Experimental Investigation on the Sound Emitted by Japanese Hanging-Bells

By **Ichiro Aoki**

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Abstract

The writer recorded the sounds of five bells which belong to temples in and near Kyoto by using a Low-Hilger audiometer. These records of sounds were analysed into partial sine-curves by the method of periodogram analysis. Taking the results of former investigations into account, the writer arrived at the following conclusions:

1. The frequencies of the fundamental tones are nearly in inverse proportion to the linear dimensions of the bells. But as the bell becomes smaller, the frequency of its fundamental tones becomes smaller than the value obtained by this relation.

2. The frequencies of the partial tones do not change when we change the position of the striking-point along the same meridian line passing through the "Tsukiza" (striking-seat). But their values change when we strike different points along the same horizontal line passing through the "Tsukiza".

3. There exists one partial tone between the fundamental tone and what was formerly considered the first overtone. This overtone seems to appear when the size of the bell becomes comparatively large.

4. The frequency of the fundamental tone and its overtones are in approximately the following ratios:

$$4^2 : 5^2 : 6^2 : 7^2 : 8^2 : \ldots\ldots\ldots$$

1. The Method of Investigation

The sound of some Japanese hanging-bells was recorded on rotating films by a Low-Hilger audiometer, and analysing curves thus obtained by periodogram analysis,[1] the frequency and amplitudes of the partial tones, which constitute the sound of bells, were determined.

The bells newly investigated are those which belong to the following temples:

A. Byôdôin-temple in Uji-chô.
B. Hônenin-temple in Kyoto.
C. Zenrinji-temple in Kyoto.
D. Kakôji-temple in Kyoto.
E. Daiunji-temple in Iwakura-mura.

We will hereafter call these five bells A, B, C, D and E respectively.

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1. Whittaker and Robinson, "The Calculus of Observation." pp. 343–360.
Carse and Shearer, "Fourier's Analysis and Periodogram Analysis." pp. 29–50.

The photographs and the dimensions expressed in cm. of these five bells are shown in Pls. I, II, and III. The results already obtained by K. Yamashita, J. Obata and the present writer are added and treated together. Thus in all, nine bells are treated in this paper.

Size	Big Bells						Small Bells		
Designation of Bell	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*
Name of Temple	Byôdôin	Hônenin	Zenrinji	Kakôji	Daiunji	Myôshin-ji			
Investigator	Aoki	Aoki	Aoki	Aoki	Aoki	Aoki[1]	Obata[2]	Aoki[3]	Yamashita[4] and Aoki

2. The Records of the Sounds

The sound-records of the five bells *A*, *B*, *C*, *D*, *E* are shown in Pls. IV, V, VI, VII, and VIII respectively. Fig. 1's in these five plates are records of the sounds taken immediately after the bell was struck, and Fig. 2's and Fig. 3's are the records taken some minutes after the striking. The last figures in these plates are the records of the sounds taken when the bell was struck with a very small wooden hand-hammer (usually used for small bells) thickly wrapped in cloth.

The bell *E* has a somewhat different form, its shape being nearly cylindrical, and the ratio of the height to its diameter being much larger than those of the other bells. So the writer recorded the sounds of this bell by varying the point of striking in order to investigate the relation between the frequency of the partial tones and the position of the striking. These records are shown in Pl. IX. Figs. 1, 2 and 3 in this plate are the records taken respectively when the bell was struck at points 10 cms. above, 20 cms. above and 10 cms. below the striking-seat along the same meridian line. Figs. 4, 5, 6 and 7 are the records made when the bell was struck at points $\frac{1}{4}90^\circ$, $\frac{1}{2}90^\circ$, $\frac{3}{4}90^\circ$ and 90° apart from the striking seat along the same horizontal line. Each of these figures in Pl. IX are the records taken 2–4 secs. after the bell was struck.

The marked portions in these six plates were measured accurately and were analysed into partial sine-curves. The results of this analysis

1. I. Aoki, These Memoirs, XV, 311–313 (1932).
2. J. Obata and T. Tesima, Japanese Journal of Physics, IX, 49–73 (1934).
3. I. Aoki, These Memoirs, XIV, 213–218 (1931).
4. K. Yamashita and I. Aoki, loc. cit. XV, 323–326 (1932).

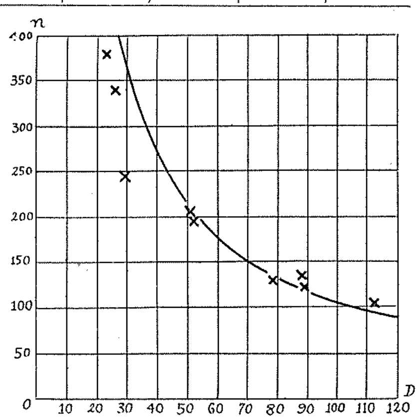
are shown in the diagrams appended to this paper. These diagrams show the frequencies and the amplitudes of the partial tones of the five bells.

The frequencies in these diagrams having noughts as their last figures were not so accurately obtained as the others, owing to the method of the periodogram analysis.

3. On the Fundamental Tones

Between the frequencies of the fundamental tones, n, and the diameters of the bells at their mouths, D, we compiled the following table and graph.

	A	*B*	*C*	*F*	*D*	*E*	*G*	*H*	*I*
D	112.0	88.5	88.0	79.5	51.0	50.5	28.5	25.6	22.7
n	108	125	135	129	197	206	247	340	380
nD	12096	11062	11880	10255	10047	10403	7039	8704	8626



It is evident theoretically that if the bells have similar forms (including thickness) and are made of the same material, n will be in inverse-proportion to D, i. e. the product nD will be constant and the graph will become a rectangular hyperbola[1]. But in our case, if we draw a rectangular hyperbola nearly through the mean positions of n, we find that the points showing the frequencies of the larger bells lie nearly on the curve, while those of the smaller bells deviate much from the curve on its lower side. It seems this is due to the fact that the forms of the bells are not quite similar, especially in the way the thickness of each bell varies.

In this graph, we see that the position of n of the bell G most deviate from the mean curve. Taking the ratios of the thickness d at the lowest ends of the bells to the diameter D at their mouths, we have the following table, which shows that d is nearly equal to 10%

1. K. Yamashita, These Memoirs, XV, 315-322 (1932).
K. Yamashita and I. Aoki, loc. cit. XV, 323-326 (1932).

of D and that there must be some remarkable deviation in the bells F and G from the others. In fact, the bell F has no specially thick portion at its bottom end usually called "Komanotsume", and the bell G has a frequency n much lower than that given by the hyperbolic curve. From this table we see that the values of $(d/D) \times 10$ for different

	A	B	C	F	D	E	G	H	I
D	112.0	88.5	88.0	79.5	51.0	50.5	28.5	25.6	22.7
d	11.5	10.0	11.0	5.0	6.0	4.8	1.6	2.6	2.3
$\frac{d}{D} \times 10$	1.03	1.13	1.25	0.63	1.18	0.95	0.56	1.02	1.01

bells are nearly equal to unity, i. e. the thickness of the bells is nearly equal to 10% of the diameter, exceptions being in the case of the bells F and G. But the bell F does not have a so-called "Komanotsume" (the lowest thick part), and the thickness along the meridian line does not decrease so rapidly as do the other bells as we pass upward from the lowest end. For this reason, it seems, the bell G will show the greatest deviation, but the bell F will not deviate so much. The deviation of smaller bells G, H, and I will be explained by the fact that in their cases the thickness along the meridian line decreases more rapidly as we pass upwards from the lowest end, and the upper parts of the "Komanotsume" are much thinner than those of the larger bells.

4. The Relation between the Position of the Striking-point and the Frequencies of the Partial Tones

The bell E is, as we have already stated, nearly cylindrical and the ratio of its height to its diameter is much larger than those for the other bells. Therefore, with it the writer tried to investigate the relation between the position of the striking-point and the frequencies of the partial tones by shifting the striking-point along the meridian line and the horizontal line passing through the striking-seat. The result is as follows:

	Position of Striking	Fundamental Tone	First Overtone
Meridian Line	Striking-seat	206	450
	10 cms. above	206	450
	20 cms. above	206	450
	10 cms. below	206	450

(Continued from the preceding page)

	Position of Striking	Fundemental Tone	First Overtone
Horizontal Line	$\frac{1}{4}$ 90°	206	450
	$\frac{1}{2}$ 90°	206	**445**
	$\frac{3}{4}$ 90°	**207**	450
	90°	206	**452**

From these findings we see that when the position of the striking-point is displaced along the same meridian line the frequency of the partial tone does not change, while when the point is displaced along the horizontal line, the frequency generally changes a little, as the writer has previously reported in the case of the small bell.[1]

5. On the Frequencies of the Partial Tones

Up to this time in regard to the ratios of the frequencies of the partial tones of the bell, the following relations were proposed:

$2^2 : 3^2 : 4^2 : 5^2 : 6^2 : \ldots\ldots\ldots$[2]

(proposed by K. Yamashita and the present writer)

$2 : 5 : 7 : 9 : 11 : \ldots\ldots\ldots$[3]

(proposed by J. Obata and T. Tesima)

But in the former relation, the fourth partial tone whose frequency may come between 4^2 and 5^2 is excluded.

Now all the frequencies of the partial tones of the nine bells we have found are shown in the following table.

Designation of Bell	Frequencies of the Partial Tones									
A	108	200	276	333	450	830	1600				
B	125	251	330	530	660	900	1250	1600	2900		
C	135	220	300	390	450	690	1000	1300	1500	1900	2400
D	197	430	720	930	1200	1500					
E	206	450	740	950	1250	1600					
F	129	335	550	700	900	1250					
G	247	624	850	1100	1380	1640	2050				
H	340	800	1100	1400	1800						
I	380	900	1250	1600	2450	3600					

1. I. Aoki, These Memoirs, XIV, 213-218 (1931).
2. K. Yamashita and I. Aoki, These Memoirs, XIV, 213-218 (1931).
3. J. Obata and T. Tesima, Japanese Journal of Physics, IX, 49-73 (1934).

If we take the frequencies of the fundamental tones as 10, the ratios of the frequencies of the higher partial tones are shown in the following table.

Designation of Bell	Ratios of Frequencies										
A	10	18	26	31	42	77	150				
B	10	20	26	42	53	72	100	128	232		
C	10	16	22	29	33	51	74	96	111	147	178
D	10	22	36	48	61	79					
E	10	22	36	46	61	78					
F	10	26	43	52	70	97					
G	10	25	34	44	56	66	83				
H	10	23	32	41	53						
I	10	24	33	42	64	95					

Rearranging these numbers under the supposition that some of the partial tones might be absent, we get:

Table A.

Order of Partial Tones / Designation of Bell	1	2	3	4	5	6	7
F	10		26		43	52	70
B	10	20	26		42	53	72
A	10	18	26	31	42		
G	10		25	34	44	56	66
I	10		24	33	42		64
H	10		23	32	41	53 →	?
C	10	16	22	29	33	51	
D	10		22		36	48	61
E	10		22		36	46	61
Mean	10	18	24	32	40	51	64

If we put the frequencies of the third partials equal to 10, the above table becomes:

Table B.

Order of Partial Tones / Designation of Bell	1	2	3	4	5	6	7
F	3.7		10		16	21	27
B	3.8	7.9	10		16	20	27
A	3.9	7.2	10	12	16		
G	4.0		10	14	18	22	26

(Continued from the preceding page)

Order of Partial Tones / Designation of Bell	1	2	3	4	5	6	7
I	4.2		10	14	18		27
H	4.2		10	14	18	22	
C	4.3	7.3	10	13	15	23 →	?
D	4.6		10		17	22	28
E	4.8		10		16	21	28
Mean	4.2	7.5	10	13	17	21	27

Similarly by putting the frequencies of the fifth partials equal to 10, we get:

Table C.

Order of Partial Tones / Designation of Bell	1	2	3	4	5	6	7
F	2.3		6.1		10	13	16
B	2.4	4.7	6.3		10	12	17
A	2.2	4.4	6.1	7.2	10		
G	2.2		5.7	7.6	10	13	15
I	2.4		5.0	7.8	10		15
H	2.4		5.7	7.9	10	13	
C	3.0	4.9	6.1	8.7	10	15 →	?
D	2.7		6.0		10	13	17
E	2.8		6.1		10	13	17
Mean	2.5	4.7	5.9	7.8	10	13	16

By these table, we see that the numbers in each column are nearly equal, a fact which may be regarded as proof that they are of the partial tones of the same order. Rendering these numbers into frequencies, we get:

Order of Partial Tones / Designation of Bell	1	2	3	4	5	6	7
A	108	200	276	333	450		
C	135	220	300	390	450	690 →	?
B	125	251	330		530	660	900
F	129		335		550	700	900
D	197		430		720	930	1200
E	206		450		740	950	1250
G	247		624	850	1100	1380	1640
H	340		800	1100	1400	1800	
I	380		900	1250	1600		2450

Taking the mean values of each columns in table A, B, C, we get:

Table \ Order of Partial Tones	1	2	3	4	5	6	7
A	10	18	24	32	40	51	64
B	4.2	7.5	10	13	17	21	27
C	2.5	4.7	5.9	7.8	10	13	16

Keeping the same ratios and changing the numbers in the first column to 10, we get:

Table \ Order of Partial Tones	1	2	3	4	5	6	7
A	10	18	24	32	40	51	64
B	10	18	24	31	40	50	64
C	10	19	24	31	40	52	64
Mean	10	18	24	31	40	51	64

If we assume the ratios

$$4^2 : 5^2 : 6^2 : 7^2 : \cdots\cdots\cdots$$

modifying Dr. Yamashita's relation a little, and the ratios

$$2 : 3 : 5 : 7 : \cdots\cdots\cdots$$

adding one partial between the first and the second in Dr. Obata's relation, and dividing the above mean values by the corresponding numbers in these ratios, we get:

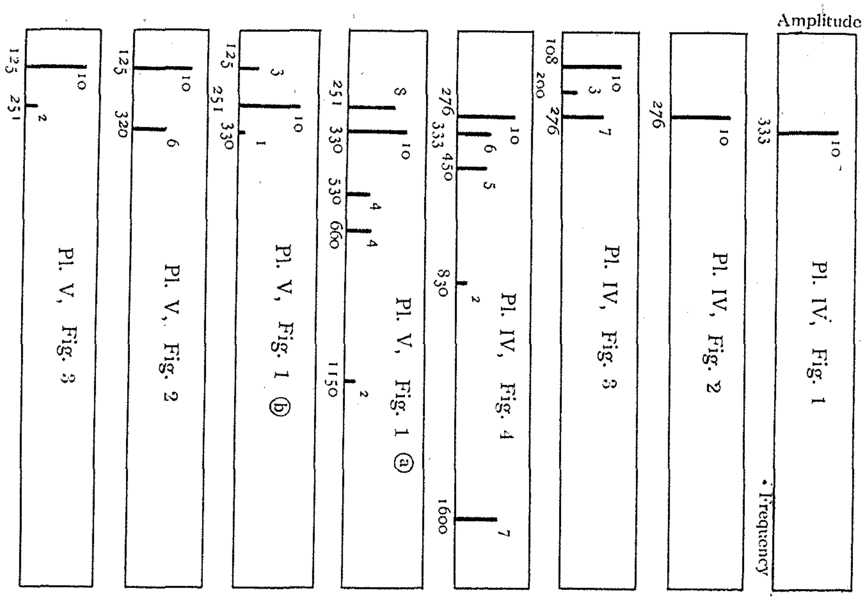
$$10 \div 4^2 = 0.63 \qquad 10 \div 2 = 5.0$$
$$18 \div 5^2 = 0.72 \qquad 18 \div 3 = 6.0$$
$$24 \div 6^2 = 0.67 \qquad 24 \div 5 = 4.8$$
$$31 \div 7^2 = 0.63 \qquad 31 \div 7 = 4.4$$
$$40 \div 8^2 = 0.63 \qquad 40 \div 9 = 4.4$$
$$51 \div 9^2 = 0.63 \qquad 51 \div 11 = 4.6$$
$$64 \div 10^2 = 0.64 \qquad 64 \div 13 = 4.9$$

These quotients are nearly constant in both cases, but the agreement is better in the former ratios. Thus the frequencies of the partial tones of the bell may be represented by the ratios

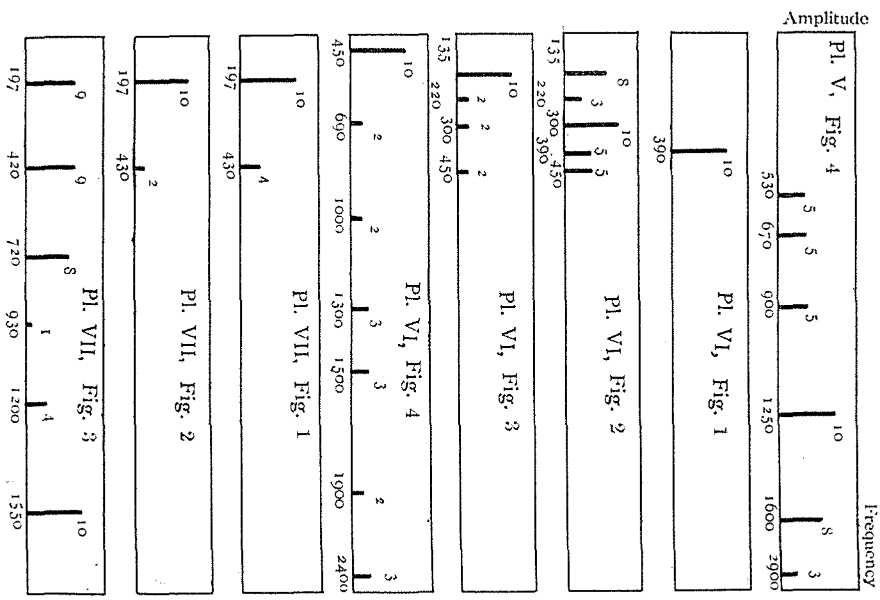
$$4^2 : 5^2 : 6^2 : 7^2 : \cdots\cdots\cdots$$

and the second partial tones only appear in larger bells *A*, *B*, and *C*.

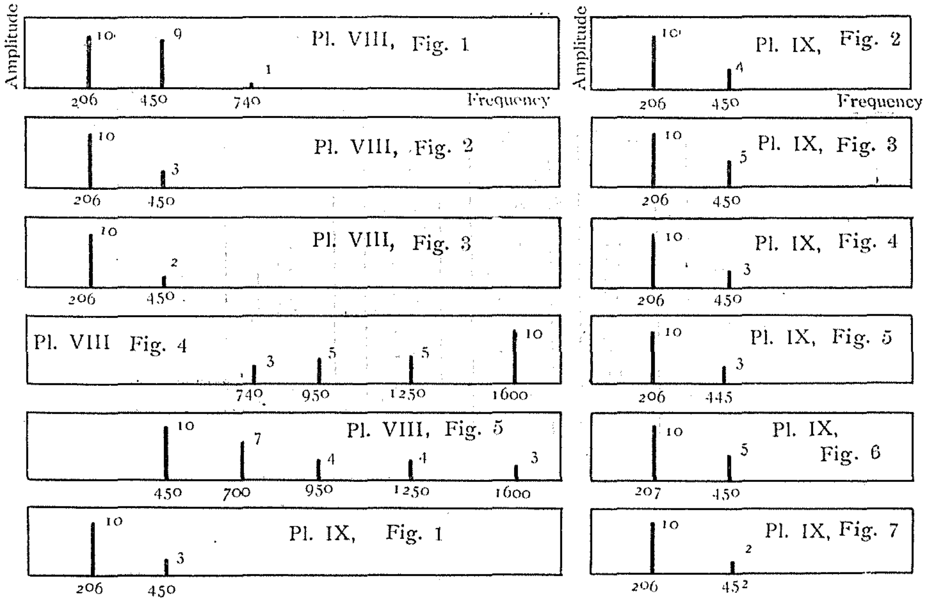
In conclusion the writer wishes to express his gratitude to Prof. K. Tamaki for his kind guidance, to Dr. K. Yamashita for his advice and to the Hattori Hôkô-kwai for the grant with which the present investigation has been carried out. The writer also cordially thanks the authorities of the temples who kindly permitted him to use the bells for the present study.



Amplitude
Frequency
Pl. IV, Fig. 1
Pl. IV, Fig. 2
Pl. IV, Fig. 3
Pl. IV, Fig. 4
Pl. V, Fig. 1 ⓐ
Pl. V, Fig. 1 ⓑ
Pl. V, Fig. 2
Pl. V, Fig. 3



Amplitude
Frequency
Pl. V, Fig. 4
Pl. VI, Fig. 1
Pl. VI, Fig. 2
Pl. VI, Fig. 3
Pl. VI, Fig. 4
Pl. VII, Fig. 1
Pl. VII, Fig. 2
Pl. VII, Fig. 3



Amplitude
Pl. VIII, Fig. 1
10
9
1
206
450
740
Frequency
Pl. VIII, Fig. 2
10
3
206
450
Pl. VIII, Fig. 3
10
2
206
450
Pl. VIII Fig. 4
3
5
5
10
740
950
1250
1600
Pl. VIII, Fig. 5
10
7
4
4
3
450
700
950
1250
1600
Pl. IX, Fig. 1
10
3
206
450
Amplitude
Pl. IX, Fig. 2
10
4
206
450
Frequency
Pl. IX, Fig. 3
10
5
206
450
Pl. IX, Fig. 4
10
3
206
450
Pl. IX, Fig. 5
10
3
206
445
Pl. IX, Fig. 6
10
5
207
450
Pl. IX, Fig. 7
10
2
206
452

Ichiro Aoki

Plate I

(Bell *A*)

Fig. 1

Fig. 1′

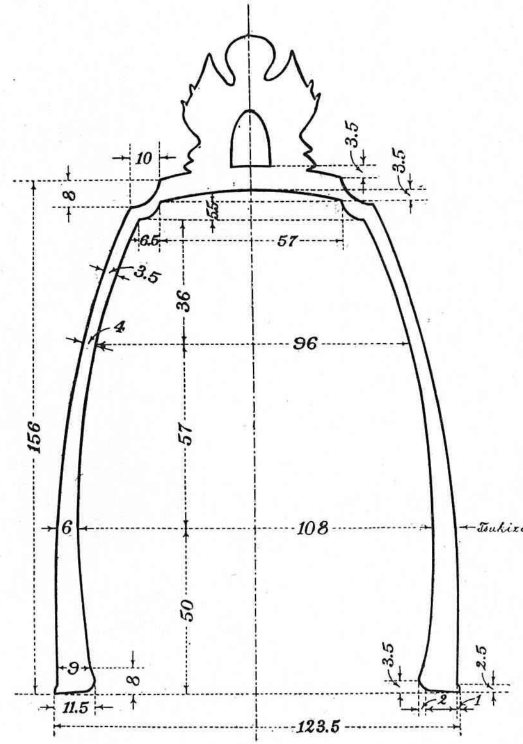


Plate II

(Bell *B*)

Fig. 1

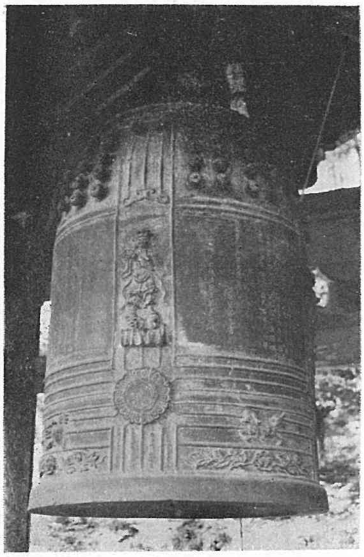


Fig. 1′

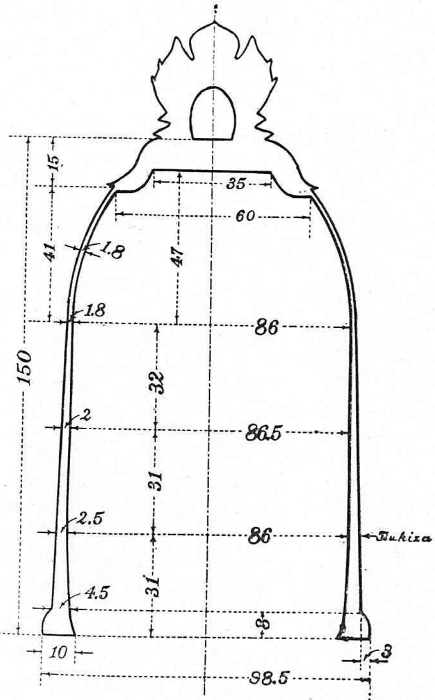


Fig. 2′ (Bell *C*)

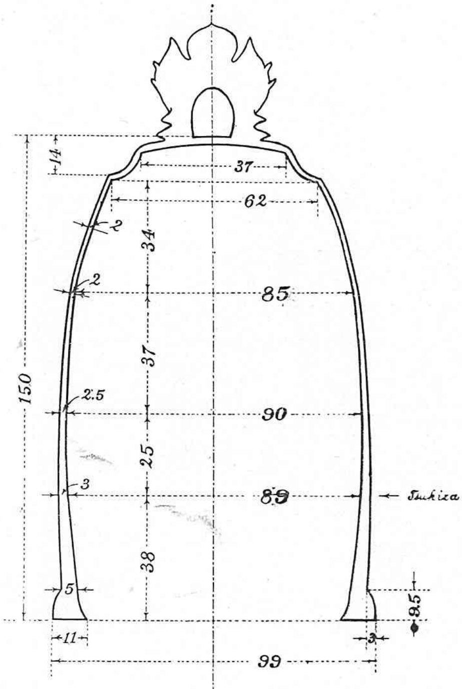


Fig. 2

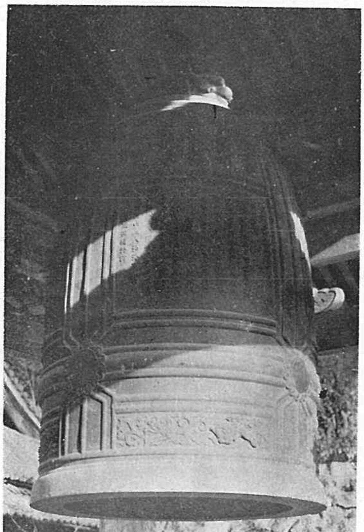


Plate III

Fig. 1 (Bell *D*)

Fig. 1′

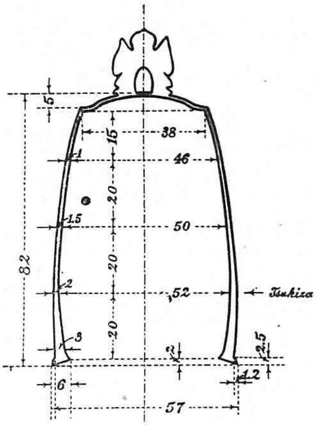


Fig. 2 (Bell *E*)

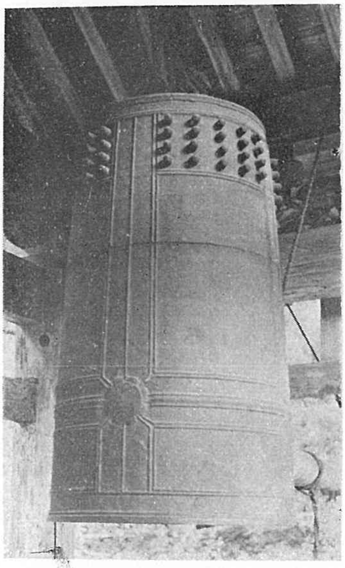


Fig. 2′

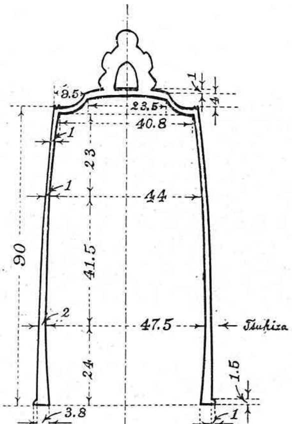


Plate IV (Bell *A*)

Fig. 1. $\longrightarrow \left| \frac{1}{50} \text{sec.} \right| \longleftarrow$

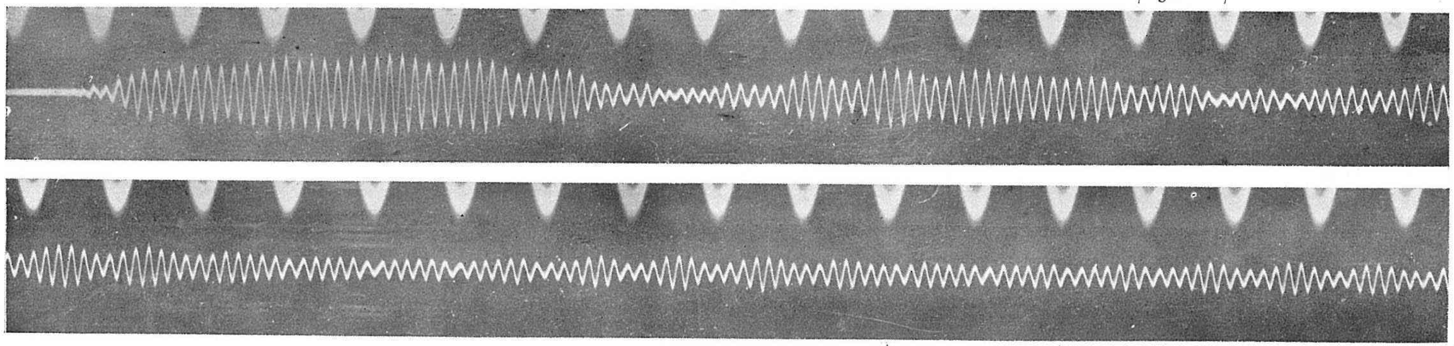


Fig. 2. 1 sec. after it is struck.

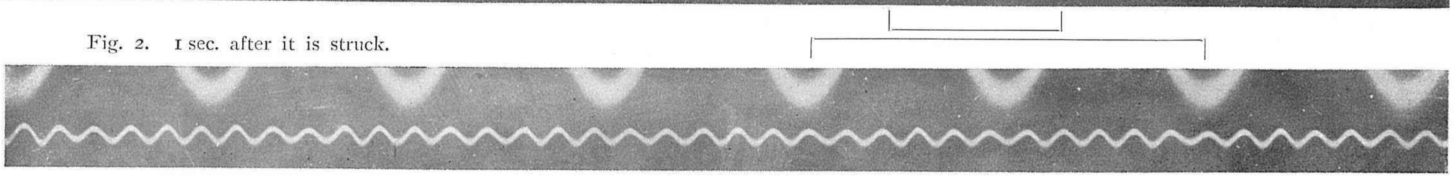


Fig. 3. 4 secs.

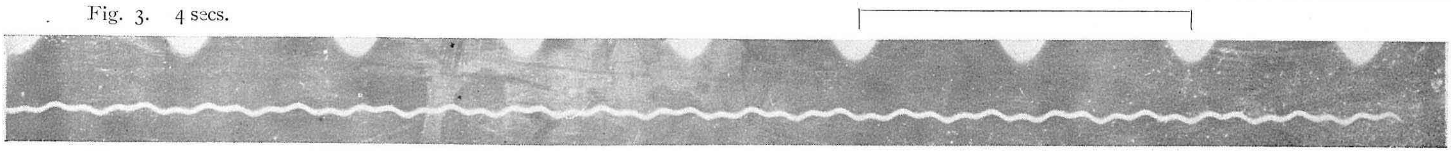


Fig. 4.

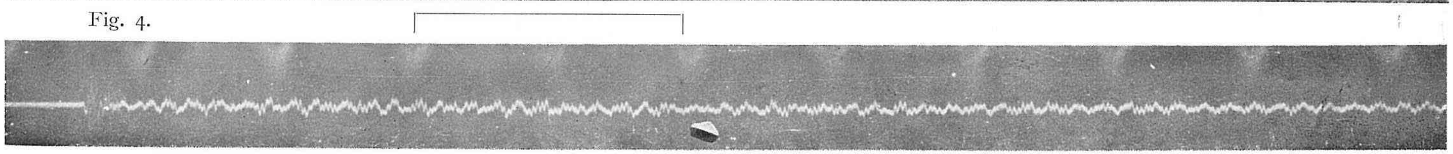


Plate V (Bell *B*)

Ichiro Aoki

Fig. 1.

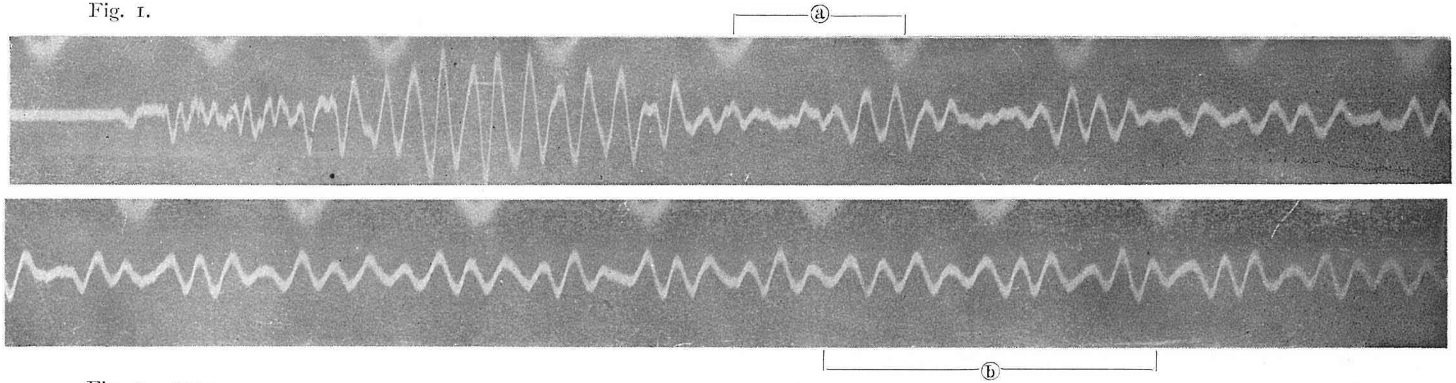


Fig. 2. 2 secs.

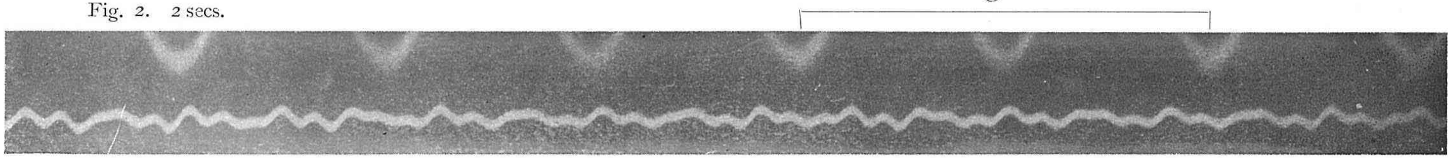


Fig. 3. 4 secs.

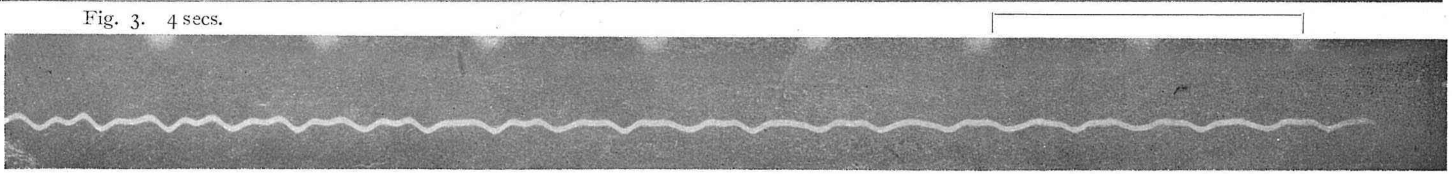


Fig. 4.

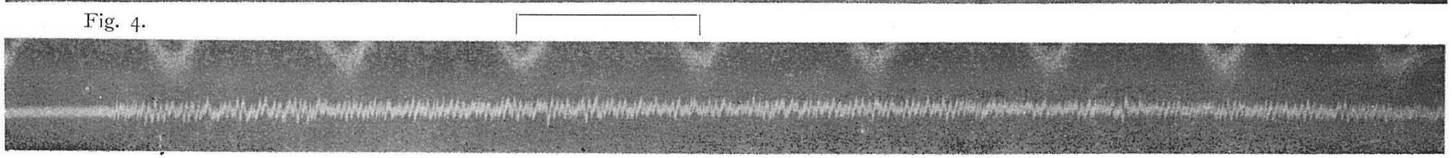


Plate VI (Bell *C*)

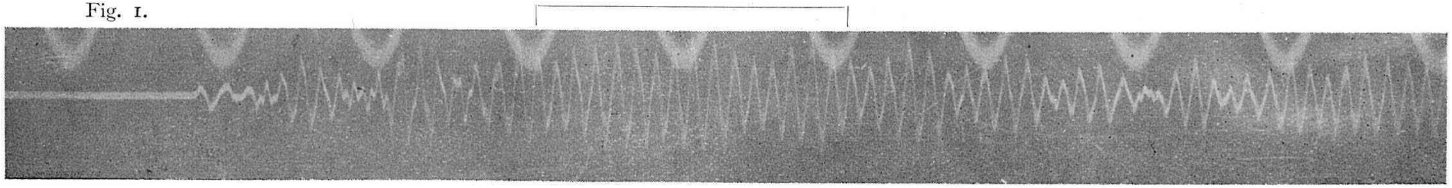


Fig. 1.

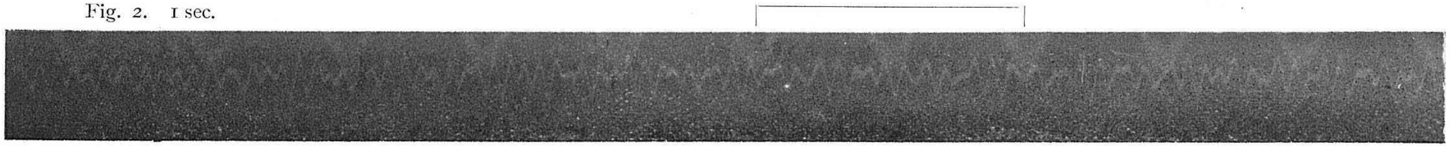


Fig. 2. 1 sec.

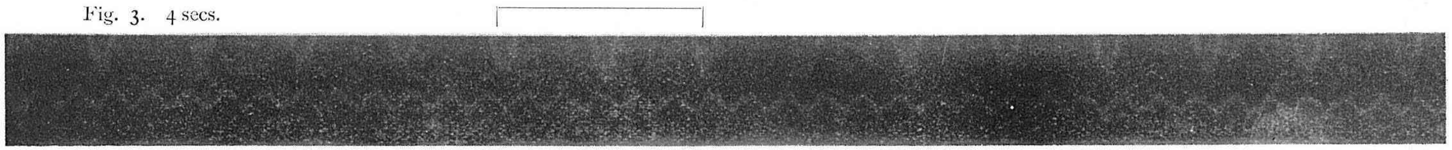


Fig. 3. 4 secs.

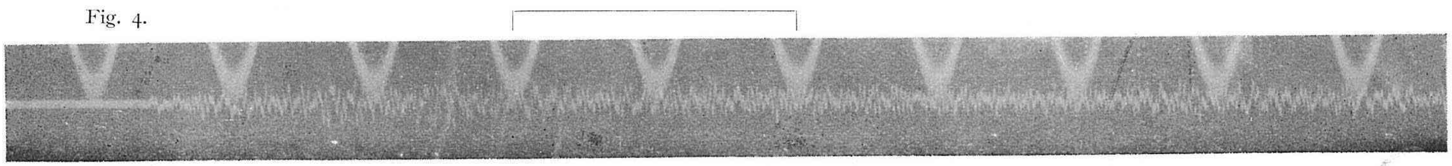


Fig. 4.

Plate VII (Bell *D*)

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Fig. 1.

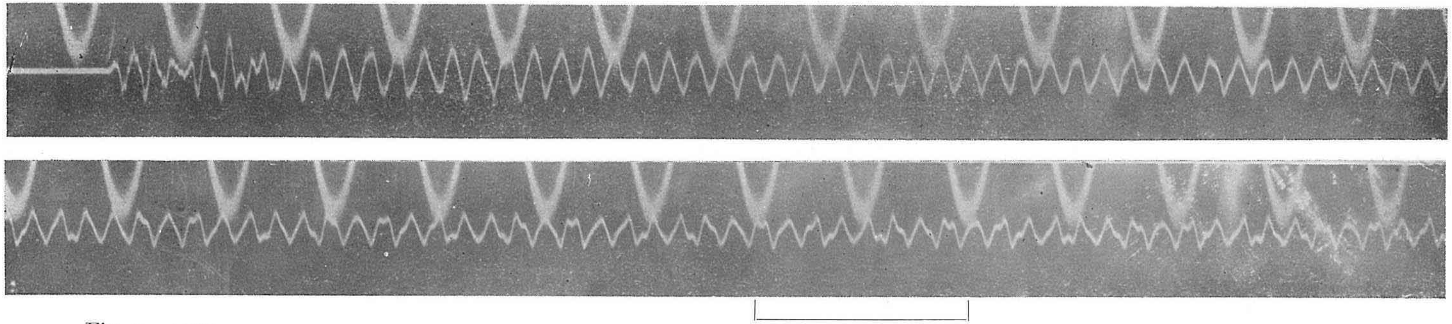

Fig. 2. 4 secs.

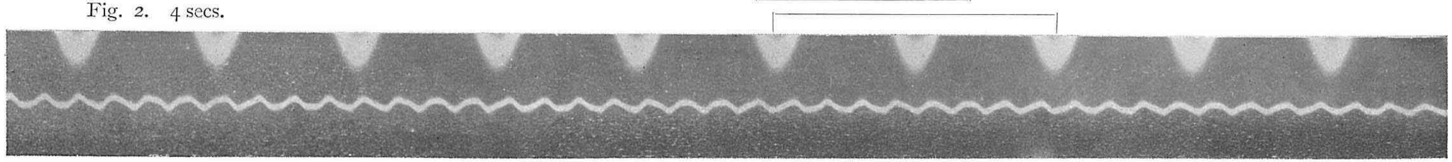

Fig. 3.

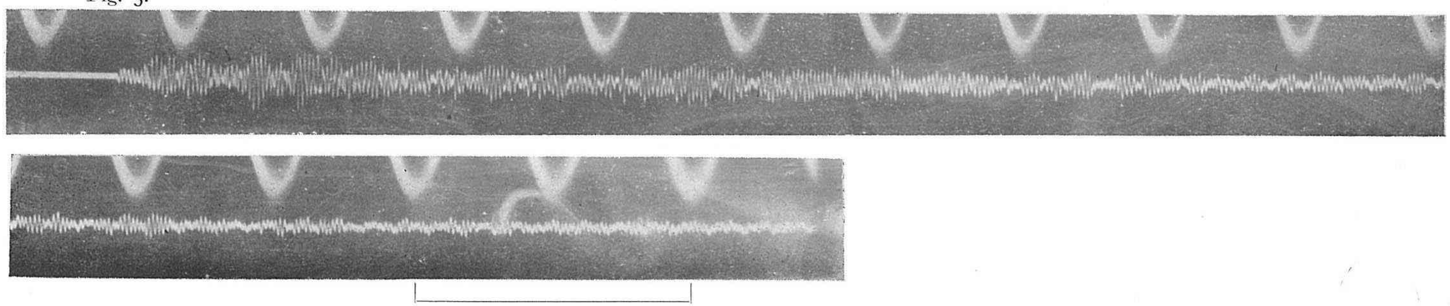

Plate VIII (Bell *E*)

Ichiro Aoki

Fig. 1.

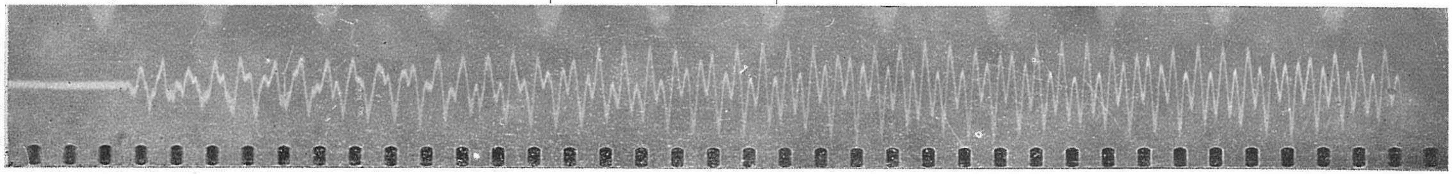

Fig. 2. 2 secs.

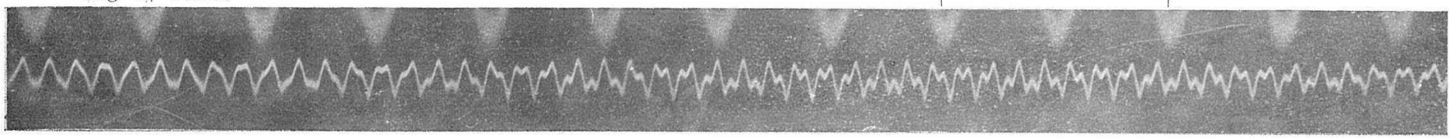

Fig. 3. 4 secs.

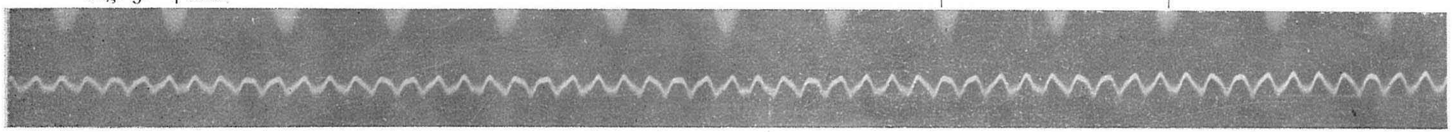

Fig. 4.

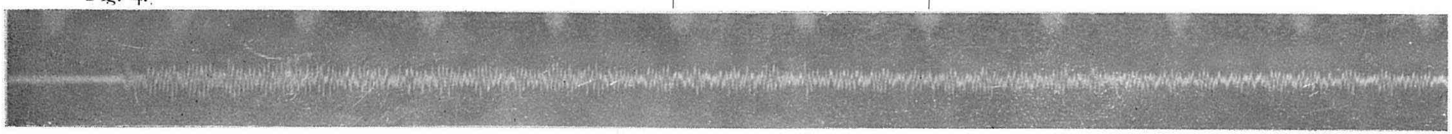

Fig. 5.

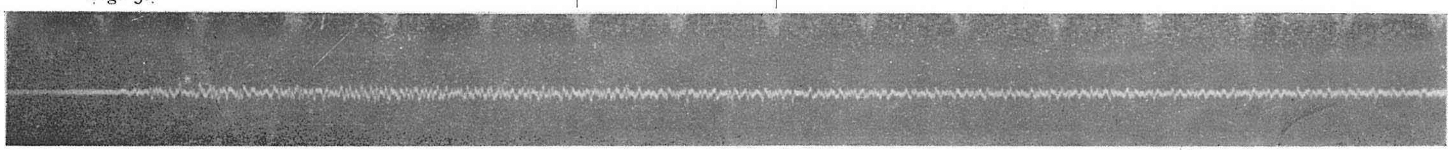

Plate IX (Bell *E*)

Fig. 1.

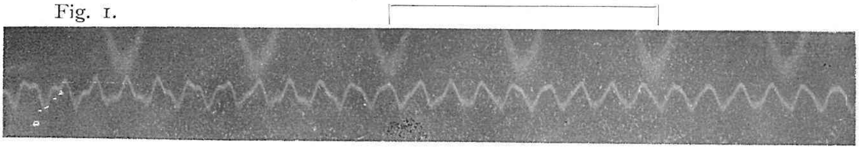

Fig. 2.

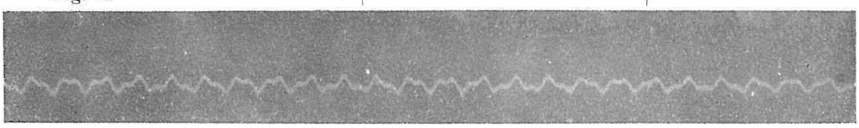

Fig. 3.

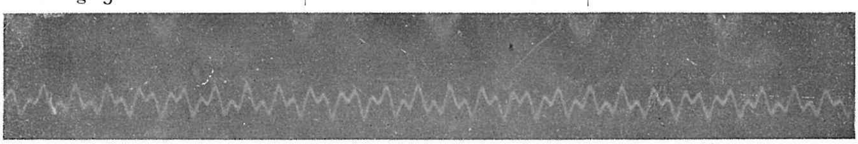

Fig. 4.

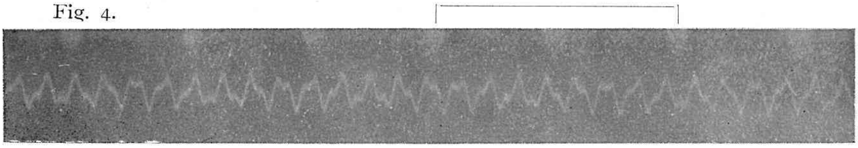

Fig. 5.

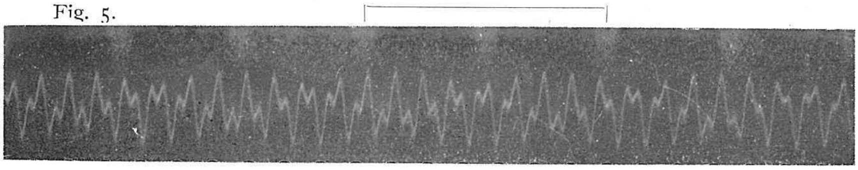

Fig. 6.

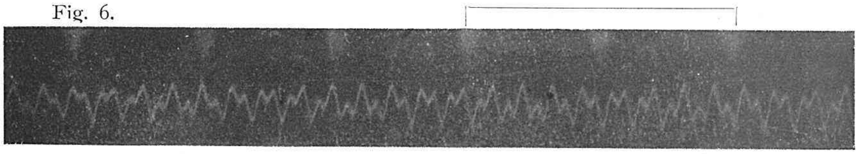

Fig. 7.