Distribution of the Electric Field in the Crookes Dark Space in a Narrow Tube

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By

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According to the investigation by Stark, the relation between the intensity of the electric field and the amount of separation of a certain spectrum line affected by it is well established; and consequently we may determine the distribution of the electric field in the Crookes dark space by utilizing Lo Surdo's[1] method. But it must be remembered that the results obtained by this method are applicable only to a vacuum tube of very small diameter, for with a larger one we encounter many experimental difficulties.

In the present study, the photographs taken in our former investigations[2] were utilized, some of which have already been published. The gas with which the vacuum tubes were filled was, in most cases, hydrogen, but, in a few cases, it was a mixture of helium and hydrogen.

The glow was never uniform in the dark space, but was mainly concentrated in the axial portion of the tube, excepting the space just in front of the cathode, where a glow of nearly equal intensity occupied the whole portion of the tube. As, in the present experiment, this intense glow in the axial portion was projected on the slit of a spectrograph, it must be remembered that the distribution of the electric field was actually measured in this portion.

For the determination of the electric field, two intense outer (p) and (s) components of the H_γ line were taken, as their separation was comparatively great, and the proportionality between the separation and the intensity of the electric field has already been proven by Stark.

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1 Rendiconti d. Lincei, **22**, 664 (1913).

2 Mem. Coll. Sci. Kyoto, **2**, 137, 321, 325, (1917).

As the amount of separation of the outer (p) and (s) components, Stark gave the values 4.6 Å.U. and 3.2 Å.U. per 10^4volt/cm. respectively. These values were employed in the following determination of the intensity of the electric field.

In some cases, the distribution of the electric field in the Crookes dark space was determined by both (p) and (s) components. The curves which represent the relation between the distance of a point in the dark space from the cathode, and the intensity of the electric field at that point were drawn by measuring the (p) and (s) components individually; and a fair agreement of these two curves was seen in every case examined. So in some photographs, where one of these two components were faint, only (p) or (s) components were measured.

With regard to the relation between the potential V of any point in the dark space and its distance x from the cathode, Schuster[1] proposed the following empirical formula.

$$V = V_0(1 - e^{-kx}),$$

where V_0 is the potential in the glow proper, and k a constant, the potential of the cathode being taken as zero. By this formula the intensity of the electric field E at any point in the dark space may be expressed by

$$E = -\frac{dV}{dx} = - V_0 k e^{-kx}.$$

Lo Surdo[2] observed that the form of the affected spectrum lines of hydrogen due to the glow in the dark space is Y shaped, and concluded that the empirical formula given by Schuster was correct as the first approximation.

Now in our photographs, shown in foregoing papers, the forms of H_β and H_γ are not Y shaped, their outer components being curved convex to the central undisplaced lines.

If Schuster's formula be applicable to the present case, the curve which represents the relation between $\log E$ and x should be a straight line. On testing the formula, the result came out negative.

Here it is to be noted that Aston[3] and Harris[4] measured the distribution of the electric field in the Crookes dark space by the deflec-

1 Proc. Roy. Soc., **47**, 541 (1890).
2 Rendiconti d. Lincei, **23**, 117 (1914).
3 Proc. Roy. Soc., **84**, 526 (1910).
4 Phil. Mag., **30**, 182 (1915).

tion of cathode rays. But the diameters of the discharge tubes used by them were very large compared with these used in the present experiment.

In a former paper[1] it has already been stated that, if we denote the field intensity at the end of the dark space by E_0, and that at any point in the dark space by E, the quantity $E-E_0$ seems to increase nearly proportionally to the square of the distance between that point and the end of the dark space. Denoting the distance of any point in the dark space from the cathode by d and the whole length of the dark space by d_0, the intensity of the electric field E at that point may be represented by the following parabolic relation.

$$E-E_0 = k(d_0-d)^2,$$

where k is a constant. Taking $E_0=0.23$ (expressed in 10^4volt/cm), $d_0=2.3$ mm., and $k=1.03$, the curve represented in Fig. 3 of the former paper[2] was drawn. It will be seen that the observed points coincided fairly well with this parabolic curve.

Since then, in the course of our further investigations on the effect of the electric field on the spectrum lines of hydrogen and helium, the formula above mentioned was tested by means of many photographs taken under different experimental conditions; and in every case, it was found that the parabolic formula expressed fairly well the distribution of the electric field in the Crookes dark space.

The three curves shown in Fig. 1 (No. 14, No. 18, and No. 99) were actually drawn by the formula above mentioned, proper constants being chosen for each curve respectively; here the marks × and ⊙ denote the observation points. Photographs No. 18[3] and No. 99[4] have already been represented in former papers.

The constants k of the parabolas and various other numerical data for each of the 12 photographs are tabulated in Table I.

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1 Mem. Coll. Sci., Kyoto, **2**, 137 (1917).

2 Ibid. **2**, 137 (1917).

3 Ibid. **2**, 325, Fig. 1, (1917).

4 Ibid. **2**, 321, Fig. 1, (1917).

TABLE I.

I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII
No. of plate	Gas	Current through the tube in milli amp.	Diameter of the tube in mm.	Current density per mm^2	Length of dark space in mm. (observed)	Length of dark space in mm. (calculated)	E_0 in 10^4volt/cm.	Maximum E in 10^4volt/cm.	k	k×(dark space)×(diameter)	Deviation from the mean value	Cathode fall in volt (Calculated)
14	He and H_2	0·6	1·72	0·26	1·4	1·59	0·29	4·88	1·76	4·8	−0·4	2820
18	He and H_2	0·6	1·72	0·26	1·7	1·75	0·41	6·53	2·00	6·0	+0·8	4300
37	H_2	1·2	2·00	0·38	2·3	2·30	0·23	5·85	1·03	4·8	−0·4	4700
77	H_2	1·7	2·4	0·38	2·5	2·64	0·28	7·30	0·86	5·5	+0·3	6240
80	H_2	1·3	2·4	0·29	2·6	2·59	0·35	5·35	0·76	4·7	−0·5	5310
88	H_2	0·7	1·5	0·40	2·1	1·94	0·26	7·00	1·89	5·5	+0·3	5100
89	H_2	0·9	1·5	0·51	1·6	1·64	0·35	5·50	1·95	4·8	−0·4	3430
90	H_2	0·7	1·5	0·44	2·0	2·03	0·30	6·90	1·64	4·9	−0·3	5190
97	H_2	0·7	1·7	0·31	1·5	1·57	0·40	5·27	1·96	5·5	−0·3	3160
99	H_2	0·7	1·7	0·31	2·0	2·13	0·29	7·60	1·61	5·8	+0·6	5810
102	H^2	0·6	1·42	0·38	1·1	1·22	0·35	4·42	2·78	4·8	+0·4	2110
104	H_2	0·4	1·77	0·16	2·5	2·31	0·25	7·80	1·28	5·2	0	5830
										mean 5·2		

From the table it will be seen that the intensity E_0 of the electric field at the end of the dark space has a value lying between 2300 volt/cm. and 4100 volt/cm.; but, as the percentage errors are large, further consideration of each value will lead to no definite conclusion.

Although the value of the parabolic constant k varies within a wide range, yet its value, multiplied by the product of the length of the dark space and the diameter of the tube, remains practically constant, as shown in the eleventh column of the table.

The deviation of each value from the mean is, roughly speaking, less than 10%; and this may be ascribed to experimental errors.

As the diameter of the tube and the length of the dark space are, in the present experiment, restricted within a narrow range, the constancy of the quantity $k \times$(dark space)$\times$(diameter) may have no general meaning. But for practical purposes, it may serve for an estimation of the intensity of the electric field in the Crookes dark space in a narrow tube.

If we take the constant value of the above quantity as 5.2, the distribution of the electric field in the Crookes dark space may be represented, in our study, by the equation

$$E = E_0 + \frac{5.2}{d_0 \times \text{diameter}} (d_0 - d)^2;$$

and, consequently, the maximum electric field $E_{\text{max.}}$ just in front of the cathode is given by

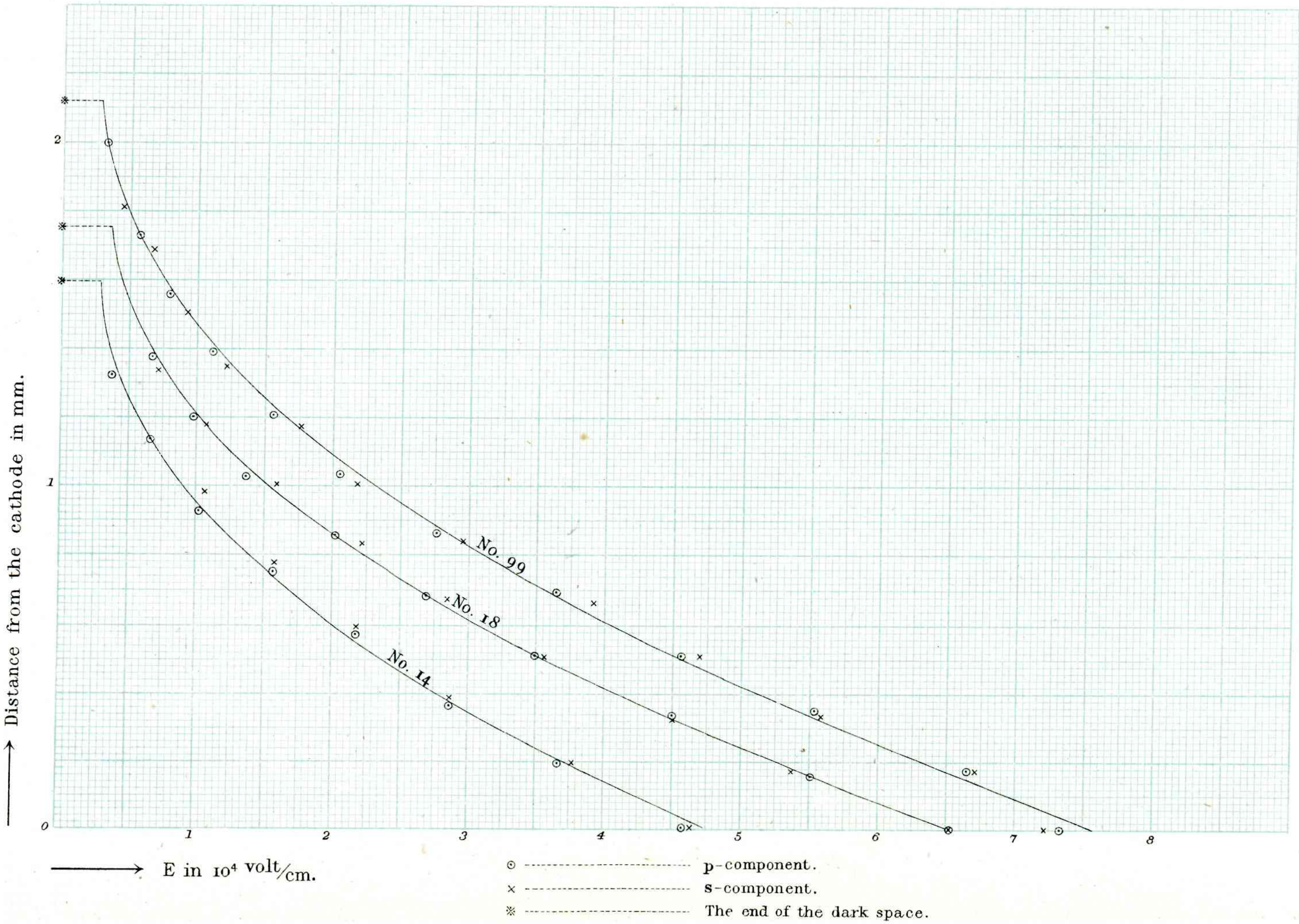
$$E_{\text{max.}} = E_0 + 5 \cdot 2 \frac{d_0}{\text{diameter}}.$$

Thus in order to increase the intensity of the etectric field just in front of the cathode we have to increase the length of the dark space and to diminish the diameter of the capillary tube. But, with such small tubes as we used, when the length of the dark space was increased, the margin of the dark space became indistinct; and at last a sudden change occurred in the appearance of the dark space. The weak glow which formerly had existed in the dark space disappeared, and a glow intense as that in the positive column occupied that portion of the dark space. In this state of discharge, no trace of the electric separation of the spectrum lines of hydrogen could be observed. This state determines the upper limit of the length of the dark space. For example, with a tube whose diameter was 1.5 mm., this sudden change occurred when the length of the dark space was increased above 2.5 mm. Though this maximum length of the dark space increased with the diameter of the tube, yet the numerical relation between them was not examined. The maximum electric field we arrived at was 7.8×10^4 volt/cm., the length of the dark space being 2.5 mm, and the diameter of the tube being 1.77 mm.

Lo Surdo states that the length of the dark space is independent of the strength of the current when the pressure is kept constant. But, in our case, a certain increment in the length of the dark space was observed when the current was increased.

In conclusion, the authors' cordial thanks are due to Prof. T. Mizuno for his kind advice.

Fig. 1.



Distance from the cathode in mm.
0
1
2
1
2
3
4
5
6
7
8
No. 99
No. 18
No. 14
E in 10⁴ volt/cm.
p-component.
s-component.
The end of the dark space.