On the so-called "Grenzfläche" in the Current due to the Difference of Density.

By

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ABSTRACT.

What difference in height of sea-level should be produced by a local difference in the density of sea water? Or, what ratio must the difference in level in the dynamical equilibrium hold to that when statical equilibrium is assumed? This ratio must obviously have an intimate connection with the "Grenzfläche" which is defined as an equipressure layer lying horizontal in the water.

The present paper treats theoretically the Grenzfläche in a basin of rectangular section, in which the density of the water differs from place to place in the upper part of some thickness, but in the remaining lower part it is homogeneous throughout.

Accoring to the result, if local inequality of density extend equally throughout the whole depth from surface to bottom, the real level difference will be only 3/8 to 4/8 of the statical one. But the thinner the part in which inequality of water exists, the nearer the level difference in dynamical equilibrium approaches to that in statical equilibrium; and when the part of heterogeneous water is, as in the actual ocean basin, less than about 1/10 of the whole depth, the sea surface will assume practically the same level as if it were in statical equilibrium.

Moreover, the calculation shows that the no-current layer does not always coincide with the Grenzfläche of constant pressure, but will exist generally somewhat lower.

By the bye, the paper involves a discussion on the inequality of the pressure upon the sea bottom which will be caused by the difference in density of water, because the bottom-pressure may have some relation to the frequency of submarine earthquakes, as pointed out by Dr. Omori.

Introduction.

The oceanography teaches us that the temperature and the salinity, and, consequently, the density, of the water in the ocean surface greatly differs according to locality, and that these local differences become rapidly less and less as the depth increases, until below a level of some hundreds of meters they are very small, and the water may be regarded as practically uniform throughout the whole ocean. Thus, although the differences in the density of sea water are limited only in the upper stratum which is very thin compared with the mean depth (about 4000 meters) of the world ocean, sea water can not hold a statical equilibrium. In a deep level, the pressure in places where the water is denser must be greater than that where the upper water is lighter, and so water will flow from the former regions towards the latter, with the result that the water surface in the latter regions rises up higher than in the former. As soon as an inclination of surface is produced, the distribution of pressure in a horizontal section of the uppermost strata becomes contrary to that in the bottom layer; i. e., the pressure in regions of a higher level of surface will be greater than that where the surface level is lower, and therefore the water in the upper layers must flow from the higher level regions to the lower in order to diminish the level difference of the surface. Thus, in the upper and lower parts of the sea there must generate two systems of currents and of pressure distributions, which are reverse to each other. Between these two systems somewhere one may conceive an equipressure layer which is horizontal, and also another layer in which no current flows. H. Mohn[1] assumed these two layers—equipressure and no-current —to be coincident and called the layer the Grenzfläche. Krümmel[2] states that for a basin of rectangular section the Grenzfläche will lie at half the depth of the bottom.

Now the prime object of the present paper is to throw a light on the problem:

(i) What difference of sea level will be produced by a local difference in the density of water when dynamical equilibrium has been attained?

But this is almost the same thing as determining

(ii) the depth of the Grenzfläche.

Here I define a Grenzfläche as such a one in which the horizontal gradient of pressure is nil.

For me it is not self-evident but to be discussed

(iii) Whether the Grenzfläche must always coincide with the no-current layer, or not.

Moreover I will, by the way, discuss the question:

(iv) What difference of pressure at the sea-bottom will be caused by the difference in the density of water?

1 M. P. Rudzki, Physik der Erde, 447 (1911).

2 Handbuch der Ozeanographie, II, 482 (1911).

For, Dr. Omori[1] investigated the frequency of earthquakes of suboceanic origin with relation to the pressure on the sea-bottom. He assumed that a seasonal fluctuation of mean sea-level, the barometric effect being removed, must produce a variation in the pressure on the sea-bottom, directly proportional to the difference in height of the surface level. But Dr. Nagaoka[2] disagreed with the opinion of Omori, and said:

"If the necessary temperature correction be introduced, it would follow that there are slight irregularities which would have no relation to seismic frequency."

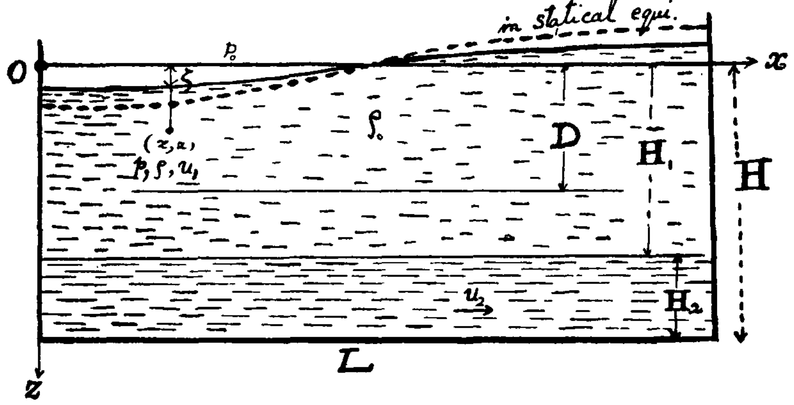
This statement seems to involve that the difference in the density of water would not affect the pressure on the sea-bottom.

The present writer is of a somewhat different opinion, which will be seen at the end of this paper.

I. Fundamental Equations.

§ 1. For simplicity, let us consider a rectangular ocean basin of length L and of uniform depth H. Let the co-ordinate axes be taken as in the figure, the x-axis being taken in the mean free surface and the z-axis vertically downward.

Fig. I.



We shall denote by:

ρ the density of the water, a function of x and z,

μ the viscosity,

u, w the component velocities of current, in the directions of x and z respectively,

p the pressure at a point (x, z),

1 Pub. Earthq. Inves. Com., **18**, 26 (1904);
Bull. Imp. Earthq. Inves. Com., **2**, 47 (1908).

2 Tokyo Sugaku-Buturigakkwai Kizi., **4**, 384 (1908).

p_0 the external pressure over the free surface, which is assumed to be constant,

ζ the depression of the free surface from the mean level,

t the time,

g the acceleration due to gravity.

Now the fundamental equations of motion of a viscous liquid in two dimensions are:

$$\frac{Du}{Dt}=X-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\mu}{\rho}\Delta_1^2 u \quad\ldots\ldots(1),$$

$$\frac{Dw}{Dt}=Z-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{\mu}{\rho}\Delta_1^2 w \quad\ldots\ldots(2),$$

where

$$\frac{D}{Dt}=\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+w\frac{\partial}{\partial z},$$

$$\nabla_1^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}.$$

Since our ocean has a vast extent compared with its depth, as if it were merely a thin lamina, the quantities w and $\frac{\partial u}{\partial x}$ may be neglected compared with its horizontal motion, unless there were a very abrupt local disturbance. In our case we have $X=0$, $Z=g$. Hence, the above equations reduce to:

$$\frac{\partial u}{\partial t}=\frac{\mu}{\rho}\frac{\partial^2 u}{\partial z^2}-\frac{1}{\rho}\frac{\partial p}{\partial x} \quad\ldots\ldots(1_a)$$

$$p-p_0=\int_\zeta^z g\rho . dz \quad\ldots\ldots(2_a)$$

viz., the pressure at any point (x, z) is sensibly equal to the statical pressure due to the depth below the surface.

Further let us assume that in the upper strata of the sea of the thickness H_1 the density of water ρ differs from place to place horizontally but not vertically, and in the remaining lower strata of the thickness H_2, the water is homogeneous throughout, and also that the difference of the density at any two points is so small that its higher orders can be neglected. In these circumstances, the equation (2_a) gives

$$-\frac{\partial p}{\partial x}=g\left(\rho\frac{\partial \zeta}{\partial x}-\frac{\partial \rho}{\partial x}z\right)$$

$$=b(D-z) \quad \text{for the upper strata } O<z<H_1$$

$$=b(D-H_1) \quad \text{for the lower strata } H_1<z<H,$$

where

$$\left.\begin{aligned} b &= g.\frac{\partial\rho}{\partial x} \text{ (known)}, \\ D &= \rho.\frac{\partial\zeta}{\partial x} \Big/ \frac{\partial\rho}{\partial x} \text{(unknown)} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3).$$

It is here to be noticed that, in our assumed ocean, both of these quantities, b and D, must be independent of z, but may depend on x and t only, and also that D corresponds to the depth of the Grenzfläche, because $\frac{\partial p}{\partial x}=0$ when $z=D$.

Thus, if we confine ourselves to the final state in which the flow has become steady, and if by u_1 and u_2 we denote the horizontal currents in the upper heterogeneons and lower homogeneous layers respectively, the dynamical equation (1_a) takes the form

$$\left.\begin{aligned} \frac{\partial^2 u_1}{\partial z^2} + \frac{b}{\mu}(D - z) &= 0 \text{ for } 0 < z < H_1 \\ \frac{\partial^2 u_2}{\partial z^2} + \frac{b}{\mu}(D - H_1) &= 0 \text{ for } H_1 < z < H \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots (4),$$

and the equation of continuity may be written

$$0 = \int_0^H u \cdot dz = \int_0^{H_1} u_1 \cdot dz + \int_{H_1}^H u_2 \cdot dz \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5).$$

As the boundary conditions, we shall take:

At the free surface

$$\left|\frac{\partial u_1}{\partial z}\right|_{z=0} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6).$$

At the intermediate surface of discontinuity $z=H_1$

$$\left|u_1 = u_2\right|_{z=H_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7),$$

and $$\left|\frac{\partial u_1}{\partial z} = \frac{\partial u_2}{\partial z}\right|_{z=H_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8).$$

At the bottom, according to the assumption,

either $$\left|u_2\right|_{z=H} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9_a),$$

or $$\left|\frac{\partial u_2}{\partial z}\right|_{z=H} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9_b).$$

At the end-walls

$$u=0 \text{ for } x=0 \text{ and } x=L \ldots\ldots\ldots\ldots\ldots\ldots (10).$$

II. Solutions.

§ 2. *Case where the bottom condition is to be taken* $|u_2|_{z=H}=0$, *i. e., no bottom-current.*

For the upper strata, the first integral of equation (4) is

$$\frac{\partial u_1}{\partial z}=\frac{b}{2\mu}(z^2-2Dz) \quad\ldots\ldots(11),$$

the integration constant being O by surface condition (6). Integrating again there results

$$u_1=\frac{b}{6\mu}(z^3-3Dz^2+B_1) \quad\ldots\ldots(12).$$

And for the lower strata, the first integral is

$$\frac{\partial u_2}{\partial z}=\frac{b}{2\mu}\left\{2(H_1-D)z-H_1^2\right\} \quad\ldots\ldots(13),$$

the integration constant being taken to satisfy condition (8). The second integral is therefore

$$u_2=\frac{b}{2\mu}\left\{(H_1-D)z^2-H_1^2z+B_2\right\} \quad\ldots\ldots(14),$$

and this, by the bottom condition (9_a), becomes

$$u_2=-\frac{b}{2\mu}\left\{(H_1-D)(H^2-z^2)-H_1^2(H-z)\right\} \quad\ldots\ldots(14_a)$$

Now the constant B_1 in equation (12) can be determined by condition (7), thus

$$B_1=-\{3(H-H_1)(HH_1-HD-H_1D)+H_1^3-3DH_1^2\}.$$

Hence, (12) finally reduces to

$$u_1=\frac{b}{6\mu}\{(z^3-3Dz^2)-(H_1^3-3DH_1^2)-3(H-H_1)(HH_1-HD-H_1D)\} \quad (12_a)$$

In order to test the equation of continuity, we first calculate the integrals $\int_0^{H_1}u_1dz$ and $\int_{H_1}^{H}u_2dz$.

$$\frac{2\mu}{b}\int_0^{H_1}u_1dz=-\frac{H_1^3}{12}(3H_1-8D)-H_1(H-H_1)(HH_1-HD-H_1D),$$

$$\frac{2\mu}{b}\int_{H_1}^{H}u_2dz=-\tfrac{1}{3}(H_1-D)(2H^3-3H^2H_1+H_1^3)+\tfrac{1}{2}H_1^2(H-H_1)^2.$$

Hence if we put

$$m \equiv \frac{D}{H_1}, \qquad n \equiv \frac{H}{H_1} \quad \dots\dots\dots\dots (15),$$

the equation of continuity gives

$$\frac{1}{12}\Big[(12n^2-4)m-(12n^2-12n+3)\Big]=\frac{1}{6}\Big[4n^3-9n^2 + 6n - 1 - 2m(2n^3-3n^2+1)\Big],$$

or, $$m \equiv \frac{D}{H_1} = \frac{8n^3-6n^2+1}{8n^3} \quad \dots\dots\dots\dots (16_a).$$

This determines the position of the Grenzfläche, which is now known to be a function of H and H_1 only, but not of x.

Next, it is required to find the position of the no-current layer.

First, in order that the no-current layer should exist just on the discontinuous surface ($z=H_1$), we must have

$$u_1 = u_2 = 0 \quad \text{at } z = H_1,$$

i. e., $$0=(H_1-D)(H^2-H_1^2)-H_1^2(H-H_1)$$

or $$m=\frac{n}{n+1},$$

which, combined with (16_a), gives

$$\left.\begin{aligned} n&=H/H_1=2{\cdot}752,\\ m&=D/H_1=0{\cdot}735\end{aligned}\right\} \quad \dots\dots\dots\dots (17_a).$$

If the given basin has $n<2{\cdot}752$, the no-current layer must lie in the upper part of heterogeneous density, and its depth can be determined by $u_1=0$, namely

$$\left(\frac{z}{H_1}\right)^3-3m\left(\frac{z}{H_1}\right)^2=(1-3m)+3(n-1)\{n-(n+1)m\}.$$

Cardan's solution of this cubic equation takes the form:

$$\frac{z}{H_1}=m+\left\{\frac{K_a}{2}+m^3+\sqrt{\frac{K_a^2}{4}+K_a m^3}\right\}^{\frac{1}{3}}+\left\{\frac{K_a}{2}+m^3-\sqrt{\frac{K_a^2}{4}+K_a m^3}\right\}^{\frac{1}{3}} \quad \dots\dots (18_a),$$

where

$$K_a=(1-3m)+3(n-1)\{n-(n+1)m\}.$$

On the contrary, if $n>2\cdot752$, the no-current layer will lie in the lower part of homogeneous density and its position can be found by

$$u_2=0=(H_1-D)(H^2-z^2)-H_1^2(H-z),$$

i. e., $$\frac{z}{H_1}=\frac{1}{1-m}-n=\frac{2n^3+n}{6n^2-1} \quad \dots\dots\dots\dots (19_a).$$

§ 2_b. *Case where the bottom condition is to be taken* $\left|\frac{\partial u_2}{\partial z}\right|_{z=H}$ $=0$, *i, e., no bottom-friction.*

Equations (11) to (14) hold good, as they stand, in the present case too. But the bottom condition (9_b) is now

$$\left|\frac{du_2}{dz}\right|_{z=H}=\frac{b}{2\mu}\{2(H_1-D)H-H_1^2\}=0,$$

which immediately gives the depth of the Grenzfläche

$$m\equiv D/H_1=1-1/2n \text{ ....}(16_b).$$

The remaining condition (7) and the equation of continuity (5) determine the two integration constants B_1 and B_2. That is, from the condition that $u_1=u_2$ at $z=H_1$, we have

$$H_1^3+B_1=3B_2,$$

and by the equation of continuity

$$B_1H_1+\frac{H_1^3}{4}(H_1-4D)=-(H_1-D)(H^3-H_1^3)+\frac{3}{2}H_1^2(H^2-H_1^2)$$
$$-3B_2(H-H_1).$$

Hence, putting $m\equiv D/H_1$, $n\equiv H/H_1$ as before, and having the relation (16_b) in mind, we obtain

$$B_1=H_1^3\cdot\frac{4n^2-4n+1}{4n}, \quad \text{and} \quad B_2=H_1^3\cdot\frac{4n^2+1}{12n}$$

and so finally

$$u_1=\frac{b}{6\mu}\left[z^3-3DZ^2+\frac{4n^2-4n+1}{4n}\cdot H_1^3\right]\text{.....................}(12_b),$$

$$u_2=\frac{b}{2\mu}\left[(H_1-D)z^2-H_1^2z+\frac{4n^2+1}{12n}\cdot H_1^3\right] \text{ ....}(14_b).$$

As to the no-current layer, it lies just at the discontinuous surface ($z=H_1$), if

$$|u_1|_{z=H_1}=0=H_1^3-3DH_1^2+\frac{4n^2-4n+1}{4n}\cdot H_1^3,$$

viz.,
$$\left.\begin{matrix}n=2\cdot207,\\ m=0\cdot774\end{matrix}\right\}\text{...................................}(17_b).$$

But if the given basin is of $n<2\cdot207$, the no-current layer will be at a depth such that

$$u_1=0=\left(\frac{z}{H_1}\right)^3-3m\left(\frac{z}{H_1}\right)^2+\frac{4n^2-4n+1}{4n},$$

which gives

$$\frac{z}{H_1}=m+\left\{m^3-\frac{K_b}{2}+\sqrt{\left(\frac{K_b}{2}\right)^2-K_b\cdot m^3}\right\}^{\frac{1}{3}}$$

$$+\left\{m^3-\frac{K_b}{2}-\sqrt{\left(\frac{K_b}{2}\right)^2-m^3\cdot K_b}\right\}^{\frac{1}{3}} \quad \cdots\cdots\cdots (18_b),$$

where

$$K_b=\frac{4n^2-4n+1}{4n}.$$

And if $n>2\cdot207$, the depth of the no-courrent layer is given by

$$u_2=0=(1-m)\left(\frac{z}{H_1}\right)^2-\frac{z}{H_1}+\frac{4n^2+1}{12n},$$

viz.,

$$\frac{z}{H_1}=n-\frac{1}{6}\sqrt{12n^2-6} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (19_b).$$

§ 3. In the preceding paragraphs we have treated two cases: in the one case we assumed the bottom-friction to be nil, and in the other the bottom-friction is taken to be great enough to keep the bottom-velocity always at zero. For the real sea, of course, the bottom condition will be between these two, and consequently all things in it may also be presumed to be between those in the above extreme cases.

III. Discussions

§ 4. *The difference of surface-level due to a difference in the density of water.*

Let $\bar{\zeta}$ = the supposed depression of the free surface from the mean sea-level if the water were in statical equilibrium,

ζ = the real deviation of level when dynamical equilibrium has been attained.

Let the distribution of density in the upper strata be given by

$$\rho=f(x)=\rho_0+\sum_{1}^{\infty} A_r \operatorname{Cos}\frac{2\pi r}{L}x,$$

ρ_0 being the mean density at the place where $\zeta=0$.

Then, in statical equilibrium, we have

$$(H_1-\bar{\zeta})\cdot\rho=H_1\rho_0,$$

which gives

$$\bar{\zeta}=\frac{\rho-\rho_0}{\rho}\cdot H_1 \doteqdot H_1\left[\sum_1^\infty r A_r \operatorname{Cos}\frac{2\pi r}{L}\cdot x\right],$$

since ρ does not much differ from 1.

Similarly, in dynamical equilibrium, the weight of the column of water above the Grenzfläche being constant, we have

$$\zeta=\frac{\rho-\rho_0}{\rho}\cdot D=D\left[\sum_1^\infty r\ A_r\cdot\ \operatorname{Cos}\frac{2\pi r}{L}x\right].$$

Hence

$$\frac{\zeta}{\bar{\zeta}}=\frac{D}{H_1}=m \cdots\cdots\cdots\cdots\cdots (20).$$

Thus, the ratio of the level difference in dynamical equilibrium to that in statical is equal to the depth of the Grenzfläche measured as a fraction of the total depth of the part of heterogeneous water.

§ 5. *The depth of the Grenzfläche.*

Using the formulae for the Grenzfläche (16_a) and (16_b), we have calculated out table I.

Table I

$n\equiv\frac{H}{H_1}$	$m\equiv D/H_1$	
	(a) No bottom-velocity	(b) No bottom-friction
1	0·375(=3/8)	0·500(=4/8)
2	0·641	0·750
3	0·755	0·833
4	0·815	0·875
5	0·851	0·900
6	0·876	0·917
7	0·893	0·929
8	0·907	0·938
9	0·917	0·944
10	0·925	0·950
20	0·963	0·975
∞	1·000	1·000

From the formulae or table I, we see:

(1). If n=1, i. e., the local inequality of density extends equally throughout the whole depth from surface to bottom, the Grenzfläche will lie at 3/8 to 4/8* of the total depth of the basin, according to the bottom

* This coincides with the statement of Krümmel, loc, cit.

conditions.

This means also that, in such a case, the level difference produced by a difference of density in dynamical equilibrium will be only 3/8 to 4/8 of the value calculated statically.

(2). m increases with n, and approaches to unity as its limiting value.

In other words, the shallower the upper part of the heterogeneous density in comparison with the total depth of the sea, the nearer the Grenzfläche approaches to, but necessarily higher than, the surface bounding the upper heterogeneous and the lower homogeneous parts of the sea-water.

(3). The thinner the part in which inequality of water exists, the nearer the level difference in dynamical equilibrium approaches to that calculated statically; and when the part of heterogeneous water is less than about 1/10 of the whole depth as in the actual ocean, the sea-surface will assume practically the same level as if it were in statical equilibrium.

§ 6. *The depth of the no-current layer.*

By formulae (17) to (19) with suffixes a and b, we have prepared table II.

Table II

$n \equiv \frac{H}{H_1}$	z/H_1	
	(a) No bottom-velocity	(b) No bottom-friction
1	0·422	0·500
2	0·764	0·920
2·207	——	1·000
2·752	1·000	——
3	1·076	1·318
4	1·389	1·726
5	1·711	2·142
6	2·037	2·559
7	2·365	2·975
8	2·695	3·400
9	3·025	3·823
10	3·356	4·240
20	6·678	8·470
∞	$\sqrt{1/3} \cdot n = \infty$	$n/3 = \infty$

Comparing this with table I, we observe that:

(1). The no-current layer will generally be lower than the Grenzfläche.

The only exceptional case, in which the no-current layer coincides with the Grenzfläche, is when the local inequality of density extends equally throughout the whole depth, and the bottom-friction may be taken as entirely zero.

(2). As $n \equiv H/H_1$ increases indefinitely, so also does the quantity z/H_1; and the no-current layer sinks down far below the depth H_1.

§ 7. *The pressure at the bottom.*

Since on the Grenzfläche the pressure is everywhere the same, so the pressure at the bottom must obviously vary according to locality.

Let Δp_H denote the excess of bottom-pressure at a place over that at the place of mean density ρ_0, then

$$\Delta p_H = g\rho(H_1 - D) - g\rho_0(H_1 - D) = g(\rho - \rho_0)(H_1 - D).$$

But we have already seen that the depression of the sea-level, ζ, is given by

$$\zeta = \frac{\rho - \rho_0}{\rho} . D$$

Introducing this into the above equation, we get

$$\Delta p_H = \frac{H_1 - D}{D} \cdot g\rho\zeta = \frac{1 - m}{m} \cdot g\rho\zeta \quad \cdots\cdots\cdots\cdots (21).$$

Thus, the pressure at the sea-bottom is greater where or when the sea-level is lower, the excess Δp_H being $\frac{1-m}{m}$ of the weight of a water-column whose height is equal to ζ.

A table for $(1 - m)/m$ is given below:

n	(1−m)/m	
	(a) No bottom-velocity	(b) No bottom-friction
1	1·667(=3/5)	1·000
2	0·560	0·333
3	0·325	0·205
4	0·227	0·143
5	0·175	0·111
6	0·141	0·091
7	0·120	0·076
8	0·107	0·066
9	0·090	0·059
10	0·081	0·053
20	0·038	0·026
∞	0	0

In the actual ocean whose n may be taken to be > 10, the difference of the bottom-pressure due to the difference in the density of water will be negligibly small, in accordance with Nagaoka's opinion.[1] But here we have two special cases worthy of notice.

The one is such that, as off the north-eastern coast of Japan, there warm and cold currents run side by side, and the local difference of density extends to the greater parts of the total depth of sea-bottom. In such a case, the part of the level difference, which is produced by the difference in density in the limited region, will affect the pressure at the sea-bottom, but in the sense opposite to Omori's assumption[2], i, e., the pressure at the bottom being rather small when the surface level is high.

The second case to be noticed is the shallow sea which covers the continental shelf fringing the deep ocean. Let h be the depth of the fringing sea, and suppose $h \ll D$, D being the depth of the Grenzfläche of the main ocean. Then, the bottom-pressure of the shallow sea will exceed the pressure at the same depth in the ocean of mean density ρ_0 by an amount:

$$\triangle p_h = g\rho_0(D-h) - g\rho(D-h) = g(\rho_0-\rho)(D-h)$$

$$= -\frac{D-h}{D} \cdot g\rho\zeta \quad \cdots\cdots\cdots (22),$$

ζ being the depression of the mean sea-level.

In a shallow sea, therefore, the pressure at the bottom will change directly proportional to the variation of the mean sea-level. Especially in a very shallow sea, only 10 or 20 meters deep, say, the rise or fall of the mean sea-level corresponds to the increase or decrease of the bottom-pressure nearly equal to the weight of a water-column of height $|\zeta|$. This is the only case in which Omori's assumption[2] on the change of pressure at the sea-bottom may be accepted.

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1. Loc. cit.
2. Loc. cit.