Coast Effect upon the Ocean Current and the Sea Level (I) : Steady State

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Coast Effect upon the Ocean Current and the Sea Level, I. Steady State

By

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Abstract

In the present paper the writers investigate the influence of the coast upon the wind-, the barometric- and the convection-current in the steady state for the cases of (a) "no bottom-current," (b) "no bottom-friction" and (c) "a finite bottom-friction." The surface slope caused by the land effect is also determined in each case.

Introduction

Having studied the drift-, the barometric- and the density-current in a boundless sea, we shall proceed to investigate the current in a limited sea and to make clear the land effect upon the current and the surface level. In this paper only the results obtained for the steady state are reported, and the changing states will be reserved for our next research.

As to the land effect on the drift current, Ekman[1] has studied the case of uniform wind; but he treated only the sea with no bottom-current and gave no definite formula for the surface slope. Jeffreys[2] investigated the same problem assuming a finite bottom-friction and elucidated the extreme cases of very shallow and of very deep water; but in the case of a sea of intermediate depth his formulae are inadequate for making numerical calculations and getting a concrete interpretation.

In both Ekman's and Jeffreys' solutions, the current is uniform in the horizontal direction and does not vanish at the coast itself. To

1. Ark. f. Mat. Astr. och Fys., Bd. **2**, Nr. 11, 1905.
2. Phil. Mag. **46**, 114 (1923).

remove this weakness Proudman[1] and Horrocks[2] recently tried to make the normal velocity at the coast zero with a sine series expansion, but they did not succeed in the case of uniform wind. In reality, the weakness is due to the neglect of vertical motion, and is unavoidable so long as the vertical motion is not taken into account, whatever method of solution may be used. In the immediate vicinity of the coast, the vertical motion may be rather more significant than the horizontal current, and so it is obviously impossible to obtain the complete solution of the problem without taking into consideration the vertical motion.

As to the effect of the land upon the density current, one[3] of the present writers showed some special cases in his previous papers. There is yet, however, no general treatment of it.

The coast effect of the barometric current was treated in Proudman's and Horrocks' papers, only when no current can exist at the bottom but not in general.

In this paper, therefore, we intend to supply some of the deficiencies in the above-mentioned works and to advance a step in the as yet unexplored domain.

Choose the co-ordinate axes such that the z-axis is directed vertically downwards and let the following notation be used:

T=the tangential stress of the wind at the sea surface,

γ=the slope of the free surface,

ϕ=the angle of γ counter-clockwise from the direction of T,

H=the depth of the sea,

ρ=the density of the sea water,

μ=the coefficient of viscosity of the water,

$\nu=\mu/\rho$,

v_x, v_y, v_z=the component velocities of the current in the direction of the co-ordinate axes,

$w=v_x+iv_y$=the horizontal velocity of the current in the form of a complex variable,

t=the time,

ω=the angular velocity of the earth's rotation,

λ=the latitude of the place under consideration,

$\bar{\omega}=\omega\sin\lambda$,

$k=\sqrt{\bar{\omega}/\nu}=\sqrt{\rho\omega\sin\lambda/\mu}$,

1. Proc. London Math. Soc., Ser. 2, **24**, 140 (1924)
2. Proc. Roy. Soc. London, **115**, 170 (1927)
3. Nomitsu, These Memoirs, A. **16**, 383 and 397 (1933)

$D=\pi/k$=the depth of the frictional influence.

Furthermore, for the present let the vertical velocity be neglected as compared with the horizontal, admitting some error in the immediate neighbourhood of the coast. The correction for the error we shall deal with in another paper.

I. Coast Effect upon the Wind Current

§ 1. Case of No Bottom-current (Case of Ekman)

This is the problem investigated by Ekman, and many diagrams of current-distribution are given in his paper[1] and reproduced also in Krümmel's Handbuch[2]. The slope of the sea surface, however, is not much discussed, and we shall here take that point as the chief subject of our study.

When a drift current caused by wind-stress T is checked by a land, the water will accumulate at one side and a surface-slope γ must arise.

If we take the direction of the wind as the y-axis and denote by ϕ the angle which γ makes with the y-axis counter-clockwise, the equation of motion in the steady state will take the form

$$\frac{\partial^2 w}{\partial z^2} - 2ik^2 w + i\frac{g\gamma}{\nu}e^{i\phi} = 0, \tag{1}$$

and the boundary conditions will be

at the surface $\quad |\partial w/\partial z|_{z=0} = -iT/\mu, \qquad (2)$

and at the bottom $\quad |w|_{z=H} = 0. \qquad (3)$

In eq. (1) γ is not yet known, but even so the solution which satisfies (1), (2) and (3) is obviously

w=" drift current "[3] + " slope current "[4]

$$= \frac{iT}{\mu a}\frac{\sinh a(H-z)}{\cosh aH} + \frac{g\rho\gamma e^{i\phi}}{2\mu k^2}\left[1 - \frac{\cosh az}{\cosh aH}\right], \tag{4}$$

where $\quad a=(1+i)k, \quad k=\sqrt{\varpi/\nu}=\sqrt{\rho\omega\sin\lambda/\mu}$.

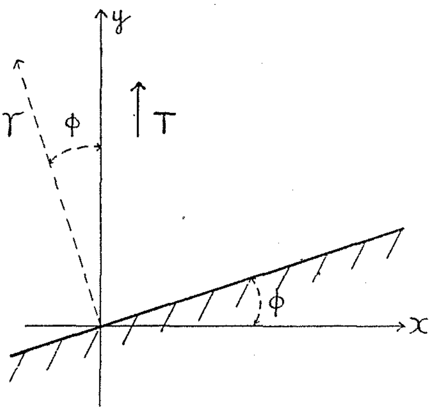
Thus, if we know the value of γ, the current w is also known. Ekman's diagrams correspond to this solution, and the numerical values

1. loc. cit., Pl. I, Figs. 1–19.
2. Handbuch der Ozeanographie, II, Figs. 150–160.
3. These Memoirs, A. **16**, 164 (1933), eq. (6), (7).
4. Ditto, p. 206, eq. (7).

are inserted in our Table 4 ($\xi=0$). To determine the slope γ we use the equation of continuity.

(A) *A long straight coast*:— When the ocean is bounded by a long straight coast and is semi-infinite in extent, and when the wind is uniform all over the sea surface or *depends only upon the distance* from the sea coast, the surface slope γ will be perpendicular to the coast, and consequently ϕ is equal to the angle which the coast makes with the x-axis (Fig. 1).

Fig. 1



Let w_n and S_n be the horizontal current and the total flow normal to the coast respectively, then the equation of continuity in the steady state may be put in the form

$$S_n=\int_0^H w_n dz=0. \qquad (5_a)$$

Calculating S_n directly from (4) or from Ekman's formulae for the total flow of drift current and of slope current, we get by (5_a)

$$\frac{T}{\mu k^2}\left[\frac{\sinh kH \sin kH}{\cosh 2kH+\cos 2kH}\cos\phi - \frac{1}{2}\,\frac{\cosh 2kH+\cos 2kH-2\cosh kH\cos kH}{\cosh 2kH+\cos 2kH}\sin\phi\right] + \frac{g\rho\gamma}{4\mu k^3}\,\frac{\sinh 2kH-\sin 2kH}{\cosh 2kH+\cos 2kH}=0$$

or

$$\gamma=-\frac{2kT}{g\rho}\times \frac{\cos\phi(2\sinh kH\sin kH)-\sin\phi(\cosh 2kH+\cos 2kH-2\cosh kH\cos kH)}{\sinh 2kH-\sin 2kH} \qquad (6)$$

It is here to be noticed that both γ and w are proportional to T. Thus, when $T=$constant as Ekman assumes, γ also must be constant so that the sea surface becomes an inclined plane; and when T is a sine function of the distance (n) from the coast as Proudman assumes, then the surface elevation $\zeta=\int\gamma dn$ will take the form of a cosine function.

The above solution, however, involves one weak point that the current w_n does not vanish at the coast itself unless the wind T vanishes there. This weakness is inevitable as long as we neglect the vertical velocity, but the correction for it will be reserved for a future investigation, as stated before.

Equation (6) indicates that the surface-slope γ generally depends upon both the component wind perpendicular to the coast, $T\cos\phi$, and the component parallel to the coast, $T\sin\phi$, and that the shallower the sea, the larger the slope becomes.

In particular, if the sea is very deep ($H>2D$), formula (6) becomes

$$\gamma \doteqdot \frac{2k}{g\rho} T\sin\phi, \qquad (6')$$

which shows that the earth's rotation is most effective in this case and the surface slope depends only upon the component wind parallel to the coast.

On the other hand, if the sea is so shallow ($H<D/10$) that $(kH)^4$ and the higher powers can be neglected compared with 1, then eq. (6) reduces to

$$\gamma \doteqdot \frac{3}{2}\,\frac{T\cos\phi}{g\rho H}. \qquad (6'')$$

That is to say, in such a shallow sea the effect of the rotation of the earth is negligible, and the surface elevation depends only upon the component wind perpendicular to the coast.

For intermediate depths, we give a few examples in Table 1, and Fig. 2 in which $2kT/g\rho$ is taken as unity.

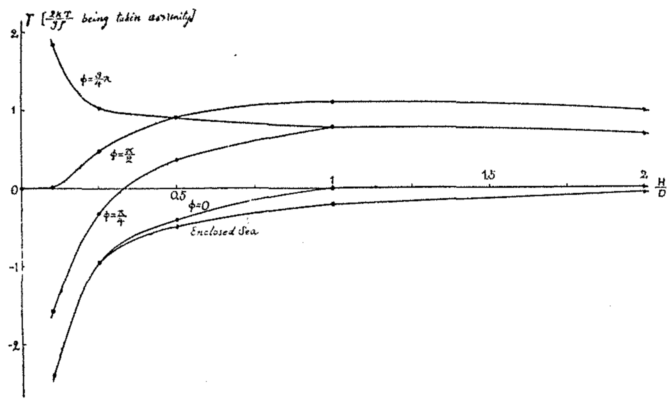
Table 1. Surface slope produced by wind (long straight coast, no bottom-current)

ϕ \ H	$D/10$	$D/4$	$D/2$	D	$2D$	∞
0°	−2.414	−0.944	−0.398	0	0	0
45°	−1.573	−0.318	+0.343	+0.771	+0.704	$1/\sqrt{2}$
90°	+0.002	+0.488	+0.917	+1.090	+0.996	1
135°	+1.841	+1.010	+0.928	+0.771	+0.704	$1/\sqrt{2}$

(B) *Enclosed sea* :— For an enclosed sea, the direction of the slope, ϕ, in eq. (1) is also unknown, whereas two eqs. of continuity

Fig. 2

Surface slope produced by wind according to the depth of sea with no bottom-current.



$$S_x=\int_0^H v_x dz=0, \qquad S_y=\int_0^H v_y dz=0 \qquad (5_b)$$

must be used, instead of the single eq. (5_a).

Calculating S_x and S_y by eq. (4) and putting them equal to zero, we have

$$S_x=\frac{T}{2\mu k^2}\frac{s_1}{\cosh 2kH+\cos 2kH}+\frac{g\rho\gamma}{4\mu k^3}\frac{s_3\cos\phi-s_4\sin\phi}{\cosh 2kH+\cos 2kH}=0,$$

$$S_y=\frac{T}{2\mu k^2}\frac{s_2}{\cosh 2kH+\cos 2kH}\cdot+\frac{g\rho\gamma}{4\mu k^3}\frac{s_3\sin\phi+s_4\cos\phi}{\cosh 2kH+\cos 2kH}=0,$$

where we put

$$\left.\begin{aligned}
s_1&=\cosh 2kH+\cos 2kH-2\cosh kH\cos kH,\\
s_2&=2\sinh kH\sin kH,\\
s_3&=2kH(\cosh 2kH+\cos 2kH)-(\sinh 2kH+\sin 2kH),\\
s_4&=\sinh 2kH-\sin 2kH.
\end{aligned}\right\} \qquad (7)$$

From the above equations we can construct the total flow in two other directions, along and perpendicular to the slope γ, and then we get

$$\gamma=-\frac{2kT}{g\rho}\frac{s_2\cos\phi-s_1\sin\phi}{s_4}=-\frac{2kT}{g\rho}\frac{s_1\cos\phi+s_2\sin\phi}{s_3}, \qquad (8)$$

and therefore $$\tan\phi=\frac{s_2s_3-s_1s_4}{s_1s_3+s_2s_4}. \qquad (9)$$

In a very deep sea

$$\left.\begin{aligned} &\tan\phi \doteqdot 1/(2kH-1), \\ &\gamma \doteqdot -\frac{2kT}{g\rho\sqrt{1+(2kH-1)^2}}, \end{aligned}\right\} \qquad (8')$$

and in very shallow water

$$\phi \doteqdot 0, \quad \gamma \doteqdot -\frac{3}{2}\frac{T}{g\rho H}. \qquad (8'')$$

The last equation coincides with that for a long straight coast and also is consistent with the experimental formula obtained by Colding[1] from the observations in the Baltic sea.

For intermediate depths, the values of γ and ϕ are given numerically in Table 2, taking $2kT/g\rho$ as unity.

Table 2. Surface slope produced by wind in an enclosed sea with no bottom-current.

H	$D/10$	$D/4$	$D/2$	D	$2D$	∞
ϕ	$0°$	$-1°$	$-4°.5$	$-10°.7$	$-5°.0$	0
γ	-2.414	-0.950	-0.469	-0.202	-0.065	0

§ 2 Case of No Bottom-friction

If, instead of (3), we assume

$$\left|\partial w/\partial z\right|_{z=H} = 0 \qquad (10)$$

as the bottom-condition, the solution that satisfies (1), (2) and (10) is clearly

w = "drift current" + "slope current" with no bottom-friction[2]

$$\left.\begin{aligned} &= \frac{iT}{\mu a}\frac{\cosh a(H-z)}{\sinh aH} + V_g e^{i\phi}, \\ \text{where} \quad &V_g \equiv g\rho\gamma/2\mu k^2 \equiv g\gamma/2\bar{\omega}. \end{aligned}\right\} \qquad (11)$$

(A) *Long straight coast*:— In a semi-infinite sea bounded by a long straight coast, the equation of continuity (5_a) becomes

1. Ekman, loc. cit., eq. (29); Krümmel, loc. cit. p. 534.
2. These Memoirs, A, **16**, 279 (1933), eqs. (6), (7); 334 (1933), eq. (2).

$$S_n = -\frac{T}{2\mu k^2}\sin\phi = 0.$$

This is possible only when ϕ is zero, that is, the wind T is perpendicular to the coast, but γ can not be determined here and it must be obtained as the limiting value when the bottom-friction in § 3 tends to zero. As explained later, it has the value

$$\gamma = -\frac{2kT}{g\rho}\cdot\frac{\sinh kH\cos kH + \cosh kH\sin kH}{\cosh 2kH - \cos 2kH}.$$

If $\phi \neq 0$, the continuity-equation is impossible. This means simply that the water accumulates to one side to produce such a large slope that the vertical motion can not be neglected.

(B) *Enclosed sea* :— In this case, substituting (11) in (5b) we have

$$S_x = \frac{T}{2\mu k^2} + \frac{g\rho\gamma}{2\mu k^2}H\cos\phi = 0,$$

$$S_y = \frac{g\rho\gamma}{2\mu k^2}H\sin\phi = 0,$$

which give

$$\left.\begin{aligned}\phi &= 0,\\ \gamma &= -T/g\rho H.\end{aligned}\right\} \qquad (12)$$

Then the current w is obtained as a pure drift current with no bottom-friction plus a constant gradient current

$$V_g = -\frac{T}{\mu k}\frac{1}{2kH}.$$

The numerical values of w in eq. (11) for various depths of sea are given in Table 4 and Figs, 4 and 8 with the indication " $\xi = \infty$ ". In all cases the direction of the bottom current is observed to lie in the third quadrant.

§ 3. Case of Finite Bottom-friction

Let the bottom condition be

$$-\mu\left|\frac{\partial w}{\partial z}\right|_{z=H} = f\rho V_H^2 e^{i\theta}, \qquad (13)$$

where f is the coefficient of bottom-friction, and V_H and θ denote the magnitude of the bottom-velocity and the angle its direction makes with the x-axis, so that

$$\left|w\right|_{z=H} = V_H e^{i\theta}. \qquad (14)$$

Then the solution of (1) which satisfies (2) and (13) is obviously $w=$ "wind current with no bottom-friction" + usual "correction term[1]"

$$\left.\begin{aligned} &= \frac{iT}{\mu a}\frac{\cosh a(H-z)}{\sinh aH} + V_g e^{i\phi} - \frac{f\rho V_H^2 e^{i\theta}}{\mu a}\frac{\cosh az}{\sinh aH}, \\ &V_g = \frac{g\gamma}{2\bar{\omega}} = \frac{g\rho\gamma}{2\mu k^2}, \qquad a=(1+i)k, \end{aligned}\right\} \quad (15)$$

and relation (14) becomes

$$V_H e^{i\theta} = \frac{iT}{\mu a}\frac{1}{\sinh aH} + V_g e^{i\phi} - \frac{f\rho V_H^2 e^{i\theta}}{\mu a}\coth aH,$$

which gives two equations when separated into real and imaginary parts. Just as in our previous paper[2] on pure drift current with a finite bottom friction, we have

$$\left.\begin{aligned} f\rho a V_H^2 + \mu kc V_H - \mu k V_g[c\cos(\theta-\phi) + d\sin(\theta-\phi)] - T\sin\theta = 0, \\ f\rho b V_H^2 + \mu kd V_H - \mu k V_g[d\cos(\theta-\phi) - c\sin(\theta-\phi)] - T\cos\theta = 0, \end{aligned}\right\} \quad (16)$$

where

$$\left.\begin{aligned} a &= \cosh kH \cos kH, \\ b &= \sinh kH \sin kH, \\ c &= \sinh kH \cos kH - \cosh kH \sin kH, \\ d &= \sinh kH \cos kH + \cosh kH \sin kH. \end{aligned}\right\} \quad (17)$$

(A) *Long straight coast* (*Case of Jeffreys*) :— This is the case investigated by Jeffreys, and the nature of the current and the surface slope in a very deep or a very shallow sea have been made clear. For a sea of intermediate depth, however, Jeffreys' formula is not complete in form and is inadequate for practical use. Hence we shall give a new form of solution which is convenient for numerical calculations in general.

Since ϕ is equal to the angle which the coast makes with the x-axis, equation of continuity (5_a) gives

$$S_n = 0 = -\frac{T}{2\mu k^2}\sin\phi + \frac{f\rho V_H^2}{2\mu k^2}\cos(\theta-\phi),$$

or

$$f\rho V_H^2\cos(\theta-\phi) = T\sin\phi = f_a\sigma Q^2\sin\phi, \quad (18)$$

where Q is the velocity of the wind, and f_a and σ are the coefficient

1. These Memoirs, A, **16**, 316 (1933), eq. (15).
2. Ditto, 313 (1933), eq. (10)

of surface friction and the density of the atmosphere respectively. This relation between V_H and θ denotes the locus of the end of the vector of the bottom velocity, corresponding to the important relation of Taylor for the surface wind.

Thus, for three unknowns V_H, θ, V_g (or γ) in eq. (15) we have three equations (16) and (18), and we can determine them completely.

For convenience, put

$$\xi=\frac{\mu k}{\sqrt{f f_a \rho\sigma}\, Q}, \qquad \eta=\sqrt{\frac{f\rho}{f_a\sigma}}\cdot\frac{V_H}{Q}, \qquad \zeta=\sqrt{\frac{f\rho}{f_a\sigma}}\cdot\frac{V_g}{Q}, \tag{19}$$

then eq. (18) becomes

$$\eta^2\cos(\theta-\phi)=\sin\phi, \tag{18'}$$

and eq. (16) reduces to

$$\left.\begin{aligned} a\eta^2+c\xi\eta-\xi\zeta[c\cos(\theta-\phi)+d\sin(\theta-\phi)]-\sin\theta=0,\\ b\eta^2+d\xi\eta-\xi\zeta[d\cos(\theta-\phi)-c\sin(\theta-\phi)]-\cos\theta=0.\end{aligned}\right\} \tag{16'}$$

To determine 3 unknowns η, ζ, θ as functions of the known quantity ξ, we put

$$\left.\begin{aligned} a&=p\cos\varphi, & b&=p\sin\varphi, & p&=\sqrt{a^2+b^2}\,,\\ c&=q\cos\psi, & d&=q\sin\psi, & q&=\sqrt{c^2+d^2}\,,\end{aligned}\right\} \tag{20}$$

then (16′) becomes

$$\left.\begin{aligned} \eta^2p\cos\varphi+\xi\eta q\cos\psi-\xi\zeta q\cos(\theta-\phi-\psi)-\sin\theta=0,\\ \eta^2p\sin\varphi+\xi\eta q\sin\psi+\xi\zeta q\sin(\theta-\phi-\psi)-\cos\theta=0.\end{aligned}\right\} \tag{16''}$$

Multiply the above two equations by $\sin(\theta-\phi-\psi)$ and $\cos(\theta-\phi-\psi)$ respectively and add, and thus eliminate ζ. Then there results

$$\eta^2p\sin(\theta-\phi-\psi+\varphi)+\xi\eta q\sin(\theta-\phi)-\cos(\phi+\psi)=0,$$

or

$$\xi=\frac{\cos(\phi+\psi)-\eta^2p\sin(\theta-\phi-\psi+\varphi)}{\eta q\sin(\theta-\phi)}. \tag{21}$$

Multiply again the two equations in (16″) by $\sin\psi$ and $\cos\psi$ respectively and subtract one from the other, and then we have

$$\eta^2p\sin(\psi-\varphi)-\xi\zeta q\sin(\theta-\phi)+\cos(\theta+\psi)=0,$$

or

$$\zeta=\frac{\cos(\theta+\psi)+\eta^2p\sin(\psi-\varphi)}{\xi q\sin(\theta-\phi)}. \tag{22}$$

Now, for every possible value of θ we can get the corresponding η by (18′), ξ by (21) and ζ by (22). Since $0 \leqq \phi \leqq 180°$, the possible limit of θ will be given by (18′); i. e.,

$$\cos(\theta-\phi)>0,$$

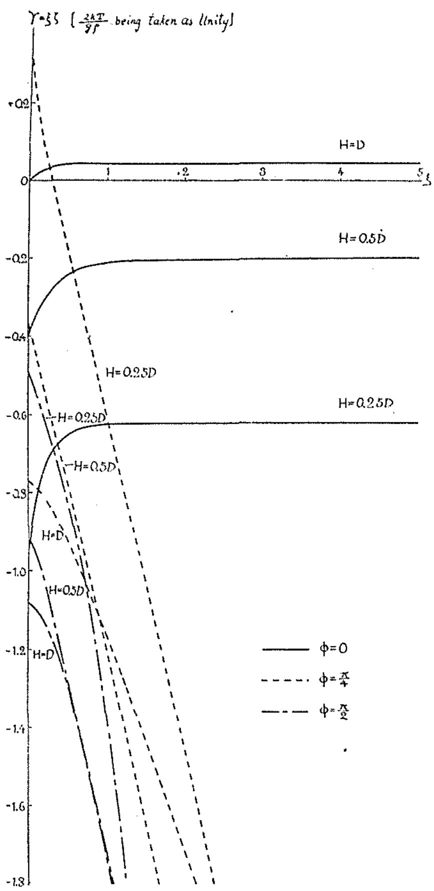
$$\text{or} \qquad -\pi/2<\theta-\phi<\pi/2,$$

which shows that the bottom current has a leeside direction as stated by Jeffreys. For more definite limits of possible θ, it is enough to obtain the values for $\xi=0$, (no bottom-current) and for $\xi=\infty$ (no bottom-friction). In the special case when $\phi=0$, eq. (18′) shows that θ has a constant value $3\pi/2$ for all values of ξ.

Table 3 gives the corresponding values of η, ζ, θ for every possible value of ξ. As ζ is a quantity proportional to the surface slope ($\gamma=\xi\zeta$, if we take $\frac{2kT}{g\rho}$ as unit as before), we plot it in Fig. 3.

Fig. 3

Surface slope produced by wind in a sea with a long straight coast according to the bottom-friction.



Having determined the values of η, ζ and θ for a given ξ, we can at once calculate the current w, as eq. (15) can be written

$$w=\frac{iT}{\mu a}\frac{\cosh a(H-z)}{\sinh aH}+\frac{T}{\mu k}\xi\zeta e^{i\phi}-\frac{T}{\mu k}\eta^2 e^{i\theta}\frac{\cosh az}{\sinh aH}. \tag{15′}$$

It will be very useful to note here that the 3 parts of the above equation are nothing but a drift- and a slope-current and a correction term respectively, and we know the numerical values of them from our previous papers, except the difference of the constant coefficients. Thus Table 4 is very easily obtained, and the graphical representation of it gives Figs. 4–6, $T/\mu k$ being taken as unit velocity.

Table 3. Corresponding values of ξ, η, ζ and θ for wind currents (long straight coast)

$\phi=0$ ($\theta=270°$)								
$H=0.25D$			$H=0.5D$			$H=D$		
ξ	η	$\xi\zeta$	ξ	η	$\xi\zeta$	ξ	η	$\xi\zeta$
0	0.698	−0.938	0.000	0.659	−0.398	0	0.294	0.000
0.056	0.600	−0.854	0.057	0.600	−0.364	0.047	0.250	0.012
0.124	0.500	−0.781	0.168	0.500	−0.314	0.117	0.200	0.023
0.213	0.400	−0.722	0.314	0.400	−0.272	0.214	0.150	0.032
0.345	0.300	−0.676	0.525	0.300	−0.240	0.383	0.100	0.038
0.584	0.200	−0.643	0.903	0.200	−0.217	0.501	0.080	0.040
1.244	0.100	−0.624	1.944	0.100	−0.204	0.691	0.060	0.042
1.567	0.080	−0.621	2.451	0.080	−0.202	1.063	0.040	0.043
2.104	0.060	−0.619	3.289	0.060	−0.201	2.155	0.020	0.043
3.160	0.040	−0.618	4.957	0.040	−0.200	4.325	0.010	0.043
6.355	0.020	−0.617	9.941	0.020	−0.199	5.409	0.008	0.043
12.700	0.010	−0.617	19.895	0.010	−0.199	7.214	0.006	0.043
15.880	0.008	−0.617	24.871	0.008	−0.199	10.823	0.004	0.043
21.200	0.006	−0.617	33.164	0.006	−0.199	21.649	0.002	0.043
∞	0.000	−0.617	∞	0.000	−0.199	43.299	0.001	0.043
						∞	0.000	0.043

Table 3. (Continued)

$\phi=\frac{\pi}{2}$											
$H=0.25D$				$H=0.5D$				$H=D$			
ξ	η	θ	$\xi\zeta$	ξ	η	θ	$\xi\zeta$	ξ	η	θ	$\xi\zeta$
0	1.006	98.°4	−0.491	0	1.072	119°5	−0.919	0	1.215	137.°4	−1.090
0.014	1.005	98°	−0.493	0.030	1.064	118°	−0.929	0.029	1.200	136°	−1.090
0.065	1.004	97°	−0.523	0.078	1.054	116°	−0.954	0.074	1.179	134°	−1.093
0.130	1.003	96°	−0.560	0.129	1.046	114°	−0.985	0.120	1.160	132°	−1.101
0.227	1.002	95°	−0.623	0.190	1.039	112°	−1.026	0.169	1.142	130°	−1.113
0.357	1.001	94°	−0.712	0.261	1.032	110°	−1.078	0.199	1.127	128°	−1.133
0.591	1.001	93°	−0.891	0.347	1.026	108°	−1.148	0.252	1.112	126°	−1.158
0.886	1.000	92°	−1.236	0.454	1.021	106°	−1.235	0.335	1.098	124°	−1.190
1.944	1.000	91°	−2.286	0.589	1.016	104°	−1.353	0.401	1.086	122°	−1.229
∞	1.000	90°	−∞	0.770	1.011	102°	−1.515	0.483	1.075	120°	−1.277
				1.023	1.008	100°	−1.746	0.556	1.064	118°	−1.333
				1.393	1.005	98°	−2.112	0.649	1.054	116°	−1.405
				2.000	1.003	96°	−2.697	0.755	1.046	114°	−1.489
				3.243	1.001	94°	−3.927	0.879	1.039	112°	−1.594
				6.971	1.000	92°	−7.637	1.025	1.032	110°	−1.726
				∞	1.000	90°	−∞	1.205	1.026	108°	−1.885
								1.422	1.021	106°	−2.087
								1.707	1.016	104°	−2.355
								2.090	1.011	102°	−2.713
								2.611	1.008	100°	−3.215
								3.393	1.005	98°	−3.999
								∞	1.000	90°	−∞

Table 3.

(Continued)

$\phi=\frac{\pi}{4}$											
$H=0.25D$				$H=0.5D$				$H=D$			
ξ	η	θ	$\xi\zeta$	ξ	η	θ	$\xi\zeta$	ξ	η	θ	$\xi\zeta$
0	0.865	26°	0.322	0	0.845	52.°5	−0.370	0	1.000	90°	−0.771
0.029	0.860	28°	0.287	0.022	0.844	52°	−0.385	0.037	0.982	88°	−0.771
0.063	0.856	30°	0.241	0.091	0.843	51°	−0.434	0.079	0.968	84°	−0.776
0.104	0.852	32°	0.183	0.183	0.842	50°	−0.513	0.125	0.955	82°	−0.788
0.167	0.848	34°	0.119	0.330	0.842	49°	−0.629	0.173	0.939	80°	−0.797
0.257	0.846	36°	0.036	0.570	0.842	48°	−0.829	0.233	0.929	78°	−0.816
0.398	0.844	38°	−0.098	1.044	0.841	47°	−1.227	0.297	0.916	76°	−0.835
0.644	0.843	40°	−0.261	2.080	0.841	46°	−2.432	0.368	0.908	74°	−0.864
1.227	0.842	42°	−0.829	∞	0.840	45°	−∞	0.442	0.900	72°	−0.898
4.000	0.841	44°	−3.275					0.541	0.892	70°	−0.939
∞	0.841	45°	−∞					0.650	0.883	68°	−0.986
								0.807	0.875	66°	−1.044
								0.929	0.870	64°	−1.120
								1.103	0.865	62°	−1.210
								1.336	0.860	60°	−1.333
								1.600	0.855	58°	−1.481
								1.958	0.852	56°	−1.681
								2.451	0.848	54°	−1.951
								3.167	0.846	52°	−2.364
								4.243	0.845	50°	−2.991
								6.260	0.843	48°	−4.156
								∞	0.841	45°	−∞

Table 4.

Wind currents in a sea with a long straight coast

$\phi=0$								
$H=0.25D$								
	$\xi=0$		$\xi=0.056$		$\xi=0.213$		$\xi=\infty$	
$\frac{z}{H}$	v_x	v_y	v_x	v_y	v_x	v_y	v_x	v_y
0	0.021	0.197	0.026	0.213	0.035	0.239	0.040	0.259
0.1	0.020	0.125	0.025	0.140	0.032	0.166	0.039	0.185
0.2	0.018	0.055	0.021	0.069	0.030	0.092	0.036	0.110
0.3	0.016	0.013	0.019	0.026	0.027	0.046	0.031	0.060
0.4	0.012	−0.020	0.014	−0.014	0.021	0.000	0.025	0.010
0.5	0.007	−0.049	0.009	−0.045	0.016	−0.037	0.019	−0.033
0.6	0.003	−0.062	0.004	−0.063	0.009	−0.065	0.012	−0.067
0.7	0.002	−0.064	0.003	−0.067	0.006	−0.084	0.007	−0.094
0.8	0.000	−0.055	0.001	−0.070	0.004	−0.094	0.004	−0.113
0.9	0.000	−0.026	0.000	−0.059	0.002	−0.095	0.001	−0.125
1.0	0·000	0.000	0.000	−0.036	0.000	−0.086	0.000	−0.128

Table 4.
(Continued)

$\phi=0$								
$H=0.5D$								
	$\xi=0$		$\xi=0.057$		$\xi=0.168$		$\xi=\infty$	
$\frac{z}{H}$	v_x	v_y	v_x	v_y	v_x	v_y	v_x	v_y
0	0.147	0.373	0.167	0.387	0.195	0.409	0.260	0.459
0.1	0.139	0.229	0.159	0.243	0.186	0.264	0.250	0.313
0.2	0.120	0.112	0.139	0.125	0.165	0.145	0.225	0.190
0.3	0.094	0.020	0.111	0.031	0.135	0.047	0.189	0.086
0.4	0.069	−0.048	0.083	−0.040	0.104	−0.027	0.150	0.002
0.5	0.046	−0.092	0.057	−0.087	0.074	−0.080	0.111	−0.064
0.6	0.026	−0.115	0.035	−0.116	0.046	−0.115	0.074	−0.116
0.7	0.011	−0.117	0.019	−0.123	0.026	−0.132	0.043	−0.154
0.8	0.004	−0.096	0.008	−0.110	0.011	−0.131	0.020	−0.179
0.9	0.001	−0.059	0.003	−0.081	0.003	−0.115	0.005	−0.194
1.0	0.000	−0.000	0.001	−0.033	0.000	−0.084	0.000	−0.199

Table 4.
(Continued)

$\phi=0$								
$H=D$								
	$\xi=0$		$\xi=0.047$		$\xi=0.117$		$\xi=\infty$	
$\frac{z}{H}$	v_x	v_y	v_x	v_y	v_x	v_y	v_x	v_y
0	0.498	0.498	0.513	0.499	0.523	0.500	0.545	0.502
0.1	0.458	0.233	0.471	0.234	0.483	0.235	0.505	0.237
0.2	0.371	0.057	0.384	0.059	0.396	0.059	0.419	0.062
0.3	0.272	−0.047	0.285	−0.045	0.295	−0.044	0.317	−0.040
0.4	0.182	−0.095	0.192	−0.093	0.202	−0.091	0.221	−0.087
0.5	0.109	−0.109	0.118	−0.106	0.127	−0.104	0.143	−0.100
0.6	0.056	−0.099	0.064	−0.097	0.071	−0.095	0.084	−0.092
0.7	0.024	−0.079	0.030	−0.079	0.035	−0.077	0.044	−0.076
0.8	0.007	−0.053	0.011	−0.055	0.013	−0.056	0.018	−0.059
0.9	0.001	−0.028	0.002	−0.033	0.003	−0.039	0.003	−0.048
1.0	0.000	−0.000	0.000	−0.012	0.000	−0.023	0.000	−0.043

Table 4.

(Continued)

$\phi=\pi/4$						
$H=0.25D$						
	$\xi=0$		$\xi=0.104$		$\xi=1.227$	
$\frac{z}{H}$	v_x	v_y	v_x	v_y	v_x	v_y
0.0	0.314	0.489	0.370	0.559	1.019	1.231
0.1	0.310	0.414	0.366	0.484	1.016	1.156
0.2	0.298	0.337	0.354	0.406	1.004	1.076
0.3	0.280	0.284	0.337	0.351	0.989	1.021
0.4	0.253	0.227	0.311	0.293	0.965	0.959
0.5	0.223	0.177	0.283	0.242	0.939	0.902
0.6	0.185	0.134	0.247	0.196	0.906	0.852
0.7	0.146	0.094	0.210	0.152	0.873	0.813
0.8	0.100	0.060	0.168	0.113	0.836	0.760
0.9	0.061	0.028	0.124	0.079	0.796	0.720
1.0	0.000	0.000	0.077	0.046	0.756	0.681

Table 4.

(Continued)

$\phi=\pi/4$						
$H=0.5D$						
	$\xi=0$		$\xi=0.183$		$\xi=1.044$	
$\frac{z}{H}$	v_x	v_y	v_x	v_y	v_x	v_y
0.0	0.691	0.920	0.804	1.020	1.319	1.526
0.1	0.680	0.774	0.789	0.874	1.305	1.380
0.2	0.640	0.648	0.749	0.749	1.265	1.257
0.3	0.580	0.540	0.688	0.642	1.205	1.150
0.4	0.507	0.448	0.616	0.547	1.132	1.060
0.5	0.424	0.369	0.533	0.474	1.048	0.986
0.6	0.337	0.296	0.443	0.404	0.959	0.919
0.7	0.251	0.228	0.354	0.338	0.867	0.857
0.8	0.158	0.160	0.263	0.273	0.775	0.795
0.9	0.076	0.086	0.179	0.203	0.689	0.729
1.0	0.000	0.000	0.102	0.123	0.608	0.655

Table 4.
(Continued)

$\phi=\pi/4$						
$H=D$						
	$\xi=0$		$\xi=0.650$		$\xi=1.600$	
$\frac{z}{H}$	v_x	v_y	v_x	v_y	v_x	v_y
0.0	1.090	1.090	1.242	1.219	1.582	1.561
0.1	1.053	0.830	1.200	0.958	1.551	1.300
0.2	0 944	0.666	1.105	0.796	1.462	1.127
0.3	0.817	0.581	0.989	0.713	1.346	1.055
0.4	0.682	0.550	0.869	0.689	1.232	1.034
0.5	0.545	0.545	0.750	0.697	1.123	1.047
0.6	0.458	0.540	0.549	0.716	1.007	1.087
0.7	0.274	0.509	0.507	0.722	0.890	1.097
0.8	0.146	0.424	0.381	0.692	0.762	1.086
0.9	0.064	0.260	0.256	0.595	0.629	1.018
1.0	0.000	0.000	0.152	0.420	0.502	0.869

Table 4.
(Continued)

$\phi=\pi/2$						
$H=0.25D$						
	$\xi=0$		$\xi=0.130$		$\xi=0.491$	
$\frac{z}{H}$	v_x	v_y	v_x	v_y	v_x	v
0.0	0.022	0.793	0.026	0.882	0.033	1.210
0.1	0.021	0.713	0.025	0.804	0.031	1.132
0.2	0.016	0.626	0.019	0.717	0.026	1.046
0.3	0.011	0.557	0.013	0.648	0.019	0.976
0.4	0.003	0.479	0.005	0.570	0.009	0.899
0.5	−0.003	0.401	−0.005	0.493	−0.002	0.822
0.6	−0.011	0.324	−0.013	0.416	−0.012	0.745
0.7	−0.018	0.245	−0.017	0.337	−0.021	0.667
0.8	−0.020	0.167	−0.020	0.261	−0.028	0.589
0.9	−0.016	0.088	−0.019	0.182	−0.028	0.511
1.0	0.000	0.000	−0.012	0.104	−0.029	0.433

Table 4.
(Continued)

$\phi=\pi/2$						
$H=0.5D$						
	$\xi=0$		$\xi=0.261$		$\xi=1.023$	
$\frac{z}{H}$	v_x	v_y	v_x	v_y	v_x	v_y
0.0	0.147	1.464	0.188	1.664	0.225	2.368
0.1	0.135	1.310	0.175	1.511	0.211	2.216
0.2	0.104	1.164	0.140	1.368	0.174	2.076
0.3	0.059	1.021	0.091	1.230	0.122	1.942
0.4	0.011	0.882	0.035	1.097	0.057	1.815
0.5	−0.035	0.743	−0.022	0.966	−0.010	1.690
0.6	−0.074	0.603	−0.074	0.834	−0.074	1.566
0.7	−0.096	0.459	−0.114	0.698	−0.129	1.437
0.8	−0.097	0.311	−0.136	0.556	−0.169	1.302
0.9	−0.066	0.158	−0.131	0.409	−0.187	1.161
1.0	0.000	0.000	−0.093	0.255	−0.176	1.008

Table 4.
(Continued)

$\phi=\pi/2$						
$H=D$						
	$\xi=0$		$\xi=0.335$		$\xi=1.025$	
$\frac{z}{H}$	v_x	v_y	v_x	v_y	v_x	v_y
0.0	0.498	1.682	0.516	1.765	0.530	2.287
0.1	0.449	1.417	0.469	1.500	0.484	2.025
0.2	0.334	1.238	0.359	1.329	0.377	1.857
0.3	0.189	1.125	0.220	1.225	0.244	1.761
0.4	0.037	1.050	0.074	1.168	0.103	1.717
0.5	−0.108	0.981	−0.067	1.122	−0.036	1.689
0.6	−0.232	0.892	−0.196	1.066	−0.169	1.657
0.7	−0.315	0.757	−0.298	0.970	−0.286	1.590
0.8	−0.332	0.563	−0.357	0.818	−0.377	1.469
0.9	−0.243	0.304	−0.342	0.594	−0.416	1.265
1.0	0.000	0.000	−0.206	0.307	−0.363	0.998

The direction (θ) and the speed (V_H) of the bottom-current can be seen clearly in the figures.

Furthermore, from the figures we observe that:

1) The effect of the wind perpendicular to the coast does not differ greatly for all values of ξ, while the wind parallel to the coast

Fig. 4

Wind currents in a sea with a long straight coast

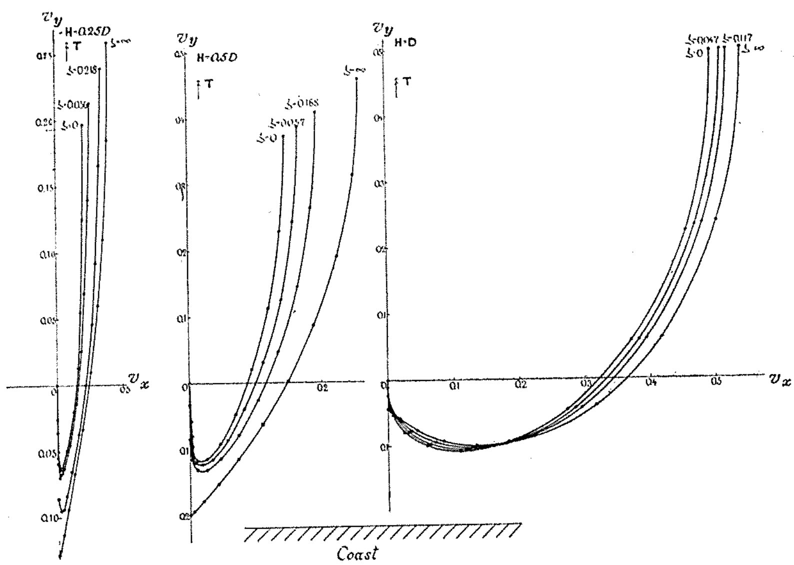
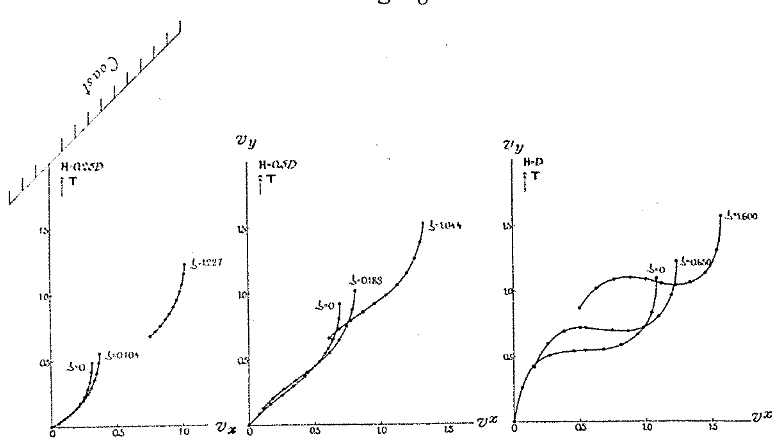


Fig. 5

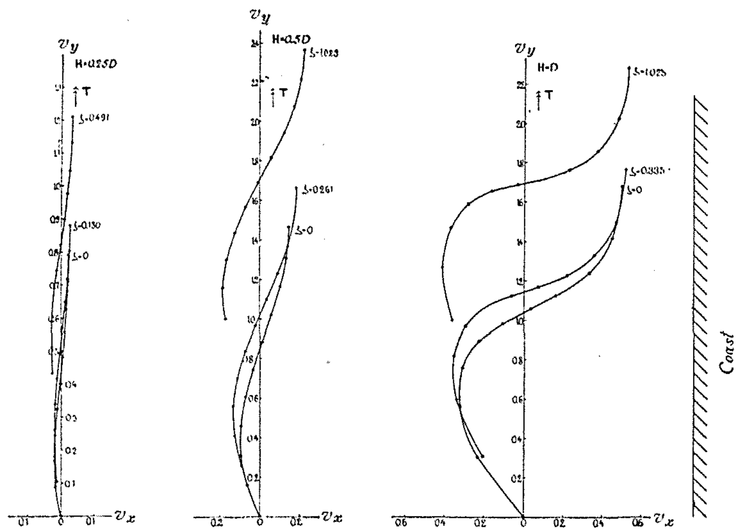


generates actively the component current in that direction and shifts the current-diagram as a whole leewards along the coast.

2) Consequently, if a component wind parallel to the coast exists, the surface slope increases indefinitely as ξ increases, and for values of ξ beyond a certain limit the vertical motion will become no longer negligible.

Fig. 6

Wind currents in a sea with a long straight coast



(B) *Enclosed sea:*— This case differs from the preceding in that ϕ is unknown and that the equation of continuity takes form (5_b) instead of (5_a).

Taking the total flow due to the three parts of w in (15) we have

$$\left.\begin{aligned} S_x &= \frac{T}{2\mu k^2} + V_g\cos\phi . H - \frac{f\rho V_H^2}{2\mu k^2}\sin\theta = 0, \\ S_y &= 0 + V_g\sin\phi . H + \frac{f\rho V_H^2}{2\mu k^2}\cos\theta = 0, \end{aligned}\right\} \tag{23}$$

or with ξ, η, ζ

$$\left.\begin{aligned} 1 + 2\xi\zeta kH\cos\phi - \eta^2\sin\theta = 0, \\ 2\xi\zeta kH\sin\phi + \eta^2\cos\theta = 0. \end{aligned}\right\} \tag{23'}$$

Eliminate ζ from these two equations, then we get a formula which corresponds to the total flow in the direction of the slope, i. e.,

$$\left.\begin{aligned} &\eta^2\cos(\theta-\phi)=\sin\phi, \\ \text{or}\quad &\tan\phi=\frac{\cos\theta}{\eta^{-2}-\sin\theta}, \end{aligned}\right\} \tag{24$_a$}$$

Secondly, to get the total flow perpendicular to the slope, multiply the upper and lower equations of (23′) by $\sin\theta$ and $\cos\theta$ respectively and subtract one from the other, then

$$\sin\theta+2\xi\zeta kH\sin(\theta-\phi)-\eta^2=0. \tag{24$_b$}$$

If, on the other hand, we multiply the two eqs. in (16′) by d and c respectively and subtract, we get

$$(ad-bc)\eta^2-\xi\zeta(c^2+d^2)\sin(\theta-\phi)+(c\cos\theta-d\sin\theta)=0,$$

which combined with (24) gives

$$(ad-bc)\eta^2-(c^2+d^2)\frac{\eta^2-\sin\theta}{2kH}+(c\cos\theta-d\sin\theta)=0,$$

or

$$\left[(ad-bc)-\frac{c^2+d^2}{2kH}\right]\eta^2=\left[\left(d-\frac{c^2+d^2}{2kH}\right)\sin\theta-c\cos\theta\right], \tag{25}$$

where

$$\left.\begin{aligned} ad-bc&=\frac{1}{2}(\sinh 2kH+\sin 2kH), \\ c^2+d^2&=\cosh 2kH-\cos 2kH, \\ c&=\sinh kH\cos kH-\cosh kH\sin kH, \\ d&=\sinh kH\cos kH+\cosh kH\sin kH. \end{aligned}\right\}$$

Thus, for any given θ we can calculate the corresponding η with (25), ϕ with (24$_a$), $\xi\zeta$ with either (23′) or (24$_b$), and ξ with (16′). The possible range of θ can be known easily from the extreme cases of no bottom-current ($\xi=0$) and no bottom-friction ($\xi=\infty$).

Table 5 gives the corresponding values of ξ, η, ζ, θ and ϕ thus obtained; and Fig. 7 represents the surface slope corresponding to ξ.

The current distribution for any given ξ can now be determined with (15′) as before. Table 6 and Fig. 8 show the results.

As Jeffreys did not touch an enclosed sea at all, we shall here enumerate its characteristic points:

1) In an enclosed sea, the effect of the bottom-friction is comparatively small, except at the bottom. More particularly, in a very deep sea it is the same for every value of ξ.

2) In a very deep sea the angle of deviation of the surface current from the direction of the wind is 45°, and in a very shallow sea it is 0°.

Fig. 7

Surface slope produced by wind in an enclosed sea

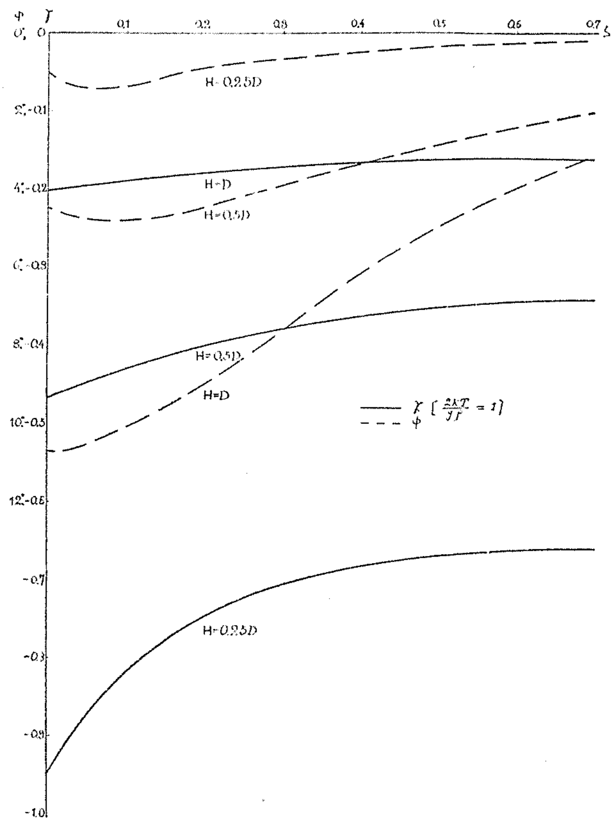


Table 5.

Corresponding values of ξ, η, ζ, θ and ϕ for wind currents in an enclosed sea

$H=0.25D$				
ξ	η	θ	ϕ	$\xi\zeta$
0.000	0.702	266°.7	−1°.0	−0.949
0.009	0.682	266°.5	−1°.2	−0.932
0.031	0.640	266°	−1°.2	−0.900
0.059	0.599	265°.5	−1°.2	−0.865
0.086	0.554	265°	−1°.2	−0.831
0.124	0.503	264°.5	−1°.2	−0.796
0.169	0.449	264°	−1°.0	−0.764
0.234	0.384	263°.5	−0°.8	−0.731
0·337	0.307	263°	−0°.7	−0.696
0.555	0.210	262°.5	−0°.3	−0.664
∞	0.000	262°.1	0°.0	−0.636

Table 5.
(Continued)

$H=0.5D$				
ξ	η	θ	ϕ	$\xi\zeta$
0.000	0.694	256°.1	−4°.5	−0.468
0.004	0.692	256°	−4°.5	−0.465
0.046	0.650	254°	−4°.8	−0.449
0.095	0.606	252°	−4°.8	−0.431
0.154	0.558	250°	−4°.7	−0.413
0.224	0.505	248°	−4°.3	−0.395
0.316	0.444	246°	−3°.8	−0.377
0.448	0.373	244°	−3°.2	−0.358
0.681	0.285	242°	−2°.1	−0.343
1.388	0.159	240°	−0°.7	−0.325
∞	0.000	239°.2	0°.0	−0.318

Table 5.
(Continued)

$H=D$				
ξ	η	θ	ϕ	$\xi\zeta$
0.000	0.583	226°.3	−10°.7	−0.202
0.005	0.581	226°	−10°.7	−0.201
0.027	0.565	224°	−10°.7	−0.198
0.052	0.549	222°	−10°.5	−0.195
0.077	0.532	220°	−10°.3	−0.191
0.103	0.514	218°	−10°.3	−0.187
0.134	0.495	216°	−9°.8	−0.185
0.166	0.474	214°	−9°.4	−0.182
0.201	0.454	212°	−9°.0	−0.179
0.241	0.431	210°	−8°.3	−0.176
0.286	0.406	208°	−7°.7	−0.173
0.335	0.382	206°	−7°.0	−0.171
0.396	0.354	204°	−6°.2	−0.168
0.469	0.323	202°	−5°.3	−0.166
0.567	0.288	200°	−4°.3	−0.164
0.699	0.249	198°	−3°.3	−0.162
0.918	0.202	196°	−2°.3	−0.161
1.386	0.142	194°	−1°.1	−0.160
∞	0.000	192°.1	0°.0	−0.159

3) In a very deep sea the slope of the free surface tends to 0, and in a very shallow sea it lies between $T/g\rho H$ and $1.5\ T/g\rho H$ according to the value of ξ.

II. Barometric Current influenced by Land

§ 4. Case of No Bottom-current

When a barometric pressure-gradient equal to $g\rho\gamma_0$ exists in the y-direction, the equation of motion, ignoring the vertical current, will be

Table 6.

Wind currents in an enclosed sea

$H=0.25D$

	$\xi=0$		$\xi=0.337$		$\xi=\infty$	
$\frac{z}{H}$	v_x	v_y	v_x	v_y	v_x	v_y
0.0	0.009	0.195	0.018	0.248	0.022	0.259
0.1	0.008	0.119	0.017	0.174	0.021	0.185
0.2	0.006	0.053	0.013	0.100	0.017	0.110
0.3	0.005	0.014	0.010	0.052	0.013	0.060
0.4	0.000	−0.031	0.004	0.005	0.006	0.010
0.5	−0.002	−0.051	−0.002	−0.035	0.001	−0.033
0.6	−0.004	−0.061	−0.007	−0.066	−0.006	−0.067
0.7	−0.005	−0.067	−0.010	−0.088	−0.011	−0.094
0.8	−0.004	−0.057	−0.013	−0.102	−0.015	−0.113
0.9	−0.003	−0.036	−0.014	−0.106	−0.017	−0.125
1.0	0.000	0.000	−0.014	−0.103	−0.020	−0.128

Table 6. (Continued)

$H=0.5D$

	$\xi=0$		$\xi=0.154$		$\xi=0.448$		$\xi=\infty$	
$\frac{z}{H}$	v_x	v_y	v_x	v_y	v_x	v_y	v_x	v_y
0.0	0.062	0.380	0.084	0.413	0.114	0.442	0.141	0.459
0.1	0.055	0.236	0.076	0.268	0.105	0.296	0.131	0.313
0.2	0.038	0.118	0.057	0.148	0.083	0.174	0.106	0.190
0.3	0.016	0.024	0.031	0.049	0.051	0.072	0.070	0.086
0.4	−0.004	−0.046	0.004	−0.027	0.019	−0.009	0.031	0.002
0.5	−0.019	−0.093	−0.018	−0.082	−0.013	−0.071	−0.008	−0.064
0.6	−0.031	−0.118	−0.037	−0.119	−0.041	−0.117	−0.045	−0.116
0.7	−0.034	−0.122	−0.047	−0.137	−0.062	−0.147	−0.076	−0.154
0.8	−0.028	−0.102	−0.049	−0.135	−0.074	−0.161	−0.099	−0.179
0.9	−0.017	−0.063	−0.044	−0.117	−0.079	−0.162	−0.114	−0.194
1.0	0.000	0.000	−0.031	−0.081	−0.074	−0.149	−0.119	−0.199

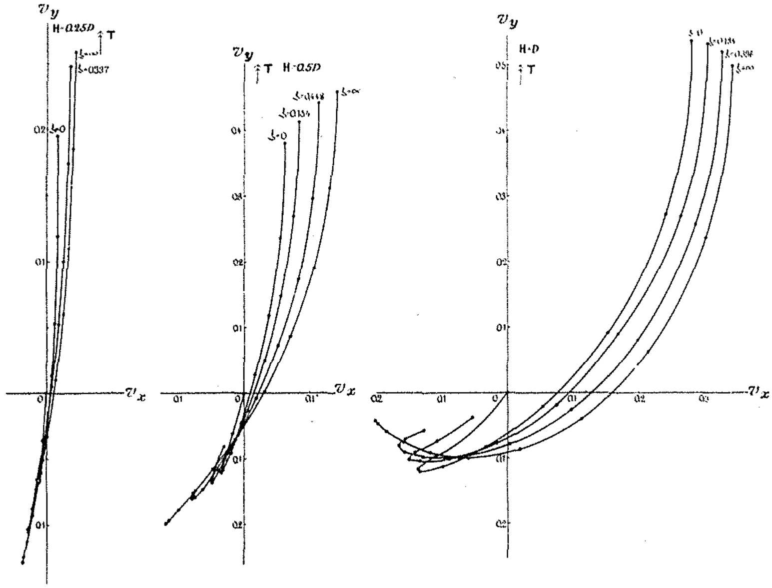
Table 6. (Continued)

$H=D$

	$\xi=0$		$\xi=0.134$		$\xi=0.396$		$\xi=\infty$	
$\frac{z}{H}$	v_x	v_y	v_x	v_y	v_x	v_y	v_x	v_y
0.0	0.281	0.539	0.305	0.536	0.328	0.523	0.343	0.502
0.1	0.241	0.273	0.265	0.270	0.288	0.257	0.303	0.237
0.2	0.153	0.091	0.177	0.090	0.199	0.080	0.215	0.062
0.3	0.054	−0.022	0.075	−0.019	0.097	−0.026	0.113	−0.040
0.4	−0.033	−0.083	−0.017	−0.077	0.003	−0.078	0.019	−0.087
0.5	−0.098	−0.113	−0.087	−0.102	−0.073	−0.099	−0.059	−0.100
0.6	−0.133	−0.121	−0.133	−0.105	−0.128	−0.099	−0.118	−0.092
0.7	−0.136	−0.117	−0.150	−0.103	−0.157	−0.092	−0.158	−0.076
0.8	−0.110	−0.098	−0.141	−0.092	−0.166	−0.081	−0.184	−0.059
0.9	−0.058	−0.064	−0.108	−0.074	−0.155	−0.072	−0.198	−0.048
1.0	0.000	0.000	−0.053	−0.038	−0.127	−0.057	−0.202	−0.043

Fig. 8

Wind currents in an enclosed sea



$$0 = \frac{\partial^2 w}{\partial z^2} - 2ik^2 w + i\frac{g\rho}{\mu}(\gamma_0 + \gamma e^{i\phi}), \tag{26}$$

and if there is no wind, the surface condition takes the form

$$|\partial w/\partial z|_{z=0} = 0. \tag{27}$$

Let the bottom-condition be taken as

$$|w|_{z=H} = 0,$$

then we have directly the following solution of (26) which satisfies the above two conditions:

$$w = \text{“ slope current ” due to } \gamma_0 \text{ and } \gamma$$

$$= \frac{g(\gamma_0 + \gamma e^{i\phi})}{2\bar{\omega}}\left\{1 - \frac{\cosh az}{\cosh aH}\right\}$$

$$= \bar{V}_g\left(1 + \frac{\gamma}{\gamma_0} e^{i\phi}\right)\left(1 - \frac{\cosh az}{\cosh aH}\right)\} \tag{28}$$

where $\bar{V} \equiv g\gamma_0/2\bar{\omega}$.

It remains only to determine the slope by the equation of continuity.

(A) *Long straight coast*:—In this case ϕ is equal to the angle between the coast and the x-axis and so it is a known quantity. The only unknown, γ, is given by (5_a), namely

$$S_n = 0 = \gamma_0(s_4\cos\phi - s_3\sin\phi) + \gamma s_4,$$

or
$$\gamma = -\gamma_0\left(\cos\phi - \frac{s_3}{s_4}\sin\phi\right). \qquad (29)$$

Substituting this γ in (28) we can calculate the current definitely.

Thus we see that γ and w are proportional to γ_0. If γ_0 is uniform all over the sea surface, then both γ and w are also uniform everywhere; and if γ_0 is a function of the distance from the coast, γ and w will show also the same variation.

Now changing the view-point, if $\phi=0$, i. e., if a barometric pressure-gradient acts perpendicularly to the coast, then $\gamma=-\gamma_0$ and consequently $w=0$, which means a hydrostatic equilibrium with a surface slope equal and opposite to the impressed barometric gradient, as in the papers of Proudman and Horrocks.

For other values of ϕ, the surface slope depends generally on the two components of γ_0 perpendicular and parallel to the coast. In a very shallow sea, however, (29) approximates to

$$\gamma \doteqdot -\gamma_0\cos\phi,$$

i. e., the component pressure-gradient perpendicular to the coast only reveals a land-effect. On the other hand in a very deep sea

$$\gamma \doteqdot -\gamma_0(\cos\phi - 2kH\sin\phi),$$

and the component pressure-gradient parallel to the coast plays the most important rôle in the land effect.

Some examples for intermediate depths are given in Table 8 and Figs. 10 and 11 with the indication "$\xi=0$".

It may be useful to remark here that the above solution contains the weakness that the current w does not vanish even at the coast itself, except when γ_0 is zero there. This is, however, due to the neglect of the vertical velocity, and it can be avoided only by taking account of the vertical motion, as stated in the case of wind current.

(B) *Enclosed sea*:—For an enclosed sea, ϕ as well as γ is unknown, while the continuity condition gives the following two equations:

$$\left.\begin{aligned} S_x &= 0 = \gamma_0 s_3 + \gamma(s_3\cos\phi - s_4\sin\phi), \\ S_y &= 0 = \gamma_0 s_4 + \gamma(s_4\cos\phi + s_3\sin\phi), \end{aligned}\right\}$$

or, turning to the total flows perpendicular and parallel to the surface slope,

$$\gamma_0(s_4\cos\phi - s_3\sin\phi) + \gamma s_4 = 0,$$
$$\gamma_0(s_3\cos\phi + s_4\sin\phi) + \gamma s_3 = 0.$$

From these equations we have

$$\phi=0, \quad \text{and} \quad \gamma=-\gamma_0,$$

i. e., an enclosed sea can be kept in the steady state only when a hydrostatic equilibrium has been reached.

§ 5. Case of No Bottom-friction

Let the bottom-condition be

$$|\partial w/\partial z|_{z=H}=0,$$

then we shall have

$$w=\frac{g}{2\varpi}(\gamma_0+\gamma e^{i\phi}), \tag{30}$$

instead of (28).

(A) *Long straight coast*:—The equation of continuity requires

$$S_n=0=-\frac{g\gamma_0}{2\varpi}H\sin\phi.$$

This is possible only if $\phi=0$; and γ may seem arbitrary here, but in reality we get $\gamma=-\gamma_0$ when considered as a limiting case of the infinitely small bottom-friction dealt with in § 6. Accordingly also $w=0$, and a hydrostatic equilibrium must be attained.

So long as $\phi\neq 0$ and γ_0 is oblique to the coast, the solution is impossible. This means that γ developes markedly owing to the effect of land and the vertical motion becomes comparable with the horizontal motion, so that the fundamental equation itself must be changed. In other words, in the range where (26) holds good a steady state can not be reached.

(B) *Enclosed sea*:—The equations of continuity in this case are

$$S_x=0=\frac{gH}{2\varpi}(\gamma_0+\gamma\cos\phi),$$

$$S_y=0=\frac{gH}{2\varpi}\gamma\sin\phi,$$

from which we have

$$\phi=0, \quad \gamma=-\gamma_0, \quad \text{and} \quad w=0.$$

Hence an enclosed sea in the steady state must be in hydrostatic equilibrium.

§ 6. Case of Finite Bottom-friction

If we take the bottom-condition as

$$\left.\frac{\partial w}{\partial z}\right|_{z=H} = -\frac{f\rho V_H^2 e^{i\theta}}{\mu},$$

the solution will obviously be

w = "barometric current" with no bottom-friction + usual correction term

$$\left.\begin{aligned} &= \overline{V}_g\left[1+\frac{\gamma}{\gamma_0}e^{i\phi}\right]-\frac{f\rho V_H^2 e^{i\theta}}{\mu a}\cdot\frac{\cosh az}{\sinh aH}, \\ \text{where}\quad &\overline{V}_g \equiv g\gamma_0/2\overline{\omega}. \end{aligned}\right\} \tag{31}$$

Since $V_H e^{i\theta}$ is the bottom-velocity and is equal to $|w|_{z=H}$, the following relation must hold:

$$V_H = \overline{V}_g e^{i\theta}\left[1+\frac{\gamma}{\gamma_0}e^{i\phi}\right]-\frac{f\rho V_H^2 \cosh aH}{\mu a \sinh aH}, \tag{32}$$

whose real and imaginary parts are

$$\left.\begin{aligned} V_H &= \overline{V}_g\left[\cos\theta+\frac{\gamma}{\gamma_0}\cos(\theta-\phi)\right]-\frac{f\rho V_H^2(\sinh 2kH-\sin 2kH)}{2\mu k(\cosh 2kH-\cos 2kH)}, \\ O &= -\overline{V}_g\left[\sin\theta+\frac{\gamma}{\gamma_0}\sin(\theta-\phi)\right]+\frac{f\rho V_H^2(\sinh 2kH+\sin 2kH)}{2\mu k(\cosh 2kH-\cos 2kH)}. \end{aligned}\right\} \tag{33}$$

If we put

$$\xi \equiv \sqrt{\frac{\mu k}{f\rho \overline{V}_g}}, \qquad \eta \equiv \frac{V_H}{\xi \overline{V}_g} \equiv V_H\sqrt{\frac{f\rho}{\mu k \overline{V}_g}}, \tag{34}$$

(33) becomes

$$\left.\begin{aligned} \xi\eta &= \left[\cos\theta+\frac{\gamma}{\gamma_0}\cos(\theta-\phi)\right]-\frac{1}{2}r\eta^2, \\ 0 &= \left[\sin\theta+\frac{\gamma}{\gamma_0}\sin(\theta-\phi)\right]+\frac{1}{2}s\eta^2, \end{aligned}\right\} \tag{33'}$$

where

$$r \equiv \frac{\sinh 2kH-\sin 2kH}{\cosh 2kH-\cos 2kH}, \qquad s \equiv \frac{\sinh 2kH+\sin 2kH}{\cosh 2kH-\cos 2kH}. \tag{34}$$

(A) *Long straight coast*:—The continuity condition is

$$S_n = 0 = -\overline{V}_g H\sin\phi+\frac{f\rho V_H^2}{2\mu k^2}\cos(\theta-\phi),$$

or

$$\eta^2\cos(\theta-\phi) = 2kH\sin\phi. \tag{35}$$

Using (33′) and (35) we can determine three unknowns V_H (or η), θ and γ corresponding to any given ξ, then the current w also is easily known by eq. (32).

Tables 7, 8 and Figs. 9, 10 and 11 show some examples.

Table 7.

Corresponding values of ξ, η, γ and θ for barometric currents (long straight coast)

$\phi=\frac{\pi}{2}$											
$H=0.5D$				$H=D$				$H=2.5D$			
ξ	η	θ	$\frac{\gamma}{\gamma_0}$	ξ	η	θ	$\frac{\gamma}{\gamma_0}$	ξ	η	θ	$\frac{\gamma}{\gamma_0}$
0.000	1.813	107°.1	1.865	0.000	2.760	124°.4	5.280	0.000	4.568	131°.2	14.711
0.045	1.808	106°	1.943	0.014	2.753	124°	5.297	0.074	4.528	130°	14.754
0.199	1.798	104°	2.125	0.123	2.720	122°	5.381	0.255	4.464	128°	14.901
0.381	1.792	102°	2.390	0.238	2.693	120°	5.524	0.433	4.406	126°	15.134
0.580	1.786	100°	2.748	0.373	2.668	118°	5.708	0.652	4.353	124°	15.465
0.865	1.781	98°	3.280	0.523	2.642	116°	5.915	0.852	4.301	122°	15.852
1.574	1.777	96°	4.333	0.691	2.621	114°	6.194	1.089	4.259	120°	16.406
2.700	1.773	94°	6.333	0.891	2.602	112°	6.571	1.349	4.219	118°	17.093
6.200	1.773	92°	12.564	1.130	2.585	110°	7.026	1.647	4.177	116°	17.867
∞	1.773	90°	∞	1.409	2.569	108°	7.601	1.983	4.144	114°	18.853
				1.761	2.557	106°	8.366	2.380	4.114	112°	20.096
				2.200	2.544	104°	9.359	2.860	4.089	110°	21.690
				2.793	2.534	102°	10.736	3.424	4.062	108°	23.853
				3.613	2.524	100°	12.650	4.129	4.043	106°	26.137
				4.871	2.520	98°	15.719	5.017	4.022	104°	29.413
				6.942	2.514	96°	20.826	6.218	4.006	102°	33.871
				10.989	2.510	94°	30.743	7.875	3.994	100°	40.174
				23.057	2.506	92°	61.228	10.421	3.984	98°	49.985
				∞	2.505	90°	∞	14.600	3.975	96°	66.403
								22.772	3.969	94°	98.243
								47.288	3.964	92°	195.886
								∞	3.962	90°	∞

Table 7. (Continued)

$\phi=\frac{\pi}{4}$											
$H=0.5D$				$H=D$				$H=2.5D$			
ξ	η	θ	$\frac{\gamma}{\gamma_0}$	ξ	η	θ	$\frac{\gamma}{\gamma_0}$	ξ	η	θ	$\frac{\gamma}{\gamma_0}$
0.000	1.525	62°	0.636	0.000	2.319	79°.3	3.041	0.000	3.841	86°.2	9.690
0.097	1.516	60°	0.730	0.056	2.300	78°	3.062	0.005	3.853	86°	9.692
0.221	1.510	58°	0.884	0.153	2.276	76°	3.145	0.148	3.780	84°	9.797
0.390	1.503	56°	1.101	0.264	2.252	74°	3.251	0.291	3.728	82°	9.902
0.637	1.500	54°	1.429	0.375	2.233	72°	3.396	0.455	3.682	80°	10.103
1.016	1.496	52°	1.959	0.512	2.213	70°	3.575	0.624	3.639	78°	10.355
1.708	1.493	50°	2.947	0.678	2.195	68°	3.798	0.814	3.599	76°	10.691
3.351	1.490	48°	5.327	0.850	2.179	66°	4.089	1.031	3.560	74°	11.103
10.852	1.490	46°	16.722	1.061	2.166	64°	4.445	1.254	3.531	72°	11.634
∞	1.490	45°	∞	1.327	2.155	62°	4.932	1.558	3.501	70°	12.243
				1.659	2.144	60°	5.541	1.854	3.471	68°	13.052
				2.090	2.135	58°	6.369	2.205	3.445	66°	14.056
				2.660	2.126	56°	7.503	2.629	3.425	64°	15.251
				3.513	2.120	54°	9.224	3.165	3.407	62°	16.870
				4.818	2.114	52°	11.877	3.831	3.390	60°	18.868
				7.107	2.112	50°	16.827	4.702	3.376	58°	21.577
				12.727	2.108	48°	28.461	5.858	3.361	56°	25.267
				39.946	2.107	46°	83.440	7.589	3.352	54°	30.846
				∞	2.107	45°	∞	10.218	3.342	52°	38.565
								15.003	3.339	50°	55.287
								26.213	3.333	48°	92.519
								79.266	3.331	46°	267.555
								∞	3.331	45°	∞

Table 8.

Barometric currents in a sea with a long straight coast

$\phi=\pi/4$						
$H=0.5D$						
	$\xi=0$		$\xi=0.390$		$\xi=1.708$	
$\frac{z}{H}$	v_x	v_y	v_x	v_y	v_x	v_y
0.0	1.256	1.077	1.655	1.402	3.031	2.708
0.1	1.240	1.072	1.640	1.399	3.015	2.707
0.2	1.194	1.058	1.595	1.389	2.969	2.703
0.3	1.119	1.030	1.518	1.370	2.892	2.692
0.4	1.014	0.986	1.412	1.339	2.786	2.673
0.5	0.882	0.919	1.278	1.288	2.648	2.637
0.6	0.729	0.825	1.120	1.213	2.486	2.580
0.7	0.553	0.695	0.941	1.107	2.301	2.496
0.8	0.371	0.520	0.741	0.956	2.090	2.369
0.9	0.184	0.294	0.542	0.757	1.871	2.196
1.0	0.000	0.000	0.342	0.496	1.648	1.962

Table 8.

(Contiuned)

$\phi=\pi/4$						
$H=D$						
	$\xi=0$		$\xi=0.512$		$\xi=1.659$	
$\frac{z}{H}$	v_x	v_y	v_x	v_y	v_x	v_y
0.0	3.422	2.336	3.800	2.655	5.195	3.991
0.1	3.403	2.363	3.786	2.682	5.188	4.018
0.2	3.342	2.438	3.743	2.759	5.159	4.087
0.3	3.222	2.551	3.672	2.878	5.095	4.225
0.4	3.024	2.678	3.492	3.021	4.969	4.389
0.5	2.723	2.776	3.236	3.154	4.751	4.606
0.6	2.299	2.789	2.864	3.228	4.407	4.691
0.7	1.750	2.631	2.390	3.167	3.911	4.713
0.8	1.109	2.197	1.705	2.570	3.262	4.553
0.9	0.474	1.360	1.005	2.212	2.493	4.054
1.0	0.000	0.000	0.375	1.057	1.704	3.075

Table 8.

(Continued)

	$\phi=\pi/4$					
	$H=2.5D$					
	$\xi=0$		$\xi=0.455$		$\xi=1.558$	
$\frac{z}{H}$	v_x	v_y	v_x	v_y	v_x	v_y
0.0	7.849	6.858	8.140	7.150	9.654	8.663
0.1	7.843	6.855	8.136	7.147	9.650	8.661
0.2	7.838	6.840	8.129	7.134	9.642	8.650
0.3	7.849	6.829	8.137	7.106	9.644	8.625
0.4	7.914	6.781	8.194	7.073	9.690	8.586
0.5	8.058	6.838	8.330	7.112	9.817	8.599
0.6	8.194	7.149	8.483	7.382	10.006	8.815
0.7	7.921	7.840	8.300	8.039	9.937	9.428
0.8	6.426	8.497	7.001	8.774	8.898	10.287
0.9	3.087	7.180	3.836	7.890	5.945	10.001
1.0	0.000	0.000	0.303	1.643	1.816	5.004

Table 8.

(Continued)

	$\phi=\pi/2$					
	$H=0.5D$					
	$\xi=0$		$\xi=0.381$		$\xi=1.574$	
$\frac{z}{H}$	v_x	v_y	v_x	v_y	v_x	v_y
0.0	0.182	2.296	0.242	2.885	0.308	4.892
0.1	0.171	2.276	0.230	2.866	0.294	4.875
0.2	0.140	2.215	0.193	2.810	0.253	4.824
0.3	0.092	2.111	0.138	2.715	0.190	4.737
0.4	0.031	1.943	0.067	2.576	0.105	4.606
0.5	−0.032	1.764	−0.017	2.387	0.007	4.430
0.6	−0.096	1.519	−0.095	2.154	−0.093	4.221
0.7	−0.131	1.219	−0.164	1.863	−0.189	3.934
0.8	−0.143	0.856	−0.208	1.513	−0.266	3.595
0.9	−0.101	0.450	−0.204	1.109	−0.306	3.202
1.0	0.000	0.000	−0.139	0.646	−0.286	2.741

Table 8.

(Continued)

$\phi=\pi/2$						
$H=D$						
	$\xi=0$		$\xi=0.238$		$\xi=1.130$	
$\frac{z}{H}$	v_x	v_y	v_x	v_y	v_x	v_y
0.0	1.086	5.739	1.115	5.954	1.173	7.397
0.1	1.041	5.747	1.072	5.960	1.136	7.414
0.2	0.902	5.760	0.944	5.987	1.023	7.455
0.3	0.671	5.755	0.722	5.998	0.824	7.502
0.4	0.343	5.681	0.405	5.953	0.531	7.511
0.5	−0.056	5.478	0.012	5.788	0.146	7.425
0.6	−0.494	5.065	−0.433	5.428	−0.316	7.170
0.7	−0.885	4.356	−0.856	4.783	−0.803	6.652
0.8	−1.086	3.283	−1.124	3.778	−1.208	5.785
0.9	−0.889	1.810	−1.042	2.358	−1.365	4.484
1.0	0.000	0.000	−0.328	0.545	−1.008	2.724

Table 8.

(Continued)

$\phi=\pi/2$						
$H=2.5D$						
	$\xi=0$		$\xi=1.089$		$\xi=2.860$	
$\frac{z}{H}$	v_x	v_y	v_x	v_y	v_x	v_y
0.0	0.989	14.720	0.990	16.409	0.992	21.694
0.1	0.988	14.705	0.989	16.402	0.990	21.689
0.2	0.998	14.685	0.994	16.384	0.991	21.671
0.3	1.040	14.666	1.027	16.361	1.017	21.645
0.4	1.132	14.701	1.111	16.376	1.097	21.645
0.5	1.218	14.903	1.218	16.512	1.218	21.769
0.6	1.043	15.347	1.144	16.941	1.216	22.155
0.7	0.085	15.758	0.395	17.455	0.613	22.745
0.8	−2.060	14.918	−1.576	17.096	−1.233	22.728
0.9	−4.057	10.281	−4.060	13.471	−4.507	19.825
1.0	0.000	0.000	−2.318	4.024	−3.995	10.947

Fig. 9

Surface slope produced by a barometric current (long straight coast)

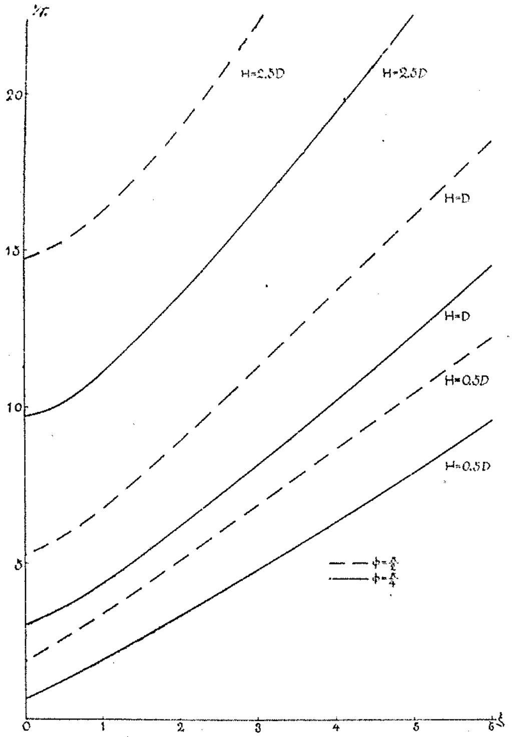


Fig. 10

Barometric currents in a sea with a long straight coast

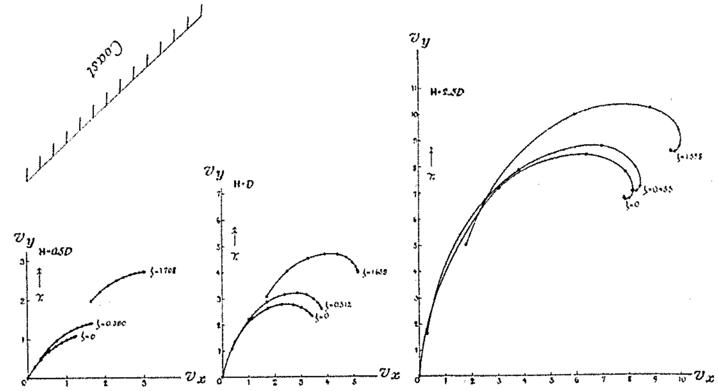
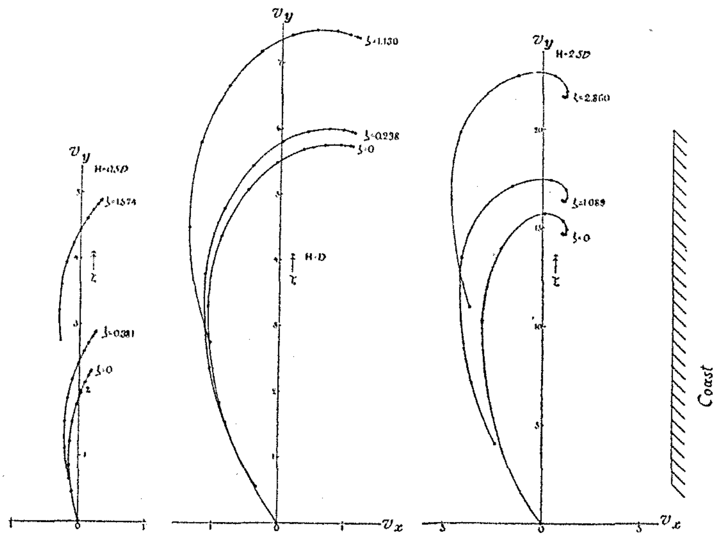


Fig. 11

Barometric currents in a sea with a long straight coast



(B) *Enclosed sea*:—ϕ is here unknown, but to compensate for that we have two equations for continuity

$$\left.\begin{aligned} S_x = 0 = \overline{V}_g H\left[1+\frac{\gamma}{\gamma_0}\cos\phi\right] - \frac{f\rho V_H^2}{2\mu k^2}\sin\theta, \\ S_y = 0 = \overline{V}_g H\frac{\gamma}{\gamma_0}\sin\phi + \frac{f\rho V_H^2}{2\mu k^2}\cos\theta, \end{aligned}\right\}$$

or

$$\left.\begin{aligned} 2kH\left[1+\frac{\gamma}{\gamma_0}\cos\phi\right] = \eta^2\sin\theta, \\ -2kH\frac{\gamma}{\gamma_0}\sin\phi = \eta^2\cos\theta. \end{aligned}\right\} \qquad (36)$$

In order to get the total flow perpendicular to the slope γ, multiply the above equation by $\sin\theta$ and $\cos\theta$ respectively and add together, then there results

$$\sin\theta + \frac{\gamma}{\gamma_0}\sin(\theta-\phi) = \frac{\eta^2}{2kH}. \qquad (37)$$

Comparing this with the lower equation of (33′) we have

$$\eta = 0,$$

and consequently from (36) and (31)

$$\phi = 0, \quad \gamma = -\gamma_0, \quad \text{and } w = 0,$$

i. e., there must be hydrostatic equilibrium here also.

§ 7. The important characteristics of the effect of land upon the barometric current are:

1) For a barometric gradient perpendicular to a straight coast or in an enclosed sea, an equal and opposite surface slope will be generated and the sea must be in hydrostatic equilibrium in the steady state.

2) If a barometric gradient acts parallel to a long straight coast, the component current parallel to it increases; and the larger ξ becomes, the more the current diagram shifts as a whole along the coast.

III. Convection Current

§ 8. Case of No Bottom-current

When the water density increases in the y-direction, the equation of motion in the steady state of the density current influenced by land is generally

$$\frac{\partial^2 w}{\partial z^2} - 2ik^2 w + i\mathfrak{a}(\mathfrak{b}e^{i\phi} - z) = 0$$

where

$$\mathfrak{a} \equiv -\frac{g}{\mu}\frac{\partial \rho}{\partial y}, \qquad \mathfrak{b} \equiv \frac{g\rho}{\mu\mathfrak{a}}\gamma, \tag{38}$$

and γ and ϕ have the same meaning as before.

Let the boundary conditions be

$\partial w/\partial z = 0$ at the surface,

and $w = 0$ at the bottom,

then the solution of (38) is obviously

$w =$ " density current "[1] with no bottom-current + " slope current "

$$= \frac{\mathfrak{a}}{4k^3}\left[\frac{(1-i)e^{-aH} + 2kH}{\cosh aH}\cosh az - (1-i)e^{-az} - 2kz\right]$$

$$+ \frac{g\gamma e^{i\phi}}{2\varpi}\left[1 - \frac{\cosh az}{\cosh aH}\right]. \tag{39}$$

The equation of continuity will serve very well to determine the unknown slope.

1. These Memoirs, A, **16**, 263 (1933), eqs. (9) and (10).

(A) *Long straight coast*:—For the sake of expressing the total flow[1] of pure density-current simply, put

$$\left.\begin{aligned} s_5 &= 2\sinh kH \sin kH + k^2H^2(\cosh 2kH + \cos 2kH) \\ &\quad - kH(\sinh 2kH - \sin 2kH), \\ s_6 &= 2\cosh kH \cos kH + kH(\sinh 2kH - \sin 2kH) \\ &\quad - (\cosh 2kH + \cos 2kH), \end{aligned}\right\} \quad (40)$$

then the total flow of the convection current perpendicular to the coast will be

$$S_n = 0 = \frac{\alpha}{4k^4}(s_5 \sin\phi - s_6 \cos\phi) + \frac{g\rho\gamma}{4\mu k^3} s_4,$$

$$\therefore \quad \gamma = \frac{\mu\alpha}{g\rho k} \frac{s_6\cos\phi - s_5\sin\phi}{s_4}. \quad (41)$$

The special case where $\phi = 0$ was treated in our previous paper[2] on the density current. For other cases, the surface slope and the current will be seen in Tables 9, 10 and Figs. 12–15 with the indication "$\xi = 0$."

(B) *Enclosed sea*:—For the convection current in an enclosed sea with no bottom-current, not only the solution formulae[3] but several numerical examples[4] were given in our previous papers, and we have no more to say here on that subject.

§ 9. Case of No Bottom-friction

This case also has been already discussed completely by one[5] of the writers in his second report on the density current, and there is no need to repeat it.

§ 10. Case of Finite Bottom-friction

The solution which satisfies the condition

$$\mu \left| \partial w / \partial z \right|_{z=H} = -f\rho V_H^2 e^{i\theta}$$

at the bottom, is obviously

w = "convection current"[6] with no bottom-friction + usual correction term

1. These Memoirs, A, **16**, 266 (1933), eq. (15).
2. Ditto, 270—273.
3. Ditto, 274.
4. Ditto, 392.
5. Ditto, 388.
6. Ditto, 390, eq. (11).

$$\left.\begin{aligned} &= \frac{\mathfrak{a}}{4k^3}\left[(i-1)\frac{\sinh a\left(\frac{H}{2}-z\right)}{\cosh\frac{aH}{2}} + 2k(\mathfrak{d}e^{i\phi}-z)\right] \\ &\qquad - \frac{f\rho V_H^2 e^{i\theta}}{\mu a}\,\frac{\cosh az}{\sinh aH}, \end{aligned}\right\} \quad (42)$$

where $\mathfrak{d} = g\rho\gamma/\mu\mathfrak{a}$.

Moreover from the obvious relation $V_H e^{i\theta} = |w|_{z=H}$, we have

$$V_H = \frac{\mathfrak{a}}{4k^3}\left[(1-i)\tanh\frac{aH}{2} + 2k(\mathfrak{d}e^{i\phi}-H)\right]e^{-i\theta}$$

$$-\frac{f\rho V_H^2}{\mu a}\coth aH,$$

of which the real and imaginary parts are

$$V_H = -\frac{\mathfrak{a}}{4k^3}\left[(p-2k\mathfrak{d}\cos\phi)\cos\theta + (q-2k\mathfrak{d}\sin\phi)\sin\theta\right]$$

$$-r\frac{f\rho V_H^2}{2\mu k},$$

$$0 = -\frac{\mathfrak{a}}{4k^3}\left[(q-2k\mathfrak{d}\sin\phi)\cos\theta - (p-2k\mathfrak{d}\cos\phi)\sin\theta\right]$$

$$+s\frac{f\rho V_H^2}{2\mu k},$$

where

$$\left.\begin{aligned} p &\equiv 2kH - \frac{\sinh kH + \sin kH}{\cosh kH + \cos kH}, \\ q &\equiv \frac{\sinh kH - \sin kH}{\cosh kH + \cos kH}, \\ r &\equiv \frac{\sinh 2kH - \sin 2kH}{\cosh 2kH - \cos 2kH}, \\ s &\equiv \frac{\sinh 2kH + \sin 2kH}{\cosh 2kH - \cos 2kH}. \end{aligned}\right\} \quad (43)$$

Or if we put

$$\left.\begin{aligned} &\xi \equiv 2k^2\sqrt{\frac{\mu}{f\rho\mathfrak{a}}} = 2k\sqrt{\frac{\omega\sin\lambda}{f\mathfrak{a}}}, \\ &\eta \equiv 2k\sqrt{\frac{f\rho}{\mu\mathfrak{a}}}V_H = \frac{V_H}{\xi V_a}, \qquad V_a \equiv \frac{\mathfrak{a}}{4k^3}, \\ &p \equiv P\cos\varphi, \qquad q \equiv P\sin\varphi, \qquad P \equiv \sqrt{p^2+q^2}, \end{aligned}\right\} \quad (44)$$

then

$$\left.\begin{aligned} \xi\eta &= -P\cos(\theta-\varphi)+2k\mathfrak{d}\cos(\theta-\phi)-\frac{1}{2}r\eta^2, \\ O &= P\sin(\theta-\varphi)-2k\mathfrak{d}\sin(\theta-\phi)+\frac{1}{2}s\eta^2, \end{aligned}\right\} \tag{45}$$

from which, on elimination of $k\mathfrak{d}$, we have

$$\xi\eta=\frac{P\sin(\phi-\varphi)+\frac{1}{2}\eta^2\Big[s\cos(\theta-\phi)-r\sin(\theta-\phi)\Big]}{\sin(\theta-\phi)}. \tag{46}$$

(A) *Long straight coast*:—The total flow perpendicular to the coast is

$$S_n=0=-\frac{\alpha}{4k^2}H^2\sin\phi+\frac{f\rho V_H^2}{2\mu k^2}\cos(\theta-\phi),$$

or $$\eta^2\cos(\theta-\phi)=-2k^2H^2\sin\phi,$$

i. e., $$\tan\phi=\frac{\cos\theta}{\sin\theta-\dfrac{2k^2H^2}{\eta^2}}. \tag{47}$$

With (45) and (47) we can determine three unknowns η, θ and $\mathfrak{d}$ for any given ξ. For instance, first calculate η for any possible θ by (47), and next get $k\mathfrak{d}$ from the lower equation of (45) and then calculate ξ with (46) or the upper equation of (45).

Table 9 shows the corresponding values of ξ, η, θ and $k\mathfrak{d}$ which are obtained by such operations.

As $\mathfrak{d}$ is a quantity proportional to γ, the surface slope, we plot it according to ξ in Fig. 12.

Table 9.

Corresponding values of ξ, η, θ and $k\mathfrak{d}$ for convection current (long straight coast)

$\phi=0$ $[\theta=270°]$								
$H=0.5D$			$H=D$			$H=2.5$		
ξ	η	$2k\mathfrak{d}$	ξ	η	$2k\mathfrak{d}$	ξ	η	$2k\mathfrak{d}$
0.000	1.065	1.307	0.000	1.476	4.110	0.000	1.415	13.710
0.060	1.000	1.367	0.188	1.300	4.348	0.120	1.300	13.865
0.163	0.900	1.454	0.309	1.200	4.473	0.234	1.200	13.990
0.281	0.800	1.532	0.441	1.100	4.588	0.361	1.100	14.105
0.420	0.700	1.602	0.590	1.000	4.693	0.500	1.000	14.210
0.589	0.600	1.661	0.763	0.900	4.788	0.661	0.900	14.305
0.809	0.500	1.711	0.961	0.800	4.873	0.852	0.800	14.390
1.114	0.400	1.753	1.209	0.700	4.948	1.080	0.700	14.465
1.529	0.300	1.785	1.515	0.600	5.013	1.366	0.600	14.530
2.498	0.200	1.808	1.927	0.500	5.068	1.750	0.500	14.585
5.144	0.100	1.821	2.526	0.400	5.113	2.300	0.400	14.630
∞	0.000	1.826	∞	0.000	5.193	∞	0.000	14.710

Table 9. (Continued)

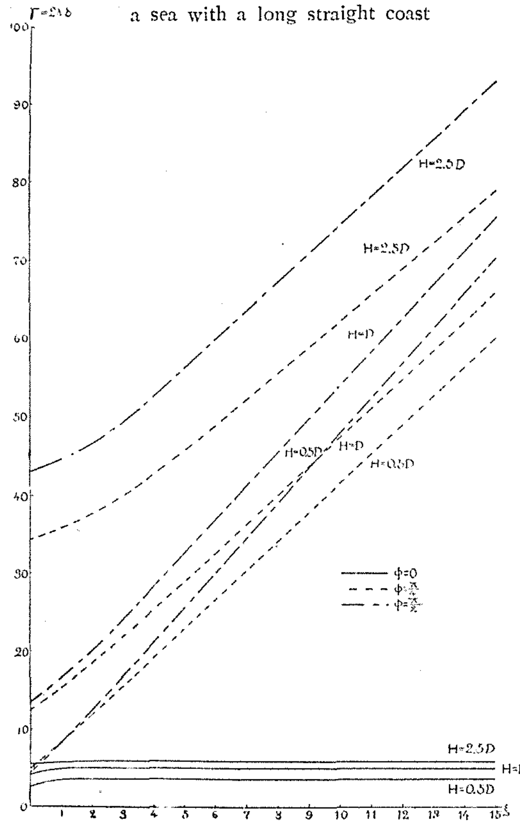
$\phi=\frac{\pi}{2}$											
$H=0.5D$				$H=D$				$H=2.5D$			
ξ	η	θ	$2k\mathfrak{d}$	ξ	η	θ	$2k\mathfrak{d}$	ξ	η	θ	$2k\mathfrak{d}$
0.000	2.242	−79.°2	2.218	0.000	4.672	−64°.7	13.485	0.000	12.442	−52°.8	107.7
0.103	2.238	−80°	2.333	0.149	4.647	−66°	13.779	0.295	12.349	−54°	108.8
0.243	2.235	−81°	2.609	0.395	4.614	−68°	14.467	0.789	12.200	−56°	110.3
0.392	2.233	−82°	2.928	0.692	4.582	−70°	15.339	1.318	12.061	−58°	112.7
0.593	2.230	−83°	3.328	1.043	4.556	−72°	16.838	1.903	11.936	−60°	116.0
0.855	2.227	−84°	3.867	1.472	4.532	−74°	18.040	2.548	11.820	−62°	120.0
1.242	2.225	−85°	4.713	2.027	4.511	−76°	20.136	3.299	11.715	−64°	125.5
1.792	2.223	−86°	5.871	2.758	4.492	−78°	23.005	4.125	11.618	−66°	131.8
2.675	2.223	−87°	8.019	3.716	4.476	−80°	27.086	5.093	11.536	−68°	140.1
4.625	2.222	−88°	12.086	5.298	4.465	−82°	33.619	6.261	11.456	−70°	150.5
9.960	2.222	−89°	23.889	7.765	4.454	−84°	44.190	7.651	11.390	−72°	163.6
∞	2.221	−90°	∞	12.807	4.447	−86°	66.157	9.357	11.330	−74°	180.3
				27.750	4.445	−88°	132.943	11.570	11.278	−76°	202.8
				∞	4.442	−90°	∞	14.493	11.232	−78°	233.1
								18.530	11.190	−80°	275.7
								24.703	11.160	−82°	342.5
								34.586	11.140	−84°	450.2
								54.747	11.120	−86°	672.4
								115.203	11.110	−88°	1343.4
								∞	11.110	−90°	∞

Table 9. (Continued)

$\phi=\frac{\pi}{4}$											
$H=0.5D$				$H=D$				$H=2.5D$			
ξ	η	θ	$2k\mathfrak{d}$	ξ	η	θ	$2k\mathfrak{d}$	ξ	η	θ	$2k\mathfrak{d}$
0.000	1.868	−47°	2.469	0.000	3.852	−25°10′	12.460	0.000	10.438	−8°.3	85.82
0.100	1.868	−46°50′	2.625	0.107	3.842	−26°	12.677	0.330	10.321	−10°	86.66
0.184	1.868	−46°40′	2.797	0.361	3.821	−28°	13.338	0.753	10.200	−12°	88.09
0.307	1.868	−46°30′	3.023	0.678	3.802	−30°	14.248	1.212	10.090	−14°	90.02
0.437	1.868	−46°20′	3.283	1.088	3.782	−32°	15.506	1.720	9.988	−16°	92.53
0.630	1.868	−46°10′	3.627	1.638	3.771	−34°	17.321	2.289	9.896	−18°	95.73
0.887	1.868	−46°	4.091	2.434	3.759	−36°	20.038	2.943	9.812	−20°	99.76
1.255	1.868	−45°50′	4.758	3.676	3.750	−38°	24.430	3.679	9.736	−22°	104.81
1.751	1.868	−45°40′	5.724	5.890	3.742	−40°	32.442	4.549	9.667	−24°	111.12
2.638	1.868	−45°30′	7.333	11.077	3.739	−42°	51.740	5.592	9.606	−26°	119.13
4.444	1.868	−45°20′	10.534	36.500	3.736	−44°	148.114	6.866	9.552	−28°	129.35
9.630	1.868	−45°10′	20.171	∞	3.736	−45°	∞	8.468	9.504	−30°	142.74
∞	1.868	−45°	∞					10.539	9.463	−32°	160.55
								13.414	9.429	−34°	185.45
								18.096	9.399	−36°	221.93
								24.644	9.376	−38°	279.77
								35.165	9.359	−40°	385.06
								61.806	9.348	−42°	632.83
								208.362	9.291	−44°	1868.20
								∞	9.341	45°	∞

Fig. 12

Surface slope produced by a convection current in a sea with a long straight coast



Having determined η (or V_H), θ and $\mathfrak{d}$, the substitution of them in (39) gives the definite value of w. Table 10 and Figs. 13–15 show the result of calculations.

(B) *Enclosed sea*:—By the continuity condition

$$S_x = 0 = -\frac{\mathfrak{a}}{4k^3}kH^2 + \frac{\mathfrak{a}}{4k^3}2kH.\mathfrak{d}\cos\phi - \frac{f\rho V_H^2}{2\mu k^2}\sin\theta,$$

$$S_y = 0 = \frac{\mathfrak{a}}{4k^3}2kH.\mathfrak{d}\sin\phi + \frac{f\rho V_H^2}{2\mu k^2}\cos\theta,$$

or

$$\left.\begin{aligned} &\eta^2\sin\theta + 2k^2H^2 - 2kH.2k\mathfrak{d}\cos\phi = 0, \\ &\eta^2\cos\theta + 2kH.2k\mathfrak{d}\sin\phi = 0, \end{aligned}\right\} \quad (48)$$

from which we get the total flows parallel and perpendicular to the slope

Table 10.

Convection currents in a sea with a long straight coast

$\phi=0$				
$H=\frac{1}{2}D$				
	$\xi=0.281$		$\xi=0.809$	
$\frac{z}{H}$	v_x	v_y	v_x	v_y
0.0	0.343	0.392	0.445	0.469
0.1	0.333	0.365	0.433	0.440
0.2	0.305	0.294	0.400	0.364
0.3	0.264	0.194	0.350	0.253
0.4	0.213	0.078	0.287	0.123
0.5	0.158	−0.041	0.218	−0.016
0.6	0.107	−0.152	0.149	−0.151
0.7	0.058	−0.236	0.088	−0.271
0.8	0.026	−0.287	0.041	−0.361
0.9	0.005	−0.289	0.011	−0.410
1.0	0.000	−0.227	0.000	−0.404

Table 10.

(Continued)

$\phi=0$						
$H=D$					$H=2.5D$	
	$\xi=0.309$		$\xi=0.763$		$\xi=0.661$	
$\frac{z}{H}$	v_x	v_y	v_x	v_y	v_x	v_y
0.0	3.319	1.027	3.697	1.055	13.305	0.999
0.1	3.220	0.934	3.554	0.964	12.734	0.644
0.2	2.938	0.712	3.264	0.750	11.374	0.208
0.3	2.517	0.434	2.826	0.481	9.731	0.004
0.4	2.009	0.139	2.293	0.197	8.064	−0.030
0.5	1.475	−0.144	1.722	−0.081	6.439	0.000
0.6	0.969	−0.399	1.166	−0.344	4.816	0.017
0.7	0.539	−0.610	0.678	−0.576	3.171	−0.056
0.8	0.227	−0.709	0.302	−0.750	1.613	−0.291
0.9	0.053	−0.661	0.071	−0.809	0.430	−0.644
1.0	0.000	−0.365	0.000	−0.682	0.000	−0.595

Table 10.

(Continued)

$\phi=3\pi/4$				
$H=0.5D$				
	$\xi=0$		$\xi=0.437$	
$\frac{z}{H}$	v_x	v_y	v_x	v_y
0.0	1.411	−1.258	1.909	−1.823
0.1	1.399	−1.263	1.917	−1.829
0.2	1.363	−1.275	1.941	−1.836
0.3	1.301	−1.268	1.879	−1.833
0.4	1.217	−1.237	1.795	−1.801
0.5	1.112	−1.168	1.683	−1.733
0.6	0.975	−1.064	1.545	−1.627
0.7	0.806	−0.894	1.370	−1.460
0.8	0.593	−0.663	1.155	−1.230
0.9	0.325	−0.360	0.899	−0.937
1.0	0.000	0.000	0.543	−0.574

Table 10.

(Continued)

$\phi=3\pi/4$						
$H=D$						
	$\xi=0$		$\xi=0.678$		$\xi=2.434$	
$\frac{z}{H}$	v_x	v_y	v_x	v_y	v_x	v_y
0.0	8.023	−8.570	9.221	−9.838	13.243	−13.965
0.1	8.001	−8.627	9.195	−9.902	13.222	−14.038
0.2	7.944	−8.719	9.134	−10.018	13.169	−14.182
0.3	7.868	−8.733	9.068	−10.071	13.112	−14.281
0.4	7.782	−8.559	9.003	−9.950	13.074	−14.223
0.5	7.634	−8.100	8.948	−9.546	13.026	−14.891
0.6	7.453	−7.243	8.688	−8.654	12.916	−13.171
0.7	6.731	−5.940	8.213	−7.492	12.595	−11.960
0.8	5.549	−4.193	7.218	−5.750	11.721	−10.217
0.9	3.454	−2.111	5.363	−3.585	10.254	−7.957
1.0	0.000	0.000	2.224	−1.254	7.449	−5.307

Table 10.

(Continued)

$\phi=3\pi/4$						
$H=2.5D$						
	$\xi=0$		$\xi=2.943$		$\xi=8.464$	
$\frac{z}{H}$	v_x	v_y	v_x	v_y	v_x	v_y
0.0	59.720	−59.637	69.603	−69.578	99.962	−99.928
0.1	59.124	−59.965	69.015	−69.902	99.380	−100.253
0.2	57.658	−60.355	67.572	−70.138	97.953	−100.597
0.3	55.786	−60.627	65.718	−70.292	96.113	−100.687
0.4	53.443	−61.120	63.762	−70.330	94.122	−100.799
0.5	52.608	−62.216	62.241	−71.014	92.434	−100.990
0.6	53.189	−65.337	62.327	−73.189	92.157	−103.545
0.7	56.759	−61.746	65.489	−76.624	95.023	−107.157
0.8	60.726	−51.315	70.615	−76.744	100.958	−104.556
0.9	51.537	−26.541	75.001	−60.500	101.296	−87.471
1.0	0.000	0.000	27.073	−10.084	69.681	−40.244

Table 10.

(Continued)

$\phi=\pi/2$						
$H=0.5D$						
	$\xi=0$		$\xi=0.392$		$\xi=1.242$	
$\frac{z}{H}$	v_x	v_y	v_x	v_y	v_x	v_y
0.0	−0.150	−2.493	−0.195	−3.251	−0.251	−5.091
0.1	−0.143	−2.494	−0.188	−3.253	−0.242	−5.094
0.2	−0.123	−2.487	−0.164	−3.248	−0.217	−5.091
0.3	−0.090	−2.456	−0.124	−3.223	−0.168	−5.072
0.4	−0.045	−2.376	−0.069	−3.154	−0.103	−5.012
0.5	0.012	−2.241	−0.001	−3.023	−0.020	−4.892
0.6	0.064	−2.046	0.067	−2.822	0.067	−4.704
0.7	0.106	−1.722	0.129	−2.524	0.149	−4.415
0.8	0.128	−1.295	0.180	−2.102	0.229	−4.008
0.9	0.101	−0.751	0.179	−1.563	0.260	−3.472
1.0	0.000	0.000	0.111	−0.868	0.238	−2.782

Table 10.

(Continued)

$\phi=\pi/2$						
$H=D$						
	$\xi=0$		$\xi=0.692$		$\xi=3.716$	
$\frac{z}{H}$	v_x	v_y	v_x	v_y	v_x	v_y
0.0	−1.541	−13.626	−1.635	−15.414	−1.796	−27.002
0.1	−1.522	−13.755	−1.625	−15.554	−1.802	−27.158
0.2	−1.442	−14.062	−1.570	−15.890	−1.790	−27.546
0.3	−1.241	−14.403	−1.403	−16.287	−1.684	−28.038
0.4	−0.848	−14.617	−1.045	−16.590	−1.386	−28.492
0.5	−0.248	−14.493	−0.461	−16.590	−0.827	−28.709
0.6	0.555	−13.797	0.371	−16.062	0.051	−28.469
0.7	1.414	−12.242	1.334	−14.710	1.194	−27.467
0.8	2.007	−9.557	2.148	−12.241	2.385	−25.371
0.9	1.812	−5.488	2.331	−8.357	3.218	−21.811
1.0	0.000	0.000	1.110	−2.922	2.982	−16.517

Table 10.

(Continued)

$\phi=\pi/2$						
$H=2.5D$						
	$\xi=0$		$\xi=1.903$		$\xi=6.261$	
$\frac{z}{H}$	v_x	v_y	v_x	v_y	v_x	v_y
0.0	−0.916	−106.746	−0.924	−115.001	−0.933	−149.480
0.1	−1.485	−107.048	−1.488	−115.309	−1.491	−149.792
0.2	−2.905	−107.333	−2.885	−115.599	−2.865	−150.090
0.3	−4.847	−107.376	−4.783	−115.623	−4.716	−150.092
0.4	−7.211	−107.632	−7.112	−115.780	−7.008	−150.145
0.5	−9.571	−109.033	−9.572	−116.971	−9.573	−151.119
0.6	−10.117	−112.377	−10.597	−120.148	−11.100	−154.119
0.7	−4.877	−115.993	−6.359	−124.246	−7.914	−158.724
0.8	9.729	−111.048	7.440	−121.603	5.040	−158.493
0.9	25.565	−78.187	25.609	−93.570	25.667	−135.533
1.0	0.000	0.000	11.347	−19.731	22.989	−65.745

Fig. 13

Convection currents in a sea with a long straight coast

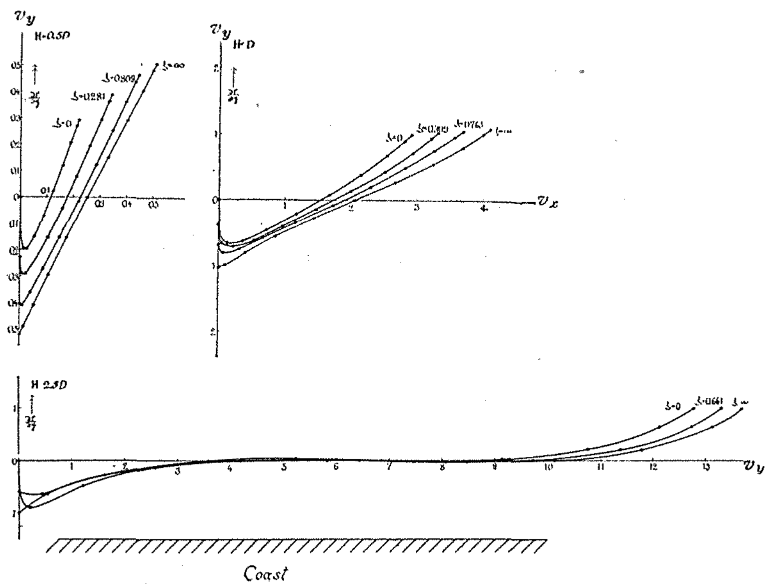


Fig. 14

Convection currents in a sea with a long straight coast

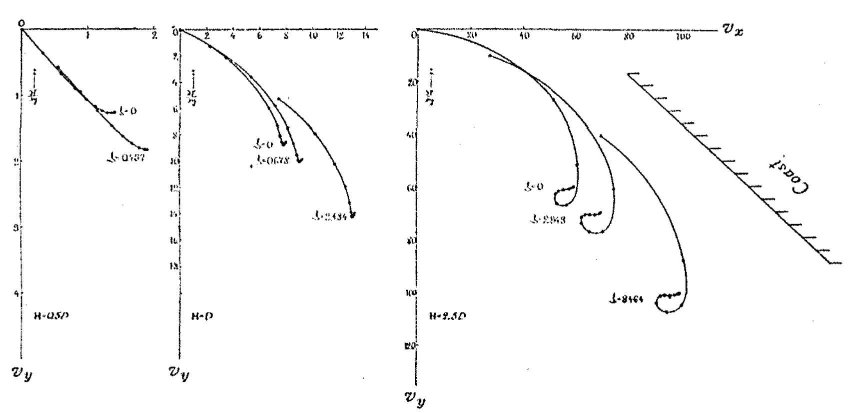


Fig. 15

Convection currents in a sea with a long straight coast

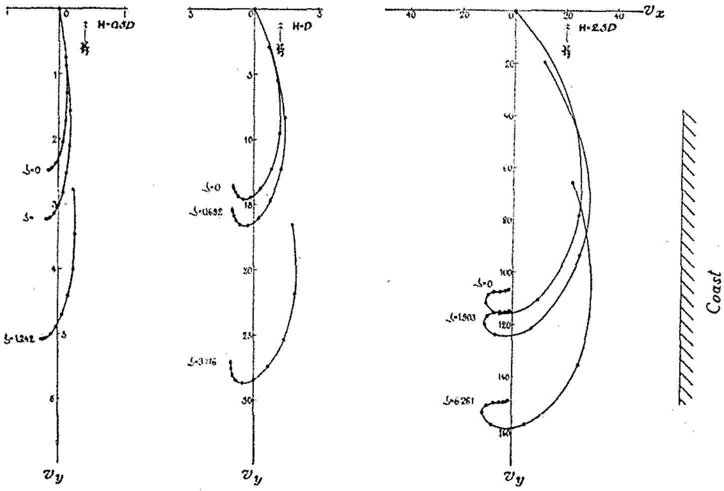
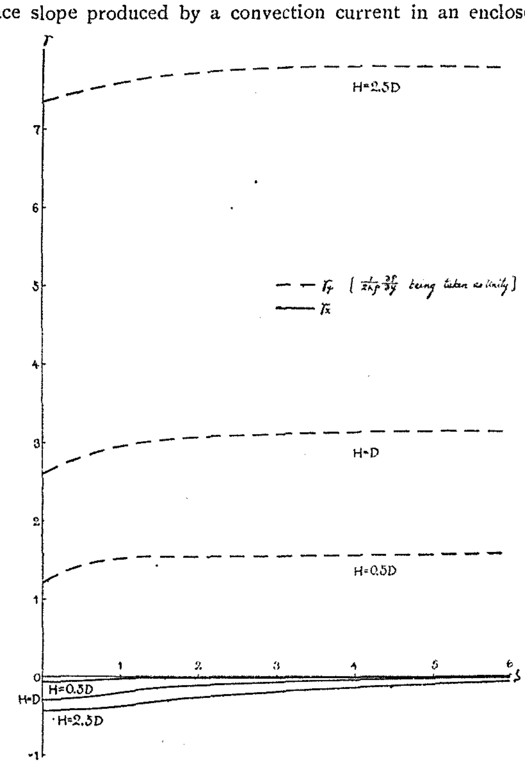


Fig. 16

Surface slope produced by a convection current in an enclosed sea



$$\left.\begin{aligned} \eta^2\cos(\theta-\phi) &= -2k^2H^2\sin\phi, \\ \eta^2+2kH[kH\sin\theta-2k\mathfrak{d}\sin(\theta-\phi)] &= 0. \end{aligned}\right\} \quad (49)$$

Now, with the 4 eqs. of (45) and (49) we can determine η, θ, $\mathfrak{d}$ and ϕ for any given ξ. For instance, substitute the lower eq. of (49) in the lower eq. of (45), then

$$\eta^2=\frac{P\sin(\theta-\varphi)-kH\sin\theta}{\dfrac{1}{2kH}-\dfrac{s}{2}} \quad (50)$$

which gives the value of η corresponding to any possible value of θ. Next determine ϕ with the upper eq. of (49), and $\mathfrak{d}$ with the lower eq. of (45); or with (48), $\mathfrak{d}\cos\phi$ and $\mathfrak{d}\sin\phi$ may be calculated. Finally ξ is obtained from the upper eq. of (45).

Table 11 shows the results thus obtained, the surface slope derived from which is shown in Fig. 16.

Table 11.

Corresponding values of ξ, η, θ, $k\mathfrak{d}$ and ϕ for convection currents in an enclosed sea

$H=2.5D$				
ξ	η	θ	$2k\mathfrak{d}\cos\phi$	$2k\mathfrak{d}\sin\phi$
0.000	3.228	229.°5	7.351	−0.430
0.091	3.175	228°	7.378	−0.429
0.207	3.110	226°	7.412	−0.428
0.335	3.038	224°	7.447	−0.422
0.465	2.963	222°	7.481	−0.415
0.605	2.882	220°	7.515	−0.405
0.756	2.795	218°	7.549	−0.392
0.910	2.709	216°	7.580	−0.378
1.077	2.617	214°	7.611	−0.361
1.255	2.521	212°	7.641	−0.343
1.453	2.418	210°	7.669	−0.322
1.680	2.304	208°	7.696	−0.298
1.918	2.192	206°	7.721	−0.275
2.204	2.061	204°	7.745	−0.247
2.536	1.926	202°	7.766	−0.219
2.906	1.786	200°	7.786	−0.191
3.395	1.624	198°	7.803	−0.160
4.040	1.443	196°	7.818	−0.127
4.928	1.244	194°	7.831	−0.096
6.377	1.005	192°	7.842	−0.063
10.087	0.664	190°	7.850	−0.028
∞	0.000	188.°3	7.855	−0.001

Table 11. (Continued)

$H=D$				
ξ	η	θ	$2k\mathfrak{d}\cos\phi$	$2k\mathfrak{d}\sin\phi$
0.000	1.967	242°.5	2.596	−0.284
0.009	1.953	242°	2.606	−0.285
0.085	1.900	240°	2.644	−0.287
0.167	1.845	238°	2.683	−0.287
0.249	1.790	236°	2.719	−0.285
0.342	1.728	234°	2.758	−0.279
0.439	1.665	232°	2.794	−0.272
0.545	1.599	230°	2.830	−0.262
0.663	1.527	228°	2.866	−0.248
0.786	1.455	226°	2.900	−0.234
0.949	1.368	224°	2.935	−0.214
1.104	1.289	222°	2.965	−0.196
1.312	1.191	220°	2.999	−0.173
1.556	1.089	218°	3.026	−0.149
1.876	0.972	216°	3.054	−0.122
2.308	0.845	214°	3.079	−0.094
2.981	0.696	212°	3.101	−0.065
4.416	0.497	210°	3.122	−0.034
∞	0.000	208°	3.142	0.000

Table 11. (Continued)

$H=0.5D$				
ξ	η	θ	$2k\mathfrak{d}\cos\phi$	$2k\mathfrak{d}\sin\phi$
0.000	1.093	261°	1.197	−0.060
0.032	1.064	260°	1.218	−0.062
0.102	0.997	258°	1.262	−0.066
0.189	0.933	256°	1.303	−0.067
0.274	0.855	254°	1.349	−0.064
0.397	0.765	252°	1.395	−0.058
0.560	0.663	250°	1.440	−0.048
0.865	0.502	248°	1.482	−0.036
1.361	0.377	246°	1.530	−0.018
∞	0.000	244°	1.571	0.000

Table 12.

Convection currents in an enclosed sea

$H=0.5D$				
	$\xi=0.189$		$\xi=0.560$	
$\frac{z}{H}$	v_x	v_y	v_x	v_y
0.0	0.113	0.376	0.176	0.455
0.1	0.105	0.349	0.166	0.426
0.2	0.083	0.278	0.137	0.350
0.3	0.052	0.179	0.094	0.240
0.4	0.015	0.063	0.041	0.111
0.5	−0.021	−0.061	−0.014	−0.025
0.6	−0.052	−0.156	−0.065	−0.155
0.7	−0.072	−0.233	−0.103	−0.267
0.8	−0.076	−0.271	−0.134	−0.347
0.9	−0.067	−0.254	−0.141	−0.390
1.0	−0.040	−0.160	−0.128	−0.351

Table 12. (Continued)

$H=D$				
	$\xi=1.556$		$\xi=0.663$	
$\frac{z}{H}$	v_x	v_y	v_x	v_y
0.0	1.863	1.248	1.632	1.330
0.1	1.756	1.153	1.527	1.229
0.2	1.452	0.913	1.232	0.954
0.3	0.998	0.637	0.792	0.658
0.4	0.447	0.315	0.271	0.296
0.5	−0.136	−0.017	−0.259	−0.081
0.6	−0.687	−0.352	−0.723	−0.453
0.7	−1.139	−0.677	−1.050	−0.790
0.8	−1.429	−0.953	−1.174	−1.027
0.9	−1.503	−1.111	−1.049	−1.066
1.0	−1.333	−1.043	−0.673	−0.755

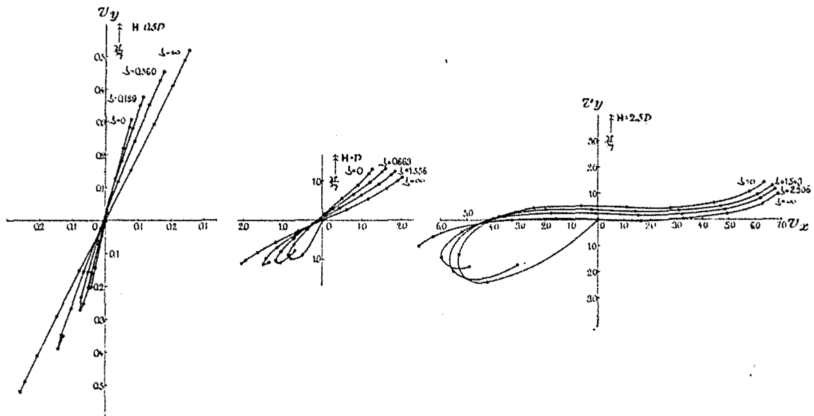
Table 12. (Continued)

$H=2.5D$				
	$\xi=2.906$		$\xi=1.453$	
$\frac{z}{H}$	v_x	v_y	v_x	v_y
0.0	6.785	1.188	6.668	1.318
0.1	6.215	0.834	6.099	0.963
0.2	4.858	0.396	4.744	0.527
0.3	3.221	0.190	3.109	0.332
0.4	1.558	0.175	1.442	0.324
0.5	−0.083	0.232	−0.225	0.392
0.6	−1.773	0.266	−1.974	0.402
0.7	−3.549	0.116	−3.804	0.125
0.8	−5.188	−0.441	−5.329	−0.715
0.9	−5.996	−1.420	−5.522	−1.953
1.0	−4.875	−1.761	−3.047	−1.746

Knowing the values of η (or V_H), θ, $\mathfrak{d}$ and ϕ for any given ξ, we can easily obtain the numerical value of w in (39). The results are shown in Table 12 and Fig. 17.

Fig. 17

Convection currents in an enclosed sea



§ **11.** The essential points in the foregoing are:

1) If a difference in water-density exists perpendicular to a long straight coast or in an enclosed sea, the land effect does not differ greatly for all values of ξ, and the surface slope produced will be 3/8 to 4/8 of that considered statically.

2) If the difference in density exists parallel to a long straight coast, a marked component current in the same direction is generated, and the current diagram will show a conspicuous shift along the coast in proportion to the increase of ξ. Thus, for large values of ξ (or small bottom-friction) beyond a certain limit, the surface slope γ becomes so large that the vertical motion can not be neglected, and the fundamental equation (31) must be modified.

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