r)\}\,. \tag{3}$$

Inserting equation (3) into equation (1), the equation for $R_{lm}(r)$ is derived

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR_{lm}}{dr}\right)-im\left(\frac{\omega\rho C}{\kappa}\right)R_{lm}-\frac{l(l+1)}{r^2}R_{lm}=0\,. \tag{4}$$

It is assumed that the temperature difference is small compared with the average temperature. The boundary condition (2) is approximated by

$$\kappa \left.\frac{\partial T}{\partial r}\right|_{r=s} + \sigma T_0^4 \{1 + 4 \sum_{l=1}^{\infty} \sum_{m=-1}^{l} Y_{lm}(\theta, \varphi) R_{lm}(r)\} = \begin{cases} (\phi/\pi s^2)(\boldsymbol{n}\boldsymbol{e}_0) & \text{for} \quad (\boldsymbol{n}\boldsymbol{e}_0) > 0 \\ 0 & \text{for} \quad (\boldsymbol{n}\boldsymbol{e}_0) < 0 . \end{cases} \tag{2$'$}$$

The integration all over the solid angle leads to the relation

$$4\pi s^2 \sigma T_0^4 = \phi \tag{5}$$

The regular solutions of equation (4) satisfying the boundary condition (2)′ are

$$\begin{aligned} R_{l0} &= \left(\frac{r}{s}\right)^l \left(\frac{\phi}{\pi s \kappa T_0}\right) \frac{\eta_{l0}}{l + (\phi/\pi s \kappa T_0)} , \\ R_{lm} &= \frac{j_l(\beta_m r)}{j_l(\beta_m s)} \cdot \left(\frac{\phi}{\pi s \kappa T_0}\right) \frac{\eta_{lm}}{[\beta_m s \cdot j_l'(\beta_m s)/j_l(\beta_m s)] + (\phi/\pi s \kappa T_0)} , \end{aligned} \tag{6}$$

where j_l is the spherical Bessel function of the order of l and

$$\beta_m^2 = -im \left(\frac{\omega \rho C}{\kappa}\right) , \tag{7}$$

and

$$\eta_{lm} = \int_{(\boldsymbol{n}\boldsymbol{e}_0) > 0} Y_{lm}^*(\theta, \phi)(\boldsymbol{n}\boldsymbol{e}_0) d\Omega . \tag{8}$$

The factor $(\phi/\pi s \kappa T_0)$ is a quantity of the same order of magnitude as $(\Delta T/T)$, so it is neglected in the denominator in each equation.

The emitted momentum from the surface of the sphere per unit time, $\Delta \boldsymbol{P}$, is

$$\Delta \boldsymbol{P} = \int \boldsymbol{n} \cdot \frac{2}{3} \frac{\sigma T^4}{c} ds \doteqdot \frac{4}{3} \frac{\phi}{C} \cdot \frac{1}{\sqrt{3\pi}} (Re(\sqrt{2} R_{1-1}(s)), \mathrm{Im}(\sqrt{2} R_{1-1}(s)), R_{10}(s)) . \tag{9}$$

Elementary calculation shows that

$$\eta_{1-1} = \sqrt{\frac{\pi}{3}} \frac{\sin \psi}{\sqrt{2}} , \qquad \eta_{10} = \sqrt{\frac{\pi}{3}} \cos \psi . \tag{10}$$

Denoting

$$\beta \equiv \beta_{-1} = (1+i)\left(\frac{\omega \rho C}{2\kappa}\right)^{1/2} , \tag{11}$$

$$\frac{j_1(\beta s)}{\beta s \cdot j_1'(\beta s)} \equiv \xi(\gamma) + i\eta(\gamma) \tag{12}$$

and

$$\gamma \equiv \left(\frac{\omega \rho C}{2\kappa}\right)^{1/2} s , \tag{13}$$

we get

$$\Delta \boldsymbol{P} = \frac{4}{9} \frac{\phi}{C} \left(\frac{\phi}{\pi s \kappa T_0}\right) (\xi(\gamma) \sin \psi, \eta(\gamma) \sin \psi, \cos \psi) . \tag{14}$$

Now we introduce the unit vector e_ω directed into the spin angular momentum and the radial unit vector e_r from the sun. The force $\boldsymbol{F}$ due to the spin is wrttien as

$$\boldsymbol{F} = -\Delta \boldsymbol{P} = \frac{K}{r^{3.5}} \{\xi e_r + \eta[e_r \times e_\omega] + (1-\xi) e_\omega(e_r e_\omega)\} , \tag{15}$$

where

$$K = \frac{4}{9\pi s\kappa c} \cdot \left(\frac{\phi_1^2}{T_1}\right) r_1^{3.5} = \frac{L_\odot}{36\pi c\kappa} \left(\frac{16\pi\sigma}{L_\odot}\right)^{1/4} \cdot s^3 . \tag{16}$$

In the expression of K, $r_1 = 1$ AU, ϕ_1 and T_1 are the values of ϕ and T at $r = r_1$ respectively and $L_\odot$ is the solar luminosity.

The functions $\xi(\gamma)$ and $\eta(\gamma)$ are shown in figure 2. If the delay time when the maximum temperature is reached after the noon is represented by a meridian angle Θ,

$$\tan \Theta = \eta(\gamma)/\xi(\gamma) . \tag{17}$$

For $\gamma \ll 1$, the functions, ξ, η and $\tan\Theta$ are written approximately as

$$\xi = 1 - \frac{8}{700}(2\gamma)^4, \quad \eta = \frac{1}{10}(2\gamma)^2, \quad \tan\Theta = \frac{1}{10}(2\gamma)^2 , \tag{18}$$

and for $\gamma \gg 1$,

$$\xi = \frac{1}{2\gamma}\left(1 + \frac{1}{\gamma^3}\right), \quad \eta = \frac{1}{2\gamma}\left(1 + \frac{1}{\gamma}\right), \quad \tan\Theta = 1 + \frac{1}{\gamma} . \tag{19}$$

The approximate formulae (19) are rather good for $\gamma > 2$. When γ increases from zero, $1-\xi$ increases monotonic and approaches 1. The maximum of η is 0.44 at abou $\gamma \sim 1.5$. The angle Θ is zero for $\gamma = 0$, then increases and reaches the maximum value 52.5° at $\gamma = 2.8$, and then approaches 45° with increasing γ.

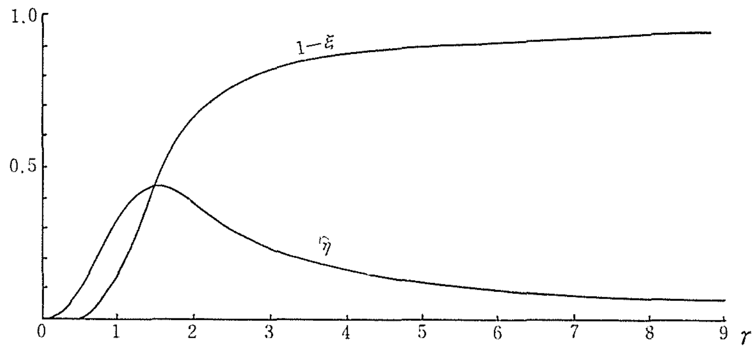

Fig. 2. Functions $1-\xi(\gamma)$ and $\eta(\gamma)$.

4. General Feature of the Spin Effect

The force due to the spin is represented by the right hand side of equation (15). The first term $(K/r^{3.5})\xi(\gamma)e_r$ is the radial, repulsive and conservative force. This term is out of consideration in the following. The second and the third terms change the energy E and the orbital angular momentum $\boldsymbol{L}$ of the dust.

Now, the power $\boldsymbol{F}\cdot\boldsymbol{v}$ and the torque $\boldsymbol{N}=\boldsymbol{r}\times\boldsymbol{F}$ are averaged over a period along the Kepler orbit. The result is found to be a complicated function including the eliptic integral of the eccentricity e. For the sake of simplicity, we choose a circular orbit with radius a. This choice is allowable, because $de/dt=0$ at $e=0$. The result is as follows,

$$\frac{dE}{dt}=\frac{Kv\eta}{a^{3.5}}(\boldsymbol{e}_\omega\boldsymbol{e}_L)\,, \tag{20}$$

$$\frac{d\boldsymbol{L}}{dt}=\frac{K\eta}{a^{2.5}}\frac{1}{2}(\boldsymbol{e}_\omega+\boldsymbol{e}_L(\boldsymbol{e}_\omega\boldsymbol{e}_L))+\frac{K(1-\xi)}{a^{2.5}}\frac{1}{2}[\boldsymbol{e}_\omega\times\boldsymbol{e}_L](\boldsymbol{e}_\omega\boldsymbol{e}_L)\,. \tag{21}$$

Decomposing the equation (21) into three equations about the magnitude L and the polar angle (i, Ω) of $\boldsymbol{L}$ and adding the contribution of Poynting-Robertson drag force, we get

$$\frac{dL}{dt}=\frac{K\cdot\eta\,\cos i}{a^{2.5}}-\frac{\alpha L}{a^2}\,, \tag{22}$$

$$\frac{d\cos i}{dt}=\frac{K\eta\sin^2 i}{2La^{2.5}}\,, \tag{23}$$

$$\frac{d\Omega}{dt}=\frac{K(1-\xi)\cos i}{2La^{2.5}}\,, \tag{24}$$

where α is a parameter of the Poynting-Robertson drag force given by

$$\alpha=\frac{\phi r^2}{mc^2}=\frac{3L_\odot}{16\pi c^2 s\rho}\,. \tag{25}$$

From the equations (22), (23) and (24), we see the spin effect has the following feature:

(i) Change of the magnitude of the orbital angular momentum

The spin effect increases the magnitude of the orbital angular momentum L when the spin angular momentum and the orbital angular momentum are in the same sense ($\cos i>0$), and decreases L when the two are in the opposite sense ($\cos i<0$).

The magnitude of this effect is proportional to $\eta(\gamma)$, so this effect is zero at zero spin and at the limit of very large spin, and becomes the maximum at certain value of ω.

(ii) Change of the direction of the orbital angular momentum

The spin effect changes the direction of $\boldsymbol{L}$. This is an important feature of the spin effect, which is quite different from the feature of the Poynting-Robertson drag force, friction of the neutral gas and the plasma friction. Since the right hand side of equation (23) is non negative, the direction of $\boldsymbol{L}$ approaches always that of the spin.

In order to describe the grobal character of motion of a dust, we introduce the characteristic time. The time t_{inc}, during which the direction of $\boldsymbol{L}$ changes appreciably, is defined by $t_{\text{inc}}\sim L\cdot a^{2.5}/K\eta(\gamma)$ except for the neighbourhood of $\sin i\sim 0$. In case of the predominant spin effect, this time is shorter than the characteristic time for the Poynting-Robertson effect $t_{PR}=mc^2/\phi$. Even if initially $\boldsymbol{L}$ is the opposite to $\boldsymbol{e}_\omega$ and so the spin effect works for the decceleration, the direction of $\boldsymbol{L}$ will tend to the direction of $\boldsymbol{e}_\omega$ and then the spin effect works for the acceleration.

(iii) The precession of the orbital angular momemtum

The spin effect induces the precession of the orbital angular momemtum $\boldsymbol{L}$. From equations (23) and (24), we get

$$-\frac{\cos i}{\sin i}\frac{di}{d\Omega}=\frac{\eta}{1-\xi}. \tag{26}$$

Integration gives

$$\frac{\sin i}{\sin i_0}=\exp\left(-\frac{\eta}{1-\xi}(\Omega-\Omega_0)\right). \tag{27}$$

During Ω changes 2π, $\sin i$ changes by a factor $\exp[-2\pi\eta/(1-\xi)]$. If the spin of the dust is so rapid as $\gamma\gg\pi$, $2\pi\eta/(1-\xi)\sim(\pi/\gamma)\ll 1$ from equation (24), and then the factor is nearly equal to unity. So the angle i is unchanged essentially. The period of the precession is approximately,

$$T_{\text{prec}}\simeq\frac{4\pi L\cdot a^{2.5}}{K(1-\xi)\cos i}. \tag{28}$$

On the contrary, if the condition $\gamma\gg\pi$ is not satisfied, the expression of equation (28) is almost meaningless.

5. Validity of the Present Discussion and the Change of the Spin

Until now the cause of changing a spin of dust has not been mentioned. Practically the spin of dust is not constant but variable. The validity of the averaging process over the Kepler orbit is justified as long as the characteristic time of the spin change t_{spin} is sufficiently longer than the Kepler Period. If this condition is satisfied, it is meaningful to trace the orbit of the dust by solving the equations (22), (23) and (24) in the time range shorter than than the characteristic time t_{spin}.

The mechanism of changing a spin of dust has not been yet established. The study of it is not the main purpose of this paper. Here we shall consider the variation of the spin, assuming the spin is given to the dust at certain time through any process.

(i) The thermal radiation emitted from the surface of rotating dust carries out the spin angular momentum. Its order is (ϕ/c) per unit time. Since the surface rotates with the velocity of the order of $s\omega$, the fraction $(s\omega/c)$ of the momentum gives the reaction along the surface, thus causing a torque $(\phi/c)(s\omega/c)s$. The characteristic time of the spin damping $t_{\text{spin}}^{(1)}$ is defined by $(\phi/c)\times(s\omega/c)st_{\text{spin}}^{(1)}=I\omega$, where I is the moment of inertia of the dust and is of the order of ms^2. Thus

$$t_{\text{spin}}^{(1)}=\frac{mc^2}{\phi}. \tag{29}$$

This time is of the same order of magnitude as the Poynting-Robertson characteristic time t_{PR}, during which the magnitude of the orbital angular momentum

L changes appreciably.

(ii) Since the photons have an angular momentum $\hbar$, a dust particle absorbing the photons suffers the Brownian motion (HARWIT 1970). When the dust absorbs N photons, the root mean square angular momentum of the dust is $\sqrt{N}\hbar$. The characteristic time $t_{\text{spin}}^{(2)}$ is defined by $\sqrt{N}\hbar t_{\text{spin}}^{(2)}=I\omega$. Then

$$t_{\text{spin}}^{(2)}=\frac{1}{N_0\pi s^2}\left(\frac{I\omega}{\hbar}\right)^2, \tag{30}$$

where N_0 is the photon flux. Using the relation $I\omega=(16\pi/15)\ (\kappa/C)\gamma^2 s^3$, and inserting the photon flux N_0 of 1.7×10^{18} photons $\text{cm}^{-2}\text{sec}^{-1}$ at 1 AU we, get

$$t_{\text{spin}}^{(2)}=\begin{cases}1.75\times10^{25}\gamma^4 s^4 r^2 \text{ years} & \text{for silicate glass,}\\ 1.75\times10^{29}\gamma^4 s^4 r^2 \text{ years} & \text{for iron,}\end{cases} \tag{31}$$

where s and r are in cm and AU unit respectivery.

(iii) The protons in the solar wind hit a dust and give angular momentum. The momentum of a proton with the velocity of 300 km sec^{-1} is 5.0×10^{-17}g cm sec^{-1}. The average angular momentum transported to the dust is $(3/4)s$ times the momentum. The characteristic time $t_{\text{spin}}^{(3)}$ is obtained in the same manner as in (ii),

$$t_{\text{spin}}^{(3)}=\begin{cases}2.3\times10^{13}\gamma^4 s^2 r^2 \text{ years} & \text{for silicate glass,}\\ 2.3\times10^{17}\gamma^4 s^2 r^2 \text{ years} & \text{for iron,}\end{cases} \tag{32}$$

where the proton flux of 10^9 protons $\text{cm}^{-2}\ \text{sec}^{-1}$ at 1 AU is used.

The characteristic times $t_{\text{spin}}^{(2)}$ and $t_{\text{spin}}^{(3)}$ are sufficiently long. For a silicate glass grain of radius 1 μ at 1 AU and for $\gamma\sim1$, $t_{\text{spin}}^{(2)}\sim10^9$ years and $t_{\text{spin}}^{(3)}\sim2\times10^5$ years. Thus the contributions of the random processes (ii) and (iii) are negligible for the motion of dust considered here.

Now the motion of dust can be described by equations (22), (23) and (24) with the variable spin angular velocity ω. Unless the effective action increasing the spin is operative, the change of ω is determined by equation,

$$\frac{1}{\omega}\frac{d\omega}{dt}=-\frac{5}{3}\frac{\phi}{mc^2}. \tag{33}$$

In order to solve the equations, numericacl calculation is needed, because the functions $\eta(\gamma)$ and $1-\xi(\gamma)$ are complicated functions of ω. However $\eta(\gamma)$ is insensitive for the change of ω for $\gamma\gtrsim1$. In this region of γ values, the solution with a constant η value has a meaning as an approximation. Substituting $L=m\sqrt{\mu'}\,a^{1/2}$ (a is the radius of the orbit and $\mu'=GM_{\odot}-\alpha c$) and eliminating t, we get a differential equation

$$\sin^2 i\cdot\frac{da}{d\cos i}-4a\cos i=-\frac{4\alpha m\sqrt{\mu'}}{k\eta}a^2. \tag{34}$$

Integration of equation with an initial condition $a=a_0$ at $i=i_0$ gives

$$\frac{1}{a}=\frac{1}{a_0}\frac{\sin^4 i}{\sin^4 i_0}+\frac{4\alpha m\sqrt{\mu'}}{K\eta}\sin^4 i(f(i)-f(i_0)), \tag{35}$$

where

$$f(i)=\frac{1}{8}\cdot\frac{\cos i}{\sin^4 i}(3\sin^2 i+2)-\frac{3}{8}\log\left|\tan\frac{i}{2}\right|. \tag{36}$$

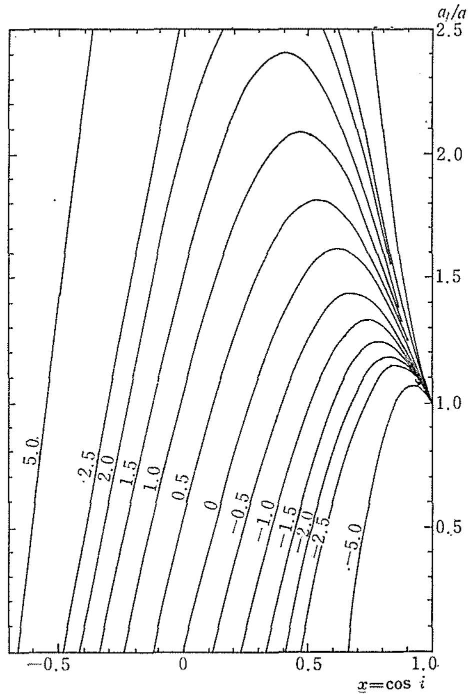

Fig. 3. Relation between i and a. A dust particle goes along the curve from left to right. Suffix p of each curve means; $p=a_1/(a_0 \sin^4 i_0)-f(i_0)$.

The figure 3 shows the relation between i and a for various initial values i_0 and a_0. After a long time the angle i approaches 0 and the radius a approaches

$$a_l=K\eta/\alpha m\sqrt{\mu'} \tag{37}$$

independent of the initial condition. This is just the distance at which the Poynting-Robertson effect and the spin effect balance each other. The minimum radius occurs at

$$a=K\eta\cos i/\alpha m\sqrt{\mu'}\ . \tag{38}$$

When $a_0>K\cdot\cos i_0\alpha m\sqrt{\mu'}$, the distance a reaches a minimum and then increases and finally approaches a_l. Thus if there exists any mechanism which maintains the spin angular momentum, the fallin of dust into the sun is prevented. But, in the absence of such a mechanism, the spin decreases and accordingly a_0 reduces appreciably as the time goes on, and finally the dust falls into the sun.

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