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A Size-Dependent Sign Reversal in the Hall Coefficient of Indium

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mean number of boundaries encountered before an electron is scattered. Assuming one conduction electron per atom, the value of a calculated from the results is close to one grain boundary encounter per scattering event.

If it is assumed that the material in the grain boundary has a liquid configuration, having a residual resistivity of about 10^{-5} ohm cm, and it is assumed that the sole cause of grain boundary scattering is the resistivity of the grain boundary material, the grain boundary thickness, t, is given by $2t = (S/a\ 10^{-5})$ cm, where, assuming spherical grains, $2t$ is the mean length of the journey of electrons through boundary material when passing from one grain to another. The thickness so calculated is 3.3×10^{-7} cm, or about twelve atomic distances. This thickness is rather large; about two atomic distances would have been expected [3]. Either the residual resistivity of copper in the liquid configuration is considerably greater than 10^{-5} ohm cm, which is unlikely, or the grain boundary region does not consist merely of material in the liquid configuration.

Grain boundaries are often considered to be aggregates of dislocations. The resistivity due to dislocations in copper has been measured by Clarebrough et al. [4], and more recently by Foxon [5]. If Foxon's figure of 1.3×10^{-19} ohm cm/dislocation line/cm^2 is taken to be the more reliable, a calculation of the dislocation population of grain boundaries, assuming that most of the resistivity due to dislocation lies in the dislocation core, would indicate that, in the present results, almost all the planes in a grain boundary consist of dislocation. This is consistent with the large grain boundary angles that might be expected in such samples, and would account for the calculated value of a, which indicates a very high order of probability of an electron being scattered on each encounter with a grain boundary.

References

1. T.H.Blewitt, R.R.Coltman and J.K.Redman, Phys. Rev. 93 (1954) 891.
2. E.M.Porbansky, Trans.Met.Soc.AIME 230 (1964) 925.
3. N.F.Mott, Proc.Phys.Soc.60 (1948) 391.
4. L.M.Clarebrough, M.E.Hargreaves and M.H. Loretto, Phil.Mag.7 (1962) 115.
5. C.T.B.Foxon, private communication.

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A SIZE-DEPENDENT SIGN REVERSAL IN THE HALL COEFFICIENT OF INDIUM

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The Hall coefficient of indium is positive in the bulk material at all magnetic fields. We find that the sign of the effect in low fields changes in sufficiently thin plates in a perpendicular magnetic field.

The indium plates were pressed from very pure bulk material as described previously [1]. They were of rectangular shape with an effective area of 1 cm by 3 cm after mounting. With this geometry the end effects reduce the measured Hall voltage not more than 1.4% [2] and can be neglected. The specimens used are listed in table 1 with their ratio of d.c. resistivity at 4.2°K (ρ_F) to room temperature resistivity (ρ_{293}) and their thickness d. The bulk resistivity ρ_B and the mean free path l were calculated by comparing the measured resistivity with the Fuchs [3] size effect theory (assuming diffuse scattering at the surface and using the value $\rho_B l = 1.27 \times 10^{-15}$ Ωm^2 found by Cotti [1]).

We may note that as pointed out by Cotti, Fryer and Olsen [4] a free path so calculated cor-

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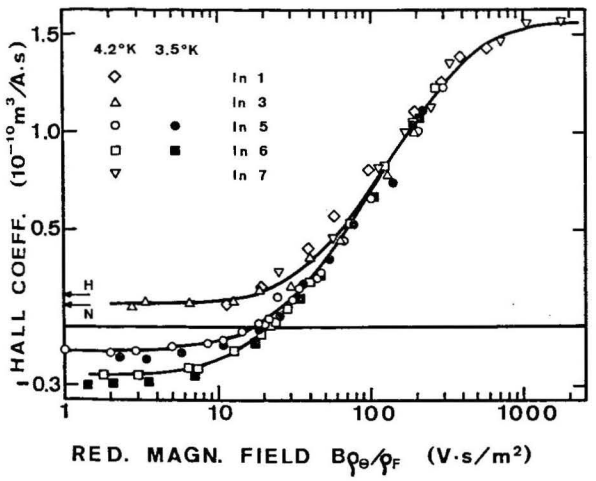

Fig. 1. Hall coefficient R_H for indium plates of different thickness in a perpendicular magnetic field. H and N indicate R_H at 20°K and 77°K, respectively. (1 V s/m^2 = 10^4 gauss).

responds to the dominating group of carriers only. In indium this covers roughly 43% of the Fermi surface. The remainder of the carriers have much shorter free paths.

In the figure we have plotted the Hall coefficient R_H against a reduced field $B\rho_\theta/\rho_F$, where ρ_θ is the resistivity at the Debye temperature, θ = 100°K. In addition to the sign reversal the following details should be noted:

(i) Although specimens In 1, In 3 and In 7 show quite different size effects in their resistivity, there is no detectable difference in the behaviour of their Hall coefficients. We therefore consider the horizontal part of the curve for In 3 the zero field limit for bulk indium.

(ii) The low field R_H in In 3 is in agreement with the value at 77°K (In 5).

(iii) At 4.2°K Borovik [5] finds a sign reversal in bulk indium at a reduced field of about 20 V s/m^2. As a size effect in Borovik's specimen can be excluded, this discrepancy is difficult to explain. At 77°K and 20°K and at high fields our data are in agreement with those of Borovik [6] except for the sign in general.

The results are best discussed by using the trajectory method described by Pippard [7]. In the low field limit it is then easy to show that with the field in the z direction the Hall angle θ_H is given by

$$\tan\theta_H = \frac{eB}{h}\,\frac{\int (v_\perp \tau(k))^2 \cos^2\alpha \, d\alpha \, dk_z}{\int (v\tau(k)) \cos^2\beta \, dS},$$

Table 1
Specimen properties.

Sample	d (mm)	$\frac{\rho_F}{\rho_{293}} \times 10^4$	$\frac{\rho_F}{\rho_B}$	l (mm)
In 1	0.080	1.4	2.5	0.27
In 3	0.083	4.31	1.25	0.044
In 5	0.026	2.75	5.1	0.28
In 6	0.013	4.50	7.8	0.26
In 7	0.63	0.77	1.17	0.23

where the integrals are taken over the entire Fermi surface. $v_\perp$ is the velocity component perpendicular to the magnetic field, τ the relaxation time, α the angle between $v_\perp$ and the electric field, β the angle between v and the electric field.

We may divide the Fermi surface into three regions. The (locally) electron-like main surfaces of the second zone surface, the hole-like corners of this surface, and the electron-like monster in the third zone. The hole-like sign in the bulk Hall effect can be understood in the light of our previous suggestion that the free path on the monster is small if we assume at the same time that the free path of the hole-like corners is not too much smaller than the free path on the main faces. How delicate the balance is, may be seen from the electron-like sign of R_H at low fields in aluminium [8, 9], where the Fermi surface is of the same general character as that of indium.

To account for the sign change we require a free path limitation that affects both regions of the second zone and allows the relative importance of the monster to increase. In this connection it should be pointed out that the explanation outlined here is not unique. The suggestion by Rayne [10] of hole-like regions in the main faces in the second zone need consideration also, but a detailed account of the parts of the Fermi surface first affected is beyond the scope of our present technique; it may emerge from more accurate work on single crystals.

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1. P. Cotti, Phys. kondens. Materie 3 (1964) 40.
2. H. J. Lippmann and F. Kurth, Z. Naturf. 13a (1958) 462.
3. K. Fuchs, Proc. Camb. Phil. Soc. 34 (1938) 100.
4. P. Cotti, E. M. Fryer and J. L. Olsen, Helv. Phys. Acta 37 (1964) 585.

5. E.S.Borovik, Dokl.Akad.Nauk SSSR 75 (1950) 639.
6. E.S.Borovik and V.G.Volotskaya, Soviet Phys. - JETP 11 (1960) 189.
7. A.B.Pippard, Rep.Progr.Phys.23 (1960) 179.
8. E.S.Borovik, Zh.Eksperim.i Teor.Fiz.23 (1952) 83.
9. K.Førsvoll and I.Holwech, Phil.Mag.10 (1964) 921.
10. J.A.Rayne, Phys.Rev.129 (1963) 652.

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LOGARITHMIC TERMS IN THE DENSITY EXPANSION OF TRANSPORT COEFFICIENTS

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It is recently noted by a number of authors [1-4] that the transport coefficients for classical systems cannot be expanded in a power series in the density. The evidence is born out the analysis of the correction terms to the Boltzmann equation. Particularly convincing is the calculation of Sengers [2] who found divergencies of a logarithmic type in the so called triple collision term for a two dimensional system of hard disks and showed that the various contributions do not compensate each other. Arguments are presented [3, 4] that these divergencies can be repaired by the introduction of higher order terms which lead to a logarithmic term in the density in the expansion of the transport coefficient. In this letter we report on calculations supporting and extending these results for a simplified model, which allows a more detailed calculation and a further analysis of the low density behaviour of the transport coefficients.

We also use a hard sphere model but instead of having equal masses, we take one sphere in a medium of identical infinite heavy spheres. By this limit the dynamics is reduced to the motion of one sphere in a random configuration of static rigid spheres.

We concentrate on the diffusion coefficient D, given by the time integral over the velocity auto-correlation function of the moving sphere:

$$D = \int_0^\infty \mathrm{d}t \, \langle v(0) \cdot v(t) \rangle \,, \tag{1}$$

where the average is taken over a canonical distribution of the static sphere configurations.

The basic function in this model is the conditional probability $\varphi_t(x, x')$ that, given the moving sphere is at $t = 0$ at the position-velocity point $x = (q, v)$, it is found at the time t later at the position-velocity point x'. Introducing the Fourier-Laplace transform $\varphi_{zk}(v, v')$, defined by

$$\varphi_{zk}(v, v') = \int \mathrm{d}^s q' \, \exp\{\mathrm{i}k \cdot (q' - q)\} \int_0^\infty \exp(-zt) \, \varphi_t(x, x') \,, \tag{2}$$

the diffusion coefficient D is given as:

$$D = \int \mathrm{d}^s v \int \mathrm{d}^s v' \, g(v) \, \varphi_{00}(v, v') \, v \cdot v' \,, \tag{3}$$

where $g(v)$ is the normalized Maxwell-Boltzmann velocity distribution and s the dimensionality of the system.

The standard way to obtain a density expansion for D is to set up a kinetic equation for $\varphi_{zk}(v, v')$ which has the general form:

$$(z - \mathrm{i}k \cdot v) \, \varphi_{zk}(v, v') = \delta(v - v') + \mathscr{B}_{zk} \, \varphi_{zk}(v, v') \,. \tag{4}$$

The collision operator $\mathscr{B}_{zk}$ operates on the velocity v only. Various techniques [5] are available to obtain a density expansion of $\mathscr{B}_{zk}$ in the form