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1987-06

The Linear Dependency Structure of Covariance Nonstationary Time Series

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NPS55-87-006

NAVAL POSTGRADUATE SCHOOL
Monterey, California

TECHNICAL REPORT

THE LINEAR DEPENDENCY STRUCTURE OF
COVARIANCE NONSTATIONARY TIME SERIES

WILL GERSCH

JUNE 1987

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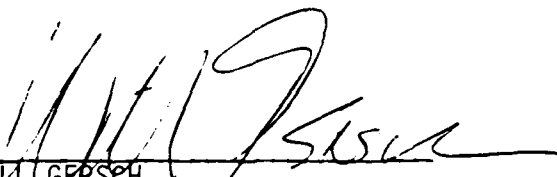
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REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION	1b RESTRICTIVE MARKINGS
UNCLASSIFIED	

2a. SECURITY CLASSIFICATION AUTHORITY	3 DISTRIBUTION AVAILABILITY OF REPORT
	Approved for public release; distribution
2b DECLASSIFICATION DOWNGRADING SCHEDULE	unlimited.

4 PERFORMING ORGANIZATION REPORT NUMBER(S)

NPS55-87-006
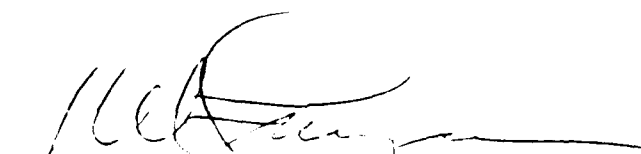
5 MONITORING ORGANIZATION REPORT NUMBER(S)

6a NAME OF PERFORMING ORGANIZATION	6b OFFICE SYMBOL (If applicable)	7a NAME OF MONITORING ORGANIZATION
Naval Postgraduate School	Code 55	

6c. ADDRESS (City, State, and ZIP Code)	7b. ADDRESS (City, State, and ZIP Code)
Monterey, CA 93943-5000	

8a. NAME OF FUNDING SPONSORING ORGANIZATION	8b OFFICE SYMBOL (If applicable)	9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER

8c. ADDRESS (City, State, and ZIP Code)

10 SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO	PROJECT NO	TASK NO	WORK UNIT ACCESSION NO

11 TITLE (Include Security Classification)

THE LINEAR DEPENDENCY STRUCTURE OF COVARIANCE NONSTATIONARY TIME SERIES

12 PERSONAL AUTHOR(S)
Gersch, Will

13a TYPE OF REPORT	13b TIME COVERED	14 DATE OF REPORT (Year, Month, Day)	15 PAGE COUNT
Technical	FROM ___ TO ___	1987, JUN	34

16 SUPPLEMENTARY NOTATION

17 COSATI CODES			18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB GROUP	Information theory, time series, time-varying model, autoregression, feedback, causality, electroencephalogram.

19 ABSTRACT (Continue on reverse if necessary and identify by block number)

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20 DISTRIBUTION AVAILABILITY OF ABSTRACT	21 ABSTRACT SECURITY CLASSIFICATION
☒ UNCLASSIFIED UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS	UNCLASSIFIED

22a NAME OF RESPONSIBLE INDIVIDUAL	22b TELEPHONE (Include Area Code)	22c OFFICE SYMBOL
Will Gersch	(408)646-2594	Code 55

DD FORM 1473, 84 MAR

THE LINEAR DEPENDENCY STRUCTURE OF
COVARIANCE NONSTATIONARY TIME SERIES *

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ABSTRACT: The linear dependence, feedback and causality structure of covariance nonstationary time series is developed. At every instant in time, the amount of linear dependence between time series vectors is expressible as the sum of the amount of feedback from the first time series vector to the second, the amount of feedback from the second time series to the first and the amount of instantaneous feedback. The parametric modeling of multivariate covariance nonstationary time series and the computation of their interdependency structure from the fitted model are also treated. The time series is modeled by a multivariate time varying autoregressive (MVTVAR) model. The fitted MVTVAR model yields an instantaneous power spectral density (IPSD) matrix. The IPSD is used in computing the linear dependency structure of nonstationary time series. An example of the modeling and the determination of instantaneous causality from a human implanted electrode seizure event EEG is shown

KEYWORDS: Information theory, time series, time-varying model, autoregression, feedback, causality, electroencephalogram.

1. INTRODUCTION

The linear dependence, feedback and causality structure of covariance nonstationary time series is developed. The parametric modeling of multivariate covariance nonstationary time series and the computation of their interdependency structure from the fitted model are also treated. The analysis is applied to a spontaneous human epileptic seizure event electroencephalogram (EEG).

The structure of multivariate stationary time series has been of interest in econometrics, oceanography, meteorology, engineering and statistics. System quantities such as the transfer function between time series and quantitative measures such as the amount of information between time series and the amount of feedback are of interest. In the analysis of econometric data and human epileptic seizure data, the presence or absence of causality are additionally of interest. The modeling of multivariate stationary time series has been treated for example in Gelfand and Yaglom 1959, Caines and Chan 1975, Gustafson , Ljung and Soderstrom 1977, Gevers and Wertz 1982 and Geweke (1982).

A statistical interpretation of causality in stationary time series from nonrepeated experiments appears in Wiener 1956, Granger 1963, Caines and Chan 1975, Geweke 1982 and references therein. In stationary time series econometric data, driving or causality is usually defined to mean the presence of a "feedback free" condition. In stationary time series epileptic event EEG data, causality has been defined operationally as the detection of a delay of one time series with respect to the causal time series, Brazier 1972, Gotman 1983. In covariance nonstationary EEG data, the objective of analysis is the estimation of the amount of information, the amount of feedback and the detection of causality, all at an instant in time. The inference of linear dependency, feedback and causality in nonstationary time series are new topics.

The determination of the interdependencies between the electrical activity at different brain sites is a subject of current interest in EEG analysis. The recent work of Gevins et al 1983, in event related potentials (ERPs) and of Mars and Lopes da Silva 1983, Saito and Harashima 1981,

Innouye et al 1983 and Gotman 1983 in ongoing or background EEGs is evidence of that interest.

The literature on multivariate nonstationary time series modeling is surprisingly sparse when we consider the fact that many natural and social science phenomena are nonstationary. Rosenberg 1973, Sarris 1973 and Swamy and Tinsley 1980 are studies in the econometrics literature on modeling time series with time-dependent stochastic parameters. Except for Bohlin 1976. (scalar nonstationary time series), these methods have not been applied to the analysis of nonstationary EEGs. The approach described here, a deterministic regression modeling of multichannel nonstationary time series, differs from the stochastic regression coefficient modeling methods described in the literature cited above.

The complex demodulation analysis by Walter 1968 was very likely the first nonstationary time series analysis of the EEG. Subsequent nonstationary analyses were by Kawabata 1973 (overlapping periodograms) and Bohlin 1976 and Isaksson 1975 (nonstationary time series models and Kalman filter algorithms). An indirect approach to the analysis of nonstationary covariance time series is to segment them into locally stationary time series segments and to model the segments separately as stationary time series. Bodenstein and Praetorious 1977. Sagan and Sanderson 1980 and Benveniste and Basseville 1984 and references therein. None of the aforementioned analyses treat multichannel EEGs.

In Section 2 the linear dependency structure of stationary time series, the linear dependency structure of covariance nonstationary time series and the concept of causality are treated. In Section 3 the modeling of covariance nonstationary time series by the MVTVAR method is described. An application of MVTVAR modeling of human implanted electrode epileptic seizure event data and the detection of causality is demonstrated in Section 4. Section 5 is a Summary and Discussion.

2. LINEAR DEPENDENCY STRUCTURE IN MULIVARIATE TIME SERIES

2.1 BACKGROUND. STRUCTURE IN STATIONARY TIME SERIES.

The quantitative measure of the amount of information between discrete random variables, Shannon 1948, is the intellectual precedent for the quantitative measure of the amount of information in one time series about time series. The measure of the the amount of information between two vectors of continuously distributed random variables X and Y. Shannon's amount of mutual information, is given by,

$$ I_{x,y} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f_{X,Y}(x,y) ln(f_{X,Y}(x,y) \; f_X(x) f_Y(y)) dx dy. \qquad (2.1.1) $$

In (2.1.1), $f_{X,Y}(x,y)$ and $f_X(x), f_Y(y)$ are respectively the joint probability distribution function of the vector random variables X and Y and the marginal probability density functions of X and Y. $I_{x,y}$ is a measure of the amount of dependence between the random vectors X and Y. Equation (2.1.1) is the negative of the entropy of the "true" distribution $f_{X,Y}$ with respect to an assumed distribution $f_X f_Y$. Also $I_{x,y}$ is the dissimilarity or information divergence between the alternative joint probability distribution $f_{X,Y}$ and the factored or independent probability descriptions $f_{X,Y} = f_X f_Y$ of the vector random variables X, Y. Kullback 1958. Formally, (2.1.1) is equal to the amount of information, on the average, per observation, to reject the null hypothesis that the random vectors X, Y are independent.

Gelfand and Yaglom 1959 computed (2.1.1) for the case of X, Y stationary, jointly normally distributed time series. Let $\{z(t)\}$ be a vector of time series that is partitioned into the two component vector time series, $z(t) = [x(t), y(t)]^T$. Let the power spectral density matrix of $\{z(t)\}$ be written in the partitioned form

$$ S_z(f) = \begin{vmatrix} S_{xx}(f) & S_{xy}(f) \\ S_{yx}(f) & S_{yy}(f) \end{vmatrix}. \qquad (2.1.2) $$

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Alternate forms of the Shannon-Gelfand-Yaglom (S-G-Y) measure of the amount of information between the two vectors of time series are, (Gelfand-Yaglom 1959),

$$I_{z,y} = \int_{-1/2}^{1/2} \ln(|S_{zz}(f)| / |S_{zz}(f)| |S_{yy}(f)|) df \qquad (2.1.3)$$

$$I_{z,y} = \int_{-1/2}^{1/2} \ln(|S_{zz}(f)| / |S_{zz}(f) - S_{zy}(f) S_{yy}^{-1}(f) S_{zy}'(f)|) df$$

$$I_{z,y} = -\int_{-1/2}^{1/2} ln[1 - W_{zy}^2(f)] df$$

In (2.1.3), $|A|$ denotes the determinant of the matrix A and the symbol $'$ denotes the complex conjugate transpose. The first formula in (2.1.3) mirrors the joint distribution versus the product of independent distributions in (2.1.1). In the second formula in (2.1.3), the term in the denominator is the residual spectral density of the time series $\{x(t)\}$ after the removal by regression of the influence of $\{y(t)\}$. If the two time series are independently distributed, $\{y(t)\}$ does not influence $\{x(t)\}$, the denominator of the second term is identical to the numerator and $I_{z,y}$ is zero, as expected. The third formula in (2.1.1) is a special case of the amount of information between two scalar time series $\{x(t)\}$ and $\{y(t)\}$, expressed in terms of the spectral coherence at frequency f, $W_{zy}^2(f)$. The last formula in (2.1.3) is particularly useful in our analysis of causality.

It should be noted that the information theoretic $I_{z,y}$ and $I_{X,Y}$ formulas hold only for the case of jointly normal time series or jointly normal random variables. Otherwise $I_{z,y}$ ($I_{X,Y}$) has an interpretation as a measure of linear dependence between time series (random variables).

Geweke 1982 obtains other useful properties of the linear dependency structure between jointly stationary time series. To illustrate those properties we need consider several alternative linear projections of the time series $\{x(t)\}$ and $\{y(t)\}$. First we assume that the time series $\{z(t)\}$ has an AR representation.

$$\tilde{z}_t = z_t - \sum_{m=1}^{\infty} A_m z_{t-m}, \qquad E[z_t] = 0, \qquad var[\tilde{z}_t] = \tilde{\Sigma} = \begin{bmatrix} \tilde{\Sigma}_{x2} & C \\ C^T & \tilde{\Sigma}_{y2} \end{bmatrix}. \qquad (2.1.4)$$

The terms $\tilde{\Sigma}_{x2}$ and $\tilde{\Sigma}_{y2}$ are defined below. Define four different linear projections of x_t on the past of x_t, upon the pasts of x_t and y_t upon the past of x_t and the present and past of y_t and upon the past of x_t and the past and future of y_t. That is, let

$$(2.1.5)$$

$$\tilde{x}_{1,t} = x_t - \sum_{m=1}^{\infty} A_{1,m} x_{t-m}, \qquad \tilde{x}_{2,t} = x_t - \sum_{m=1}^{\infty} A_{2,m} x_{t-m} - \sum_{m=1}^{\infty} B_{2,m} y_{t-m}.$$

$$\tilde{x}_{3,t} = x_t - \sum_{m=1}^{\infty} A_{3,m} x_{t-m} - \sum_{m=0}^{\infty} B_{3,m} y_{t-m}, \qquad \tilde{x}_{4,t} = x_t - \sum_{m=1}^{\infty} A_{4,m} x_{t-m} - \sum_{m=-\infty}^{\infty} B_{4,m} y_{t-m}.$$

$$var[x_{i,t}] = \tilde{\Sigma}_{x,i}.$$

Also, identify linear projections of $\{y(t)\}$ in a similar manner.

$$(2.1.6)$$

$$\tilde{y}_{1,t} = x_t - \sum_{m=1}^{\infty} C_{1,m} y_{t-m}, \qquad \tilde{y}_{2,t} = x_t - \sum_{m=1}^{\infty} C_{2,m} y_{t-m} - \sum_{m=1}^{\infty} C_{2,m} y_{t-m}.$$

$$\tilde{y}_{3,t} = x_t - \sum_{m=1}^{\infty} C_{3,m} y_{t-m} - \sum_{m=0}^{\infty} C_{3,m} y_{t-m}, \qquad \tilde{y}_{4,t} = x_t - \sum_{m=1}^{\infty} C_{4,m} y_{t-m} - \sum_{m=-\infty}^{\infty} C_{4,m} y_{t-m}.$$

$$var[y_{i,t}] = \tilde{\Sigma}_{yi}.$$

Now, in a natural way, define: (i) The amount of feedback from the time series $\{y_t\}$ to the time series. $\{x_t\}$ $I_{y \cdot x}$, (ii) The amount of feedback from the time series $\{x_t\}$ to the time series. $\{y_t\}$, $I_{x \cdot y}$ and (iii) The amount of instantaneous feedback between the time series $\{x_t\}$ and $\{y_t\}$. I_{xy} by.

$$I_{y \to z} = \ln(|\dot{\Sigma}_{z1}| / |\dot{\Sigma}_{z2}|), \qquad I_{z \to y} = \ln(|\dot{\Sigma}_{y1}| / |\dot{\Sigma}_{y2}|), \qquad I_{zy} = \ln(|\dot{\Sigma}_{z2}| / |\dot{\Sigma}_{z3}|).$$

From (2.1.5), these feedback quantities also have specific interpretations as the amount of information in the jointly Gaussian distributed time series case or as amounts of linear dependencies in the general case. More emphatically, in the jointly normal time series case, feedback is information!

A theorem proved in Geweke 1982, (the measure of linear dependence between stationary time series is the sum of the measure of linear feedback from the first series to the second, linear feedback from the second to the first and instantaneous feedback), is,

Theorem:

$$I_{z,y} = I_{z \to y} + I_{y \to z} + I_{zy}. \tag{2.1.8}$$

with

$$i) \quad I_{z,y} = ln(|\dot{\Sigma}_{z1}| / |\dot{\Sigma}_{z4}|) = ln(|\dot{\Sigma}_{y1}| / |\dot{\Sigma}_{y4}|) \tag{2.1.9}$$

$$ii) \quad I_{y \to z} = ln(|\dot{\Sigma}_{z1}| / |\dot{\Sigma}_{z2}|) = ln(|\dot{\Sigma}_{y3}| / |\dot{\Sigma}_{y4}|)$$

$$iii) \quad I_{z \to y} = ln(|\dot{\Sigma}_{z3}| / |\dot{\Sigma}_{z4}|) = ln(|\dot{\Sigma}_{y1}| / |\dot{\Sigma}_{y2}|)$$

$$iv) \quad I_{zy} = ln(|\dot{\Sigma}_{z2}| / |\dot{\Sigma}_{z3}|) = ln(|\dot{\Sigma}_{y2}| / |\dot{\Sigma}_{y3}|)$$

That is, each of the amounts of linear dependence and amounts of feedback can be computed from the residual matrices of the different projections of the time series $x(t)$ and $y(t)$.

Proof: (Similar to Geweke 1982.) From (2.1.3)-(2.1.7), we observe that $ln(|\dot{\Sigma}_{z4}|) = ln(|\dot{\Sigma}| |\dot{\Sigma}_{y1}|)$. Then, $ln(|\dot{\Sigma}_{z1}| |\dot{\Sigma}_{z4}|) = ln(|\dot{\Sigma}_{z1}| |\dot{\Sigma}_{y1}| |\dot{\Sigma}|)$, and $ln(|\dot{\Sigma}_{y1}| |\dot{\Sigma}_{y4}|) = ln(|\dot{\Sigma}_{z1}| |\dot{\Sigma}_{y1}| |\dot{\Sigma}|)$, by symmetry.

(ii) By construction of (2.1.8) $\dot{\Sigma}_{z3} = \dot{\Sigma}_{z2} + C\dot{\Sigma}_{y2}^{-1}C^T$, so $|\dot{\Sigma}_{z3}| |\dot{\Sigma}_{y2}| = |\dot{\Sigma}|$ which leads to (ii).

(iii) follows by symmetry from (ii).

(iv) Follows from $|\tilde{\Sigma}_{z3}| |\tilde{\Sigma}_{y2}| = |\tilde{\Sigma}|$ and symmetry.

Another important result in Geweke 1982 is that the feedback quantities can be additively decomposed in frequency. For example.

$$I_{y\to z} \geqslant \int_{-1/2}^{1/2} I_{y\to z}(f)df. \tag{2.1.10}$$

Similar additive relationships hold for dependence and the other feedbacks. Equality holds in (2.1.13) under mild technical conditions on the regression coefficients that are straightforward to verify. see Geweke 1982 for details. (The invertibility of the time series model is the issue involved.).

Consider the first and fourth projections of the time series $\{x(t)\}$ in (2.1.5). Let $S_z(f)$, be the power spectral density computed from the regression of $\{x(t)\}$ on its own past. Let $S_{z|z,y}(f)$, be the power spectral density computed from the regression of $\{x(t)\}$ on its own past and the past and future of $\{y(t)\}$. Then, $I_{y\to z}(f)$. the feedback from $\{y(t)\}$ to $\{x(t)\}$ is,

$$I_{y\to z}(f) = \ln(|S_z(f)|/|S_{z|z,y}(f)|). \tag{2.1.11}$$

For example. when $\{y(t)\}$ does not influence or feedback on $\{x(t)\}$. the spectral density $S_{z\,z,y}(f)$ will be identical to $S_z(f)$. the spectral density computed from $\{x(t)\}$ alone. Then the ratio in (2.1.11) is one and hence $I_{y\to z}(f)$ is zero. again as expected.

There are two important extensions of the results in (2.1.8)-(2.1.11). 1) The results hold under conditioning or partial regression of the series $\{x(t),y(t)\}$ on another vector of time series $\{w(t)\}$. Geweke 1984. 2) The results hold for nonstationary time series. Some of those results for nonstationary time series are discussed in Section 2.2. A MVTVAR coefficient model which permits estimation of these results is discussed in Section 3.

2.2 THE LINEAR STRUCTURE OF COVARIANCE NONSTATIONARY TIME SERIES

Here we extend the notation and results on the structure of stationary time series in Section 2.1 to nonstationary covariance time series. Assume an MVTVAR representation for the time series $z(t)$, again with $z(t) = [x(t)y(t)]^T$.

$$z_t = \sum_{m=1}^{\infty} A_{m,t} z_{t-m} + \epsilon_t, \quad E[\epsilon_t] = 0, \quad E[\epsilon_{t+k}\epsilon_t^T] = \tilde{\Sigma}_t \delta_{t+k,t} \qquad (2.2.1)$$

with

$$\tilde{\Sigma}_t = \begin{bmatrix} \tilde{\Sigma}_{2,t} & C_t \\ C_t^T & \tilde{T}_{2,t} \end{bmatrix}.$$

where $\tilde{\Sigma}_{2,t}$ and $\tilde{T}_{y2,t}$ are defined below.

Define $A(f,t)$, the polynomial operator,

$$A(f,t) = I - \sum_{m=1}^{\infty} A_{m,t} \exp[-2\pi mf]. \qquad (2.2.2)$$

Then in a natural way, as an extension of the definition of power spectral density for stationary time series, define the instantaneous power spectral density, (IPSD), matrix in terms of the MVTVAR model,

$$S(f,t) = A(f,t)^{-1} \tilde{\Sigma}_t A(f,t)^{-1*} \qquad (2.2.3)$$

(The concept of an instantaneous power spectral density was introduced in Page 1952. The most notable development since Page 1952 appears to be by Priestly 1965,1967. Priestly 1981, Chapter 11 is a review of the literature. By the way of contrast, Page and Priestly are both frequency domain spectral methods, while the MVTVAR IPSD is a parametric model method.)

As we did in the case of stationary time series, identify linear projections of $\{x_t\}$

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$$\tilde{x}_{1,t} = x_t - \sum_{m=1}^{\infty} A_{1m,t} x_{t-m}, \quad \tilde{x}_{2,t} = x_t - \sum_{m=1}^{\infty} A_{2m,t} x_{t-m} - \sum_{m=1}^{\infty} B_{2m,t} y_{t-m},$$

$$\tilde{x}_{3,t} = x_t - \sum_{m=1}^{\infty} A_{3m,t} x_{t-m} - \sum_{m=0}^{\infty} B_{3m,t} y_{t-m}, \quad \tilde{x}_{4,t} = x_t - \sum_{m=1}^{\infty} A_{4m,t} x_{t-m} - \sum_{m=-\infty}^{\infty} B_{4m,t} y_{t-m}.$$

$$var[\tilde{x}_{1,t}] = \dot{\Sigma}_{1,t}.$$

Also in a similar manner, identify linear projections of $\{y_t\}$

$$\tilde{y}_{1,t} = y_t - \sum_{m=1}^{\infty} C_{1m,t} y_{t-m}, \quad \tilde{y}_{2,t} = y_t - \sum_{m=1}^{\infty} C_{2m,t} x_{t-m} - \sum_{m=1}^{\infty} D_{2m,t} y_{t-m},$$

$$\tilde{y}_{3,t} = y_t - \sum_{m=1}^{\infty} C_{3m,t} x_{t-m} - \sum_{m=0}^{\infty} D_{3m,t} y_{t-m}, \quad \tilde{y}_{4,t} = y_t - \sum_{m=1}^{\infty} C_{4m,t} x_{t-m} - \sum_{m=-\infty}^{\infty} D_{4m,t} y_{t-m}.$$

$$var[\tilde{y}_{1,t}] = \dot{T}_{1,t}.$$

Then using algebra, we obtain results for nonstationary time series that are analogous to those obtained for stationary time series. The amount of linear dependence at time t between $\{x_t\}$ and $\{y_t\}$, the amount of feedback at time t from $\{y_t\}$ to $\{x_t\}$, the amount of feedback at time t from $\{x_t\}$ to $\{y_t\}$, and the instantaneous amount of feedback at time t from $\{x_t\}$ to $\{y_t\}$ satisfy:

$$I_{x,y,t} = I_{x \to y,t} + I_{y \to x,t} + I_{xy,t} \tag{2.2.6}$$

$$I_{x,y,t} \quad ln(|\dot{\Sigma}_{1,t}| |\dot{\Sigma}_{4,t}|) \quad ln(|\dot{T}_{1,t}| |\dot{T}_{4,t}|)$$

$$I_{y \to x,t} \quad ln(|\dot{\Sigma}_{1,t}| |\dot{\Sigma}_{2,t}|) \quad ln(|\dot{T}_{3,t}| |\dot{T}_{4,t}|)$$

$$I_{x \to y,t} \quad ln(|\dot{\Sigma}_{3,t}| |\dot{\Sigma}_{4,t}|) \quad ln(|\dot{T}_{1,t}| |\dot{T}_{2,t}|)$$

$$I_{xy,t} \quad ln(|\dot{\Sigma}_{2,t}| |\dot{\Sigma}_{3,t}|) \quad ln(|\dot{T}_{2,t}| |\dot{T}_{3,t}|)$$

The main result in (2.2.6) is that at each instant in time, the amount of linear dependence between time series is the sum of the amount of linear feedback from the first series to the second, linear feedback from the second to the first and instantaneous feedback. Also, as in stationary time series, in nonstationary time series, there is an additive decomposition of the feedbacks with frequency. For example

$$I_{y \to x, t} \geqslant \int_{-} I_{y \to x, t}(f) df. \qquad (2.2.7)$$

An example of a relationship between the linear dependency measures for nonstationary and stationary time series is

$$I_{x,y} = \frac{1}{T} \int_0^T I_{x,y,t} dt \qquad (2.2.8)$$

The other nonstationary linear dependencies satisfy similar relationships.

2.3 CAUSALITY IN TIME SERIES

The emphasis of the applications of structure in stationary time series in Geweke 1982 is on feedback between economic time series. As shown in Section 2.2 instantaneous feedback (linear dependency) between nonstationary time series, can be expressed in simple formulas. Indications of ways of estimating those linear dependencies from nonstationary time series are shown in Section 3. The emphasis here is on a concept of causality between time series. The motivation for this section is the determination of causality in implanted electrode EEG data in humans observed during epileptic seizures. Our concept of causality between time series is different than the "feedback free" condition definition used in econometrics (Geweke 1982 and references therein) and the time precedence definition used in the analysis of epileptic event EEG data. (Brazier 1972. Gotman 1983) We define causality in terms of the S-G-Y and conditional S-G-Y measures between time series. Consider the 3 time series or vectors of time series, $\{x(t)\}$, $\{y(t)\}$ and $\{w(t)\}$. Assume

that there is pairwise linear dependence between the series. That is,

$$I_{x,y} \neq 0, \qquad I_{x,w} \neq 0, \qquad I_{y,w} \neq 0. \tag{2.3.1}$$

Now assume that each pair of time series is conditioned on the excluded third time series and the following results are obtained.

$$I_{x,y|w} \neq 0, \qquad I_{x,w|y} \neq 0, \qquad I_{y,w|x} = 0. \tag{2.3.2}$$

From (2.3.2), removing the influence of $\{w(t)\}$ from $\{x(t)\}$ and $\{y(t)\}$ leaves that pair of time series linearly related. Similarly, removing the influence of $\{y(t)\}$ from $\{x(t)\}$ and $\{w(t)\}$ leaves that pair of time series linearly related. However, removal of the influence of $\{x(t)\}$ from $\{y(t)\}$ and $\{w(t)\}$ leaves that pair uncorrelated. Thus the series $\{x(t)\}$ uniquely explains the linear relationships between three series. We say that under the conditions in (2.3.1) and (2.3.2) that the time series $\{x(t)\}$ is causal to the time series $\{y(t)\}$ and $\{w(t)\}$. (A mathematical model that exhibits the properties in (2.3.1)-(2.3.2) is in Gersch 1972.)

Epileptic event EEGs tend to be characterized by concentrations of spectral energy in a narrow frequency band and causality or driving may not be present during an entire seizure. We are thus motivated to exploit the instantaneous additive decomposition of linear dependence in (2.2.6) and adapt the definition of causality in (2.3.1) and (2.3.2) to a definition of causality at some particular frequency f_A and the time instant t.

Let the IPSD matrix of the three nonstationary time series $\{x(t), y(t), w(t)\}$ be expressed in component form.

$$S(f,t) = \begin{bmatrix} S_{xx}(f,t) & S_{xy}(f,t) & S_{xw}(f,t) \\ S_{yx}(f,t) & S_{yy}(f,t) & S_{yw}(f,t) \\ S_{wx}(f,t) & S_{wy}(f,t) & S_{ww}(f,t) \end{bmatrix}. \tag{2.3.3}$$

Several additional ingredients are needed for our causality analysis. The following are defined: $W_{xy}^2(f,t)$, the instantaneous spectral coherence between the two generic time series $\{x(t), y(t)\}$ at time t and frequency f, and the instantaneous partial spectral coherence $S_{xx\ w}(f,t)$, $S_{yy\ w}(f,t)$ and $S_{xy\ w}(f,t)$, respectively the instantaneous spectral density of $\{x(t)\}$ conditioned on $\{w(t)\}$, the instantaneous spectral density of $\{y(t)\}$ conditioned on $\{w(t)\}$, and the the instantaneous cross spectrum between $\{x(t)\}$ and $\{y(t)\}$ at time t and frequency f conditioned on $\{w(t)\}$. In terms of the components of the spectral density matrix in (2.3.3) these are given by

$$W_{xy}^2(f,t) = |S_{xy}(f,t)|^2 \ S_{xx}(f,t) S_{yy}(f,t) \qquad (2.3.4)$$

$$S_{xx\ w}(f,t) = S_{xx}(f,t)(1 - W_{xw}^2(f,t))$$

$$S_{yy\ w}(f,t) = S_{yy}(f,t)(1 - W_{yw}^2(f,t))$$

$$S_{xy\ w}(f,t) \quad S_{xy}(f,t) S_{yz}(f,t) \ S_{ww}(f,t)$$

The instantaneous partial spectral coherence between the generic time series $\{x(t), y(t)\}$ conditioned on the time series $\{w(t)\}$ at time t and t and frequency f, computed from the instantaneous partial spectra in (2.3.4) is

$$W_{xy\ w}^2 \quad |S_{xy\ w}(f,t)|^2 \ S_{xx\ w}(f,t) S_{yy\ w}(f,t). \qquad (2.3.5)$$

Now, using the additive frequency domain decomposition property of feedback between time series (2.2.7), and our definition of causality in (2.3.1) and (2.3.2) we define the series $\{x(t)\}$ to be causal to the series $\{y(t), w(t)\}$ at frequency f_1 at time t if the following conditions are satisfied.

$$I_{x\ y,t}(f_1) \quad 0, \qquad I_{x\ w,t}(f_1) \quad 0, \qquad I_{y\ w,t}(f_1) \quad 0, \qquad (2.3.6)$$

$$I_{x,y\ w,t}(f_1) \quad 0, \qquad I_{x,w\ y,t}(f_1) \quad 0, \qquad I_{y,w\ x,t}(f_1) \quad 0,$$

From the third equation in (2.1.3) we seen that the definition of causality in (2.3.6) is equivalent

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to the following conditions on spectral coherences and partial coherences

$$W_{x,y}^2(f_A,t) \neq 0, \qquad W_{x,w}^2(f_A,t) \neq 0, \qquad W_{y,w}^2(f_A,t) \neq 0,\tag{2.3.7}$$

$$W_{x,y\ w}^2(f_A,t) \neq 0, \qquad W_{x,w\ y}^2(f_A,t) = 0, W_{y,w\ x}^2(f_A,t) = 0.$$

Thus the identification of causality at frequency f_A at time t is determined by the detection of

zero partial instantaneous coherence. This definition of causality at an instant in time is a general-

ization of a related concept for stationary time series introduced in Gersch and Goddard 1970 and

used in Gersch 1972, Gersch and Tharp 1976 and Brillinger 1976.

The distribution of coherence and partial coherence is treated in Hannan 1970, Brillinger

1974 and Koopmans 1974. For our purposes the most important result is the distribution of a

transformed version of the partial spectral coherence in the vicinity of zero partial coherence. A

convenient form of that result, Koopmans 1974, is given by

$$F = \frac{(\nu-1)\,W_{xy\ z}^2(f)}{[1 - W_{xy\ z}^2(f)]} \sim F_{2,2(\nu-1)}.\tag{2.3.8}$$

In (2.3.10), ν is the number of degrees of freedom in the F distribution with 2 and $2(\nu-1)$

degrees of freedom in the numerator and denominator respectively.

3. A MULTIVARIATE TIME VARYING AR COEFFICIENT MODEL AND LINEAR DEPENDENCY COMPUTATIONS

A parametric model is assumed for the time series. The successful use of autoregressive (AR) models in stationary time series analysis motivates consideration of a time-varying AR coefficient model for nonstationary covariance time series modeling. Denote the D component row vector at time n by $x(n) = (x_1(n),...,x_D(n))^T$. Then, the multivariate time-varying AR, (MVTVAR), model is

$$x(n) - A_{1,n}x(n-1) - a_{2,n}x(n-2) - ... - A_{p,n}x(n-p) - \epsilon(n). \tag{3.1.1}$$

$$E(\epsilon(n)) = 0, \quad E(\epsilon(n+k)\epsilon(n)^T) = V(n)\delta_{n-k,n}.$$

In (3.1.1), the $\{A_{i,n}i=1,...p; \ n=1,...,N\}$ are $D \times D$ coefficient matrices. The $p \times D \times D \times N$ unknown AR model parameters and the $N \times D(D+1)/2$ parameters in $V(n)$, $n=1,.....N$ in the model in (3.1.1) are to be estimated. There are more parameters than data, so least squares or maximum likelihood methods for estimating the unknown parameters will not yield useful results. The unknown parameters can be modeled implicitly. A strategy for economizing on the number of parameters to be estimated is to consider the vector of time series one component at a time and to regress that time series upon a lagged version of itself and upon the other components of the vector of time series in an orthogonal polynomial least squares method of modeling. That is, express each of the elements in the matrix of time varying AR coefficients as a linear combination of say J orthogonal polynomial functions of time, where J is a small number compared to N, the number of observations. Using this method as many as $p \times D \times J$ coefficients are fitted to each of the D component time series. If this number is small compared to N, a reasonable model can be fitted. The total number of fitted coefficients in the model is then $p \times D \times D \times J$. That number is considerably smaller than the number of implicit AR coefficients in (3.1.1), $p \times D \times D \times N$. The MVTVAR model was introduced in Gersch and Kitagawa 1983. More specifically let

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$$A_{i,n} = [a_{rk}(i,n)], \quad r,k = 1,...,D \qquad (3.1.2)$$

$$a_{rk}(i,n) = \sum_{j=0}^{J-1} c_{rk}(i,j) f(j,n), \quad i = 1,...,p$$

$$f(j,n) = \sum_{s=0}^{N} (-1)^s \binom{s-j}{s} \binom{s}{j} n^{(s)} / N^{(s)}, \quad n = 0,1,......N.$$

In (3.1.2), $a_{rk}(i,n)$ $r,k = 1,...,D$ are the elements of the time varying AR coefficient matrix $A_{i,n}$ $i = 1,...,p; n = 1,.....,N$. Also in (3.1.2) $n^{(s)} = n! / (n-s)!$, $N^{(n)} = N! / (N-s)!$ where $n!$ is n factorial and N is the number of data points. The functions $f(j,n)$ that we use in (3.1.2), are the discrete orthogonal Legendre polynomials.

The orthogonal polynomials satisfy

$$\sum_{n=0}^{N} f(j,n) f(k,n) = 0, \quad k \neq j. \qquad (3.1.3)$$

The first three discrete orthogonal Legendre polynomials are

$$f(0,n) = 1, \quad f(1,n) = 1 - 2n/N, \quad f(2,n) = 1 - 6n/N + 6n(n-1)/N(N-1). \qquad (3.1.4)$$

From (3.12.4), fitting the MVTVAR model (3.1.1), with the zeroth degree Legendre polynomial is equivalent to fitting a stationary coefficient multivariate autoregressive (MVAR) model.

Orthogonal polynomial-least squares modeling of scalar time series using time-varying AR coefficient models appears in Kozin 1977 and Grenier 1983. Gersch and Kitagawa 1983 is an extension of that method to the fitting of multivariate time series using multivariate time-varying AR coefficient (MVTVAR) models to economic data. In that modeling, the orthogonal polynomial least squares computations were realized in a Householder transformation algorithm. Akaike's AIC statistic, Akaike 1974, orthogonal polynomial model degree and regression subset selection (For more details on the modeling method, see Gersch and Kitagawa 1983.)

Each of the $\hat{\Sigma}_{i,t}$ and $\hat{T}_{i,t}$ $i = 1,...,4$ quantities that are required to compute the instantaneous linear dependency relations in (2.3.6) are computable by fitting of MVTVAR models. That is, $\hat{\Sigma}_t, \hat{\Sigma}_{2,t}, \hat{T}_{2,t}$ are obtained from the fit of the MVTVAR model to the $z(t) = [x(t) y(t)]$

Then $\tilde{\Sigma}_{3,t} = \tilde{\Sigma}_{2,t} - C_t \tilde{T}_{2,t}^{-1} C_t^T$. From (2.2.6), $|\tilde{\Sigma}_{4,t}| = |\tilde{\Sigma}_{2,t}| |\tilde{\Sigma}_{3,t}| / |\tilde{T}_{1,t}|$. The terms $\tilde{\Sigma}_{1,t}, \tilde{T}_{1,t}$ can be obtained by the fit of MVTVAR models to $x(t)$ and $y(t)$ respectively. Using these quantities in (2.2.6) each of the terms $I_{x \, y \, t}$, $I_{x \cdot y,t}$, $I_{y \cdot t}$, and $I_{xy,t}$ can be computed.

Also, the IPSD can be estimated by adapting the definition in (2.2.2)-(2.2.3) to the finite lag order fitted MVTVAR model. For the fitted MVTVAR model of lag order M, define the polynomial operator

$$A(f,t) = I - \sum_{m=1}^{M} A_{m,t} \exp(-2\pi m f) \qquad (3.1.4)$$

Then, define the estimated instantaneous power spectral density matrix in terms of the fitted MVTVAR model.

$$S(f,t) = A(f,t)^{-1} \tilde{\Sigma}_t A(f,t)^{-1*} \qquad (3.1.5)$$

The spectral coherence and partial spectral coherence at frequency f at the time instant t can be computed using the estimated IPSD.

4. AN EXAMPLE, ANALYSIS OF A HUMAN EPILEPTIC EVENT EEG

We illustrate a case in which an epileptic focus was located and determined to be present during a short time interval from an analysis of an epileptic EEG event from deeply implanted electrodes in a human. (About 60% of individuals with epilepsy have seizures initiated from an anatomically localized brain region or seizure focus. The focus initiated seizure propagates through the brain, Hauser and Kurland 1975. Many of these individuals do not respond to drug treatment. In such individuals, if a unique anatomical site from which the seizures emanate can be localized to an operable site, they may be suitable candidates for surgical treatment to remove the seizure focus and potentially relieve the individual from seizures, Anderman 1987.) Figure 1 illustrates data from a 7-second 6 site data record of the electrical activity at the dramatic onset of a spontaneous seizure event. (This data was supplied by Jeffrey P. Lieb, UCLA Reed Neurological Research Center. The original data was obtained at a 200 sample second rate. The data used for analysis was rate reduced to 50 samples per second.) The objective of the analysis was to determine whether any one of the observed anatomical recording sites could be interpreted as "driving" or being causal to the electrical activity at the other recording sites. The data is complex, and there is no obvious visual clue that might unambiguously identify an initiating or driving site or identify when driving is present.

The appearance of the individual EEG traces in Figure 1 reveals that the local structure of the time series changes with time. The analysis involves the fitting of a MVTVAR model to the data and a subsequent spectral analysis using the IPSD.

The data analysis will be described in terms of the analysis of the simultaneous EEG's in the channels labeled 1 2 and 3 in Figure 1. In Figure 2 we show a "birds-eye" view of the estimated IPSD's versus frequency computed at successive 1 2 second intervals from the fitted MVTVAR model for each of channels 1 2 and 3. Initially there are relatively sharp concentrations of energy in each of channels 1 2 and 3 at 7 25 Hertz and 14 5 Hertz. As the EEG evolves these peaks become diminished in amplitude and the distribution of spectral energy becomes more diffuse

Later in the record the emergence of a spectral peak at 4.25 Hertz can be observed in each channel. A current practice is to analyze epileptic event EEG's of no less than 6.25 second duration in overlapping 2.5 second intervals by classical windowed periodogram spectral analysis methods as if the time series in each 2.5 second interval were stationary, Gotman 1983,1987, Lieb et al. 1987. (The windowed periodogram methods used by Gotman and Lieb do not have the the time-frequency resolution properties necessary to capture the transitory characteristics of the rapidly changing epileptic event EEG data.)

We identify causality or driving at an instant in time by a frequency domain analysis using the estimated IPSD via the concept described in Section 3. Computational results of both the evolving spectral coherence and evolving partial spectral coherence for each of the three possible pairs of three different data channels at successive one second time instants are shown in Figure 3. The computational results are used to determine whether and when one channel drives the other two.

A view down the columns of the one second apart "snapshots" in Figure 3 illustrates the changing with time structure of pairwise coherence versus frequency between pairs of channels and the corresponding changing with time partial coherence versus frequency. At one and two seconds into the record, there are sharp coherence peaks between each of the of the channel pairs in the vicinity of both 7.25 and 14.5 Hertz. The partial coherence between channels 1 and 2 and between channels 1 and 3 with the partialing done on the excluded third channel remain significantly corre-lated. That is channels 2 and 3 do not have much explanatory power. On the other hand, the partial coherence between channels 2 and 3 with the influence of channel 3 removed is statistically indistinguishable from zero over the frequency interval between 7 and 11 Hertz. (An MV IV AR model order M 7, Legendre polynomial order J 1 model was fitted to the N 350 data. Using that data in (2.3 8) ν 350 14 and the 95% confidence interval for zero partial coherence is 0.05. That is a computed partial coherence of less than 0.05 is statistically indistinguishable from zero partial coherence.)

We conclude that the pairwise coherence between the three channels is a consequence of the fact that channel 1 was driving channels 2 and 3 for the first 2 seconds of the record in the vicinity of 7.25 Hertz. The pairwise coherences and partial coherences at three seconds and subsequently reveals a decrease in the pairwise coherence between each of the channel pairs and cessation of driving by channel 1 of channels 2 and 3. Later in the record, the locations of the peaks of the spectral coherence shifted to a lower frequency and the partial coherence does not indicate driving. The relatively low partial coherence between channels 2 and 3 partialed on channel 1 throughout the epoch is compatible with evidence cited by Gotman 1987

Figure 4 is a birds eye view of the evolution of the pairwise coherences and the partial coherences. This illustration was computed from the same data as Figure 3. In general, partial coherences are smaller than coherences. The relatively flat partial coherence of channels 2 and 3 partialed on channel 1 suggests that this kind of illustration may be a useful diagnostic aid to identify candidate driving channels. (Gotman 1983,1987 is an alternative method of detecting driving.

For the purposes of comparison, in Figure 5 we show the coherence and partial coherence results, computed from a multivariate AR model, as if the time series were stationary. Such analyses yield a blurred version of the sharp time-frequency resolution features available from an MVTVAR-IPSD analysis. Also, these results do not definitively implicate channel 1 as driving the other two channels.

In addition to the data analyzed here, several epileptic EEG episodes were analyzed from each of three different patients. In each case the focus was unambiguously localized for approximately three seconds. The MVTVAR-IPSD determined location of the epileptic foci were consistent with analyses done earlier and with the successful outcomes of surgical temporal lobectomy seizure elimination procedures Lieb et al 1987

5. SUMMARY AND DISCUSSION

Measures of linear dependence and feedback at each instant in time for multiple covariance nonstationary time series have been proposed. Several new ideas are introduced. The development is heuristic. For jointly normally distributed stationary time series, linear dependence is equivalent to the Shannon-Gelfand-Yaglom measure of the amount of information between time series. The concept of information at an instant in time appears to be new. The time series are assumed to be represented by a multivariate time varying autoregressive (MVTVAR) model. The MVTVAR model is the key to the computation of linear dependency and linear feedback at each instant in time. At each instant in time, the measure of linear dependence is the sum of the measure of linear feedback from the first series to the second, linear feedback from the second series to the first and instantaneous feedback. The measures of linear feedback from one series to another can be additively decomposed by frequency.

The time evolution of AR parameters in the MVTVAR model is expressed as linear combinations of discrete Legendre orthogonal polynomial functions of time. The MVTVAR model is fitted by a Householder transformation-Akaike AIC method. The MVTVAR model is exploited to introduce the concept of an instantaneous power spectral density (IPSD). This is done in a natural way as an extension of the power spectral density computed from multivariate AR models for stationary time series. The time-frequency resolution properties of the MVTVAR-IPSD computations are sharper than those obtained by segmenting nonstationary time series into successive stationary segments and using windowed periodogram spectral analysis methods or by Priestly's evolutionary spectra method.

The decomposition of linear dependence is an analysis of variance, but at this stage, a theory of inference has not been developed. Potentially the MVTVAR and IPSD can be useful for investigating the instantaneously changing interrelationships in oceanographic, meteorological and other multivariate times series phenomena as well as investigations into the nature of the instantaneously changing intra and inter cerebral propagation of epilepsy.

A new concept of causality at an instant in time was proposed. A related definition of causality for stationary time series is different than the familiar "feedback free" Granger causality in econometrics or the time precedence concept by Brazier in experimental neurophysiology. This concept of causality at an instant in time appears to be particularly well suited for the determination of driving or causality in epileptic event EEGs. To date, the results of epileptic focus location obtained by our method have been consistent with analyses done earlier and with the successful outcomes of surgical temporal lobectomy seizure elimination procedures. The perspective or birds-eye views of coherence and partial coherence versus frequency and time appears to be a potentially useful diagnostic for the identification of candidate driving sites. We anticipate doing computations on other epileptic event data sets in humans and animals and comparing results obtained by other methods.

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LEGENDS

FIGURE 1. Intracerebral EEG during a spontaneous 7 second seizure episode. From the top to bottom the data channels are: right (rt.) rt. amygdala, rt. ant.pes hippocampi, rt post. pes hippocampi., rt, ant. parahippocampal gyrus, rt. mid. parahippocampal gyrus, rt. parahippocampal gyrus. Analysis of the EEG on the left side of the brain did not indicate very substantial involvement in epileptic activity in this epoch and is not shown here. The discussion in the text s confined to the analysis of the channels marked 1,2 and 3 in Figure 1
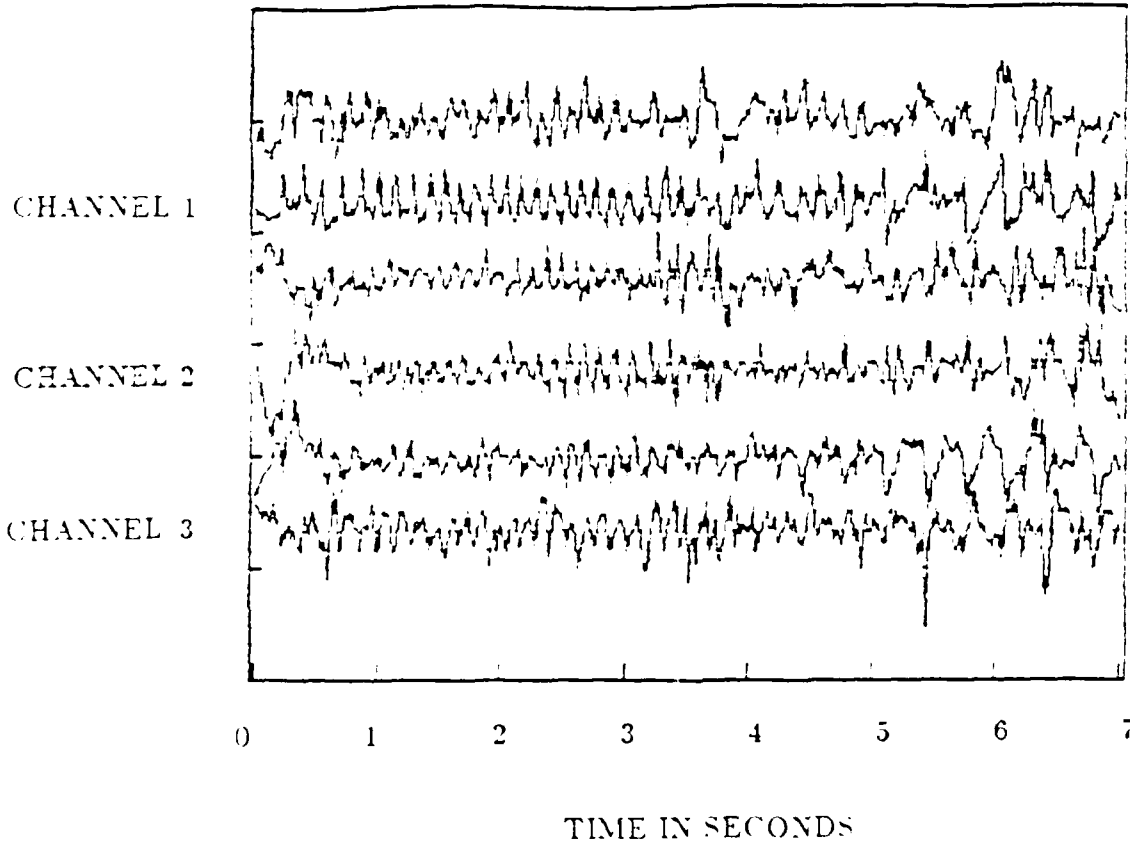
FIGURE 2. Instantaneous power spectral densities in decibels of channels 1,2 and 3 versus frequency and time in Hertz at successive 1 2 second intervals

FIGURE 3. Instantaneous coherence (solid line) and instantaneous partial coherence (dotted line) versus frequency in Hertz at successive 1 second intervals for 7 seconds. The number pairs before each column of graphs indicate the channel pairs. The number triples indicate the partial coherence between the first pair of numbers with the influence of the third channel removed from the first two channels by regression.
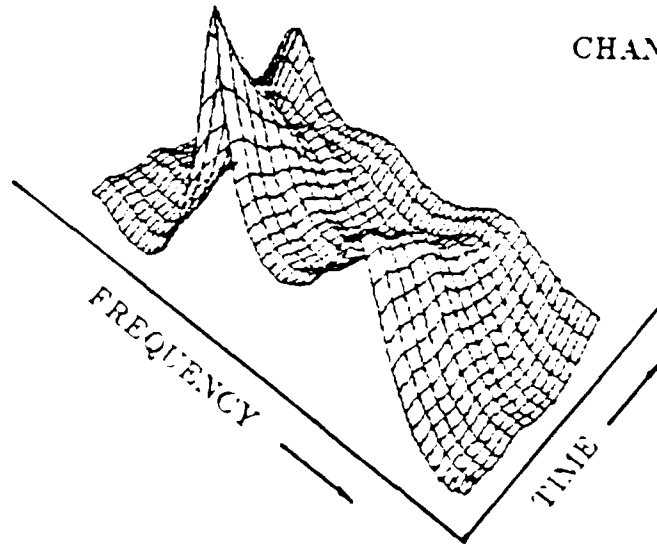
FIGURE 4. Instantaneous coherence and instantaneous partial coherence versus frequency in Hertz and time at successive 1 second intervals for 7 seconds.

FIGURE 5. Coherence and partial coherence (dotted lines) computed as if the time series were stationary. The number pairs before each column of graphs indicate the channel pairs. The number triples indicate the partial coherence between the first pair of numbers with the influence of the third channel removed from the first two channels by regression

LEGENDS

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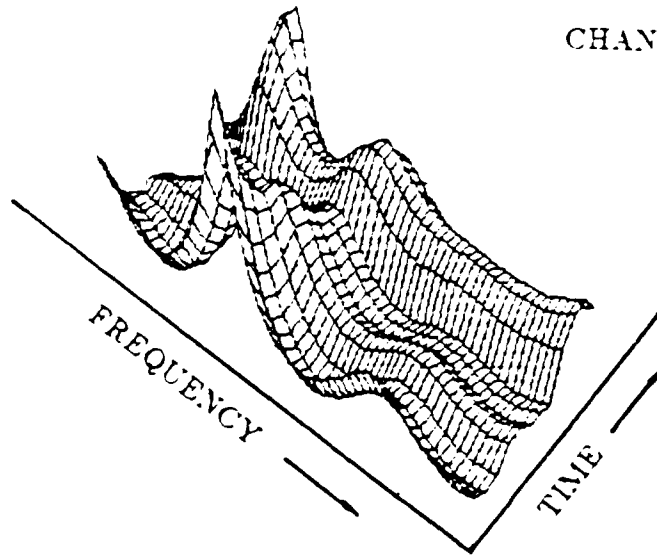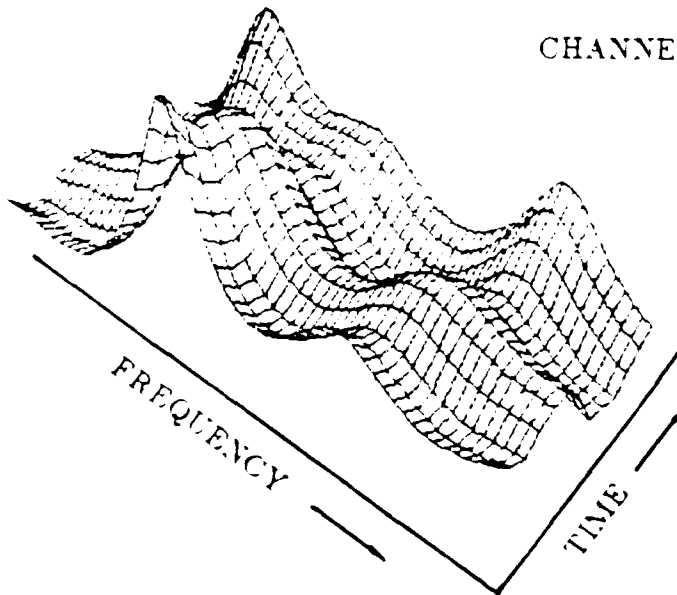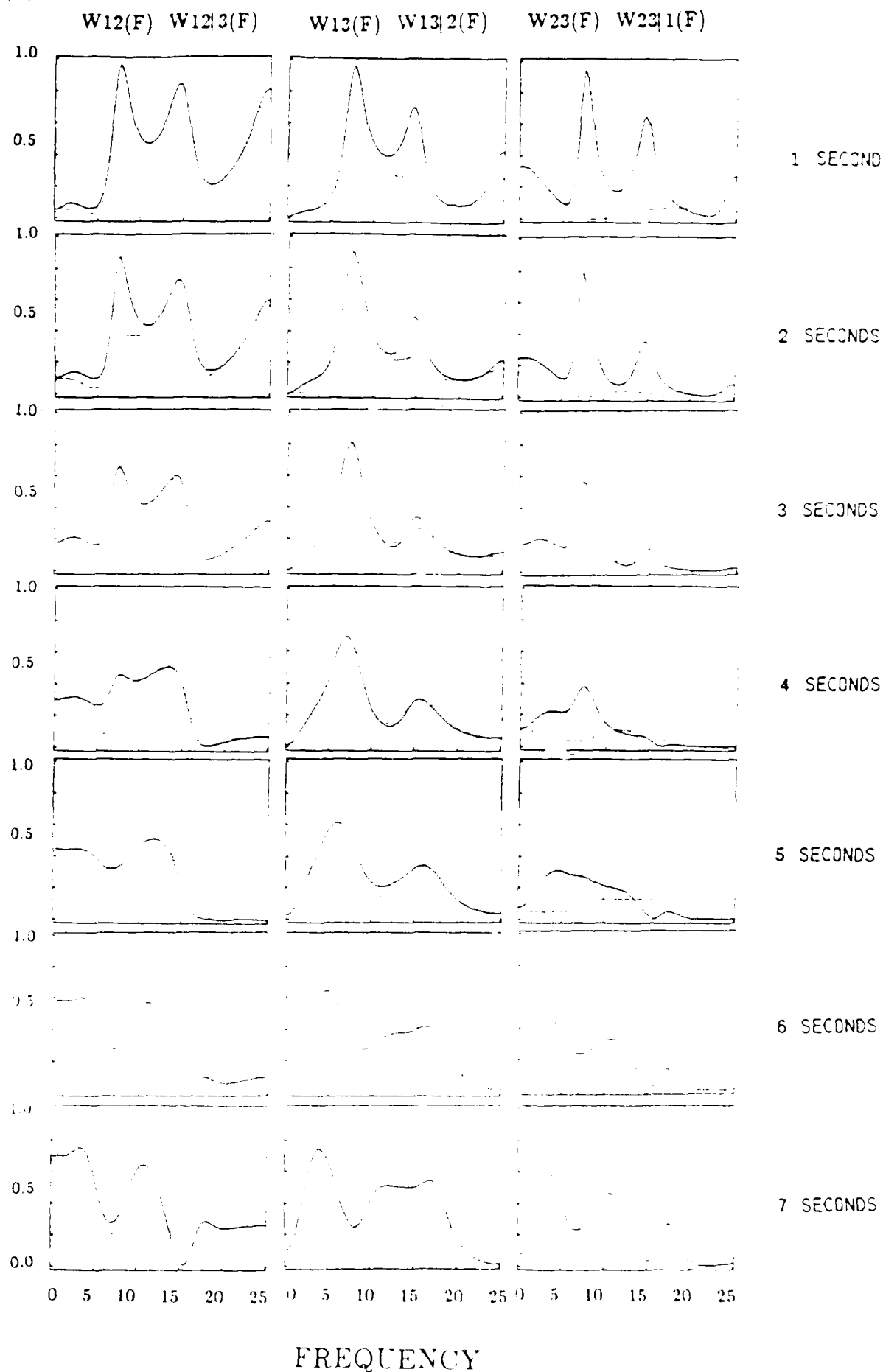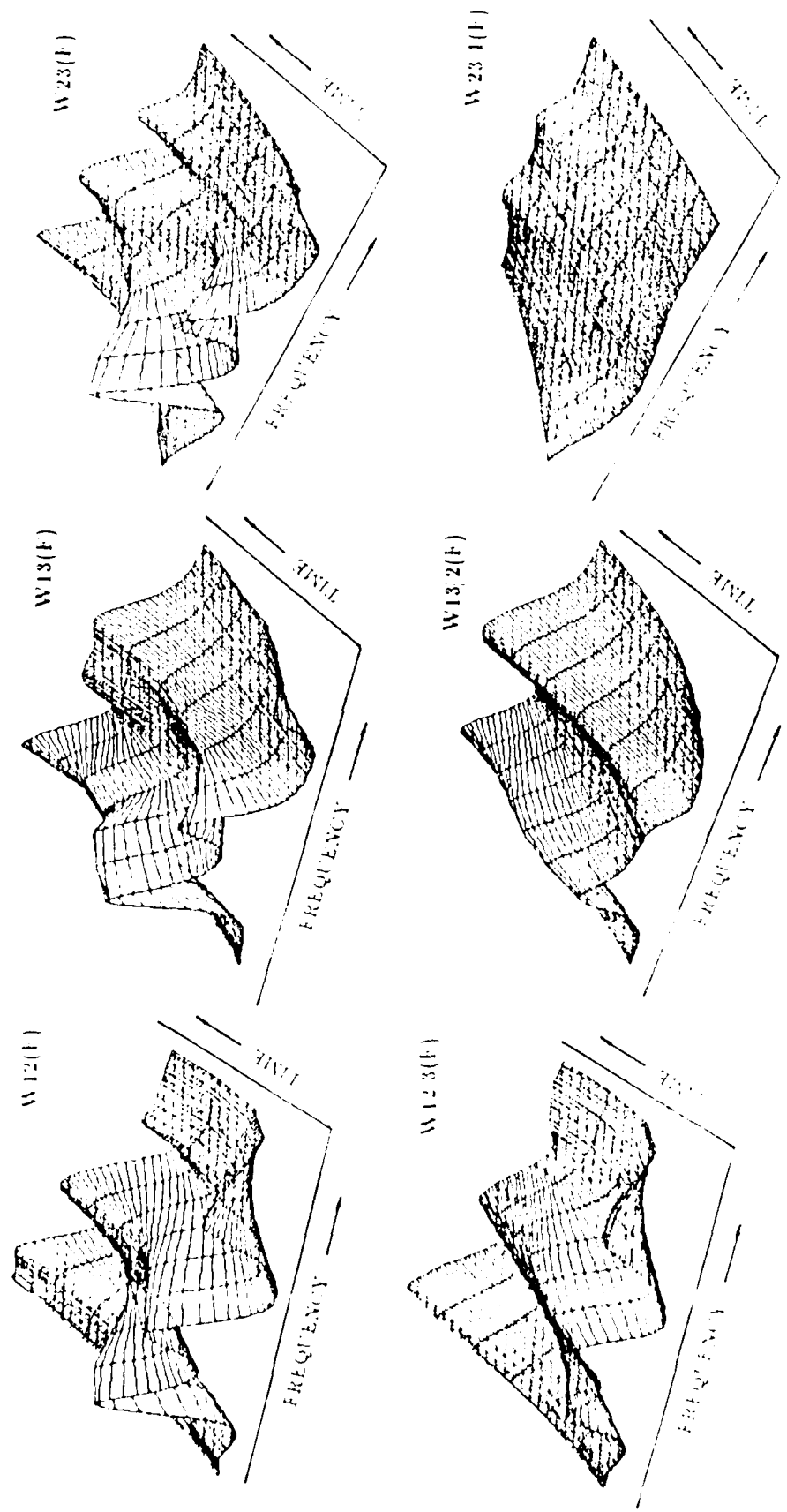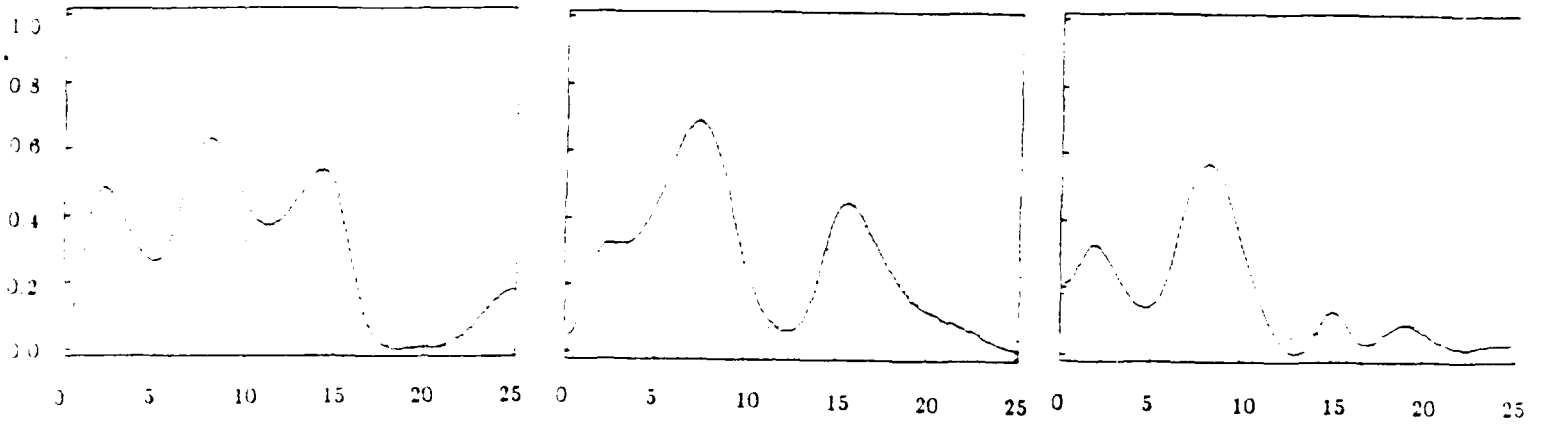
FIGURE 1

CHANNEL 1

CHANNEL 2

CHANNEL 3

TIME IN SECONDS

FIGURE 2

CHANNEL 1

CHANNEL 2

CHANNEL 3

FIGURE 5

FREQUENCY

FIGURE 4

W23(F)

W13(F)

W12(F)

W23 1(F)

W13 2(F)

W12 3(F)

FREQUENCY

FREQUENCY

FREQUENCY

FREQUENCY

FREQUENCY

FREQUENCY

TIME

TIME

TIME

TIME

TIME

TIME

FIGURE 5

FREQUENCY

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