Limit of the Elasticity of an Aluminium Rod Composed of Comparatively Small Crystal Grains

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Abstract

When cold-worked commercial aluminium rods are treated by heat for a constant time at several temperatures below aluminium's melting point, the crystal grains grow larger with the increase of the temperature. The growth of the grains is not remarkable below about 200°C; but after exceeding that temperature, the growth becomes considerable. *The limit of elasticity decreases slowly at a lower temperature, then* rapidly and again slowly with the increase of annealing temperature. The relation between the limit of elasticity and the grain number per unit length is expressed very simply by an empirical formula.

Experimental Method

Fifteen commercial cold-worked aluminium rods about 20 cm. in length, and about 3 mm. in diameter were placed together in an electric furnace, and heated at a given temperature below the melting point for 2 days. After cooling to the room temperature, they were taken out of the furnace. The diameter of each rod was measured at several points by a micrometer screw gauge and the average value was taken, from which the cross-sectional area was calculated. Nine different groups of rods were subjected separately to different temperatures below the melting point, and afterwards each rod was tested for elongation.

In order to support the test-rod vertically, its upper end was fixed to a support and the lower end was clamped by a chuck of known weight. The upper end of a spring balance of known weight was hung by a hook fastened to the chuck and the lower end was clamped to a movable screw by which the spring balance might be pulled downwards. In this way the rod was loaded in various amounts which we intended. The extensometer[1] recently designed by the writer was attached to the rod whose elongation under the load was measured with two telescopes and scales which were 165 cm. apart from the mirrors.

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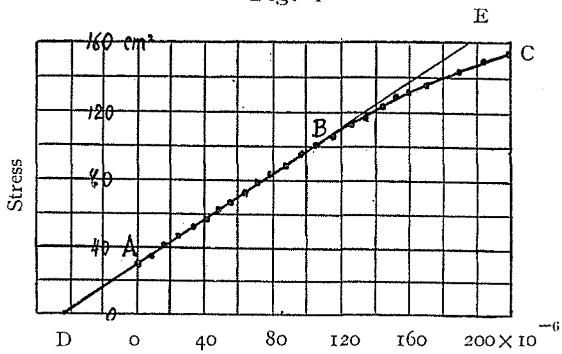
1) These Memoirs, A, **20**, 19, (1937).

In this way we took the mean value of the two scale readings made when the load was gradually increased.

The load applied to the rod was the sum of the readings of the spring balance, its weight, and the weights of the chuck, the rod and the extensometer. The weight of the rod and the extensometer might be neglected in comparison with the other weights owing to their smallness, the extensometer especially having been constructed as lightly as possible. The stress per unit area was easily calculated as the quotient of the total load divided by the average area of the rod.

We took, as the temporary zero point, the scale reading which was taken when the least possible load was initially applied to the rod. From the scale reading the elongation per unit length was calculated by using the constant[1] which had been determined for this extensometer.

Fig. 1



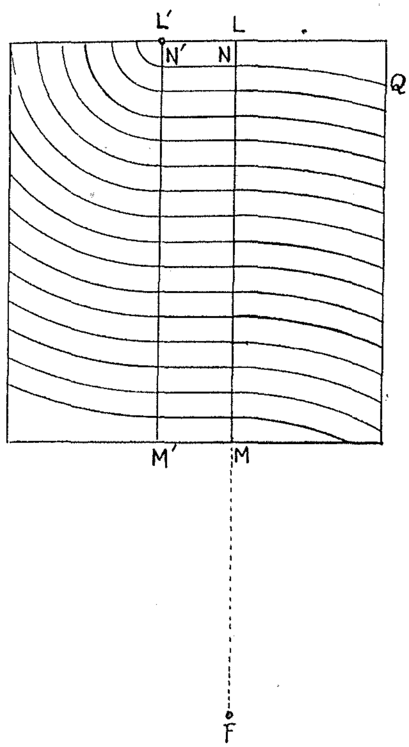
In order to show the relation between the stress per unit area and the elongation per unit length, the writer plotted the curve by taking the former as ordinate and the latter as abscissa. For example, one of the curves obtained is shown in Fig. 1. In this figure, if the curve ABC is produced to meet the abscissa at D, the point D will give a true zero point when no load was applied to the rod; and the elongation per unit length of the rod was obtained by taking D as a zero point. As is seen in Fig. 1, the part AB is a straight line on which Hooke's Law holds, and the curved part BC is beyond the elastic limit. If we draw a straight line ABE in such a way as to coincide with AB, and find the contact point of the line ABE with the curve BC, then this point gives the elastic limit required.

In order to determine such a contact point as accurately as possible, the writer employed an apparatus which may be called a "contact point finder". It is convenient in using this apparatus to consider the curve BC in Fig. 1 a circular arc in the vicinity of the elastic limit.

1) These Memoirs, A, **20**, 19, (1937).

As is shown in Fig. 2, two parallel lines LM and L′M′ are drawn on a thin transparent celluloid plate, and a number of parallel lines between them are drawn perpendicularly to the above two lines. One of them is denoted N′N. The line LM is prolonged to any point P, and an arc NQ is described with a radius PN with the point P as its centre. Similarly a number of arcs of different radii are described with P as the centre, by making them coincide with the straight lines between LM and L′M′ on the line LM. In the same way, another point is taken on the line L′M′ or on its prolongation, and many arcs are drawn with this point as the centre. For instance the point L′ is taken as such a point, and a number of arcs are described with L′ as the centre, so that these arcs meet the lines between LM and L′M′ on the line L′M′.

Fig. 2



Put the above-mentioned celluloid plate on the stress-strain curve as is shown in Fig. 1 in such a way that one of the lines between LM and L′M′, for example N′N, coincides with the line AB, and then displace the plate in the direction parallel to the line N′N and AB, and see whether the arc NQ coincides with the curve BC or not. When the coincidence is imperfect, apply another arc to BC. Thus if we find out a position of perfect coincidence on the curve, we are able to determine the point where the straight line ABE contacts with the curve BC, that is, the limit of elasticity. The result obtained by this method is, of course, slightly ambiguous, but it is convenient to determine easily and simply the limit of elasticity.

The writer used 8 to 12 aluminium rods for each group annealed at a certain temperature, and determined the limit of elasticity for each rod from the stress-strain curve by the above method, and then took the average value. The values of Young's modulus were calculated from the curve as shown in Fig. 1, within the limit of elasticity, by dividing the stress per unit area by the elongation per unit length. One of the results is shown in Table I.

Experimental Result

The result shown in Table I was that of the specimens which were annealed at 130°C for 2 days. The table shows that the elastic limits are almost the same within the experimental error, and that the values of Young's modulus are also the same.

The results obtained with various annealing temperatures are given in Table II. The annealing temperatures are shown in the first column in the table. The temperature 25°C in this column is the room temperature at which the elastic limits of cold-worked aluminium rods were measured. As the grain number per centimetre of the specimen at this temperature is very great, it is assumed to be infinitely great and is denoted by the sign ∞ in the last column in

Table I

Limit of elasticity		Young's modulus
Stress per unit area	Elongation per unit length	in C. G. S unit
540 $\frac{kg}{cm^2}$	7.90×10^{-4}	6.70×10^{11}
530	7.80 ,,	6.66 ,,
520	7.65 ,,	6.66 ,,
510	7.52 ,,	6.63 ,,
565	8.22 ,,	6.73 ,,
565	8.30 ,,	6.66 ,,
540	7.95 ,,	6.66 ,,
530	7.73 ,,	6.72 ,,
575	8.33 ,,	6.76 ,,
535	7.90 ,,	6.64 ,,
Average 541	7.93×10^{-4}	6.68×10^{11}

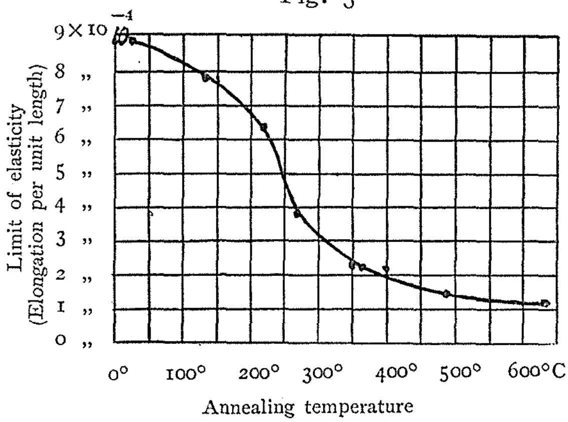
Table II

Annealing temperature	Limit of elasticity		Young's modulus	Grain number per unit length
in centigrade	Stress per unit area	Elongation per unit length	in C. G. S unit	per one cm.
25	603 $\frac{kg}{cm^2}$	8.88×10^{-4}	6.65×10^{11}	∞
130	541	7.93 ,,	6.68 ,,	21220
220	439	6.44 ,,	6.68 ,,	9084
270	264	3.87 ,,	6.67 ,,	3200
350	150	2.21 ,,	6.67 ,,	1515
370	150	2.20 ,,	6.70 ,,	1490
400	148	2.17 ,,	6.70 ,,	1303
480	98	1.44 ,,	6.68 ,,	1049
630	81	1.19 ,,	6.68 ,,	773

the table. The grain number was measured by examining the etched surface of the specimen through a microscope.

Table II shows that according to the increase of the annealing temperature the elastic limit as well as the grain number of each specimen decreases gradually, but that Young's modulus has almost the same value independent of crystal grain sizes. The writer has already reported that Young's modulus of aluminium rod composed of large crystal grains had almost the same value independent of the grain sizes[1]. The writer confirmed this point in the present research, the values of Young's modulus being almost the same in both of his experiments.

Fig. 3



The relation between the annealing temperature and the limit of elasticity is shown in Fig. 3 by taking the former as abscissa and the latter as ordinate. The curve in this figure shows that with the increase of annealing temperature the limit of elasticity decreases rather slowly at first from about 25°C to about 200°C, then rapidly from about 200°C to about 300°C, and again slowly from about 300°C to about 630°C.

In order to show the relation between the grain number per unit length and the limit of elasticity, a curve was plotted by taking the former as abscissa and the latter as ordinate. From the curve the writer approximately obtained an empirical formula as follows:

$$L = L_0(1 - e^{-\alpha z}),$$

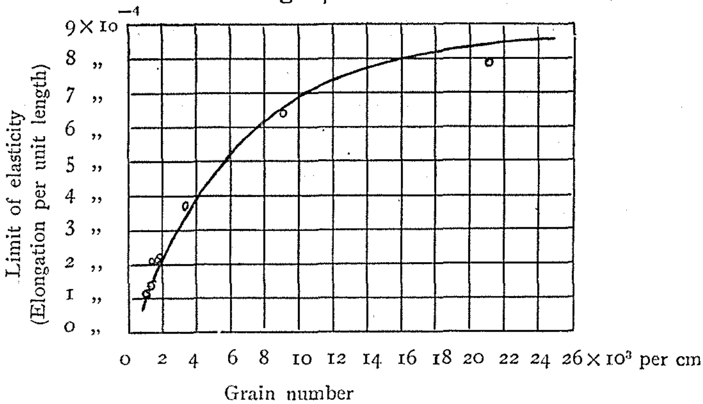
where z denotes the grain number per unit length, L the limit of elasticity represented by elongation per unit length, L_0 the value of L when z becomes infinity, e the base of natural logarithm and α a certain constant. For the constants α and L_0, the following values are found to accord with the results of the present experiment.

$$\alpha = 0.000151, \qquad L_0 = 8.88 \times 10^{-4}.$$

The curve in Fig. 4 is drawn with the above values of α and L_0

1) These Memoirs, A, **17**, 389, (1934).

Fig. 4

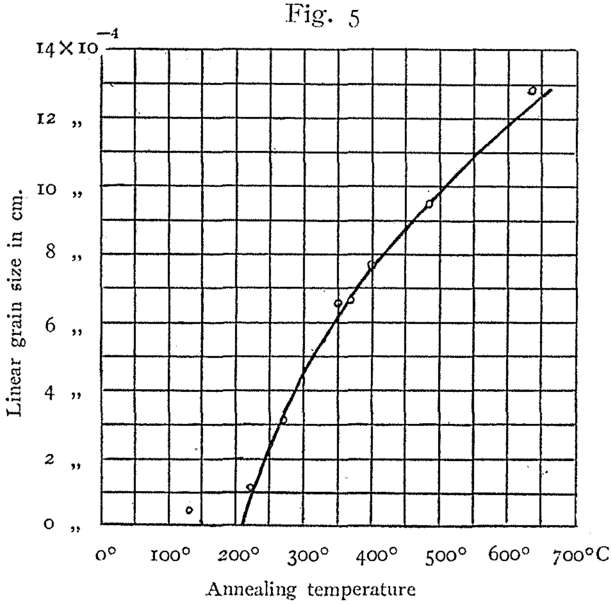


for various values of z, and the small circles for the observed values. The agreement between the calculation and the observation is almost perfect, except at the point of the largest grain number, where the exact counting of the grain number is very difficult.

So far as the present research is concerned, the limit of elasticity increases with increasing grain number rather rapidly at first, and then slowly asymptotically approaches a definite saturation value of 8.88×10^{-4}.

Further, the writer plotted a curve which shows the relation between the annealing temperature and the linear grain size, that is, the reciprocal of the grain number per unit length, by taking the former as abscissa and the latter as ordinate. From the curve the writer obtained an empirical formula as follows:

Fig. 5



$$Y = A(T - 200)^{\frac{1}{2}} + B,$$

where Y denotes

the linear grain size, T the annealing temperature, and A and B are constants which take 0.73 and -2.7×10^{-4} respectively.

The curve drawn in Fig. 5 is that calculated from the above empirical formula, and the small circles denote the observed values. The coincidence seems to be satisfactory in this kind of experiment. From the curve it is clear that the crystal grains almost cease to grow at the temperature below about 200°C, but after exceeding 200°C they grow rapidly. This indicates that the recrystallization temperature is higher than about 200°C in the case of the test pieces of aluminium examined in the present experiment. By calculating the grain size at the melting point 657°C of aluminium from the above formula, the value 12.86×10^{-4} cm. was obtained.

Note: The relation between the grain size and the annealing temperature already has been studied by Professor U. Yoshida and Mr. S. Nagata[1]. They preliminarily annealed cold-worked aluminium plate, then elongated it by a certain small amount and at last annealed it again at a higher temperature below its melting point. That they annealed the specimen in two stages makes their experiment different from the present one. The discrepancy between their research and the present one is due to a different heat treatment and the degree of cold working.

In conclusion the writer wishes to express his sincere thanks to Professor U. Yoshida for his kind guidance in this experiment.

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1) These Memoirs, A, **19**, 169, (1936).