1992-05

Error probabilities of fast frequency-hopped FSK with self-normalization combining in a fading channel with partial-band interference

Robertson, R. Clark; Ha, Tri T.

IEEE

R.C. Robertson, T.T. Ha, "Error probabilities of fast frequency-hopped FSK with self-normalization combining in a fading channel with partial-band interference," IEEE Journal of Selected Areas in Communications, v.10, no.4, (May 1992), pp. 714-723.
http://hdl.handle.net/10945/56458

Error Probabilities of Fast Frequency-Hopped FSK with Self-Normalization Combining in a Fading Channel with Partial-Band Interference

R. Clark Robertson, *Senior Member, IEEE,* and Tri T. Ha

Abstract—**An error probability analysis is performed for a binary orthogonal frequency-shift-keying (FSK) receiver employing fast frequency-hopped spread-spectrum waveforms transmitted over a frequency-nonselective slowly fading channel with partial-band interference. Diversity is performed using multiple hops per data bit. A nonlinear combination procedure referred to as self-normalization combining is employed by the receiver to minimize partial-band interference effects. Each diversity reception is assumed to fade independently according to a Rician process. The partial-band interference is modeled as a Gaussian process. Thermal noise is also included in the analysis.**

Diversity is found to completely negate degradation of the self-normalized receiver caused by partial-band interference regardless of the strength of the direct signal component; although, for signals with a large bit energy-to-interference noise density ratio and a very strong direct component, nonlinear combining losses dominate receiver performance and negate any enhancement obtained with diversity if the level of thermal noise is too high. In addition, diversity offers definite receiver performance improvement when the direct signal component is weak. The self-normalized receiver is very sensitive to fading channels, evincing a significant performance degradation as compared with its nonfaded performance. For severe channel fading, the performance of a conventional noncoherent binary FSK receiver with diversity is generally either equivalent or superior to the performance of the self-normalized receiver with the same order of diversity.

I. Introduction

THIS paper presents an error probability analysis of a fast frequency-hopped binary orthogonal frequency-shift-keying (FFH/FSK) system with noncoherent detection for communications over channels with both fading and partial-band interference. The FFH/FSK transmitter is assumed to perform L hops per data bit, thus achieving a diversity of L levels. At the receiver, the dehopped signals are demodulated by a bandpass filter followed by a quadratic detector. Self-normalization combining is used to nonlinearly combine the outputs of the quadratic detectors of the two branches of the FSK demodulator to form the L diversity signals, which are then combined to obtain the decision statistics. The self-normalized, noncoherent quadratic detector is examined in [1] for channels with no fading. In self-normalization combining, the reciprocal of the sum of the outputs of the two quadratic detectors is used to normalize the output of each detector before the L hop receptions are combined. As a result, the output of each detector when a hop contains a large amount of interference will be smaller than the output when interference is not present, and the hops without interference will have a greater influence on the decision statistics than hops containing interference. A block diagram of the FFH/FSK receiver with self-normalization combining is shown in Fig. 1.

We assume that each dehopped signal fades independently; that is, we assume that the smallest spacing between frequency hop slots for a bit is larger than the coherent bandwidth of the channel [2]–[4]. We also model the channel for each hop as a frequency-nonselective slowly fading Rician process. Hence, we assume that the signal bandwidth is much smaller than the coherence bandwidth of the channel and that the hop rate is much greater than the Doppler spread of the channel [2], [3]. As a result, the dehopped signal amplitude is a Rician random variable, and the dehopped signal can be considered as the sum of two components: a nonfaded (direct) component and a Rayleigh-faded (diffuse) component.

The interference that we consider in this paper is partial-band interference, which may be due to a partial-band jammer as well as other unintended narrowband interferences. The interference is modeled as additive Gaussian noise and is assumed to be present in both detectors of the FSK demodulator for any reception of the dehopped signal with probability γ. Thus, γ represents the fraction of the spread bandwidth being jammed [1]. The probability that the interference is not present in both detectors is $1 - \gamma$. If $N_I/2$ is the average power spectral density of interference over the entire spread bandwidth, then $\gamma^{-1}N_I/2$ is the power spectral density of partial-band interference when it is present. In addition to partial-band interference, we assume that the spread bandwidth is also corrupted by thermal noise and other wideband interferences which we model as additive white Gaussian noise. The power spectral density of this wideband noise is defined as $N_0/2$. Hence, the power spectral density of the total

Manuscript received May 3, 1990; revised May 15, 1991. This work was supported by the Naval Postgraduate School Research Council and the Naval Ocean Systems Center.

The authors are with the Department of Electrical and Computer Engineering, Code EC/RC, Naval Postgraduate School, Monterey, CA 93943.

IEEE Log Number 9107004.

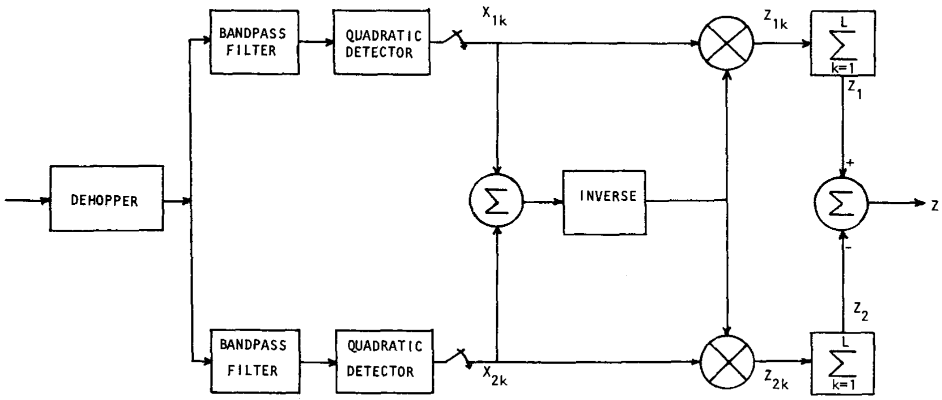

Fig. 1. Self-normalization combining FFH/FSK receiver.

noise $N_T/2$ is $\gamma^{-1}N_I/2 + N_0/2$ when interference is present and $N_0/2$ when interference is not present. The equivalent noise bandwidth of the bandpass filters in both branches of the self-normalized FSK receiver is taken to be B Hz. As a result, for each hop the signal is received with noise of power $\sigma_k^2 = (\gamma^{-1}N_I + N_0)B$ with probability γ when interference is present and with noise of power $\sigma_k^2 = N_0 B$ with probability $1 - \gamma$ when interference is not present.

The bit rate is defined as $R_b = 1/T_b$ where the duration of a bit interval is taken to be T_b seconds. Hence, for a diversity of L hops per bit, the duration of a hop interval is $T_h = T_b/L$ and the hop rate is $R_h = 1/T_h = LR_b$. As a result, the average energy per hop $E_h = ST_h$ where S is the average signal power, and the average energy per bit $E_b = LE_h$. The signal power-to-noise power ratio is related to E_h/N_T and to E_b/N_T by:

$$\frac{S}{\sigma_k^2} = \frac{E_h R_h}{N_T B} = \frac{E_b R_h}{L N_T B}. \tag{1}$$

Since there is no other modulation of the carrier, the equivalent noise bandwidth of the signal is equal to the hop rate. Hence, the minimum equivalent noise bandwidth of the bandpass filters in each branch of the self-normalized FSK demodulator is the hop rate, and in this paper we use $B = R_h$.

II. Analysis

Our analysis concerns the derivation of the bit error probability versus the bit energy-to-interference density ratio for the receiver in Fig. 1, given the Rician statistics of the fading channel. The analysis thus requires the knowledge of the statistics of the sampled outputs x_{ik}, $i = 1, 2$ of the quadratic detector for a given hop k of a bit and, furthermore, of the normalized samples z_{ik}, $i = 1, 2$ before diversity combining. We will show that the probability density functions of the random variables X_{ik} and Z_{ik} that represent the samples x_{ik} and z_{ik} can be derived analytically.

Let σ_k^2 represent the noise power in a given hop k of a bit. We assume that the signal is present in branch 1 of the FSK demodulator. Then, the conditional density of the random variable X_{1k} at the output of the quadratic detector given a signal amplitude $\sqrt{2}a_k$ is given by [5]:

$$f_{X_{1k}}(x_{1k} \mid a_k) = \frac{1}{2\sigma_k^2} \exp\left(-\frac{x_{1k} + 2a_k^2}{2\sigma_k^2}\right) \cdot I_0\left(\frac{a_k\sqrt{2x_{1k}}}{\sigma_k^2}\right) u(x_{1k}) \tag{2}$$

where $u(\cdot)$ is the unit step function, a_k is a Rician random variable representing the fading of hop k of a bit, $I_0(\cdot)$ represents the modified Bessel function of zero order, and:

$$\sigma_k^2 = N_0 B \quad \text{with probability } 1 - \gamma$$
$$\sigma_k^2 = (\gamma^{-1}N_I + N_0)B \quad \text{with probability } \gamma. \tag{3}$$

The probability density function of the Rician random variable a_k is [5]:

$$f_{A_k}(a_k) = \frac{a_k}{\sigma^2} \exp\left(-\frac{a_k^2 + \alpha^2}{2\sigma^2}\right) I_0\left(\frac{a_k \alpha}{\sigma^2}\right) u(a_k) \tag{4}$$

where α^2 is the average power of the unfaded (direct) component of the signal, and $2\sigma^2$ is the average power of the Rayleigh-faded (diffuse) component of the signal. Hence, the total average signal power of hop k of a bit is $\alpha^2 + 2\sigma^2$ and, in this paper, is assumed to remain constant from hop to hop. Note that if $\alpha^2 = 0$, the channel model is a Rayleigh fading model, and if $2\sigma^2 = 0$ there is no fading.

The probability density function of the random variable X_{1k} is given by:

$$f_{X_{1k}}(x_{1k}) = \int_0^\infty f_{X_{1k}}(x_{1k} \mid a_k) f_{A_k}(a_k)\, da_k. \tag{5}$$

Substituting (2) and (4) into (5), we get:

$$f_{X_{1k}}(x_{1k}) = \frac{1}{2\sigma^2\sigma_k^2} \exp\left[-\frac{1}{2}\left(\frac{\alpha^2}{\sigma^2} + \frac{x_{1k}}{\sigma_k^2}\right)\right] \times \int_0^\infty a_k \exp\left[-a_k^2\left(\frac{1}{2\sigma^2} + \frac{1}{\sigma_k^2}\right)\right] \cdot I_0\left(\frac{a_k\sqrt{2x_{1k}}}{\sigma_k^2}\right) I_0\left(\frac{a_k\alpha}{\sigma^2}\right) da_k \quad (6)$$

which can be evaluated analytically to yield [6, p. 718]:

$$f_{X_{1k}}(x_{1k}) = \frac{1}{2(\sigma_k^2 + 2\sigma^2)} \exp\left[-\frac{1}{2}\left(\frac{x_{1k} + 2\alpha^2}{\sigma_k^2 + 2\sigma^2}\right)\right] \cdot I_0\left(\frac{\alpha\sqrt{2x_{1k}}}{\sigma_k^2 + 2\sigma^2}\right) u(x_{1k}). \quad (7)$$

The probability density function of the random variable X_{2k} of hop k of a bit that corresponds to the sampled outputs of branch 2 of the demodulator (Fig. 1) that contains no signal is obtained from (7) by replacing X_{1k} with X_{2k} and letting $\alpha^2 = 2\sigma^2 = 0$ to yield:

$$f_{X_{2k}}(x_{2k}) = \frac{1}{2\sigma_k^2} \exp\left[-\frac{x_{2k}}{2\sigma_k^2}\right] u(x_{2k}). \quad (8)$$

Since no signal is present in branch 2, fading has no effect on X_{2k}.

A. Probability Density Function of the Normalized Random Variables

As illustrated in Fig. 1, the normalized random variable Z_{ik}, $i = 1, 2$ is given by:

$$Z_{ik} = \frac{X_{ik}}{X_{1k} + X_{2k}} \quad (9)$$

where it is apparent that $0 \le Z_{ik} \le 1$. Introducing the auxiliary random variable $V_k = X_{1k} + X_{2k}$ and using the Jacobian of the transformation $J = v_k^{-1}$, we obtain the probability density function of Z_{1k} as:

$$f_{Z_{1k}}(z_{1k}) = \int_0^\infty v_k f_{X_{1k}X_{2k}}[v_k z_{1k}, v_k(1 - z_{1k})]\, dv_k \quad (10)$$

where $f_{X_{1k}X_{2k}}(x_{1k}, x_{2k})$ is the joint probability density function of the independent random variables X_{1k} and X_{2k}. Since X_{1k} and X_{2k} are independent random variables, $f_{X_{1k}X_{2k}}(x_{1k}, x_{2k})$ is obtained from the product of (7) and (8) as:

$$f_{X_{1k}X_{2k}}(x_{1k}, x_{2k}) = \frac{1}{4\sigma_k^2\beta_k^2} \exp\left[-\left(\frac{x_{1k} + 2\alpha^2}{2\beta_k^2} + \frac{x_{2k}}{2\sigma_k^2}\right)\right] \cdot I_0\left(\frac{\alpha\sqrt{2x_{1k}}}{\beta_k^2}\right) u(x_{1k})u(x_{2k}) \quad (11)$$

where $\beta_k^2 = \sigma_k^2 + 2\sigma^2$. Combining (10) and (11), we get:

$$f_{Z_{1k}}(z_{1k}) = \frac{1}{4\sigma_k^2\beta_k^2} \exp\left[-\frac{\alpha^2}{\beta_k^2}\right] \times \int_0^\infty v_k \exp\left[-\frac{v_k}{2}\left(\frac{\beta_k^2 - 2\sigma^2 z_{1k}}{\sigma_k^2\beta_k^2}\right)\right] \cdot I_0\left(\frac{\alpha\sqrt{2z_{1k}v_k}}{\beta_k^2}\right) dv_k \quad (12)$$

which can be evaluated to yield [6, p. 721]:

$$f_{Z_{1k}}(z_{1k}) = \frac{\sigma_k^2}{(\beta_k^2 - 2\sigma^2 z_{1k})^2}\left[\beta_k^2 + \frac{\alpha^2\sigma_k^2 z_{1k}}{\beta_k^2 - 2\sigma^2 z_{1k}}\right] \cdot \exp\left[-\frac{\alpha^2(1 - z_{1k})}{\beta_k^2 - 2\sigma^2 z_{1k}}\right] \quad 0 \le z_{1k} \le 1. \quad (13)$$

We now define $\rho_k = \alpha^2/\sigma_k^2$ as the signal-to-noise ratio of the nonfaded (direct) component of hop k of a bit and $\xi_k = 2\sigma^2/\sigma_k^2$ as the signal-to-noise ratio of the Rayleigh-faded (diffuse) component of hop k of a bit. Thus, (13) can be written as:

$$f_{Z_{1k}}(z_{1k}) = \frac{\rho_k z_{1k} + (1 + \xi_k)[1 + \xi_k(1 - z_{1k})]}{[1 + \xi_k(1 - z_{1k})]^3} \cdot \exp\left[\frac{-\rho_k(1 - z_{1k})}{1 + \xi_k(1 - z_{1k})}\right] \quad 0 \le z_{1k} \le 1. \quad (14)$$

For the special case of Rayleigh fading, $\rho_k \to 0$ and (14) reduces to:

$$f_{Z_{1k}}(z_{1k}) = \frac{(1 + \xi_k)}{[1 + \xi_k(1 - z_{1k})]^2} \quad 0 \le z_{1k} \le 1. \quad (15)$$

B. Probability of Bit Error

The bit error probability for the receiver in Fig. 1 in the presence of partial-band interference is [1]:

$$P_b = \sum_{i=0}^{L} \binom{L}{i} \gamma^i(1 - \gamma)^{L-i} P_b(i) \quad (16)$$

where $P_b(i)$ is the conditional bit error probability given that i hops of a bit have interference. Let the combined sampled outputs of each branch of the FSK demodulator in Fig. 1 be represented by the random variables:

$$Z_i = \sum_{k=1}^{L} Z_{ik}, \qquad i = 1, 2. \quad (17)$$

Since $Z_{2k} = 1 - Z_{1k}$, then from (9) we have:

$$Z_2 = \sum_{k=1}^{L} (1 - Z_{1k}) = L - Z_1. \quad (18)$$

Therefore, the conditional bit error probability given that i hops of a bit have interference is:

$$P_b(i) = \Pr(Z_1 < Z_2 | i) = \Pr(Z_1 < L/2 | i). \quad (19)$$

Let $f_{Z_{1k}}^{(1)}(z_{1k})$ be the probability density function of Z_{1k} assuming that hop k of a bit has interference. Hence, $f_{Z_{1k}}^{(1)}(z_{1k})$ is obtained from (14) with $\sigma_k^2 = (\gamma^{-1}N_I + N_0)B$. Similarly, let $f_{Z_{1k}}^{(2)}(z_{1k})$ be the probability density function of Z_{1k} when hop k of a bit has no interference. Then $f_{Z_{1k}}^{(2)}(z_{1k})$ is given by (14) with $\sigma_k^2 = N_0 B$. From (17), since the random variables corresponding with each hop are assumed to be independent, the conditional probability density of Z_1 given that i hops of a bit have interference is given as:

$$f_{Z_1}(z_1 | i) = [f_{Z_{1k}}^{(1)}(z_{1k})]^{\otimes i} \otimes [f_{Z_{1k}}^{(2)}(z_{1k})]^{\otimes(L-i)} \quad (20)$$

where $\otimes i$ represents an i-fold convolution. Now, from (19), we can use (20) to obtain the conditional bit error probability given that i hops of a bit have interference as:

$$P_b(i) = \int_0^{L/2} f_{Z_1}(z_1 | i)\, dz_1. \quad (21)$$

III. Numerical Procedure

We have obtained numerical results for $L = 1, 2, 3$, and 4 hops/bit. For slow hopping ($L = 1$), the change of variables

$$w_k = \frac{1 - z_{1k}}{1 + \xi_k(1 - z_{1k})} \quad (22)$$

in (14) allows (21) to be evaluated analytically to obtain:

$$P_b(i) = \frac{1}{2 + \xi_k} \exp\left[\frac{-\rho_k}{2 + \xi_k}\right]. \quad (23)$$

As expected, (23) is identical to the bit error probability of a conventional noncoherent FSK demodulator with no self-normalization for a Rician fading channel. The probability of bit error for $L = 1$ is now obtained by evaluating (23) for the two cases of the hop jammed and the hop free of interference and the results used in (16). The probability of bit error for $L = 2$ is obtained by integrating

$$P_b(i) = \int_0^{L/2} \int_0^1 f_{Z_{1k}}^{(j)}(u) f_{Z_{1k}}^{(k)}(z_1 - u)\, du\, dz_1 \quad (24)$$

numerically for each of the three cases of both hops jammed ($j = k = 1$), both hops free of interference ($j = k = 2$), and one hop jammed and the other unjammed ($j = 1$, $k = 2$). The results of these computations are then used in (16).

The probability of bit error for $L = 3$ and $L = 4$ is obtained by expressing the probability density function of the sum of the self-normalized diversity outputs with a Gram–Charlier series [5]. The advantage of the Gram–Charlier series is that a pseudoanalytic solution for the conditional probability of bit error $P_b(i)$ can be obtained that is much more computationally efficient as well as more elegant than a discrete Fourier transform approach. The Gram–Charlier series representation of the probability density function of the sum of the self-normalized diversity outputs is:

$$f_{Z_1}(z_1 | i) = \frac{1}{\sigma_{Z_1}}\left[\phi\left(\frac{z_1 - \bar{Z}_1}{\sigma_{Z_1}}\right) + \sum_{n=3}^{\infty} c_n \phi^{(n)}\left(\frac{z_1 - \bar{Z}_1}{\sigma_{Z_1}}\right)\right] \quad (25)$$

where $\bar{Z}_1$ and $\sigma_{Z_1}^2$ are the expected value and variance of Z_1, respectively, and:

$$\phi^{(n)}(x) = \frac{(-1)^n}{2^{n/2}\sqrt{2\pi}} H_n\left(\frac{x}{\sqrt{2}}\right) \exp\left(-\frac{x^2}{2}\right) \quad (26)$$

is the nth derivative of the zero-mean unity variance Gaussian probability density function. The function $H_n(\cdot)$ is a Hermite polynomial of order n defined by:

$$H_n(y) = (-1)^n \exp(y^2) \frac{d^n}{dy^n} \exp(-y^2). \quad (27)$$

The coefficients of the infinite series in (25) are obtained from:

$$c_n = \frac{(-1)^n}{2^{n/2} n!} \int_{-\infty}^{\infty} H_n(y) f_Y(y | i)\, dy \quad (28)$$

where the probability density function $f_Y(y | i)$ is obtained from $f_{Z_1}(z_1 | i)$ by the linear transformation

$$Y = \left(\frac{Z_1 - \bar{Z}_1}{\sqrt{2}\sigma_{Z_1}}\right). \quad (29)$$

Hence, the coefficients of the infinite series can be evaluated analytically and expressed in terms of the moments of the random variable Y which, in turn, can be expressed in terms of the moments of the random variable Z_1. Since the random variable Z_1 is the sum of the independent Z_{1k}'s, characteristic functions can be used to express the moments of Z_1 in terms of the moments of the Z_{1k}'s.

Substituting (25) into (21), we obtain the "analytic" result:

$$P_b(i) = \frac{1}{2}\operatorname{erf}\left(\frac{\bar{Z}_1}{\sqrt{2}\sigma_{Z_1}}\right) - \frac{1}{2}\operatorname{erf}\left(\frac{\bar{Z}_1 - L/2}{\sqrt{2}\sigma_{Z_1}}\right) + \frac{1}{\sqrt{\pi}} \sum_{n=3}^{\infty} \frac{(-1)^{(n-1)} c_n}{2^{n/2}} \times \left\{ H_{n-1}\left(\frac{L/2 - \bar{Z}_1}{\sqrt{2}\sigma_{Z_1}}\right) \exp\left[-\left(\frac{L/2 - \bar{Z}_1}{\sqrt{2}\sigma_{Z_1}}\right)^2\right] - H_{n-1}\left(\frac{-\bar{Z}_1}{\sqrt{2}\sigma_{Z_1}}\right) \exp\left[-\left(\frac{-\bar{Z}_1}{\sqrt{2}\sigma_{Z_1}}\right)^2\right]\right\} \quad (30)$$

where erf $(\cdot)$ is the error function. Unfortunately, the infinite series in (30) converges very slowly for large signal-to-noise ratios. Expressing (30) symbolically by the infinite series

$$P_b(i) = \sum_{n=0}^{\infty} b_n \quad (31)$$

where the b_n's are obtained by comparing (31) with (30) and $b_1 = b_2 = 0$, we can write:

$$h(w|i) = \sum_{n=0}^{\infty} b_n w^n \tag{32}$$

where $P_b(i) = h(1|i)$. Now we can perform an Euler transformation on (32) to obtain a much more rapidly converging, alternative form of (30) as [7]:

$$P_b(i) = \frac{1}{2} \sum_{n=0}^{\infty} \frac{(-1)^n d_n}{2^n} \tag{33}$$

where

$$d_n = \sum_{j=0}^{n} (-1)^{j+n} \binom{n}{j} b_j. \tag{34}$$

Equation (33) must be evaluated once for each of the four possible combinations of jammed and unjammed hops that occur for $L = 3$ and once for each of the five possible combinations of jammed and unjammed hops that occur for $L = 4$. This requires the moments of Z_{1k} for both the jammed and unjammed cases, which are evaluated by numerical integration. The first twenty-three terms in the series are used for $L = 3$, while the first eighteen terms are used for $L = 4$. All results presented in this paper are obtained by assuming that the ratio of direct-to-diffuse signal energy $\alpha^2/2\sigma^2$ is the same for each hop k of a bit.

IV. Numerical Results

The probability of bit error as a function of the ratio of the direct-to-diffuse signal energy with only thermal noise present is shown in Fig. 2. The ratio of bit energy-to-thermal noise density is $E_b/N_0 = 13.35$ dB. This value of E_b/N_0 corresponds to $P_b = 10^{-5}$ when there is no fading or interference and $L = 1$. This corresponds to the signal-to-thermal noise density ratio used in [1] and allows our results to be compared directly to the nonfaded results presented in [1]. As can be seen, for $\alpha^2/2\sigma^2 > 10$, nonlinear combining losses begin to dominate receiver performance while for $\alpha^2/2\sigma^2 < 3$ receiver performance for each diversity order is relatively unchanged as the ratio of the direct-to-diffuse signal energy approaches the Rayleigh limit. It is interesting to note that, for $L = 1$, the direct signal component must be very large relative to the diffuse signal component before receiver performance approaches the nonfaded limit. For $\alpha^2/2\sigma^2 = 100$ and $L = 1$, receiver performance is nearly three times worse than the nonfaded limit. Consequently, it is very important to take channel fading into account when comparing the performance of the self-normalized receiver for various orders of diversity.

Receiver performance for specific fractions of partial-band interference are compared with worst-case performance for a relatively strong direct signal ($\alpha^2/2\sigma^2 = 10$) in Figs. 3–6 for diversities of $L = 1$, 2, 3, and 4, respectively. Worst-case performance implies a composite performance obtained by computing the probability of bit error for a fixed value of E_b/N_I as a function of the ratio of

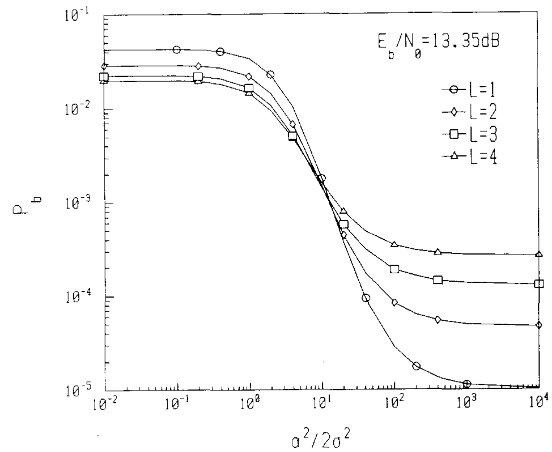

Fig. 2. Performance of the self-normalized receiver with no partial-band interference as a function of the amount of fading with $E_b/N_0 = 13.35$ dB and $L = 1$, 2, 3, and 4.

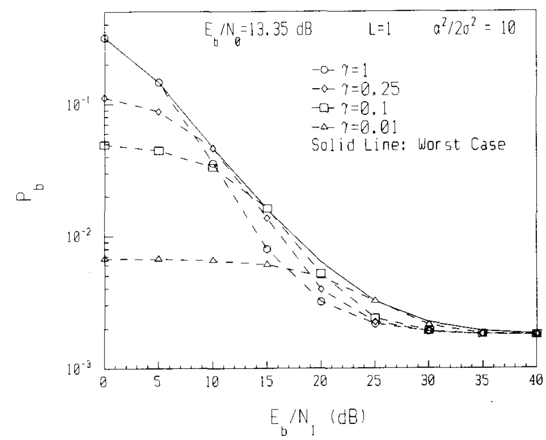

Fig. 3. Performance of the self-normalized receiver for partial-band jamming fractions of $\gamma = 1$, 0.25, 0.1, and 0.01 compared with worst-case performance for a strong direct signal with $E_b/N_0 = 13.35$ dB and $L = 1$.

partial-band interference γ to obtain the largest probability of bit error at that particular signal-to-interference energy ratio. The ratio of bit energy-to-thermal noise density is again taken to be $E_b/N_0 = 13.35$ dB. As can be seen in Fig. 3, when no diversity is used partial-band interference results in a fairly significant degradation in receiver performance as compared with uniform interference from bit energy-to-interference noise density ratios of about 10–30 dB. Below bit energy-to-interference noise density ratios of about 10 dB and above about 30 dB worst-case receiver performance is approximately equal to receiver performance with broadband (uniform) interference; that is, partial-band interference does not have a significant negative effect on receiver performance. It is interesting to note that for bit energy-to-interference noise density ratios less than about 10 dB, receiver performance improves dramatically when the interference is partial-band rather than uniform. This is particularly true for L

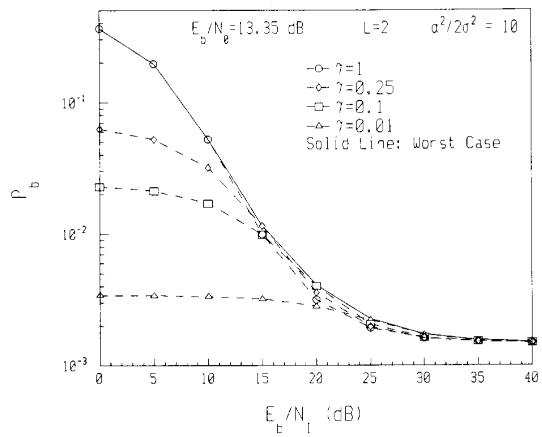

Fig. 4. Performance of the self-normalized receiver for partial-band jamming fractions of $\gamma = 1$, 0.25, 0.1, and 0.01 compared with worst-case performance for a strong direct signal with $E_b/N_0 = 13.35$ dB and $L = 2$.

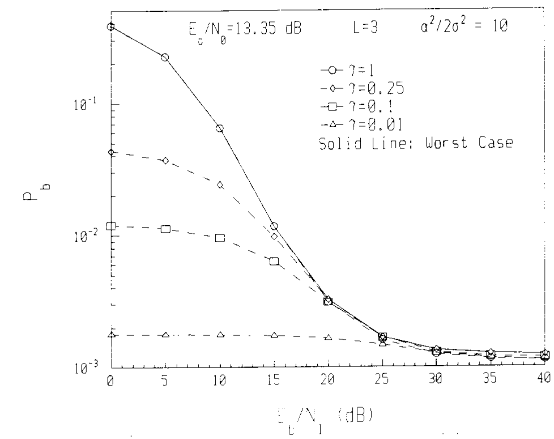

Fig. 5. Performance of the self-normalized receiver for partial-band jamming fractions of $\gamma = 1$, 0.25, 0.1, and 0.01 compared with worst-case performance for a strong direct signal with $E_b/N_0 = 13.35$ dB and $L = 3$.

> 1. As can be seen in Fig. 4, a diversity of two is sufficient to virtually eliminate any degradation due to partial-band interference. Except for a small range in the neighborhood of a bit energy-to-interference noise density ratio of about 20 dB, worst-case performance and the performance for uniform interference are nearly the same. For diversities of three and four, degradation due to partial-band interference is virtually eliminated. As can be seen in Figs. 5 and 6, for $L = 3$ and $L = 4$ worst-case performance is obtained for all bit energy-to-interference noise density ratios when the interference is uniform. Hence, for the case of a relatively strong direct component, the performance of the fast frequency-hopped, self-normalized receiver is immune to degradation caused by partial-band interference.

A comparison of the worst-case performance shown in Figs. 4–6, with the worst-case performance with no diversity ($L = 1$), reveals that in all three cases the performance of the self-normalized receiver with no diversity is superior when relatively high values of interference are present. For ease of comparison, the worst-case performance with no diversity is plotted in Fig. 6. Similarly, in all three cases, for bit energy-to-interference noise density ratios greater than roughly 15 dB there is a distinct improvement when diversity is used. This can be contrasted with the nonfaded results [1], where diversity offers an improvement only for bit energy-to-interference noise density ratios greater than roughly 15 dB and less than roughly 32–37 dB, depending on the order of diversity. It can be seen that, for a relatively strong direct component, the worst-case performance for $L > 1$ is about the same. In other words, there is not a dramatic improvement in performance as L increases; although, above around 20 dB, $L = 3$ is somewhat better that either $L = 2$ or $L = 4$.

The performance of the self-normalized receiver with no diversity for specific fractions of partial-band interference is compared with worst-case performance for a very

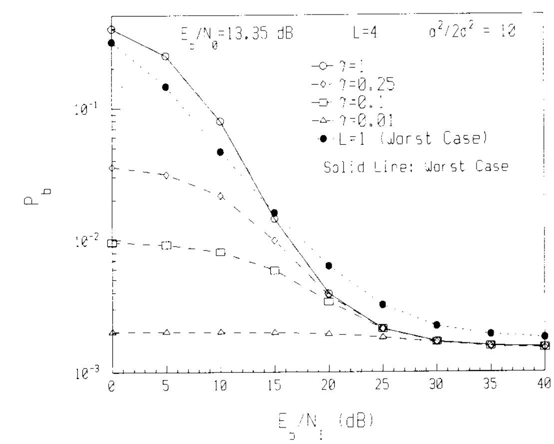

Fig. 6. Performance of the self-normalized receiver for partial-band jamming fractions of $\gamma = 1$, 0.25, 0.1, and 0.01 compared with worst-case performance for a strong direct signal with $E_b/N_0 = 13.35$ dB and $L = 4$.

strong direct signal ($\alpha^2/2\sigma^2 = 100$) in Fig. 7, and in Fig. 8 the performance of the self-normalized receiver with diversities of $L = 2$ and $L = 4$ for broadband interference are compared with the corresponding worst-case performances for a very strong direct signal. Also plotted in Fig. 8, for ease of comparison, is the worst-case performance with no diversity. As before, the ratio of bit energy-to-thermal noise density is taken to be $E_b/N_0 = 13.35$ dB. As can be seen in Fig. 7, when no diversity is used, partial-band interference is very effective in degrading receiver performance. As in the case of a relatively strong direct signal, we see in Fig. 8 that a diversity of four is sufficient to completely negate the degradation caused by partial-band interference. Also, as in the case of a relatively strong direct signal, for bit energy-to-interference noise density ratios greater than roughly 13 dB there is a distinct improvement when diversity is used. However, in contrast, for a very strong direct signal receiver, performance with no diversity is markedly supe-

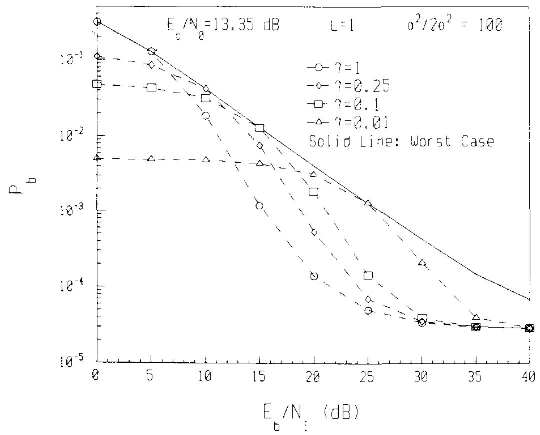

Fig. 7. Performance of the self-normalized receiver for partial-band jamming fractions of $\gamma = 1$, 0.25, 0.1, and 0.1 compared with worst-case performance for a very strong direct signal with $E_b/N_0 = 13.35$ dB and $L = 1$.

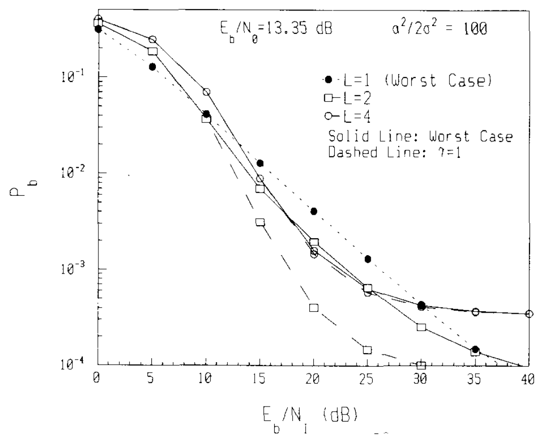

Fig. 8. Performance of the self-normalized receiver for broadband interference compared with the corresponding worst-case performances for a very strong direct signal with $E_b/N_0 = 13.35$ dB and $L = 2$ and 4.

rior for bit energy-to-interference noise density ratios greater than 30 dB for a diversity of four and 35 dB for a diversity of two. This is because nonlinear combining losses are a factor for signals with a strong direct component with the result that there is an upper limit of the ratio of bit energy-to-thermal noise density above which the use of diversity degrades system performance. These conclusions are consistent with the results obtained for a nonfaded channel [1]. Also in contrast to the performance obtained for a relatively strong direct signal, a diversity of two is not sufficient to eliminate most of the negative impact of partial-band interference when the signal has a very strong direct component. Nevertheless, in Fig. 8 we see that the worst-case performance with a diversity of two is either roughly equivalent or superior to that obtained with a diversity of four over the entire range of E_b/N_I. This is because a receiver with a diversity of two suffers much less from nonlinear combining losses than receivers with higher orders of diversity. This is clear in Fig. 2. As a result of the foregoing, recalling the performance obtained with a relatively strong direct signal, it seems reasonable to conclude that receiver performance is not substantially enhanced by using diversities greater than two for signals with a direct component ranging from relatively strong to very strong.

Worst-case receiver performance for a relatively weak direct signal ($\alpha^2/2\sigma^2 = 3$) is shown in Fig. 9 for diversities of $L = 1$, 2, 3, and 4. The ratio of bit energy-to-thermal noise density is again taken to be $E_b/N_0 = 13.35$ dB. In this case, receiver performance continues to improve as the order of diversity increases; although, the improvement from $L = 3$ to $L = 4$ is much less than the improvement from $L = 2$ to $L = 3$. As expected, receiver performance is much worse when there is no strong direct signal component as compared with the case where the signal contains a relatively strong direct component.

As might be expected from the results shown in Fig. 2, there is not a significant difference in performance as the ratio of direct-to-diffuse signal power varies from about three down to the Rayleigh limit. When there is not a strong direct component to the signal, partial-band interference results in virtually no degradation of receiver performance. In this instance, worst-case receiver performance of the self-normalization combining receiver is virtually identical to receiver performance when the interference is uniform. As in the case of a strong direct signal, performance is dramatically improved for bit energy-to-interference noise density ratios less than about 10 dB when the interference is partial-band in nature.

The curves in Fig. 10, with the exception of the curve represented by the broken line, are also composites, and provide a comparison between the case of no diversity and a diversity of two when E_b/N_0 is increased from 13.35 to 16 dB for a weak direct signal ($\alpha^2/2\sigma^2 = 3$), a relatively strong direct signal ($\alpha^2/2\sigma^2 = 10$), and a very strong direct signal ($\alpha^2/2\sigma^2 = 100$). The broken line in Fig. 10 represents receiver performance for broadband interference with no diversity and a very strong direct signal. As can be seen by comparing Fig. 10 with previous results, diversity improves receiver performance substantially more when thermal noise is less significant. It is interesting to note that at this level of signal-to-thermal noise density, receiver performance for a relatively strong and a very strong direct signal are roughly comparable for $E_b/N_I < 25$ dB. Also note that when the signal contains a very strong direct component and no diversity is used, partial-band interference results in a substantial degradation in receiver performance as compared with the performance obtained for broadband interference, especially for $E_b/N_I > 10$ dB. For the case where the effects of fading are significant, receiver performance is still relatively poor even when interference is negligible. This indicates that the self-normalized receiver should be operated in conjunction with some form of forward error correction coding when strongly fading channels are expected.

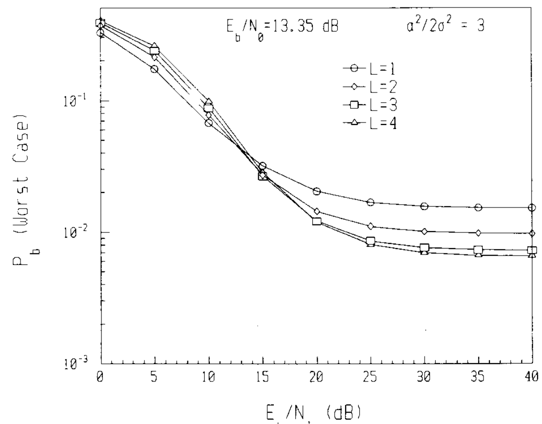

Fig. 9. Worst-case performance of the self-normalized receiver with diversity combining and partial-band interference in a fading channel for a strong direct signal with $E_b/N_0 = 13.35$ dB.

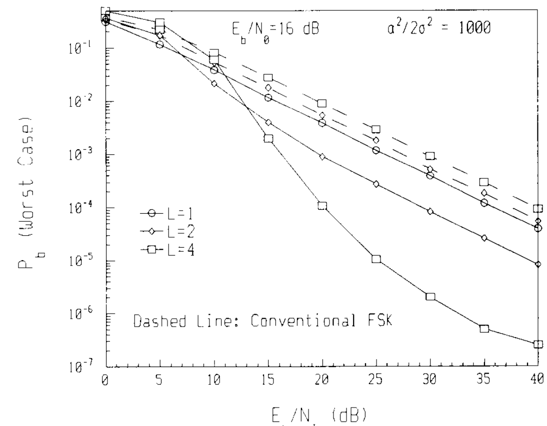

Fig. 11. Worst-case performance of the self-normalized receiver with partial-band interference for a Gaussian channel with $E_b/N_0 = 16$ dB and $L = 1$, 2, and 4 as compared with the performance of a conventional noncoherent binary FSK receiver with diversity operating under the same conditions of fading and E_b/N_0.

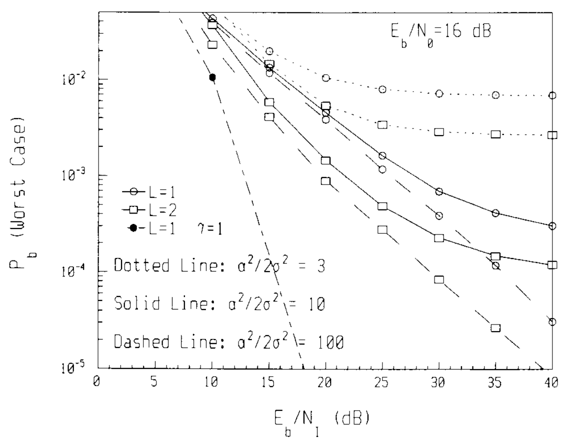

Fig. 10. Worst-case performance of the self-normalized receiver with partial-band interference for a relatively weak, a relatively strong, and a very strong direct signal with $E_b/N_0 = 16$ dB and $L = 1$ and 2. Also shown is receiver performance for broadband interference with no diversity for a very strong direct signal.

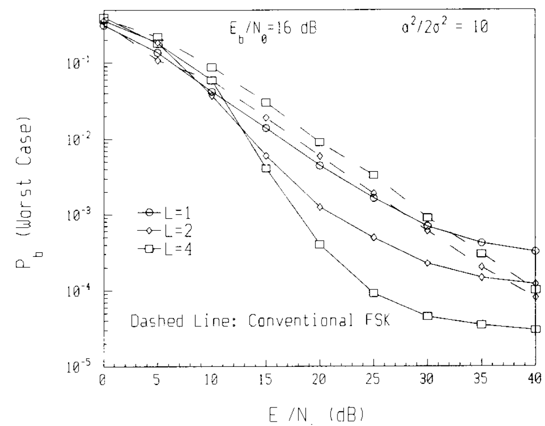

Fig. 12. Worst-case performance of the self-normalized receiver with partial-band interference for a relatively strong direct signal with $E_b/N_0 = 16$ dB and $L = 1$, 2, and 4 as compared with the performance of a conventional noncoherent binary FSK receiver with diversity operating under the same conditions of fading and E_b/N_0.

Worst-case receiver performance for essentially a Gaussian channel ($\alpha^2/2\sigma^2 = 1000$), a relatively strong direct signal ($\alpha^2/2\sigma^2 = 10$), and a weak direct signal ($\alpha^2/2\sigma^2 = 1$) are shown in Figs. 11–13 for diversities of $L = 1$, 2, and 4, respectively. The ratio of bit energy-to-thermal noise density is 16 dB. Also shown is the performance under the same conditions of fading and E_b/N_0 of a conventional noncoherent binary FSK receiver employing diversity. As mentioned previously, the performance for slow hopping is identical for both receivers. Numerical results for a conventional noncoherent FSK receiver with fast hopping are obtained by an adaptation of the procedure used in [8]. For a Gaussian channel, the use of diversity degrades the performance of the conventional FSK receiver, while the self-normalized receiver with diversity demonstrates significant improvement as diversity increases. The performance degradation of the conventional receiver is consistent with results previously reported for Gaussian channels [9]. For a relatively strong direct signal, the performance of a conventional FSK receiver with diversity is degraded as compared with no diversity for E_b/N_I less than about 30 dB, depending on the order of diversity. On the other hand, the performance of the self-normalized receiver is always enhanced by diversity for E_b/N_I greater than about 10 dB. The self-normalized receiver with a diversity of four is significantly better than the conventional FSK receiver with a diversity of four for E_b/N_I greater than about 10 dB, especially for E_b/N_I between about 20 and 30 dB. Comparing Figs. 4, 6, and 11, we can also see that the performance improvement obtained with the self-normalized receiver with diversity is dramatically better when thermal noise is less

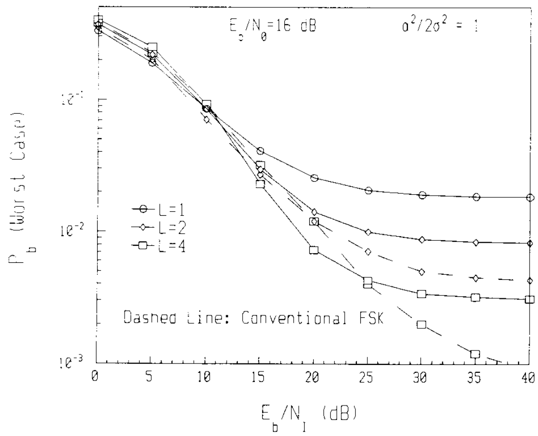

Fig. 13. Worst-case performance of the self-normalized receiver with partial-band interference for a weak direct signal with $E_b/N_0 = 16$ dB and $L = 1$, 2, and 4 as compared with the performance of a conventional noncoherent binary FSK receiver with diversity operating under the same conditions of fading and E_b/N_0.

significant. In contrast to the performance obtained for the Gaussian channel and the strong direct signal, the performance of both the conventional FSK receiver and the self-normalized FSK receiver is enhanced by diversity when channel fading is strong. In this particular case, for a diversity of four the self-normalized receiver is superior to the conventional FSK receiver only for E_b/N_I between 11 and 24 dB, while for E_b/N_I greater than 25 dB the conventional FSK receiver is significantly better than the self-normalized receiver. Hence, when channel fading is strong the use of self-normalization to improve receiver performance is a questionable strategy.

V. Conclusions

The self-normalized receiver with diversity is seen to neutralize receiver performance degradation due to partial-band interference as well as improve performance as compared with the self-normalized receiver with no diversity when fading is present provided the signal-to-interference density ratio is about 13 dB or more. This is particularly true when the signal does not contain a strong direct component. Even a diversity of only two provides a significant advantage. In addition, even when the signal contains a very strong direct component, diversity is seen to entirely negate any degradation in receiver performance due to partial-band interference. However, for signals with a very strong direct component, the performance of the self-normalized receiver with diversity is superior to the self-normalized receiver with no diversity only for a limited range of the signal-to-interference density ratio unless thermal noise is also low. This is because, for large signal-to-interference density ratios, receiver degradation due to nonlinear combining losses outweighs the enhancement obtained with diversity when the signal contains a very strong direct component if thermal noise is not sufficiently small. Consequently, in this case a diversity of two provides the best balance between minimizing partial-band interference effects and minimizing nonlinear combining losses for fading channels such that the direct component of the signal ranges from relatively strong to very strong. When thermal noise is less significant, a diversity of four provides significantly better performance than a diversity of two for all conditions of channel fading. For strongly fading channels, additional diversity is advantageous regardless of the signal-to-thermal noise ratio. Finally, superior receiver performance is obtained when diversity is not present for signal-to-interference density ratios of less than about 10 dB regardless of the strength of the direct signal component and the signal-to-thermal noise ratio.

An interesting result is the sensitivity of the self-normalized receiver to the presence of fading. The self-normalized receiver experiences significant degradation in performance with even moderate fading as compared with the nonfaded performance. For severe fading, the performance of a conventional noncoherent FSK receiver employing diversity is roughly the same as or better than the performance of the self-normalized receiver with the same level of diversity.

The use of error correction coding should significantly improve the performance of the self-normalized receiver as should the use of a higher-order modulation scheme such as MFSK ($M > 2$).

References

[1] L. E. Miller, J. S. Lee, and A. P. Kadrichu, "Probability of error analysis of a BFSK frequency-hopping system with diversity under partial-band jamming interference—Part III: Performance of a square-law self-normalizing soft decision receiver," *IEEE Trans. Commun.*, vol. COM-34, no. 7, pp. 669-675, July 1986.

[2] C. M. Keller and M. B. Pursley, "Diversity combining for channels with fading and partial-band interference," *IEEE J. Select. Areas Commun.*, vol. 5, no. 2, pp. 248-260, Feb. 1987.

[3] J. G. Proakis, *Digital Communications*, 2nd Ed. New York: McGraw-Hill, 1989.

[4] C. M. Keller and M. B. Pursley, "Frequency spacing in FSK frequency-hopped spread spectrum communications over selective fading channels," in *Proc. 17th Conf. Inform. Sci. Syst.*, Johns Hopkins Univ., Baltimore, MD, Mar. 1983, pp. 749-754.

[5] A. D. Whalen, *Detection of Signals in Noise.* New York: Academic, 1971.

[6] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products, Corrected and Enlarged Ed.* New York: Academic, 1980.

[7] J. Mathews and R. L. Walker, *Mathematical Methods of Physics*, 2nd Ed. Menlo Park, CA: W. A. Benjamin, Inc., 1970.

[8] R. C. Robertson and T. T. Ha, "Error probabilities of fast frequency-hopped MFSK with noise-normalization combining in a fading channel with partial-band interference," *IEEE Trans. Commun.*, vol. 40, no. 2, Feb. 1992.

[9] J. S. Lee, R. H. French, and L. E. Miller, "Probability of error analysis of a BFSK frequency-hopping system with diversity under partial-band jamming interference—Part I: Performance of square-law linear combining soft decision receiver," *IEEE Trans. Commun.*, vol. COM-32, no. 6, pp. 645-653, June 1984.

R. Clark Robertson (S'83–M'83–SM'88) received the B.S. degree in electrical engineering (with honors) from Texas Tech University and both the M.S. degree in engineering and the Ph.D. degree from the University of Texas, Austin.

He has worked for the IBM Corporation, where he designed analog electronic circuits, and for Virginia Polytechnic Institute and State University as an Assistant Professor. He joined the faculty of the Electrical and Computer Engineering Department at the Naval Postgraduate School in June 1989 as an Associate Professor. His current research interests include spread-spectrum communications and mobile radio communications.

Tri T. Ha received the B.S.E.E. and the M.S.E.E. degrees from Ohio University, Athens, and the Ph.D. degree from the University of Maryland, College Park.

He has worked for Fairchild Industries, GTE, and Virginia Polytechnic Institute and State University. Since 1987, he has been with the Naval Postgraduate School. He is the author of *Digital Satellite Communications*, 2nd Edition (New York: McGraw-Hill, 1990), *Digital Satellite Communications* (New York: MacMillan, 1986), and *Solid-State Microwave Amplifier Design* (New York: Wiley-Interscience, 1981). He has authored and coauthored more than fifty technical papers. His current research interests include satellite communications and mobile radio communications.