Distortion of Single Crystals of Aluminium in the Form of Thin Plate by Plastic Elongation

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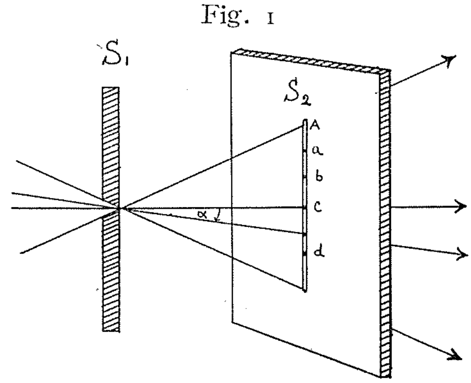
Abstract

Thin and flat aluminium plate consisting of a single crystal is stretched by several percent, and the manner of the crystal slipping and of its destruction into crystallites is examined with X-rays by an improved long slit method. It is found that different parts of the crystal slip along the slip direction in a certain number of slip planes, which is about 6～10 per mm in the stretching direction. With a thin and flat plate, the slip direction [110] is not necessarily the one nearest to the stretching direction, but that which is closely parallel to the surface of the plate.

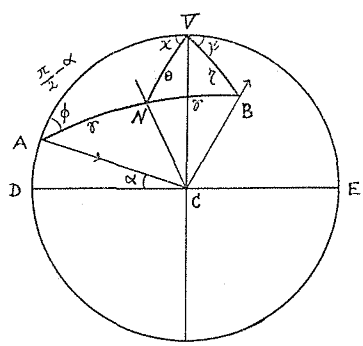
It is known that, when a metal piece composed of a single crystal is stretched plastically under an external force, different parts of the crystal slip along a certain direction in a slip plane. As the slip direction in this case does not coincide necessarily with the stretching direction, a rotation of the crystal axes in reference to the outer form of the specimen arises. Moreover the crystal is broken into crystallites by slipping. In order to examine these points more minutely, the writer devised a method of X-ray diffraction photography by improving the long slit method proposed by U. Yoshida and K. Tanaka. The principle of the method is shown in Fig. 1. In this figure, S_1 is a very narrow circular slit 0.1 mm in diameter; S_2 is a long and narrow slit placed just in front of the specimen, and confines the diverging X-rays within the plane containing its direction and the pin hole S_1. The crevice of the long slit is 0.1 mm and its length is 15 mm, and it is closed at a few places (a, b, c, d) by small pieces of thin lead wire. The presence of these thin wires is of great benefit to know the location in the test piece, which corresponds to a point in the diffracted image impressed on a photographic plate. With such a long slit system, every point in an atomic plane

Fig. 1



of the single crystal plate which is exposed to the X-rays just behind the long slit S_2, receives the X-rays with a different angle of incidence. Now let us cosider that an X-ray beam impinges on the surface of the specimen at an angle of $\left(\frac{\pi}{2}-\alpha\right)$ degree, and we shall find how the direction of the X-ray beam reflected from a certain atomic plane in the specimen changes with the direction of the normal to the atomic plane. We consider a unit sphere (Fig. 2) having a point in the single crystal as its center C, the intersections of the incident and the reflected X-ray beams to and from the atomic plane with the surface of the unit sphere are denoted A, B respectively, and the intersections of the direction of the long slit which is vertical in the present case and of the normal to the atomic plane with the surface of the unit sphere are denoted V, N respectively. Now put

Fig. 2



$$\left\{\begin{array}{l}\text{arc } AN=\text{arc } NB\equiv\gamma\\ \text{arc } VN\equiv\theta\\ \text{arc } VB\equiv\eta\end{array}\right.$$

$$\left\{\begin{array}{l}\angle NAV\equiv\phi\\ \angle NVA\equiv x\\ \angle BVE\equiv\psi\end{array}\right.$$

Then

$$\cos 2\gamma=\cos\eta.\sin\alpha-\sin\eta.\cos\alpha.\cos\psi \qquad (1)$$

$$\sin\psi=\sin\phi\frac{\sin 2\gamma}{\sin\eta} \qquad (2)$$

$$\cos\theta=\cos\gamma.\sin\alpha+\sin\gamma.\cos\alpha.\cos\phi \qquad (3)$$

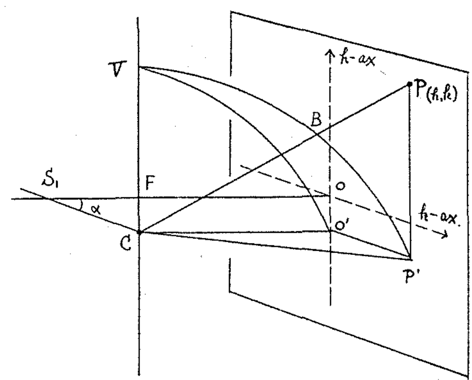
$$\sin x=\sin\gamma\frac{\sin\phi}{\sin\theta} \qquad (4)$$

In Fig. 3, P represents a Laue spot impressed on a photographic plate which is caused by a certain atomic plane in a part of the single crystal, which is situated at a point C just behind the long slit S_2.

Let us consider that the direct X-ray beam which is perpendicular both to the surface of the specimen and to the photographic plate strikes the photographic plate at the point O, and draw a horizontal and a vertical coordinate-axis on the photographic plate by making the point O their origin. Let the coordinates of the Laue spot P be (h, k) in reference to the rectangular coordinate-axes stated above, the distance between the first slit S_1 and specimen F be d, and the distance between the specimen F and the photographic plate O be D; then we have

$$\begin{cases} h=\dfrac{D}{\cos\psi}\cdot\cot\eta-d.\tan a & (5) \\ k=D.\tan\psi & (6) \end{cases}$$

Fig. 3



The values of D and d are determined from the experimental condition, and the values of a, h and k can be measured for a point in a diffracted line on the photographic plate ; and thus from these values the values of θ and x are obtained by the aid of equations (5), (6), (1), (2), (3) and (4). In such a manner the orientation of the normal to an atomic plane which is responsible for the diffraction line under consideration is obtained, and it can be plotted readily on the surface of the crystal globe. When a thin plate of aluminium composed of a single crystal is stretched slightly, the single crystal breaks into crystallites and their several orientations move a little from that of the mother single crystal. Such movement of the orientation of the crystallites can be measured by comparing the values of θ and x for the crystallites with those of the mother single crystal. With the long slit method described above, the writer obtained an interesting X-ray diffraction pattern with an aluminium single crystal plate which was stretched slightly, as is shown by Fig. 1, Plate I. The stretching was 2.0 percent in the direction parallel to that of long slit. As is seen in the photograph each Laue-line presents a striped pattern. The cause of such splitting of a continuous Laue-line into stripes by stretching is to be explained as follows : when an aluminium single crystal plate is stretched slightly, different parts of the single crystal slip on each other along a slip-direction in a certain slip-plane ; thus it breaks into many blocks of crystallites by confining the slipping mainly along the slip plane between two neighbouring blocks ;

and the crystallites in such a slip plane are broken down into finer ones and their orientation becomes much more scattered than in the interior of each block, a fact acts to weaken the intensity in that part of a Laue-line which corresponds to the slip plane between the two neighbouring blocks. The slipping of different parts of the crystal along a certain slip direction which is not generally parallel to the stretching direction, causes a rotation of the crystal axes in reference to the outer form of the specimen.

In Fig. 2, let α, θ and x be known for an atomic plane of a single crystal, and let the angle α be denoted by $\Delta\alpha$ by making it very small. Moreover if we put $\gamma=\gamma_0$, $\phi=\phi_0$ for the values when $\alpha=0$, we get for the first approximation

$$\cos\gamma=\Delta\alpha\cdot\cos\theta+\cos\gamma_0$$
$$\sin\gamma=\sin\gamma_0-\cos\gamma_0\cdot\cos\phi_0\cdot\Delta\alpha$$
$$\sin\phi=\sin\phi_0\left(1+\frac{\cos\theta\cdot\cos\gamma_0}{\sin^2\gamma_0}\cdot\Delta\alpha\right)$$
$$\cos\phi=\cos\phi_0\left(1-\frac{\cos\gamma_0\cdot\sin^2\phi_0}{\cos\theta}\cdot\Delta\alpha\right)$$
$$\sin 2\gamma=\sin 2\gamma_0-\frac{2\cos\theta\cdot\cos 2\gamma_0}{\sin\gamma_0}\cdot\Delta\alpha$$
$$\cos 2\gamma=\cos 2\gamma_0\left(1+\frac{4\cdot\cos\theta\cdot\cos\gamma_0}{\cos 2\gamma_0}\cdot\Delta\alpha\right)$$
$$\cos\eta=\cos\eta_0-(1-2\cos^2\theta)\cdot\Delta\alpha$$
$$\sin\eta=\sin\eta_0+\cot\eta_0(1-2\cos^2\theta)\cdot\Delta\alpha$$
$$\sin\psi=\sin\psi_0\left(1-\frac{\cos\theta\cdot\cos 2\gamma_0}{\cos\gamma_0\cdot\sin^2\eta_0}\cdot\Delta\alpha\right)$$

$$\left\{\begin{aligned}
&\cos\psi=\cos\psi_0\left(1+\frac{\cos\theta\cdot\cos 2\gamma_0\cdot\tan^2\psi_0}{\cos\gamma_0\cdot\sin^2\eta_0}\cdot\Delta\alpha\right)\equiv\cos\psi_0(1+A\cdot\Delta\alpha) && (7)\\
&\tan\psi=\tan\psi_0\left(1-\frac{\cos\theta}{\cos\gamma_0\cdot\cos 2\gamma_0}\cdot\Delta\alpha\right)\equiv\tan\psi_0(1+B\cdot\Delta\alpha) && (8)\\
&\cot\eta=\cot\eta_0\left(1-\frac{1-2\cos^2\theta}{\cos\eta_0\cdot\sin^2\eta_0}\cdot\Delta\alpha\right)\equiv\cot\eta_0(1+C\cdot\Delta\alpha), && (9)
\end{aligned}\right.$$

where A, B and C represent respectively

$$A=\frac{\cos\theta\cdot\cos 2\gamma_0\cdot\tan^2\psi_0}{\cos\gamma_0\cdot\sin^2\eta_0}$$
$$B=-\frac{\cos\theta}{\cos\gamma_0\cdot\cos 2\gamma_0}$$
$$C=-\frac{1-2\cos^2\theta}{\cos\eta_0\cdot\sin^2\eta_0}$$

Fig. 4

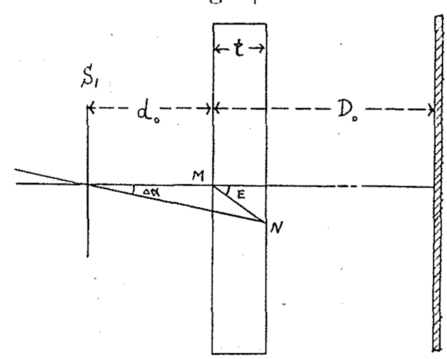
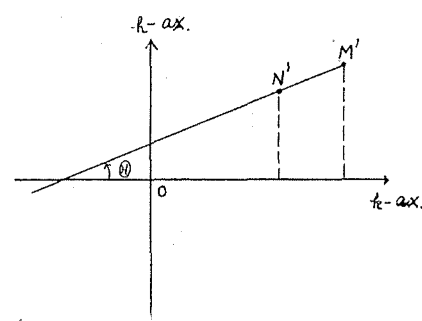


Fig. 5



In Fig. 4, let the distance between the first slit S_1 and the front surface of the specimen be denoted by d_0, the distance between the front surface of the specimen and the photographic plate by D_0, and the thickness of the specimen be denoted by t.

In Fig. 4, the straight line MN represents the intercepting line of a slip plane with the plane containing the direction of the long slit S_2 and the incident X-rays, and the inclination of the line MN against the direction normal to the surface of the specimen and the photographic plate is represented by the angle E. Let the positions of the images of the X-rays reflected at the points M and N, and impressed on the photographic plate be respectively $M'(h_0, k_0)$, $N'(h_1, k_1)$ in Fig. 5, then we have

$$\begin{cases} h_0 = \dfrac{D_0}{\cos\psi_0}\cdot\cot\eta_0 \\ k_0 = D_0\cdot\tan\psi_0 \end{cases}$$

$$\begin{cases} h_1 = \dfrac{D_0 - t}{\cos\psi}\cdot\cot\eta - \Delta a \\ \quad = h_0\left(1 - \dfrac{t}{D_0}\right) + \left\{h_0(C-A) - d_0\right\}\cdot\Delta a \\ k_1 = k_0\left(1 - \dfrac{t}{D_0}\right) + k_0\cdot B\cdot\Delta a \\ \Delta a \doteqdot \dfrac{t}{d_0}\cdot\tan E \end{cases}$$

$$\therefore \quad \tan\Theta \equiv \frac{h_0 - h_1}{k_0 - k_1}$$

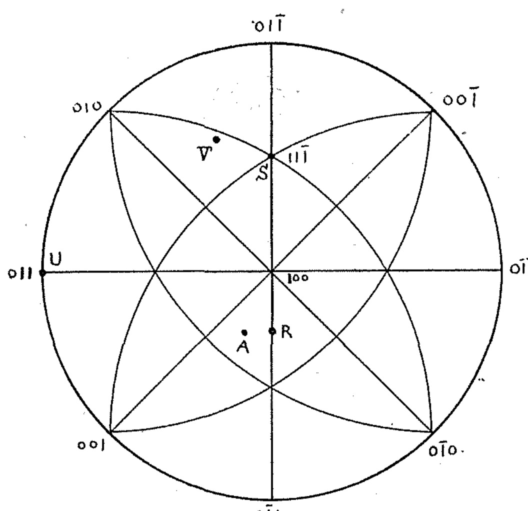
$$
= \frac{h_0 - \left\{ h_0 (C - A) - d_0 \right\} \dfrac{D_0}{d_0} \cdot \tan E}{k_0 \left\{ 1 - \dfrac{D_0}{d_0} \cdot B \cdot \tan E \right\}} \qquad (10)
$$

The angle Θ is the inclination of the parallel stripes in a Laue-line against the horizontal axis, and we can measure it approximately on the photographic plate, and all other values except E can also be obtained from the position of a stripe on the photographic plate; consequently by means of (10) we can find the value of E for the parallel stripes in any Laue-line under consideration.

Now let us take one of the atomic planes (111) as the slip plane and one of the crystallographic axes [110] in the slip plane which will be the slip direction as we have found to be the case in stretching of this kind and investigate their orientations. As is stated before, the straight line MN in Fig. 4 is the intercepting line of the slip plane with the plane containing the direction of the long slit S_2 and the incident X-rays; thus the direction of the normal to the slip plane (111) must lie in the plane of the great circle which is perpendicular to the

Specimen No. 1

Fig. 6



Thickness of the plate; 0.2 mm; stretching: 2.5%; angle of rotation of crystallites: 1.5°; number of blocks: 8 per mm; angle $\widehat{UV}$: 72°; angle $\widehat{VS}$: 20°.

Table I

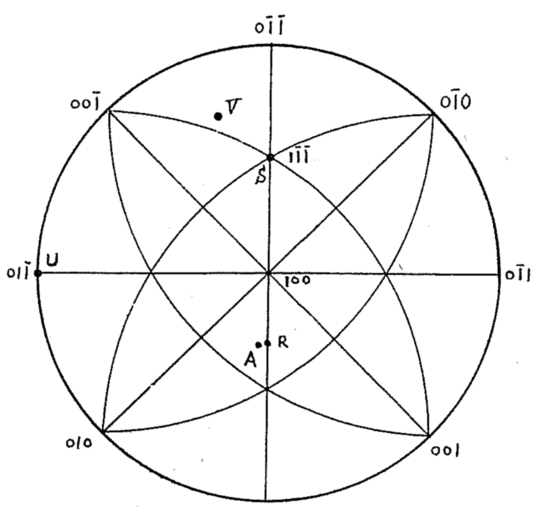
Indices of Laue-line	Θ	E	Mean value of E
$1\bar{1}\bar{1}$	11.5°	3.2°	
$1\bar{3}\bar{1}$	−19.0°	2.8°	3.0°
$10\bar{1}$	46.0°	3.0°	

direction MN, and the slip direction must be the one of the directions [1 1 0] in the slip plane, which is nearest to the stretched direction. By the process of slipping, the crystallites come to rotate in reference to the outer form of the specimen and the axis of such rotation of crystallites which is perpendicular to the directions of slipping and stretching is determined from the displacement of a particular point in a Laue-line by stretching as will be stated later.

The results obtained by stretching 2.5 percent a single crystal plate of aluminium, 0.2 mm thick, are shown in Fig. 6 by stereographic projection. In this figure, R represents the axis of rotation of the crystallites, S the normal to the slip plane, V the stretched direction which in this experiment was made vertical, U the slip direction, and A represents the direction of the incident X-rays which are perpendicular to the plane of the specimen and the photographic plate. Here it must be noted that the angles $\widehat{AV}$, $\widehat{SU}$, $\widehat{VR}$ and $\widehat{UR}$ are all right angles. The determination of the slip plane, slip direction and the axis of rotation of the crystallites is performed as is stated before. The results of measurements for the same specimen are tabulated in Table I.

Specimen No. 2

Fig. 7



Thickness of the plate: 0.2 mm; stretching: 1.5%; angle of rotation of crystallites: 2.0°; number of blocks: 9 per mm; angle $\widehat{UV}$: 80°; angle $\widehat{VS}$: 18°.

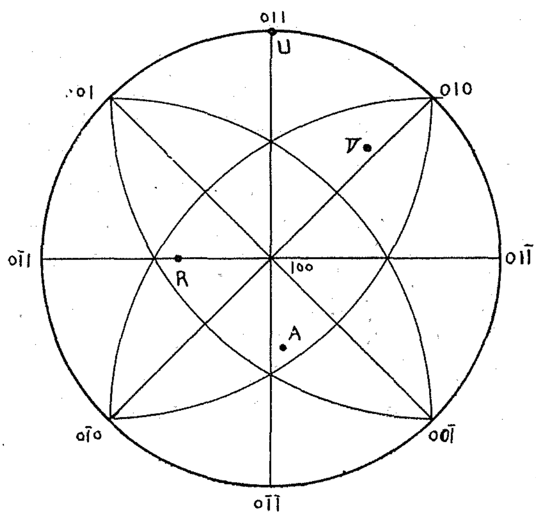
Table II

Indices of Laue-line	Θ	E	Mean value of E
001	68.5°	11.0°	
$1\bar{1}3$	45.0°	11.5°	11.3°
010	49.0°	11.7°	

The results obtained with two other specimens are shown in Fig. 7 and Fig. 8 and the numerical values for these specimens are tabulated in Table II and Table III.

Specimen No. 3

Fig. 8



Thickness of the plate: 0.4 mm; stretching: 2.0%; angle of rotation of crystallites: 1.5°; number of blocks: 7 per mm; angle $\widehat{UV}$: 46°; angle $\widehat{VS}$: 41°.

Table III

Indices of Laue-line	Θ	E	Mean value of E
113	32.0°	−54.5°	
$1\bar{1}1$	−48.5°	−54.0°	−54.3°
311	−67.0°	−54.5°	

By the experiment, the single crystal plate of aluminium whose thickness was 0.02∼0.04 cm, breadth 3 cm and length 8 cm, was obtained by the stress-annealing method; and it was stretched in its lengthwise direction. The percent of stretching was calculated from the increase of the distance between two parallel lines drawn on the surface of the specimen. By stretching the deformation of the specimen is not necessarily uniform over the whole surface of the plate, but by drawing square mesh on its whole surface, the portion of the plate which was deformed uniformly was selected and this portion was examined by X-rays. Fig. 1, Plate I, is an X-ray diffraction pattern obtained with a single crystal plate which is not stretched. Fig. 2, Plate I, is of 0.4 percent stretching, and we can already detect the occurrence of stripes by stretching in the Laue-lines. Fig. 3 and 4 in Plate I are of 2.0 % and 4.0 % stretching respectively. Fig. 5 in Plate I is an enlarged photograph of a portion of a Laue-line. With the ordinary Laue method using a circular slit system, the presence of such stripes in each Laue-spot can hardly be detected with the stretching of 2.0 % or 3.0 %.

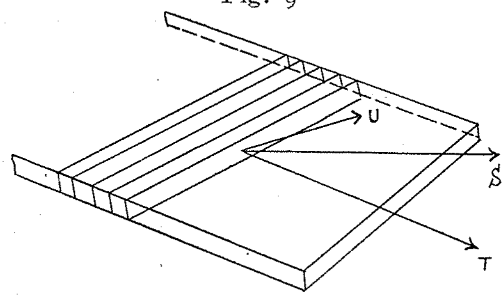
Next the manner of determining the axis of rotation of the crystallites will be described. First of all, we take an X-ray diffraction photograph with an unstretched specimen, and take it again after stretching to some degree. Then we calculate the values of θ and x of two corresponding specified points in the corresponding Laue-line and find the difference of θ, x (i. e. $\Delta\theta$, Δx) caused by stretching. Such values of $\Delta\theta$ and Δx are obtained for many Laue-lines having different indices. As the observed values of $\Delta\theta$ and Δx are very small even with the stretching of several percents, the writer magnified all the observed values of $\Delta\theta$ and Δx in the same proportion, and the changes of the orientations of the normals to various atomic planes having different indices were marked in magnified scale on the surface of the crystallographic globe at their respective positions. Next by considering that such changes of orientations of the normals are due to a rotation around a certain axis, the direction of this axis of rotation and the amount of rotation in the magnified scale were readly found by trial with the aid of the spherical cap; and the actual value of the angle of rotation was obtained by reducing the magnified amount of rotation in the same proportion. The axis of rotation of the crystallites was found to be always at right angles to the stretched direction and the slip direction [110] as it theoretically should be. Thus the experimental results obtained prove reversely the correctness of taking the slip direction as one of the crystallographic axes [110] as we did in the previous statement.

Table V

Stretching in %	1.1	1.2	1.5	2.0	0.9*	2.0*	4.0*
Angle of rotation of crystallites	45′	2.0°	18.5′	52′	1°21′	3°48′	4°30′

In Table V, the degree of rotation of the crystallites by stretching for every specimen tested is tabulated. In this table the specimen marked by * is the same specimen stretched differently and all other specimens are different ones. So far the writer has examined about twenty thin plates of aluminium single crystal of 0.2 mm~0.4 mm in thickness, and it is found that the plate breaks into the blocks of crys-

Fig. 9



U: Slip direction [110]
S: Normal to the slip plane (111)
T: Dirction of stretching

tallites of 6～10 per 1 mm length by the stretching of 0.5～5.0 %, and that even if it is stretched by 2.0 % or 3.0 % more than that, the number of blocks of the crystallites, that is the number of stripes, in a Laue-line does not increase further. From the experiments described above the slip direction [110] in the slip plane (111) is found not to be necessarily the one nearest to the stretching direction with thin and flat plate, but the one which is closely parallel to the surface of the plate, as is illustrated by Fig. 9.

In conclusion the writer wishes to express his sincere thanks to Prof. Dr. U. Yoshida and Dr. K. Tanaka for their kind guidance throughout this experiment.

Plate I

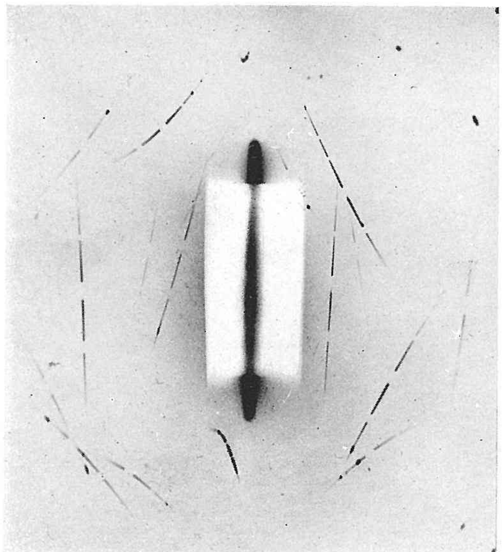


Fig. 1

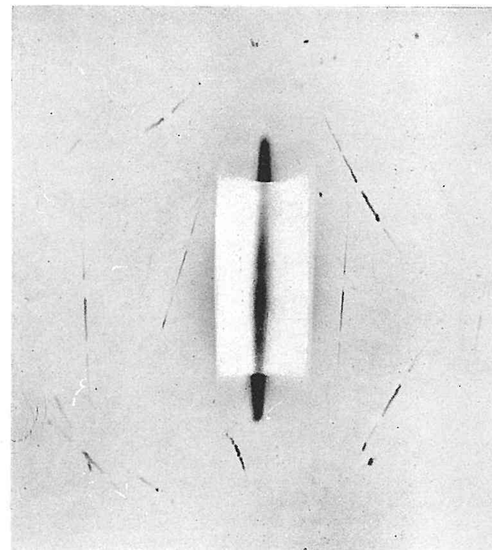


Fig. 2

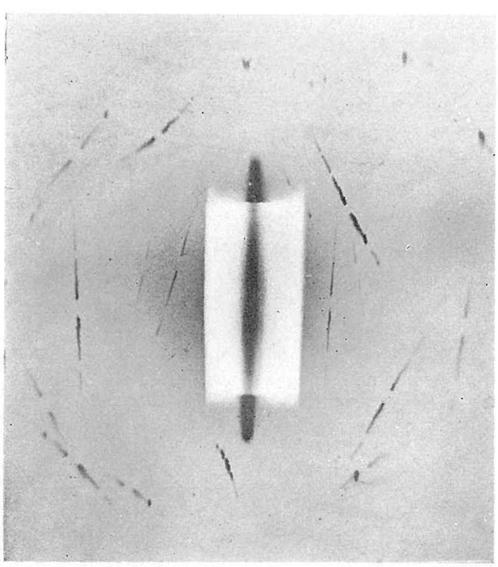


Fig. 3

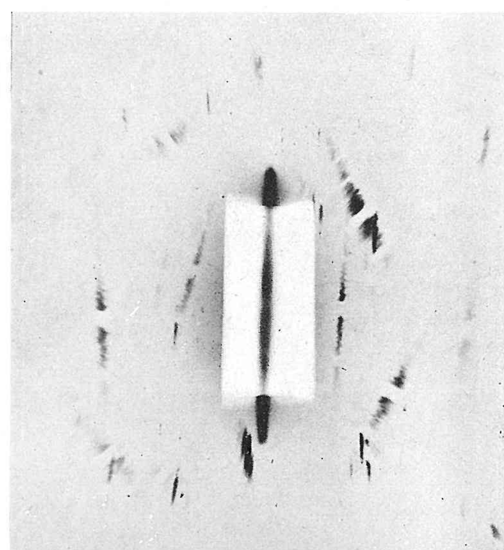


Fig. 4

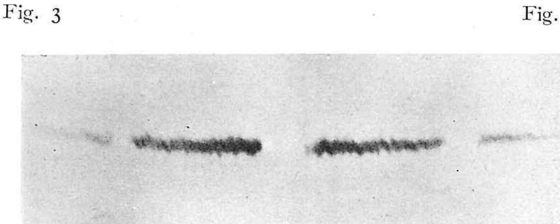


Fig. 5