The Electric Field due to a System of Two Linear Oscillators similar in Form to a Bent Antenna

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The Electric Field due to a System of Two Linear Oscillators similar in Form to a Bent Antenna.

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As the treatment of the electric field due to a bent antenna is not easy, let us here consider a simple case in which an antenna is made up of two linear oscillators, one of which being vertical and the other horizontal, both being so near situated to take an inverted L shape.

For the sake of simplicity, let us assume that the length of the horizontal part of the antenna is twice that of the vertical and that oscillation currents with the same phase are excited in each part.

Now, as is well known, the horizontal component dY and the vertical component dZ of the electric force due to a Hertzian doublet oscillator at a point distant r from its centre are given by

$$\left\{\begin{aligned} dY &= -\phi\left\{\left(\frac{3}{r^3}-\frac{m^2}{r}\right)\sin n\left(t-\frac{r}{v}\right)+\frac{3m}{r^2}\cos n\left(t-\frac{r}{v}\right)\right\}\sin\theta\cos\theta \qquad \cdots\cdots\cdots (1), \\ dZ &= \phi\left[-2\left\{\frac{1}{r^3}\sin n\left(t-\frac{r}{v}\right)+\frac{m}{r^2}\cos n\left(t-\frac{r}{v}\right)\right\}\right. \\ &\quad \left.+\left\{\left(\frac{3}{r^3}-\frac{m^2}{r}\right)\sin n\left(t-\frac{r}{v}\right)+\frac{3m}{r^2}\cos n\left(t-\frac{r}{v}\right)\right\}\sin^2\theta\right] \quad (2), \end{aligned}\right.$$

where ϕ is the electric moment of the doublet and m and n are so related that $m=\frac{2\pi}{\lambda}$ and $\frac{n}{m}=v$, λ being the wave length of the emitted wave and v the velocity of light in vacuum.

If dz be the length of the doublet and Q denote its maximum charge measured in *E. S. U.*, we have

$$\phi = Qdz.$$

Let us assume that the oscillatory current in the oscillator is given by

$$i = Q\frac{n}{v}\sin nt,$$

then (1) and (2) may be written

$$\begin{cases} dY = -\dfrac{v}{n}\dfrac{\partial^2}{\partial y \partial z}\left(\dfrac{i_{t-\frac{r}{v}}}{r}\right)dz & \dots\dots\dots (3), \\ dZ = \left(\dfrac{v}{n}\dfrac{\partial^2}{\partial y^2} + \dfrac{v}{n}\dfrac{1}{p}\dfrac{\partial}{\partial y}\right)\left(\dfrac{i_{t-\frac{r}{v}}}{r}\right)dz & \dots\dots\dots (4). \end{cases}$$

The fundamental oscillation in a linear oscillator of length $2l$ is given by

$$i = a\cos\left(\frac{\pi z}{2l}\right)\sin\left(\frac{\pi v}{2l}\right),$$

where a is the amplitude of the current at its centre.

Since we may consider a long linear oscillator as the combination of many Hertzian doublet oscillators in series, substituting the above value of i in (3) and (4) we get

$$\begin{cases} dY = \dfrac{a}{r}\dfrac{\pi}{2l}\cos\left(\dfrac{\pi z}{2l}\right)\sin\dfrac{\pi}{2l}(vt-r)\sin\theta\cos\theta\, dz \\ \qquad - \dfrac{3a}{r^2}\cos\left(\dfrac{\pi z}{2l}\right)\cos\dfrac{\pi}{2l}(vt-r)\sin\theta\cos\theta\, dz \\ \qquad - \dfrac{3a}{r^3}\dfrac{2l}{\pi}\cos\left(\dfrac{\pi z}{2l}\right)\sin\dfrac{\pi}{2l}(vt-r)\sin\theta\cos\theta\, dz \dots (5), \\ dZ = -\dfrac{a}{r}\dfrac{\pi}{2l}\cos\left(\dfrac{\pi z}{2l}\right)\sin\dfrac{\pi}{2l}(vt-r)\sin^2\theta\, dz \\ \qquad + \dfrac{a}{r^2}\cos\left(\dfrac{\pi z}{2l}\right)\left(3\sin^2\theta-2\right)\cos\dfrac{\pi}{2l}(vt-r)\, dz \\ \qquad + \dfrac{a}{r^3}\dfrac{2l}{\pi}\left(3\sin^2\theta-2\right)\cos\left(\dfrac{\pi z}{2l}\right)\sin\dfrac{\pi}{2l}(vt-r)\, dz \dots (6). \end{cases}$$

These expressions give the y, z components of the electric force due to an elementary portion dz of the long linear oscillator. Therefore, to find the components due to the whole oscillator, we must integrate the above expressions with respect to z from $-l$ to $+l$.

Let us consider the electric force at a point P at a great distance r_0 from the centre of the oscillator, where we may neglect terms containing the fourth power of its reciprocal.

As shown in Fig. I we have

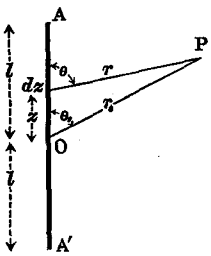


Fig. I.

$$r\cos\theta = r_0\cos\theta_0 - z \quad \ldots\ldots\ldots\ldots\ldots\ldots (7),$$

$$r\sin\theta = r_0\sin\theta_0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (8),$$

$$r = \{r_0^2 + z^2 - 2r_0 z\cos\theta_0\}^{\frac{1}{2}}$$

$$= r_0\left\{1 + \frac{z^2}{r_0^2} - 2\frac{z}{r_0}\cos\theta_0\right\}^{\frac{1}{2}} \quad \ldots\ldots\ldots (9)',$$

$$= r_0\left\{1 - \frac{z}{r_0}\cos\theta_0 + \frac{1}{2}\frac{z^2}{r_0^2}\sin^2\theta_0 + \frac{1}{2}\frac{z^3}{r_0^3}\sin^2\theta_0\cos\theta_0\right\} \quad \ldots\ldots (9).$$

By the relations (7) and (8) we get

$$\frac{\sin\theta\cos\theta}{r} = \frac{r_0^2}{r^3}\sin\theta_0\cos\theta_0 - \frac{r_0}{r^3}z\sin\theta_0$$

$$= \frac{\sin\theta_0\cos\theta_0}{r_0}\left\{1 + \frac{z^2}{r_0^2} - 2\frac{z}{r_0}\cos\theta_0\right\}^{-\frac{3}{2}}$$

$$- \frac{z\sin\theta_0}{r_0^2}\left\{1 + \frac{z^2}{r_0^2} - 2\frac{z}{r_0}\cos\theta_0\right\}^{-\frac{3}{2}} \qquad \text{by (9)}'$$

$$= \frac{\sin\theta_0\cos\theta_0}{r_0}\left\{1 - 3\frac{z}{r_0}\cos\theta_0 + \frac{3}{2}\frac{z^2}{r_0^2}(5\cos^2\theta_0 - 1)\right\}$$

$$- \frac{z\sin\theta_0}{r_0^2}\left\{1 + 3\frac{z}{r_0}\cos\theta_0\right\} \quad \ldots\ldots\ldots (10).$$

Similarly we have

$$\frac{\sin\theta\cos\theta}{r^2} = \frac{\sin\theta_0\cos\theta_0}{r_0^2}\left\{1 + \frac{4z}{r_0}\cos\theta_0\right\} - \frac{z\sin\theta_0}{r_0^3} \quad \ldots (11),$$

$$\frac{\sin\theta\cos\theta}{r^3} = \frac{\sin\theta_0\cos\theta_0}{r_0^2} \quad \ldots\ldots (12),$$

$$\frac{\sin^2\theta}{r} = \frac{\sin^2\theta_0}{r_0}\left\{1 + 3\frac{z}{r_0}\cos\theta_0 + \frac{3}{2}\frac{z^2}{r_0^2}(5\cos^2\theta_0 - 1)\right\} \quad \ldots\ldots (13),$$

$$\frac{\sin^2\theta}{r^2} = \frac{\sin^2\theta_0}{r_0^2}\left\{1 + 4\frac{z}{r_0}\cos\theta_0\right\} \quad \ldots\ldots (14),$$

$$\frac{\sin^2\theta}{r_0^3} = \frac{\sin^2\theta_0}{r_0^3} \quad \ldots\ldots (15),$$

$$\frac{1}{r^2}=\frac{1}{r_0^2}\left(1+2\frac{z}{r_0}\cos\theta_0\right) \qquad \ldots\ldots(16),$$

and

$$\frac{1}{r^3}=\frac{1}{r_0^3} \qquad \ldots\ldots(17).$$

Introducing the above several relations into the expressions (5) and (6) we obtain

$$\begin{aligned}
dY= a\Bigg[&\frac{\pi}{2l}\cos\left(\frac{\pi z}{2l}\right)\sin\frac{\pi}{2l}\left\{vt-r_0+z\cos\theta_0-\frac{1}{2}\frac{z^2}{r_0}\sin^2\theta_0\right.\\
&\left.-\frac{1}{2}\frac{z^3}{r_0^2}\sin^2\theta_0\cos\theta_0\right\}\left\{\frac{\sin\theta_0\cos\theta_0}{r_0}+\frac{3z\sin\theta_0\cos^2\theta_0}{r_0^2}\right.\\
&\left.+\frac{3}{2}\frac{z^2(5\cos^2\theta_0-1)\sin\theta_0\cos\theta_0}{r_0^3}-\frac{z\sin\theta_0}{r_0}-\frac{3z^2\sin\theta_0\cos\theta_0}{r_0^3}\right\}\\
&-3\cos\left(\frac{\pi z}{2l}\right)\cos\frac{\pi}{2l}\left\{vt-r_0+z\cos\theta_0-\frac{1}{2}\frac{z^2}{r_0}\sin^2\theta_0\right.\\
&\left.-\frac{1}{2}\frac{z^3}{r_0^2}\sin^2\theta_0\cos\theta_0\right\}\left\{\frac{\sin\theta_0\cos\theta_0}{r_0}+4z\frac{\sin\theta_0\cos^2\theta_0}{r_0^3}-\frac{z\sin\theta_0}{r_0^3}\right\}\\
&-3\frac{2}{\pi}\cos\left(\frac{\pi z}{2l}\right)\sin\frac{\pi}{2l}\left\{vt-r_0+z\cos\theta_0-\frac{1}{2}\frac{z^2}{r_0}\sin^2\theta_0\right.\\
&\qquad\left.-\frac{1}{2}\frac{z^3}{r_0^2}\sin^2\theta_0\cos\theta_0\right\}\frac{\sin\theta_0\cos\theta_0}{r_0^3}\Bigg]dz \qquad \ldots\ldots(18),
\end{aligned}$$

$$\begin{aligned}
dZ= a\Bigg[&-\frac{\pi}{2l}\cos\left(\frac{\pi z}{2l}\right)\sin\frac{\pi}{2l}\left\{vt-r_0+z\cos\theta_0-\frac{1}{2}\frac{z^2}{r_0}\sin^2\theta_0\right.\\
&\left.-\frac{1}{2}\frac{z^3}{r_0^2}\sin^2\theta_0\cos\theta_0\right\}\left\{\frac{\sin^2\theta_0}{r_0}+3z\frac{\sin^2\theta_0\cos\theta_0}{r_0^2}+\frac{3}{2}z^2\frac{(5\cos^2\theta_0-1)\sin^2\theta_0}{r_0^3}\right\}\\
&+3\cos\left(\frac{\pi z}{2l}\right)\cos\frac{\pi}{2l}\left\{vt-r_0+z\cos\theta_0-\frac{1}{2}\frac{z^2}{r_0}\sin^2\theta_0\right\}\\
&-\frac{1}{2}\frac{z^3}{r_0^2}\sin^2\theta_0\cos\theta_0\Big\}\left\{\frac{\sin^2\theta_0}{r_0}+4z\frac{\sin^2\theta_0\cos\theta_0}{r_0^3}\right\}\\
&-2\cos\left(\frac{\pi z}{2l}\right)\cos\frac{\pi}{2l}\left\{vt-r_0+z\cos\theta_0-\frac{1}{2}\frac{z^2}{r_0}\sin^2\theta_0\right.\\
&\left.-\frac{1}{2}\frac{z^3}{r_0^2}\sin^2\theta_0\cos\theta_0\right\}\left\{\frac{1}{r_0^2}+2z\frac{\cos\theta_0}{r_0^3}\right\}\\
&+3\frac{2l}{\pi}\cos\left(\frac{\pi z}{2l}\right)\sin\frac{\pi}{2l}\left\{vt-r_0+z\cos\theta_0-\frac{1}{2}\frac{z^2}{r_0}\sin^2\theta_0\right.\\
&\left.-\frac{1}{2}\frac{z^3}{r_0^2}\sin^2\theta_0\cos\theta_0\right\}\frac{\sin^2\theta_0}{r_0^3}
\end{aligned}$$

$$-2\frac{2l}{\pi}\cos\left(\frac{\pi z}{2l}\right)\sin\frac{\pi}{2l}\left\{vt-r_0+z\cos\theta_0-\frac{1}{2}\frac{z^2}{r_0}\sin^2\theta_0\right.$$
$$\left.-\frac{1}{2}\frac{z^3}{r_0^2}\sin^2\theta_0\cos\theta_0\right\}\frac{1}{r_0^3}\Bigg]dz \qquad \text{......(19).}$$

Putting

$$\begin{cases}\dfrac{\pi z}{2l}=u,\\ \sin\dfrac{\pi}{2l}(vt-r_0)=S,\\ \cos\dfrac{\pi}{2l}(vt-r_0)=C,\end{cases}$$

and

$$\frac{\pi}{2l}\left(z\cos\theta_0-\frac{1}{2}\frac{z^2}{r_0}\sin^2\theta_0-\frac{1}{2}\frac{z^3}{r_0^2}\sin^2\theta_0\cos\theta_0\right)=A,$$

the equation (19) simplifies to

$$dZ=-adz\frac{\pi}{2l}\cos u$$
$$\times\left\{S\cdot\cos A\left(\frac{\sin^2\theta_0}{r_0^2}+\frac{3z\sin^2\theta_0\cos\theta_0}{r_0^2}+\frac{3}{2}\frac{z^2(5\cos^2\theta_0-1)\sin^2\theta_0}{r_0^3}\right)\right.$$
$$\left.+C\cdot\sin A\left(\frac{\sin^2\theta_0}{r_0^2}+\frac{3z\sin^2\theta_0\cos\theta_0}{r_0^2}+\frac{3}{2}\frac{z^2(5\cos^2\theta_0-1)\sin^2\theta_0}{r_0^2}\right)\right\}$$
$$+3adz\cos u\left\{C\cdot\cos A\left(\frac{\sin^2\theta_0}{r_0^2}+4z\frac{\sin^2\theta_0\cos\theta_0}{r_0^3}\right)\right.$$
$$\left.-S\cdot\sin A\left(\frac{\sin^2\theta_0}{r_0^2}+4z\frac{\sin^2\theta_0\cos\theta_0}{r_0^3}\right)\right\}$$
$$+3adz\frac{2l}{\pi}\cos u\left\{S\cdot\cos A\frac{\sin^2\theta_0}{r_0^3}+C\cdot\sin A\frac{\sin^2\theta_0}{r_0^3}\right\}$$
$$-2adz\frac{2l}{\pi}\cos u\left\{S\cdot\cos A\frac{1}{r_0^3}+C\cdot\sin A\frac{1}{r_0^3}\right\}$$
$$=-a\cdot S\left\{\frac{\pi}{2l}\frac{\sin^2\theta_0}{r_0}\cos u\cdot\cos A+\frac{\pi}{2l}\frac{3\sin^2\theta_0\cos\theta_0}{r_0^2}z\cos u\cos A\right.$$
$$+\frac{\pi}{2l}\cdot\frac{3}{2}\cdot\frac{(5\cos^2\theta_0-1)\sin^2\theta_0}{r_0^3}\cdot z^2\cdot\cos u\cdot\cos A+3\frac{\sin^2\theta_0}{r_0^2}\cos u\cdot\sin A$$
$$+3\cdot4\frac{\sin^2\theta_0\cos\theta_0}{r_0^3}\cdot z\cdot\cos u\sin A-2\frac{1}{r_0^2}\cos u\cdot\sin A$$
$$\left.-2^2\frac{\cos\theta_0}{r_0^3}z\cos u\cdot\sin A-3\frac{2l}{\pi}\frac{\sin^2\theta_0}{r_0^3}\cos u\cdot\cos A-2\frac{2l}{\pi}\frac{1}{r_0^3}\cos u\cos A\right\}dz$$
$$-a\cdot C\left\{\frac{\pi}{2l}\frac{\sin^2\theta_0}{r_0}\cos u\sin A+\frac{\pi}{2l}\frac{3\sin^2\theta_0\cos\theta_0}{r_0^2}z\cos u\sin A\right.$$

$$+\frac{\pi}{2l}\frac{3}{2}\frac{(5\cos^2\theta_0-1)\sin^2\theta_0}{r_0^3}z^2\cos u\sin A-3\frac{\sin^2\theta_0}{r_0^2}\cos u\cos A$$

$$-3\cdot4\frac{\sin^2\theta_0\cos\theta_0}{r_0^3}z\cos u\cos A+2\frac{1}{r_0^2}\cos u\cos A$$

$$+2^2\frac{\cos\theta_0}{r_0^3}z\cos u\cos A-3\frac{2l}{\pi}\frac{\sin^2\theta_0}{r_0^3}\cos u\sin A$$

$$+2\frac{2l}{\pi}\frac{1}{r_0^3}\cos u\sin A\Big\}dz \qquad \ldots\ldots(20).$$

Now let us find the integral

$$\int_{-l}^{+l}\frac{\cos u\cos A}{r_0}dz=\int_{-l}^{+l}\frac{\cos u}{r_0}\left\{\cos\left(\frac{\pi}{2l}z\cos\theta_0\right)\cos\left[\frac{\pi}{2l}\frac{z}{2}\sin^2\theta_0\left(\frac{z}{r_0}+\frac{z^2}{r_0^2}\cos\theta_0\right)\right]\right.$$
$$\left.+\sin\left(\frac{\pi}{2l}z\cos\theta_0\right)\sin\left[\frac{\pi}{2l}\frac{z}{2}\sin^2\theta_0\left(\frac{z}{r_0}+\frac{z^2}{r_0^2}\cos\theta_0\right)\right]\right\}dz.$$

If we put

$$\frac{\pi}{2l}z\cos\theta_0=u_0$$

and expand the cosine and sine of the small angle

$\left[\frac{\pi}{2l}\frac{z}{2}\sin^2\theta_0\left(\frac{z}{r_0}+\frac{z^2}{r_0^2}\cos\theta_0\right)\right]$ in series, the integral becomes

$$\int_{-l}^{+l}\frac{\cos u}{r_0}\left[\cos u_0\left\{1-\frac{1}{2}\left[\frac{\pi}{2l}\frac{z}{2}\sin^2\theta_0\left(\frac{z}{r_0}+\frac{z^2}{r_0^2}\cos\theta_0\right)\right]^2\right\}\right.$$
$$\left.+\sin u_0\left\{\frac{\pi}{2l}\frac{z}{2}\sin^2\theta_0\left(\frac{z}{r_0}+\frac{z^2}{r_0^2}\cos\theta_0\right)\right\}\right]dz$$

$$=\int_{-l}^{+l}\frac{\cos u\cos u_0}{r_0}dz-\frac{1}{2}\frac{\pi^2}{4^2l^2}\frac{\sin^4\theta_0}{r_0^3}\int_{-l}^{+l}z^4\cos u\cos u_0\,dz$$

$$+\frac{\pi}{4l}\frac{\sin^2\theta_0}{r_0^2}\int_{-l}^{+l}z^2\cos u\sin u_0\,dz+\frac{\pi}{2l}\frac{\sin^2\theta_0\cos\theta_0}{r_0^3}\int_{-l}^{+l}z^3\cos u\sin u_0\,dz.$$

But since we have

$$\int_{-l}^{+l}\cos u\cos u_0\,dz=\int_{-l}^{+l}\cos\left(\frac{\pi z}{2l}\right)\cos\left(\frac{\pi}{2l}\cos\theta_0\,z\right)dz=\frac{4l}{\pi}\frac{\cos\left(\frac{\pi}{2}\cos\theta_0\right)}{\sin^2\theta_0},$$

$$\int_{-l}^{+l}z^4\cos u\cos u_0dz=\frac{4l^5}{\pi\sin^2\theta_0}\left\{\cos\left(\frac{\pi}{2}\cos\theta_0\right)+\frac{4^2\sin\left(\frac{\pi}{2}\cos\theta_0\right)\cos\theta_0}{\pi\sin^2\theta_0}\right.$$

$$-\frac{3\cdot4^2\cos\left(\frac{\pi}{2}\cos\theta_0\right)(1+3\cos^2\theta_0)}{\pi^2\sin^4\theta_0}-\frac{3\cdot4^2\sin\left(\frac{\pi}{2}\cos\theta_0\right)(1+\cos^2\theta_0)\cos\theta_0}{\pi^3\sin^6\theta_0}$$

$$\left.+\frac{2^7\cdot3\cos\left(\frac{\pi}{2}\cos\theta_0\right)(1+10\cos^2\theta_0+5\cos^4\theta_0)}{\pi^4\sin^8\theta_0}\right\},$$

$$\int_{-l}^{+l}z^2\cos u\sin u_0\,dz=0$$

and

$$\int_{-l}^{+l}z^3\cos u\cdot\cos u_0dz=\frac{4l^4}{\pi\sin^2\theta_0}\left\{\sin\left(\frac{\pi}{2}\cos\theta_0\right)-\frac{3\cdot4\cos\left(\frac{\pi}{2}\cos\theta_0\right)\cos\theta_0}{\pi\sin^2\theta_0}\right.$$

$$\left.-\frac{2^3\cdot3\sin\left(\frac{\pi}{2}\cos\theta_0\right)(1+3\cos^2\theta_0)}{\pi^2\sin^4\theta_0}+\frac{2^4\cdot3\cdot4\cos\left(\frac{\pi}{2}\cos\theta_0\right)(1+\cos^2\theta_0)\cos\theta_0}{\pi^3\sin^6\theta_0}\right\},$$

the required integral can be written

$$\frac{1}{r_0}\frac{4l}{\pi}\frac{\cos\left(\frac{\pi}{2}\cos\theta_0\right)}{\sin^2\theta_0}-\frac{\pi^2}{2\cdot4^2l^2}\frac{\sin^4\theta_0}{r_0^3}\frac{4l^5}{\pi\sin^2\theta_0}\left\{\cos\left(\frac{\pi}{2}\cos\theta_0\right)\right.$$

$$+\frac{4^2\sin\left(\frac{\pi}{2}\cos\theta_0\right)\cos\theta_0}{\pi\sin^2\theta_0}-\frac{3\cdot4^2\cos\left(\frac{\pi}{2}\cos\theta_0\right)(1+3\cos^2\theta_0)}{\pi^2\sin^4\theta_0}$$

$$-\frac{3\cdot4^2\sin\left(\frac{\pi}{2}\cos\theta_0\right)(1+\cos^2\theta_0)\cos\theta_0}{\pi^3\sin^6\theta_0}$$

$$+\frac{2\cdot3\cdot4^2\cos\left(\frac{\pi}{2}\cos\theta_0\right)(1+10\cos^2\theta_0+5\cos^4\theta_0)}{\pi^8\sin^8\theta_0}$$

$$+\frac{\pi}{4l}\frac{\sin^2\theta_0\cos\theta_9}{r_0^3}\frac{4l^4}{\pi\sin^2\theta_0}\left\{\sin\left(\frac{\pi}{2}\cos\theta_0\right)-\frac{3\cdot4\cos\left(\frac{\pi}{2}\cos\theta_0\right)\cos\theta_0}{\pi\sin^2\theta_0}\right.$$

$$\left.-\frac{2^3\cdot3\sin\left(\frac{\pi}{2}\cos\theta_0\right)(1+3\cos\theta_0)}{\pi^2\sin^4\theta_0}+\frac{2^4\cdot3\cdot4\cos\left(\frac{\pi}{2}\cos\theta_0\right)(1+\cos^2\theta_0)\cos\theta_0}{\pi^3\sin^6\theta_0}\right\},$$

i.e.

$$\int_{-l}^{+l}\frac{\cos u\cos A}{r_0}dz=\frac{1}{r_0}\frac{4l}{\pi}\frac{\cos\left(\frac{\pi}{2}\cos\theta_0\right)}{\sin^2\theta_0}-\frac{l^3}{r_0^3}\left\{\frac{\pi}{2^3}\sin^2\theta_0\cos\left(\frac{\pi}{2}\cos\theta_0\right)\right.$$

$$+\sin\left(\frac{\pi}{2}\cos\theta_0\right)\cos\theta_0-2\cdot 3\,\frac{\cos\left(\frac{\pi}{2}\cos\theta_0\right)(1+\cos^2\theta_0)}{\pi\sin^2\theta_0}$$

$$+2\cdot 3\,\frac{\text{s n}\left(\frac{\pi}{2}\cos\theta_0\right)\cos\theta_0(3+11\cos^2\theta_0)}{\pi^2\sin^4\theta_0}$$

$$+2^4\cdot 3\cdot\frac{\cos\left(\frac{\pi}{2}\cos\theta_0\right)(1+6\cos^2\theta_0+\cos^4\theta_0)}{\pi^3\sin^6\theta_0}\Bigg\} \qquad \text{.........} \ (21).$$

Similarly we get

$$\int_{-l}^{+l}\frac{z\cos u\cos A}{r_0^2}dz=\frac{l^3}{r_0^3}\left\{\sin\left(\frac{\pi}{2}\cos\theta_0\right)-2^2\cdot 3\frac{\cos\left(\frac{\pi}{2}\cos\theta_0\right)\cos\theta_0}{\pi\sin^2\theta_0}\right.$$

$$\left.-2^3\cdot 3\frac{\sin\left(\frac{\pi}{2}\cos\theta_0\right)(1+3\cos^2\theta_0)}{\pi^2\sin^4\theta_0}+2^6\cdot 3\frac{\cos\left(\frac{\pi}{2}\cos\theta_0\right)(1+\cos^2\theta_0)\cos\theta_0}{\pi^3\sin^6\theta_0}\right\}$$

$$\text{.........} \ (22),$$

$$\int_{-l}^{+l}\frac{z^2\cos u\cos A}{r_0^3}dz=\frac{4l^3}{r_0^3}\,\frac{1}{\pi\sin^2\theta_0}\left\{\cos\left(\frac{\pi}{2}\cos\theta_0\right)+2^3\,\frac{\cos\theta_0\sin\left(\frac{\pi}{2}\cos\theta_0\right)}{\pi\sin^2\theta_0}\right.$$

$$\left.-2^3\,\frac{(1+3\cos^2\theta_0)\cos\left(\frac{\pi}{2}\cos\theta_0\right)}{\pi^2\sin^4\theta_0}\right\} \ \text{.........} \ (23),$$

$$\int_{-l}^{+l}\frac{\cos u\sin A}{r_0}dz=-\frac{l^2}{r_0^0}\left\{\cos\left(\frac{\pi}{2}\cos\theta_0\right)+2^3\,\frac{\cos\theta_0\sin\left(\frac{\pi}{2}\cos\theta_0\right)}{\pi\sin^2\theta_0}\right.$$

$$\left.-2^3\,\frac{(1+3\cos^2\theta_0)\cos\left(\frac{\pi}{2}\cos\theta_0\right)}{\pi^2\sin^4\theta_0}\right\} \ \text{.........} \ (24),$$

$$\int_{-l}^{+l}z\frac{\cos u\sin A}{r_0^2}dz=\frac{l^2}{r_0^2}\,\frac{4}{\pi\sin^2\theta_0}\left\{\sin\left(\frac{\pi}{2}\cos\theta_0\right)-2^2\,\frac{\cos\theta_0\cos\left(\frac{\pi}{2}\cos\theta_0\right)}{\pi\sin^2\theta_0}\right\}$$

$$\text{.........} \ (25)$$

and

$$\int_{-l}^{+l}\frac{z^2\cos u\sin A}{r_0^3}dz=0 \qquad \text{.........} \ (26).$$

Using these integrals, we get from (20)

$$Z = \int_{-l}^{+l} dZ$$

$$= -aS\Bigg[\frac{\pi}{2l}\sin^2\theta_0\Bigg\{\frac{1}{r_0}\frac{4l}{\pi}\frac{\cos\left(\frac{\pi}{2}\cos\theta_0\right)}{\sin^2\theta_0} - \frac{l^3}{r_0^3}\left(\frac{\pi}{2^3}\sin^2\theta_0\cos\left(\frac{\pi}{2}\cos\theta_0\right)\right.$$

$$+\sin\left(\frac{\pi}{2}\cos\theta_0\right)\cos\theta_0 - 2\cdot 3\frac{\cos\left(\frac{\pi}{2}\cos\pi_0\right)(1+\cos^2\theta_0)}{\pi\sin^2\theta_0}$$

$$+2\cdot 3\frac{\sin\left(\frac{\pi}{2}\cos\theta_0\right)\cos\theta_0(3+11\cos^2\theta_0)}{\pi^2\sin^4\theta_0}$$

$$\left.+2^4\cdot 3\frac{\cos\left(\frac{\pi}{2}\cos\theta_0\right)(1+6\cos^2\theta_0+\cos^4\theta_0)}{\pi^3\sin^6\theta_0}\right)\Bigg\} + \frac{\pi}{2l}\cdot 3\sin^2\theta_0\cos\theta_0\frac{l^3}{r_0^3}\times$$

$$\times\Bigg\{\sin\left(\frac{\pi}{2}\cos\theta_0\right) - 2^2\cdot 3\frac{\cos\left(\frac{\pi}{2}\cos\theta_0\right)\cos\theta_0}{\pi\sin^2\theta_0}$$

$$-2^3\cdot 3\frac{\sin\left(\frac{\pi}{2}\cos\theta_0\right)(1+3\cos^2\theta_0)}{\pi^2\sin^4\theta_0} + 2^6\cdot 3\frac{\cos\left(\frac{\pi}{2}\cos\theta_0\right)(1+\cos^2\theta_0)\cos\theta_0}{\pi^3\sin^6\theta_0}\Bigg\}$$

$$+\frac{\pi}{2l}\cdot\frac{3}{2}(5\cos^2\theta_0-1)\sin^2\theta_0\frac{4l^3}{r_0^3}\frac{1}{\pi\sin^2\theta_0}\Bigg\{\cos\left(\frac{\pi}{2}\cos\theta_0\right)$$

$$+2^3\frac{\cos\theta_0\sin\left(\frac{\pi}{2}\cos\theta_0\right)}{\pi\sin^2\theta_0} - 2^3\frac{(1+3\cos^2\theta_0)\cos\left(\frac{\pi}{2}\cos\theta_0\right)}{\pi^2\sin^4\theta_0}\Bigg\}$$

$$-3\sin^2\theta_0\cdot\frac{l^2}{r_0^3}\Bigg\{\cos\left(\frac{\pi}{2}\cos\theta_0\right) + 2^3\frac{\cos\theta_0\sin\left(\frac{\pi}{2}\cos\theta_0\right)}{\pi\sin^2\theta_0}$$

$$-2^3\frac{(1+3\cos^2\theta_0)\cos\left(\frac{\pi}{2}\cos\theta_0\right)}{\pi^2\sin^4\theta_0}\Bigg\} + 3\cdot 4\sin^2\theta_0\cos\theta_0\frac{l^2}{r_0^3}\frac{4}{\pi\sin^2\theta_0}\times$$

$$\times\Bigg\{\sin\left(\frac{\pi}{2}\cos\theta_0\right) - 4\frac{\cos\theta_0\cos\left(\frac{\pi}{2}\cos\theta_0\right)}{\pi\sin^2\theta_0}\Bigg\} + 2\frac{l^2}{r_0^3}\Bigg\{\cos\left(\frac{\pi}{2}\cos\theta_0\right)$$

$$+2^3\frac{\cos\theta_0\sin\left(\frac{\pi}{2}\cos\theta_0\right)}{\pi\sin^2\theta_0} - 2^3\frac{(1+3\cos^2\theta_0)\cos\left(\frac{\pi}{2}\cos\theta_0\right)}{\pi^2\sin^4\theta_0}\Bigg\}$$

$$-2^2\cos\theta_0\frac{l^2}{r_0^3}\frac{4}{\pi\sin^2\theta_0}\left\{\sin\left(\frac{\pi}{2}\cos\theta_0\right)-\frac{4}{\pi}\frac{\cos\theta_0\cos\left(\frac{\pi}{2}\cos\theta_0\right)}{\sin^2\theta_0}\right\}$$

$$-3\cdot\frac{2l}{\pi}\sin^2\theta_0\frac{1}{r_0^3}\frac{4l}{\pi}\frac{\cos\left(\frac{\pi}{2}\cos\theta_0\right)}{\sin^2\theta_0}+2\cdot\frac{2l}{\pi}\frac{1}{r_0^3}\frac{4l}{\pi}\frac{\cos\left(\frac{\pi}{2}\cos\theta_0\right)}{\sin^2\theta_0}\Bigg]$$

$$-a\cdot C\left[\frac{\pi}{2l}\sin^2\theta_0\left\{-\frac{l^2}{r_0^2}\left(\cos\left(\frac{\pi}{2}\cos\theta_0\right)+2^3\cos\theta_0\frac{\sin\left(\frac{\pi}{2}\cos\theta_0\right)}{\pi\sin^2\theta_0}\right.\right.\right.$$

$$\left.\left.-2^3\frac{(1+3\cos^2\theta_0)\cos\left(\frac{\pi}{2}\cos\theta_0\right)}{\pi^2\sin^4\theta_0}\right)\right\}+\frac{\pi}{2l}\cdot 3\cdot\sin^2\theta_0\cos\theta_0\frac{l^2}{r_0^2}\frac{4}{\pi\sin^2\theta_0}\times$$

$$\times\left\{\sin\left(\frac{\pi}{2}\cos\theta_0\right)-4\frac{\cos\theta_0\cos\left(\frac{\pi}{2}\cos\theta_0\right)}{\pi\sin^2\theta_0}\right\}-3\sin^2\theta_0\frac{1}{r_0^2}\frac{4l}{\pi}\frac{\cos\left(\frac{\pi}{2}\cos\theta_0\right)}{\sin^2\theta_0}$$

$$\left.+2\frac{1}{r_0^2}\frac{4l}{\pi}\frac{\cos\left(\frac{\pi}{2}\cos\theta_0\right)}{\sin^2\theta_0}\right].$$

This reduces to

$$Z=-a\sin\frac{\pi}{2l}(vt-r_0)\left[\frac{1}{r_0}\frac{2\cos\left(\frac{\pi}{2}\cos\theta_0\right)}{\sin\theta_0}\sin\theta_0\right.$$

$$-\frac{l^2}{r_0^3}\left\{\frac{\pi^2}{4^2}\cos\left(\frac{\pi}{2}\cos\theta_0\right)\sin^3\theta_0\sin\theta_0-\pi\sin\left(\frac{\pi}{2}\cos\theta_0\right)\sin^2\theta_0\cos\theta_0\right.$$

$$-\cos\left(\frac{\pi}{2}\cos\theta_0\right)(2-3\sin^2\theta_0)-\frac{45}{\pi}\frac{\sin\left(\frac{\pi}{2}\cos\theta_0\right)}{\sin^3\theta_0}(1+\cos^2\theta_0)\sin\theta_0\cos\theta_0$$

$$\left.\left.+\frac{48}{\pi^2}\frac{\cos\left(\frac{\pi}{2}\cos\theta_0\right)\cos^2\theta_0}{\sin^4\theta_0}\right\}\right]$$

$$+a\cos\frac{\pi}{2l}(vt-r_0)\left[\frac{l}{r_0^2}\left\{\frac{\pi}{2}\cos\left(\frac{\pi}{2}\cos\theta_0\right)\sin\theta_0\sin\theta_0\right.\right.$$

$$\left.\left.-2\sin\left(\frac{\pi}{2}\cos\theta_0\right)\cos\theta_0\right\}\right] \qquad \text{.........} (27).$$

Similarly we obtain

$$Y=a\sin\frac{\pi}{2l}(vt-r_0)\left[\frac{1}{r_0}\frac{2\cos\left(\frac{\pi}{2}\cos\theta_0\right)}{\sin\theta_0}\cos\theta_0\right.$$

$$-\frac{l^3}{r_0^3}\left\{\frac{\pi^2}{4^2}\cos\left(\frac{\pi}{2}\cos\theta_0\right)\sin^3\theta_0\cos\theta_0+\frac{\pi}{2}\sin\left(\frac{\pi}{2}\cos\theta_0\right)\sin\theta_0(1-2\cos^2\theta_0)\right.$$

$$+\cos\left(\frac{\pi}{2}\cos\theta_0\right)\cos\theta_0\sin\theta_0+\frac{1}{\pi}\frac{\sin\left(\frac{\pi}{2}\cos\theta_0\right)}{\sin^3\theta_0}(61\cos^2\theta_0+29)\cos^2\theta_0$$

$$\left.+\frac{48}{\pi^2}\frac{\cos\left(\frac{\pi}{2}\cos\theta_0\right)}{\sin^3\theta_0}\cos\theta_0(1+3\cos^2\theta_0)\right\}\Bigg]$$

$$-a\cos\frac{\pi}{2l}(vt-r_0)\left[\frac{l}{r_0^2}\left\{\frac{\pi}{2}\cos\left(\frac{\pi}{2}\cos\theta_0\right)\sin\theta_0\cos\theta_0\right.\right.$$

$$\left.\left.+2\sin\left(\frac{\pi}{2}\cos\theta_0\right)\sin\theta_0\right\}\right] \quad\ldots\ldots\ldots (28).$$

These are the expressions for the component electric forces due to a single linear oscillator.

Now in the present problem it is, of course, necessary to take image effects into account; in Fig. II OA is the vertical part of the

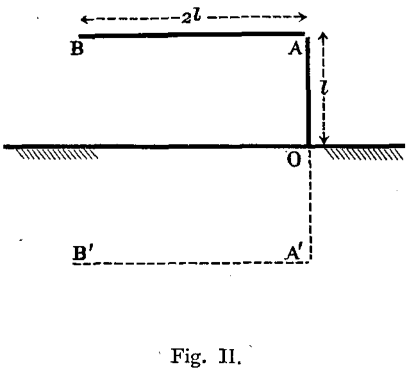


Fig. II.

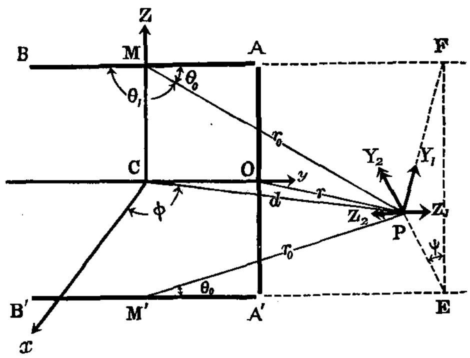


Fig. III.

antenna, AMB its horizontal part, while OA' and $A'M'B'$ are their respective images.

Let us choose the rectangular coordinate system whose origin C is the projection of M, the centre of AB, on the equatorial plane of $A'OA$, the y-axis being taken in the direction of MA and the z-axis in the direction of OA as shown in Fig. III.

Let us find the electric force at a point P, situated on the equatorial plane i.e. on the xy-plane, at a distance d from C, r from O and r_0 from M and its image M'.

Denoting the angles AMP, BMP and XCP by θ_0, θ_1 and ϕ respectively, we have

$$\left.\begin{aligned} r_0 \cos\theta_0 &= d\sin\phi \\ d^2+l^2 &= r_0^2 \end{aligned}\right\} \qquad \text{......... (29)},$$

$$\begin{aligned} r^2 &= d^2+l^2-2dl\sin\phi \\ &= r_0^2-2lr_0\cos\theta_0, \qquad \text{by (29)}, \end{aligned}$$

∴

$$\begin{aligned} r &= r_0\left\{1-2\frac{l}{r_0}\cos\theta_0\right\}^{\frac{1}{2}} \\ &= r_0 - l\cos\theta_0 - \frac{1}{2}\frac{l^2}{r_0}\cos^2\theta_0 - \frac{1}{2}\frac{l^3}{r_0^2}\cos^3\theta_0 \qquad \text{...... (30)}, \end{aligned}$$

$$\frac{1}{r} = \frac{1}{r_0}\left\{1+\frac{l}{r_0}\cos\theta_0+\frac{3}{2}\frac{l^2}{r_0^2}\cos^2\theta_0\right\} \qquad \text{......... (31)},$$

$$\frac{1}{r^2} = \frac{1}{r_0^2}\left\{1+2\frac{l}{r_0}\cos\theta_0\right\} \qquad \text{......... (32)},$$

$$\frac{1}{r^3} = \frac{1}{r_0^3} \qquad \text{......... (33)}.$$

Putting $\theta_0=\pi/2$ in (27) we get the electric force at P due to the part $A'OA$ in the direction of the z-axis, viz.

$$Z_0 = -a\left\{\frac{2}{r}-\frac{l^2\left(1+\frac{\pi^2}{4^2}\right)}{r^3}\right\}\sin\frac{\pi}{2l}(vt-r)+a\frac{2l\cdot\frac{\pi}{4}}{r^2}\cos\frac{\pi}{2l}(vt-r).$$

By means of (30)—(33) this may be expressed in terms of r_0 and θ_0, namely,

$$\begin{aligned} Z_0 = &-a\left[\frac{2}{r_0}\cos\omega+\frac{2l}{r_0^2}\left\{\frac{\pi}{4}\sin\omega\sin^2\theta_0+\cos\omega\cos\theta_0\right\}\right. \\ &\left.-\frac{l^2}{r_0^3}\left\{\frac{\pi^2}{4^2}\cos\omega\sin^4\theta_0-\pi\sin\omega\cos\theta_0\sin^2\theta_0+(1-3\cos^2\theta_0)\cos\omega\right\}\right]\sin\Omega \\ &-a\left[\frac{2}{r_0}\sin\omega-\frac{2l}{r_0^2}\left\{\frac{\pi}{4}\cos\omega\sin^2\theta_0-\sin\omega\cos\theta_0\right\}\right. \\ &\left.-\frac{l^2}{r_0^3}\left\{\frac{\pi^2}{4^2}\sin\omega\sin^4\theta_0+\pi\cos\omega\cos\theta_0\sin^2\theta_0+(1-3\cos^2\theta_0)\sin\omega\right\}\right]\cos\Omega \end{aligned}$$

.... (34),

where ω and Ω are written for $\frac{\pi}{2}\cos\theta_0$ and $\frac{\pi}{2l}(vt-r_0)$ respectively. But the horizontal component of the electric force evidently vanishes i.e.

$$Y_0 = 0.$$

Next putting $\theta_0 = \theta_1$ in (27) and (28) we get the electric force due to the part *AMB*, namely,

$$Z_1 = -a\sin\Omega\left[\frac{2}{r_0}\frac{\cos\left(\frac{\pi}{2}\cos\theta_1\right)}{\sin\theta_1}\sin\theta_1 - \frac{l^2}{r_0^3}\left\{\ldots\ldots\right\}\right]$$

$$+a\cos\Omega\frac{2l}{r_0^2}\left\{\frac{\pi}{4}\cos\left(\frac{\pi}{2}\cos\theta_1\right)\sin\theta_1\sin\theta_1 - \sin\left(\frac{\pi}{2}\cos\theta_1\right)\cos\theta_1\right\}$$

$$= -a\sin\Omega\left[\frac{2^2}{r_0}\frac{\cos\omega}{\sin\theta_0}\sin\theta_0 - \frac{l^2}{r_0^3}\left\{\ldots\ldots\right\}\right]$$

$$+a\cos\Omega\frac{2l}{r_0^2}\left\{\frac{\pi}{4}\cos\omega\sin\theta_0\sin\theta_0 - \sin\omega\cos\theta_0\right\}$$

in the direction of the y-axis, and

$$Y_1 = -a\sin\Omega\left[\frac{2}{r_0}\frac{\cos\omega}{\sin\theta_0}\cos\theta_0 - \frac{l^2}{r_0^2}\left\{\ldots\ldots\right\}\right]$$

$$+a\cos\Omega\frac{2l}{r_0^2}\left\{\frac{\pi}{4}\cos\omega\sin\theta_0\cos\theta_0 + \sin\omega\sin\theta_0\right\}$$

in the plane *AMP* at right angles to *AMB*.

From (27) and (28) we have at once the electric force due to the part *B'M'A'*, namely

$$Z_2 = -a\sin\Omega\left[\frac{2}{r_0}\frac{\cos\omega}{\sin\theta_0}\sin\theta_0 - \frac{l^2}{r_0^3}\left\{\ldots\ldots\right\}\right]$$

$$+a\cos\Omega\frac{2l}{r_0^3}\left\{\frac{\pi}{4}\cos\omega\sin\theta_0\sin\theta_0 - \sin\omega\cos\theta_0\right\}$$

in the direction of the y-axis, and

$$Y_2 = a\sin\Omega\left[\frac{2}{r_0}\frac{\cos\omega}{\sin\theta_0}\cos\theta_0 - \frac{l^2}{r_0^3}\left\{\ldots\ldots\right\}\right].$$

$$+a\cos\Omega\frac{2l}{r_0^2}\left\{\frac{\pi}{3}\cos\omega\sin\theta_0\cos\theta_0 + \sin\omega\sin\theta_0\right\}$$

in the plane *A'M'P'*, directed perpendicularly to the part *B'M'A'*.

Since the part *A'M'B'* is the image of the part *AMB*, Z_1, Z_2 are equal to one another in magnitude but opposite in direction, while

Y_1, Y_2 are also equal to one another in magnitude, equally inclined by an angle PEF, ψ say, to, but on the opposite sides of the plane passing through P parallel to the plane of yz, as shown in Fig. III. It is to be noticed that the line PE is drawn perpendicular to the line $M'A'$ and the line EF parallel to the line OA.

Hence, if we assume that the current in the part AB flows from A to B when that in the part OA flows from O to A, the resultant electric force due to AMB and $B'M'A'$ is given by

$$Z_3 = 2\cdot\cos\psi\, Y_2.$$

But we have the geometrical relation

$$\cos\psi = \frac{l}{r_0 \sin\theta_0},$$

and therefore we get

$$Z_3 = 2\,\frac{l}{r_0\sin\theta_0}\,Y_2$$

$$= 2\cdot\frac{2al}{r_0^2}\,\frac{\cos\omega}{\sin^2\theta_0}\cos\theta_0\sin\Omega - 2\cdot\frac{2al^2}{r_0^3}\left\{\frac{\pi}{4}\cos\omega\cos\theta_0 + \sin\omega\right\}\cos\Omega \qquad \cdots\cdots\cdots (35),$$

where small quantities of the higher orders are neglected.

Finally by adding (34) and (35) we obtain the following electric force at the point P under consideration due to the bent antenna:

$$\begin{aligned}
Z &= Z_0 + Z_3 \\
&= -a\Bigg[\frac{2}{r_0}\cos\omega - \frac{2l}{r_0^2}\left\{\frac{\cos\omega}{\sin^2\theta_0}(1+\cos\theta_0) - \frac{\pi}{4}\sin\omega\sin^2\theta_0\right\} \\
&\quad - \frac{l^2}{r_0^3}\Bigg\{\left\{\left(1+\frac{\pi^2}{4^2}\right) - \frac{\pi^2}{4^2}\cos^2\theta_0(1+\sin^2\theta_0) - 3\cos^2\theta_0\right\}\cos\omega \\
&\qquad - \pi\cos\theta_0\sin^2\theta_0\sin\omega\Bigg\}\Bigg]\sin\Omega \\
&\quad - \Bigg[\frac{2}{r_0}\sin\omega + \frac{2l}{r_0^2}\left\{\sin\omega\cos\theta_0 - \frac{\pi}{4}\cos\omega\sin^2\theta_0\right\} \\
&\quad - \frac{l^2}{r_0^3}\Bigg\{\left\{\left(1+\frac{\pi^2}{4^3}\right) - \frac{\pi^2}{4^2}\cos^2\theta_0(1+\sin^2\theta_0) - 3\cos^2{}_0\right\}\sin\omega \\
&\qquad - \pi\cos^2\theta_0\cos\omega - 4\sin\omega\Bigg\}\Bigg]\cos\Omega.
\end{aligned}$$

Denoting the coefficients of $\sin\Omega$ and $\cos\Omega$ by p and q respectively, in the above expression, the amplitude of the electric force is

$$A = \sqrt{p^2+q^2}$$

$$= \frac{2a}{r_0}\sqrt{1+2\frac{l}{r_0}\cos\theta_0\left\{1-2\frac{\cos^2\omega}{\sin^2\theta_0}\right\}+\frac{4l^2}{r_0^2}\left\{\frac{\cos^2\omega}{\sin^4\theta_0}\cos^4\theta_0+\cos^2\theta_0+\sin^2\omega-\frac{1}{4}\right\}}$$

where we have neglected small quantities of higher orders.

Therefore the intensity of the electric force at a point on the equatorial plane at a distance r_0 from M or $d=\left(1-\frac{l^2}{r_0^2}\right)^{\frac{1}{2}}$ from C is proportional to

$$I = 1+2\frac{l}{r_0}\cos\theta_0\left\{1-2\frac{\cos^2\left(\frac{\pi}{2}\cos\theta_0\right)}{\sin^2\theta_0}\right\}+\frac{4l^2}{r_0^2}\left\{\frac{\cos^2\left(\frac{\pi}{2}\cos\theta_0\right)}{\sin^4\theta_0}\cos^4\theta_0\right.$$

$$\left.+\cos^2\theta_0+\sin^2\left(\frac{\pi}{2}\cos\theta_0\right)-0.25\right\}.$$

In the positive direction of the y-axis the second term is positive, while in its negative direction this term is negative. Hence the intensity of the electric force at the head-side of the antenna is greater than at its tail side, and the longer the antenna, the greater this directive property.

In order to compare intensities of the electric field at different points lying on a circle of the radius d with its centre at C, it is necessary and sufficient to consider the change of the value I accompanying the change of the angle ϕ.

The relation between ϕ and θ_0 is given by (29) i.e.

$$\sin\phi = \frac{r_0}{d}\cos\theta_0$$

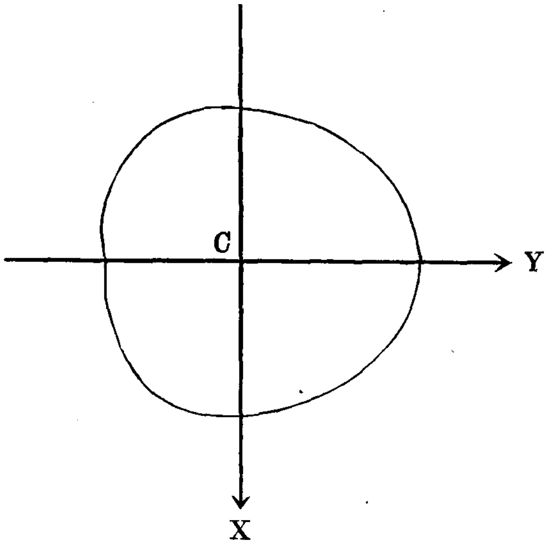
$$= \frac{r_0}{\sqrt{r_0^2-l}}\cos\theta_0$$

$$= \left(1-\frac{l^2}{r_0^2}\right)^{-\frac{1}{2}}\cos\theta_0.$$

For the sake of convenience, taking the value of d to be $r_0\left\{\left(1-\frac{1}{10}\right)^2\right\}^{\frac{1}{2}}$ (this means the ratio l/r_0 is equal to $1/10$) and using the above relation, the values of I for different values of ϕ are calculated as follows:

θ_0		ϕ		I
5°	44′	90°		1.30
7	37	85		1.30
9	3	83		1.30
10		81	48′	1.30
20		70	41	1.24
30		60	30	1.20
40		50	21	1.12
50		40	15	1.05
60		30	10	0.99
70		20	6	0.96
80		10	3	0.96
85		5	2	0.97
89		1		0.99
89	50	0	10	0.99
90		0		0.99
91		1	10	0.99
95		5	2	1.01
100		10	3	1.03
110		20	6	1.05
120		30	10	1.06
130		40	15	1.03
140		50	21	1.00
150		60	30	0.97
160		70	41	0.94
170		81	48	0.90
170	57	83		0.90
172	23	85		0.90
174	16	90		0.90

In the annexed figure, obtained by plotting the above result, the radius vector of any point on the curve shows the relative intensity of the field in that direction.



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