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THE SHOCK PROPAGATION THROUGH AN ATMOSPHERIC MODEL OF AN RR LYRAE TYPE STAR

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ABSTRACT

The propagation of shock waves through a model atmosphere of an RR Lyrae type star is examined on the basis of BRINKLEY-KIRKWOOD method. The effects of gravity, radiation pressure and variation of specific heat ratio with depth are taken into consideration. It is supposed that a shock wave is generated below the convective region with the shock strength and shock energy taken as parameters. The numerical results show that the shock strength remains almost constant in the convective region while it abruptly increases from the top of this region toward the outer surface. The results are generally in good agreement with those of Sachdev and others. A brief discussion on the validity of our calculations of shock propaga- is also presented.

1. Introduction

The problems of shock wave propagation have been investigated by numerous astronomers in connection with the problems of novae and supernovae, Cepheids, and the solar chromosphere and corona. Among several methods of treating the shock propagation, the BRINKLEY-KIRKWOOD method (BRINKLEY-KIRKWOOD 1947) has proved its utility in wide applications. A generalization of the method to the case of inhomogeneous medium has been made, for example, by ONO, SAKASHITA and OHYAMA (1961) and KOGURE and OSAKI (1962). Applications of the method have been made, for example, by SCHATZMAN (1949) and SAITO (1964) to the solar chromosphere, by ODGERS and KUSHWAHA (1960) and BHATNAGAR and KUSHWAHA (1961) to pulsating variable stars, and by NADEZHIN and FRANK-KAMENETSKII (1965) and SACHDEV (1968) to stellar envelopes.

RR Lyrae type stars are population II variables with periods between 0.3 and 1 days and show various interesting phenomena, such as, the discontinuity of radial velocities and the appearence of emission and double absorption lines, at the phases of the ascending branch of the light curve. Therefore, in connection with these phenomena it is generally accepted that shock waves are formed in the photosphere and propagate outward.

IROSHINIKOV (1962) considered a shock wave model in an RR Lyrae atmosphere and examined the relation between the variation of $H\alpha$ line profile and a shock wave propagation. PRESTON, SMAK and PACZYNSKI (1965) made a spectral analysis of RR Lyrae by use of extensive observations, particularly at the phases of the ascending branch of the light curve and showed that the results can be interpreted qualitatively as a shock wave phenomenon. CHRISTY (1964, 1966) and HILL (1972) made nonlinear hydrodynamic model atmospheres for RR Lyrae type stars. HILL's results give excellent agreements between radial velocity curves from the model calculation and observed radial velocities of X Ari.

In this paper, following the current view that a shock wave is generated in the outer region of an RR Lyrae type star, we study the shock propagation through a static atmospheric model of such a star on the basis of the B-K method. In section 2 the expressions for the propagation of shock waves are presented. A static atmospheric model of RR Lyrae is constructed in section 3, and shock propagation through the atmosphere is then examined in section 4 with numerical results and discussions.

2. Equations of Shock Propagation

We consider one dimensional spherical shock propagation through a stellar atmosphere in hydrostatic equilibrium. In deriving the equations of shock propagation based on the B-K method, we take into account the radiation pressure, gravity and the variations with depth of specific heat and of mean molecular weight. Viscosity, conductivity and magnetic field are not considered.

The hydrodynamic equations specialized for the shock front are (e.g., BHRINKLEY and KIRKWOOD)

$$\frac{\rho}{\rho_0}\frac{\partial u}{\partial r}+\frac{1}{\rho c^2}\frac{\partial P}{\partial t}+\frac{2u}{r}=0, \tag{1}$$

and

$$\frac{\partial u}{\partial t}+\frac{1}{\rho_0}\frac{\partial P}{\partial r}=-g, \tag{2}$$

where ρ is the density, P the total pressure, u the particle velocity, r the Lagrangian coordinate of the shock front at time t, c the velocity of sound and g the gravitational acceleration. The subscript "o" denotes quantities in the undistrubed medium.

The Rankine-Hugoniot relations at the shock front are

$$\rho_0 u U_s=p(=P-P_0),$$

and

$$\rho(U_s-u)=\rho_0 U_s, \tag{3}$$

$$\Delta H=\frac{1}{2}p\left(\frac{1}{\rho_0}+\frac{1}{\rho}\right),$$

where U_s is the shock front velocity, ΔH the increment of specific enthalpy across the shock front and p the excess pressure behind the shock front.

Applying the derivative $\frac{\mathrm{d}}{\mathrm{dr}}=\frac{\partial}{\partial r}+\frac{1}{U_s}\frac{\partial}{\partial t}$ to the first of equations (3), we get

$$\frac{\partial u}{\partial t}+U_s\frac{\partial u}{\partial r}-\frac{1}{\rho_0}\left(\frac{\partial p}{\partial r}+\frac{1}{U_s}\frac{\partial p}{\partial t}\right)=-\frac{uU_s}{\rho_0}\frac{d\rho_0}{dr}-u\frac{dU_s}{dr}, \tag{4}$$

where U_s is considered as $U_s=U_s(p, \rho, \rho_0)$ through the equations (3).

The shock energy $D(r)$ when the front is at a point r at time t is defined by

$$D(r)=\int_t^\infty r^2p'u'dt'=r^2put_c\nu, \tag{5}$$

where u' and p' denote particle velocity and excess pressure behind the shock front, $\nu\sim 1$ and t_c is defined by

$$\frac{1}{t_c}=-\left[\frac{\partial \ln (r^2p'u')}{\partial t'}\right]_{t'=t}=-\frac{1}{p}\frac{\partial p}{\partial t}-\frac{1}{u}\frac{\partial u}{\partial t}-\frac{2u}{r} \tag{6}$$

and expresses the steepness of the shock wave behind the shock front. Elimination of t_c from equations (5) and (6) gives

$$\frac{1}{p}\frac{\partial p}{\partial t}+\frac{1}{u}\frac{\partial u}{\partial t}+\frac{2u}{r}=-\frac{r^2pu\nu}{D(r)}\,. \tag{7}$$

Solving the expressions for $\partial p/\partial t$ and $\partial p/\partial r$ from equations (1)$\sim$(4) and (7), and substituting them into

$$\frac{dp}{dr}=\frac{\partial p}{\partial r}+\frac{1}{U_s}\frac{\partial p}{\partial t}\,, \tag{8}$$

we get an ordinary differential equation for the excess pressure p as a function of r.

It is convenient to write down the differential equation in a nondimensional form. To do so, we define the shock strength Z, y and the non-dimensional shock energy δ as follows:

$$Z=\frac{P}{P_0}\,,\ y=\frac{\rho_0}{\rho}\,,\ \delta=\frac{D}{R^3P_{00}}\,, \tag{9}$$

where R is the stellar radius, P_{00} the value of P_0 at the initial shock position. Then, combinations of equations (3) under the assumption that the specific heat ratio γ remains unchanged across the shock front lead

$$U_s{}^2=\frac{P_0}{\rho_0}\frac{Z-1}{1-y}\,,\qquad u^2=\frac{P_0}{\rho_0}\,(Z-1)(1-y)\,,$$

and

$$yZ[7\lambda^2+\beta(1-7\lambda^2)]=\lambda^2(Z-y)+7\lambda^2+\beta_0(1-7\lambda^2)\,, \tag{10}$$

where β is the ratio of the gas pressure to the total pressure and $\lambda^2=\dfrac{\gamma-1}{\gamma+1}$. Taking into account the above relations, we finally have the following non-dimensional differential equation for Z.

$$\begin{aligned}\mathrm{F1}\frac{d\ln Z}{dx}=&-\frac{P_0}{P_{00}}\frac{x^2(Z-1)(1-y)(1-a)\nu}{\delta}+F2\frac{d\ln P_0}{dx}+F3\frac{d\ln\rho_0}{dx}\\&+F4\frac{d\ln\mu}{dx}+F5\frac{d\lambda^2}{dx}-\frac{2[2y+(1-y)(1-a)]}{x}\,,\end{aligned} \tag{11}$$

where $x=r/R$. The first and last terms on the right hand side of equation (11) represent pulse and spherical dampings, respectively, while the second through the fifth terms express, in order, the effects of the variations of pressure, density, mean molecular weight and specific heat ratio upon the propagation. The non-dimensional coefficients *F1*, *F2*, *F3*, *F4* and *F5* expressed in terms of Z and y are:

$$F1=\frac{2Z}{Z-1}+\frac{y}{\Gamma(1-y)}+\frac{1}{1-y}\frac{\phi_1}{\phi_2}Z\,,$$

$$F2=-\left(2+\frac{y(Z-1)}{\Gamma Z(1-y)}+\frac{3k}{1-y}\right),$$

$$F3=1+\frac{4k}{1-y}, \qquad F4=-\frac{4k}{1-y}, \tag{12}$$

and $$F5=\{7[(1-\beta_0)-(1-\beta)yZ]+Z-y\}/(1-y)\phi_2,$$

where Γ is the adiabatic coefficient given by

$$\Gamma=\beta+(4-3\beta)^2(\gamma-1)/[\beta+12(1-\beta)(\gamma-1)],$$

and $$k=(1-7\lambda^2)\{f(\beta_0)-yZf(\beta)\}/\phi_2, \qquad a=\frac{y(Z-1)}{\Gamma Z(1-y)},$$

$$\phi_1=\beta y-\lambda^2+7\lambda^2 y(1-\beta)-3(1-7\lambda^2)yf(\beta), \tag{13}$$

and $$\phi_2=\beta Z+\lambda^2+7\lambda^2 Z(1-\beta)-4(1-7\lambda^2)Zf(\beta),$$

$$f(\beta)=\beta(1-\beta)/(4-3\beta).$$

Next the variation of the shock energy $D(r)$ with depth is given by

$$\frac{dD(r)}{dr}=-r^2\rho_0\{(h_*+\Phi_*)-(h_0+\Phi_0)\}, \tag{14}$$

where h is the specific enthalpy, Φ the gravitational potential energy, and the subscript "$*$" denotes the final state to which the gas particle will return after passage of the shock front.

By adopting the "*Schatzman's path*" which states that after the passage of the shock front each gas particle returns adiabatically to the same pressure as it had before the passage of the shock wave, we have

$$h_*-h_0=\Delta H-\left\{\frac{\gamma}{\gamma-1}\left(\frac{P_g}{\rho}-\frac{P_{g*}}{\rho_*}\right)+4\left(\frac{P_r}{\rho}-\frac{P_{r*}}{\rho_*}\right)\right\}, \tag{15}$$

where P_g is the gas pressure, and P_r the radiation pressure. The potential energy increment is calculated in the same manner as in SAITO (1964), and is given by

$$\rho_0(\Phi_*-\Phi_0)=-\frac{2D}{r^2(P/P_0-1)}\frac{\partial \ln P_0}{\partial r}. \tag{16}$$

Accordingly we obtain from equations (14)–(16) a differential equation for $D(r)$ in the non-dimensional form:

$$\frac{d\delta}{dx}=\frac{2\delta}{(Z-1)}\frac{d\ln P_0}{dx}-\frac{P_0}{P_{00}}x^2\left\{\frac{1}{2}(Z-1)(y+1)-g(\beta)yZ(1-Z^{1/\Gamma-1})\right\}, \tag{17}$$

where $g(\beta)=(4-3\beta)+\frac{\beta}{\gamma-1}$.

The two differential equations (11) and (17) must be solved simultaneously to determine the variations of the shock strength and the shock energy with depth.

Let us consider the case of δ infinite, i.e., a quasi-stationary shock, and discuss the relations between the B-K and Chisnell-Whitham (C-W) method (CHISNELL 1955; WHITHAM 1958). The C-W method is based on the ordinary differential equation valid along a positive characteristic curve. This fundamental equation, which is applied to the motion behind the shock front, is given by

$$\frac{dP}{dr}+c\rho\frac{du}{dr}+\frac{c\rho}{u+c}\left(g+\frac{2cu}{r}\right)=0\,, \tag{18}$$

and must be combined with the shock conditions (3).

Taking into account the variations of β, γ and μ we obtain the following final equation for the shock strength Z (SAKASHITA and TANAKA 1962):

$$F1'\frac{d\ln Z}{dx}=F2'\frac{d\ln P_0}{dx}+F3'\frac{d\ln \rho_0}{dx}+F4'\frac{d\ln \mu}{dx}+F5'\frac{d\lambda^2}{dx}-\frac{4}{M+1}\frac{1}{x}\,, \tag{19}$$

where

$$F1'=\frac{Z}{Z-1}+\frac{2}{\Gamma M}+\frac{Z}{1-y}\frac{\phi_1}{\phi_2}\,,$$

$$F2'=-\left(1+\frac{2}{\Gamma M}+\frac{3k}{1-y}-\frac{2}{\Gamma M(M+1)}\frac{1}{yZ}\right),$$

$$F3'=1+\frac{4k}{1-y}\,,\qquad F4'=-\frac{4k}{1-y}\,,$$

$$F5'=\frac{1}{1-y}\{7[(1-\beta_0)-(1-\beta)yZ]+Z-y\}/\phi_2,\quad M^2=\frac{(1-y)(Z-1)}{yZ}\,, \tag{20}$$

and ϕ_1, ϕ_2 and k are the same as in the B-K method.
When compared with the results of the B-K method, the equation (19) coincides with equation (11) when $D\rightarrow\infty$ and $u+c\rightarrow U_s$, that is, when the shock is stationary and weak. On the other hand, in the limit of strong stationary shock, we have $y\rightarrow\lambda^2$, and both $F2/F1$ and $F2'/F1'\rightarrow 1$ if $\beta=\beta_0=1$, but $F1>F1'$ since $F1\rightarrow 2+\frac{1}{\gamma}\frac{\gamma-1}{2}$ and $F1\rightarrow 1+\frac{2}{\gamma}\sqrt{\frac{\gamma-1}{2}}$. In other words, when $d\rho_0/dx<0$, the damping of the shock strength due to the density stratification is larger in the C-W method than in the B-K method.

3. Construction of a Static Atmospheric Model

A static atmospheric model of an RR Lyrae star is constructed following a computer program given by MIZUNO (1971). In this program the convection is treated with the mixing length theory as is usually done (e.g., MIHALAS 1967). The ratio of mixing length to pressure scale height is taken to be 1.5. The chemical abundance adopted is $X=0.5$, $Y=0.499$ and $Z=0.001$ where X, Y and Z are the abundances of hydrogen, helium and heavy elements by mass, respectively. The atmospheric mean opacity used is that published by COX and STEWART (1971). Following the observational data of HARDIE (1955), STEBBINS (1953) and ALBADA and BAKER (1971), we have adopted an RR Lyrae model:

$$\frac{L}{L_\odot}=81,\quad \frac{M}{M_\odot}=1.45,\quad T_e=6500°\mathrm{K},\quad \frac{R}{R_\odot}=7.2,$$

where L, M, Te and R are the limunosity, the mass, the effective temperature and the radius of the star, respectively.

The numerical integration for the construction of atmospheric model is made from the point of optical depth $\tau=10^{-4}$ up to $\tau=10^{5}$. In the upper layers ($\tau<1$) the following temperature-optical depth relation derived for the Sun by KRISHNA SWAMY (1966) has been used,

$$T^4=\frac{3}{4}T_e^4[\tau+q(\tau)], \qquad q(\tau)=1.39-0.815\times e^{-2.54\tau}-0.025\times^{-30\tau}. \quad (21)$$

Since the spectral type of an RR Lyrae star is not so far from that of the Sun, we except that there is no drastic difference between atmospheric structures in both type stars. Thus we can approximately apply the above T–τ relation to RR Lyrae atmospheres.

The numerical results of the distributions of temperature T_0, pressure P_0, density ρ_0 and specific heat ratio γ with optical depth are shown in Figures 1 and 2. In our model the convective region extends from $\tau=0.5$ to $\tau=2.26\times10^3$ and a density inversion occurs near $\tau=1.01$. The density increases from 1.94×10^{-10} to 3.75×10^{-8} g/cm^3 between

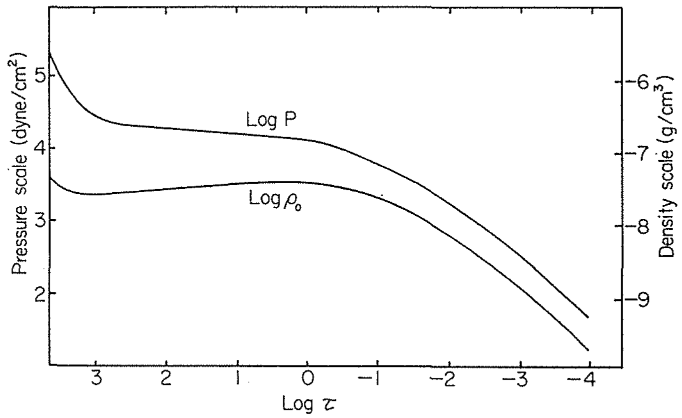

Fig. 1. Initial distributions of density ρ_0 and pressure P_0 for a static RR Lyrae atmospheric model.

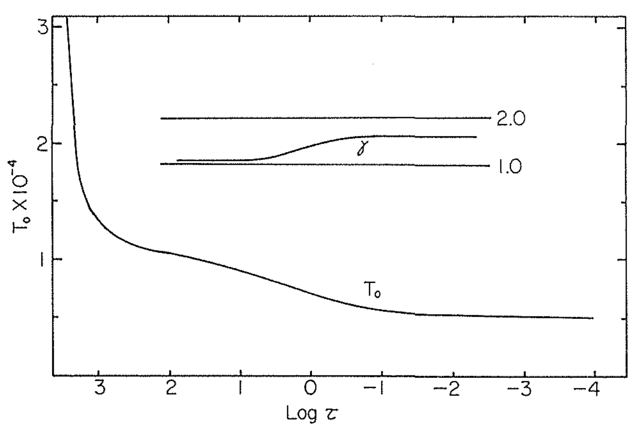

Fig. 2. Initial distributions of temperature T_0 and specific heat ratio γ for a static RR Lyrae atmospheric model.

Table 1. Physical quantities of the atmospheric model at $\tau=1.01\times10^3$, 1.01 and 10^{-4}.

τ	P dyne/cm^2	ρ g/cm^3	T °K	γ	μ	β
1.01×10^3	2.77×10^4	2.43×10^{-8}	1.33×10^4	1.14	0.97	0.997
1.01	1.34×10^3	3.75×10^{-8}	6.52×10^3	1.42	1.59	0.999
10^{-4}	54.1	1.94×10^{-10}	5.21×10^3	1.61	1.60	0.999

$\tau=10^{-4}$ and $\tau=1.01$, while decreases from 3.75×10^{-8} to 2.07×10^{-8} between $\tau=1.01$ and $\tau=2.26\times10^3$ and again increases sharply for $\tau>2.26\times10^3$. The values of temperature, pressure, specific heat ratio and other parameters at $\tau=10^{-4}$, 1.01 and 1.01×10^3 are given in Table 1.

4. Numerical Results and Discussion

(a) Initial Values

The radial dependence of pulsational velocity calculated by CHRISTY (1964, 1966) for RR Lyrae type stars suggests that the shock waves are generated below the convective region in such stars. Based on this suggestion we start our numerical integration for the shock propagation from the optical depth $\tau=1.01\times10^3$, near the bottom of the convective region.

If the particle velocity behind the shock front at $\tau=1.01\times10^3$ were known, the initial shock strength could be given as in the case of ODGERS and KUSHWAHA (1960) for BW Vulpecula. We have, however, no observational data on the velocity in such deep layers. Therefore the numerical integration is carried out in the three cases of initial shock strength $Z_0=3.0$, 5.0 and 7.0.

Next we consider the initial shock energy δ_0, which is related to the characteristic time t_c through equation (6). Hence, in order to have the shock energy, one has to evaluate t_c, which should be, at least, smaller than the pulsation period of the star.

The shock energy may also be estimated by another way. Following ODGERS and KUSHWAHA (1960), we approximately have

$$\delta_0=\frac{1}{R^3P_{00}}\int_t^\infty r^2p'u'dt'=h\int_t^\infty\left(\frac{r}{R}\right)^2dx\sim h\int_t^\infty dx\,, \tag{22}$$

where h is the average ratio of the excess pressure after passage of the shock front to the equilibrium pressure and $\int_t^\infty dx$ is the fraction of the stellar radius traversed by the gas behind the front. For RR Lyrae stars we may take $\int_t^\infty dx$ to be ~0.1, and then

$$\delta_0\sim\frac{1}{10}h\,. \tag{23}$$

In any case, however, the evaluation of t_c or h is difficult in general. We thus tentatively adopt the following three cases $t_c=2\times10^2$, 2×10^3 and 2×10^4 sec with $\nu=1$. The average ratio h of the excess pressure to the equilibrium pressure is in the range $10^{-2}\sim10$ if the above values of t_c are adopted.

With various combinations of the initial values of Z_0 and δ_0 mentioned above, equations (11) and (17) are integrated simultaneously.

(b) Results and Discussion

Table 2 gives the initial values adopted and the values of Z, δ, U_s and u calculated at $\tau=10^{-4}$. Further, in order to make a comparison, the results obtained through the C-W method are given in Table 3.

Strictly speaking we can not compare these results directly with the observational data because of the roughness of the atmospheric model adopted and of approximations involved in the calculation of shock propagation. But let us assume here that the velocity derived from the observed $H\alpha$ emission line of RR Lyrae expresses the particle velocity behind the shock front at the phase when the front reaches the region of $\tau_{H\alpha}\sim 1$ in our model. The symbol $\tau_{H\alpha}$ means the optical depth in $H\alpha$ line center. The velocity of the observed $H\alpha$ emission line is $50\sim 60$ *km/sec* (SANFORD 1949), and $\tau_{H\alpha}\sim 1$ corresponds $\tau\sim 10^{-4}$ in our model. Thus the results given in Table 2 show that the allowable ranges of initial values of Z_0 and t_c are approximately 5 to 7 for Z_0 and 2×10^2 to 2×10^3 for t_c, respectively.

In the following the results of shock propagation calculated by use of the B-K method are shown for a typical case of $Z_0=5$, together with the complementary results obtained by the C-W method. The variation of the shock strength Z against the optical depth τ is given in Figure 3. The ratio T/T_0 of temperature behind the shock front to that in the undisturbed medium is shown in Figure 4. Further the shock velocity U_s and the particle velocity u behind the shock front are given in Figure 5.

Table 2. Initial values and the values of Z, δ, U_s and u calculated at $\tau=10^{-4}$ on the B-K method.

Initial values					Values calculated at $\tau=10^{-4}$			
Z_0	δ_0	t_c sec	U_{s0} km/sec	u_0 km/sec	Z	δ	U_s km/sec	u km/sec
3.0	1.02×10^{-3}	2×10^2	19.4	12.8	23.4	3.25×10^{-4}	28.5	21.7
	1.02×10^{-2}	2×10^3			59.1	6.85×10^{-3}	44.6	36.0
	1.02×10^{-1}	2×10^4			67.2	7.48×10^{-2}	47.5	38.5
5.0	3.16×10^{-3}	2×10^2	25.0	19.8	71.3	1.49×10^{-3}	47.5	40.5
	3.16×10^{-2}	2×10^3			147.0	2.54×10^{-2}	71.0	61.0
	3.16×10^{-1}	2×10^4			182.4	2.70×10^{-1}	79.8	67.9
7.0	5.63×10^{-3}	2×10^2	29.5	23.4	91.9	2.98×10^{-3}	55.3	45.4
	5.63×10^{-2}	2×10^3			196.1	4.79×10^{-2}	80.2	67.1
	5.63×10^{-1}	2×10^4			215.5	5.05×10^{-1}	84.1	70.4

Table 3. Initial values and the values of Z, U_s and u calculated at $\tau=10^{-4}$ on the C-W method.

Initial values			Values calculated at $\tau=10^{-4}$		
Z_0	U_{s0} km/sec	u_0 km/sec	Z	U_s km/sec	u km/sec
3.0	19.4	12.8	45.1	39.1	31.1
5.0	25.0	19.0	94.8	56.1	46.1
7.0	29.5	23.4	168.0	74.3	61.9

(i) Shock Strength and T/T_0

It is noticed in Figure 3 that the propagation of shock waves reveals rather different behaviours in the convective region ($10^3 \gtrsim \tau \gtrsim 1$) and in the outer region ($\tau \lesssim 0.1$). In the convective region the shock strength remains practically unchanged, especially so for the shock waves with small energy. This behaviour is readily attributed to the density inversion in the convective region. In contrast, when the shock reaches the transition zone between the convective and radiative regions, the shock strength Z turns to steep increase toward outer surface. This is again explained as a result of density variation in outer layers as is seen in Figure 1. For example, the shock wave with $\delta_0 = 3.16 \times 10^{-2}$ varies its strength only from $Z = 5.0$ to 10.5 in the range $10^3 \gtrsim \tau \gtrsim 1$, but increases as much as $Z = 1.47 \times 10^2$ at $\tau = 10^{-4}$. Similarly, for the shock wave with $\delta_0 = 3.16 \times 10^{-3}$, Z varies

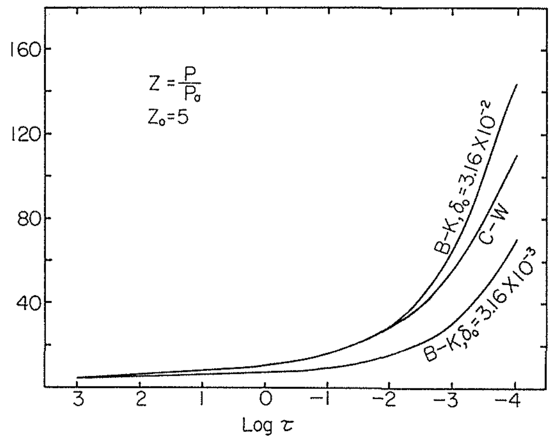

Fig. 3. Variation of shock strength Z with the initial strength $Z_0 = 5.0$ against the optical depth τ. The results by the B-K and C-W method aredenoted by "B-K" and "C-W", respectively. δ_0 is the non-dimensional initial shock energy.

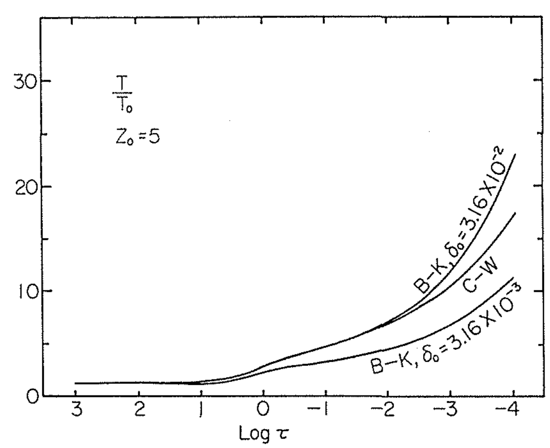

Fig. 4. Ratio of the temperature T behind the shock front to the undisturbed temperature T_0.

from 5.0 to 7.52 for $10^3 \gtrsim \tau \gtrsim 1$, whereas it attains 91.3 at $\tau=10^{-4}$. These results show also that the shock strength at the outer layers strongly depends on the initial shock energy.

The quantity T/T_0 is given by

$$\frac{T}{T_0}=\frac{\Gamma}{\Gamma_0}Zy\,. \tag{24}$$

Since the density jump ratio $1/y$ is found to be $3\sim6$ throughout the shock propagation, the optical depth dependence of T/T_0 has the behavior similar to that of Z. That is, in the case $\delta_0=3.16\times10^{-2}$, T/T_0 varies from 1.31 to 2.79 in the range $10^3 \gtrsim \tau \gtrsim 1$ and attains the value 23.0 at $\tau=10^{-4}$. Besides, the temperature T behind the shock front decreases first until it reaches the minimum value 1.33×10^{4}°K at $\tau=6.8$ and then increases outward reaching the value 1.2×10^{5}°K at $\tau=10^{-4}$. Similarly in the csae $\delta_0=3.16\times10^{-3}$, T/T_0 varies from 1.31 to 2.26 for $10^3 \gtrsim \tau \gtrsim 1$ and then increases to 11.3 at $\tau=10^{-4}$. The temperature T decreases down to the minimum value 1.22×10^{4}°K at $\tau=6.0$ and the increases to 5.9×10^{4}°K at $\tau=10^{-4}$.

Let us compare the above results of the B-K method with those calculated by the C-W method. From Figure 3 we see that the shock strength obtained by the latter method is larger than that obtained by the former one when the shock energy is as small as $\delta_0=3.16\times10^{-3}$. This is because the pulse damping is absent in the C-W method. This also shows that the C-W method is not a good approximation in this case. Figure 3 shows further that when $\delta_0=3.16\times10^{-2}$, the shock strength obtained by the C-W method coincides with the one by the B-K method in the range $10^3 \gtrsim \tau \gtrsim 10^{-2}$, while becomes smaller for $\tau<10^{-2}$. The pulse damping is not effective when the initial shock energy is as much as $\delta_0=3.16\times10^{-2}$. Thus the difference mentioned above is related to the damping of shock strength due to density stratification. In the C-W method, the damping is overestimated compared with in the B-K method when the shock wave becomes rather strong in the outer region $\tau<10^{-2}$, as stated at the end of section 2. These characteristics are also found in results obtained by NADEZHIN and FRANK-KAMENETSKII (1965).

Finally the effects of the variations of γ, β and μ with depth on shock strength are noted. We have found that while the variations of β_0 and μ have no large influence on the shock strength, the variation of γ influences Z. On the other hand, the variation of β due to shock propagation has non-negligible effects on the shock strength where the shock grows in the outer atmosphere. It loweres the shock strengjh and particularly the ratio T/T_0 compared with that in the case where the radiation pressure is neglected. In shock propagation problems in the stellar atmosphere it will be important, in general, to take account of the effects of radiation pressure.

(ii) Shock Velocity U_s and Particle Velocity u

As is shown in Figure 5, the variations of U_s and u with optical depth resemble that of temperature T in the point that these quantities attain minimum values near the upper layer of convective region. The minimum occurs at $\tau\sim3$ for U_s and at $\tau=0.5\sim0.7$ for u. The abrupt increases of U_s and u in the outer region are also remarkable. These behaviors are well explained principally as a result of density inversion in the convective region. The particle velocity u in the numerical examples here considered never attains the escape velocity of 278 *km/sec* at $\tau=10^{-4}$. The duration of shock propagation from $\tau=10^3$ to $\tau=10^{-4}$ is found to be 798 *sec* for $\delta_0=3.16\times10^{-2}$ and 992 *sec* for $\delta_0=3.16\times10^{-3}$.

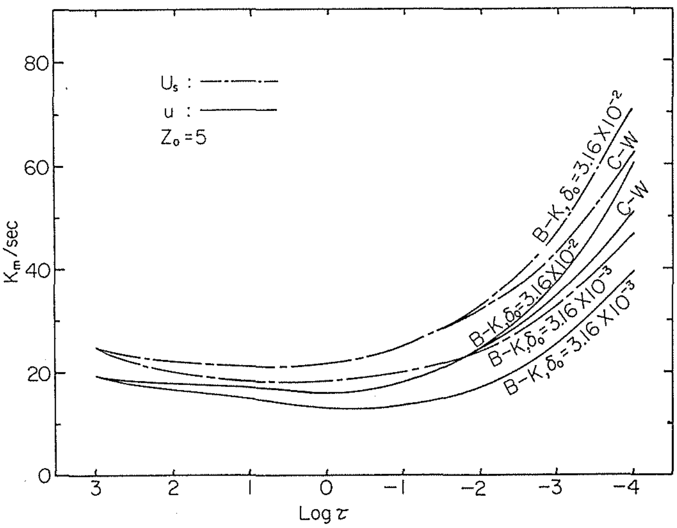

Fig. 5. Variations of the shock velocity U_s and the particle velocity u behind the shock front against the optical depth τ.

(iii) Shock Energy

The decrease of shock energy with height is shown in Table 4 for two cases $\delta_0 = 3.16\times10^{-2}$ and $\delta_0=3.16\times10^{-3}$ with $Z_0=5.0$. The Table shows that the decrease is slow: slower when the initial shock energy is larger and the shock is stronger.
The decrease of shock energy given in this table, however, might be underestimated because in the small τ region, considerable energy would be lost from the shock front by radiation.

Recently, NARITA (1973) investigated the radiative energy losses from the shock front in the case of a plane stationary shock, under the assumption that the medium ahead the shock front is optically thin for Balmer continuum radiation. He showed that the radiative energy losses are considerable when the shock is strong and the gas density is low. According to him, the energy flux F_0 of radiation passing through the shock front is, in unit of kinetic energy flux $\frac{1}{2}\rho_0 U_s^3$ of the preshock gas,

$$F_0\Big/\left(\frac{1}{2}\rho_0 U_s^3\right)=\begin{cases}0.88\sim0.75 \text{ for } M_s\sim14,\\ 0.65\sim0.45 \text{ for } M_s\sim5,\end{cases}$$

when $\rho_0=10^{-10}\sim10^{-8}\, g/cm^3$, where M_s expresses the Mach number of shock velocity. Let us apply this to our present problem and estimate roughly the radiative energy losses through the region with a small $\tau\lesssim10^{-2}$, where the shock wave becomes strong. We take $\rho_0=10^{-9}\, g/cm^3$, and $U_s=50$ and 30 km/sec for $\delta_0=3.16\times10^{-2}$ and 3.16×10^{-3}, respectively.

Table 4. Variation of the non-dimensional shock energy δ due to shock propagation.

τ	10^3	10^2	10^1	1	10^{-1}	10^{-2}	10^{-3}	10^{-4}
$t_c=2\times10^2$	3.16	2.46	2.30	2.19	1.92	1.68	1.54	1.49 ($\times10^{-3}$)
$t_c=2\times10^3$	3.16	2.96	2.90	2.86	2.75	2.64	2.57	2.54 ($\times10^{-2}$)

Further we take 500 *sec* as the time for the shock to propagate through the region. Then, the radiative energy losses are given in non-dimensional forms δ_R by

$$\delta_R=\begin{cases} \dfrac{0.84\times\left(\frac{1}{2}\rho_0 U_s^3\right)\times 500}{R\cdot P_{00}} \sim 1.9\times 10^{-3} & \text{for } U_s=50\,\text{km/sec}\,, \\ \dfrac{0.55\times\left(\frac{1}{2}\rho_0 U_s^3\right)\times 500}{R\cdot P_{00}} \sim 2.6\times 10^{-4} & \text{for } U_s=30\,\text{km/sec}\,. \end{cases}$$

If we take $\rho_0=10^{-8}$, we have $U_s \lesssim 30$ *km/sec* from Figure 5, and then $\delta_R \lesssim 2.6\times 10^{-3}$. Comparison of δ_R given above with δ shows that our results concerning shock propagation would not be affected seriously by the radiative energy losses in the case $\delta_0=3.16\times 10^{-2}$, but would be appreciably influenced in the case $\delta_0=3.16\times 10^{-3}$.

In addition to this, HILL (1972) showed that the total amount of dissipation of pulsational energy due to the shocks was 1.3×10^{38} *ergs* per period in RR Lyrae type stars. In our case the shock energy D for $\delta_0=3.16\times 10^{-3}$ is 1.13×10^{37} *ergs* and seems to be too small. Accordingly we rather prefer $\delta_0=3.16\times 10^{-2}$ to $\delta_0=3.16\times 10^{-3}$ as the value of initial shock energy.

(iv) Comparison with other Results

Our results are compared with SACHDEV's and VIRGOPIA's. SACHDEV (1968) calculated shock propagation through the envelopes of a hot star and a giant star by the B-K method, neglecting the variations of γ and β_0. He took account of the radiative energy losses from the shock front by assuming "*Weymann.s path*" (WEYMANN 1960) instead of "*Sachatzman*'s *path*". His results can not be compared in detail with ours because of the differences of the initial static model. However we observe that the characteristics concerning the variations of shock energy and strength with depth are similar to ours.

VIRGOPIA (1970) examined shock propagation through the atmosphere of a giant star with $M/M_\odot=3$, $L/L_\odot=224$, $T_e=5440°K$ and $R/R_\odot=17$ by the C-W method. He treated very strong shocks. The effects of the radiation pressure and the dissipation of the shock energy were not taken into account. Although the characteristics of the atmospheric model used by him are similar to ours, his results are quite different from ours: the shock strength reaches the maximum near $\tau=1$ and then decay rapidly outward. In his case, the damping of shock strength is mainly due to the spherical geometry. This damping is, however, smaller in general by several orders fo magnitude than the gravitational growth.

Finally our results are summarized in the following way. The shock waves generated below the convective region by the pulsation of an RR Lyrae star propagate through the region with almost constant strength but grow rapidly toward the outer atmosphere from the top of the convective region. If we take $\delta_0 \sim 10^{-2}$ and $Z_0=5\sim 7$ at $\tau=10^3$ as initial shock energy and shock strength, the particle velocities become consistent with the observed radial velocities. Shock strength calculated in this paper may be a little lowered if the radiative energy losses from the shock front are correctly taken into account. If we had more appropriate atmospheric models for the upper layer of small optical depth in RR Lyrae type stars and better treatment for the radiative energy losses, we could investigate the shock propagation in more detail. Further, it is also important to treat a nonlinear hydrodynamic model atmosphere as done by HILL (1972). These problems will be treated in future.

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