1992-02

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R.C. Robertson, T.T. Ha, "Error probabilities of fast frequency-hopped MFSK with noise normalization combining in a fading channel with partial-band interference," IEEE Transactions on Communications, v.40, no.2 (February 1992), pp. 404-412.
http://hdl.handle.net/10945/56457

Error Probabilities of Fast Frequency-Hopped MFSK with Noise-Normalization Combining in a Fading Channel with Partial-Band Interference

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***Abstract*—An error probability analysis is performed for an M-ary orthogonal frequency-shift keying (MFSK) communication system employing fast frequency-hopped (FFH) spread spectrum waveforms transmitted over a frequency-nonselective, slowly fading channel with partial-band interference. Diversity is obtained using multiple hops per data bit. A procedure referred to as noise-normalization combining is employed by the system receiver to minimize partial-band interference effects. Each diversity reception is assumed to fade independently according to a Rician process. The partial-band interference is modeled as a Gaussian process. Thermal noise is also included in the analysis. Forward error correction coding is applied using convolutional codes and Reed–Solomon codes.**

Diversity is found to dramatically reduce the degradation of the noise-normalization receiver caused by partial-band interference regardless of the strength of the direct signal component. In addition, diversity offers significant performance improvement when channel fading is strong. Further significant performance improvement is obtained for higher modulation orders ($M > 2$). Uncoded receiver performance with a diversity of four is roughly comparable to coded receiver performance with no diversity for ratios of bit energy-to-interference noise density in the range between roughly 12 and 22 dB. Substantial improvements in receiver performance are obtained by combining diversity, higher modulation orders, and coding.

I. Introduction

THIS paper presents an error probability analysis of a fast frequency-hopped M-ary orthogonal frequency-shift keying (FFH–MFSK) system with noncoherent, noise-normalized detection for communications with fading and partial-band interference. The FFH–MFSK transmitter is assumed to perform L hops per data symbol which results in a diversity of L levels. At the receiver the dehopped signals are demodulated by a bandpass filter followed by a quadratic detector. This problem was initially investigated for standard noncoherent MFSK demodulators in [1]. In this paper, noise-normalization combining (also referred to as adaptive gain control) [2]–[4] is used to combine the outputs of the quadratic detectors of the M branches of the MFSK demodulator to form the decision statistics. In a noise normalized receiver, the reciprocal of the noise power of a noise-only channel estimator is used to normalize the output of each detector before the L hop receptions are combined. A block diagram of the FFH–MFSK receiver with noise-normalization combining is shown in Fig. 1.

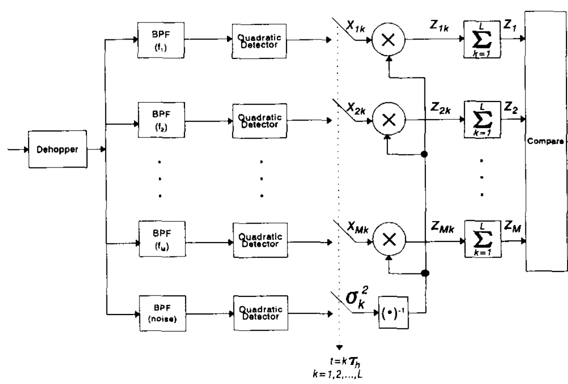

Fig. 1. FH/MFSK square law receiver with noise-normalization.

We assume that each dehopped signal fades independently; that is, we assume that the smallest spacing between frequency hop slots is larger than the coherence bandwidth of the channel [5]–[7]. We also model the channel for each hop as a frequency-nonselective, slowly fading Rician process. Hence, we assume that the signal bandwidth is much smaller than the coherence bandwidth of the channel and that the hop duration is much smaller than the coherence time of the channel [5], [6]. The latter assumption is equivalent to requiring the hop rate to be large compared to the Doppler spread of the channel. As a result, the dehopped signal amplitude is a Rician random variable, and the dehopped signal can be considered as the sum of two components, a nonfaded (direct) component and a Rayleigh-faded (diffuse) component.

The interference that we consider in this paper is partial-band interference which may be due to either a partial-band jammer or some unintended narrowband interference. The interference is modeled as additive Gaussian noise and is assumed to be present in each branch of the MFSK demodulator for any reception of the dehopped signal with probability γ. Thus, γ represents the fraction of the spread bandwidth being jammed, and the probability that narrowband interference is not present in all M detectors is $1 - \gamma$. If $N_I/2$ is the average power spectral density of interference

Paper approved by the Editor for Fading/Equalization of the IEEE Communications Society. Manuscript received December 27, 1990; revised April 9, 1991. This work was supported by the Naval Postgraduate School Research Council and the Naval Ocean Systems Center. This paper was presented in part at the 1990 IEEE Global Telecommunications Conference, San Diego, CA, December 2–5, 1990.

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IEEE Log Number 9106309.

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over the entire spread bandwidth, then $\gamma^{-1}N_I/2$ is the power spectral density of partial-band interference when it is present. In addition to partial-band interference, we assume that the signal is also corrupted by thermal noise and other wideband interferences which we model as additive white Gaussian noise. The power spectral density of this wideband noise is defined as $N_0/2$. Thus, the power spectral density of the total noise is $\gamma^{-1}N_I/2 + N_0/2$ when interference is present and $N_0/2$ otherwise. If the equivalent noise bandwidth of each bandpass filter in the noise-normalized MFSK demodulator is B Hz, then for each hop the signal is received with noise of power N_0B with probability $1-\gamma$ when interference is not present and with noise of power $(\gamma^{-1}N_I + N_0)B$ with probability γ when interference is present.

We assume that the bit rate is R_b. Thus, the corresponding symbol rate is $R_s = R_b/\log_2 M$ where M is the order of the MFSK modulation. The MFSK signal is assumed to perform L hops per symbol. Therefore, the hop rate is $R_h = LR_s$. The equivalent noise bandwidth of each bandpass filter in the noise-normalized MFSK demodulator must be at least as wide as the hop rate, and in this paper we use $B = R_h$. The overall system bandwidth is assumed to be very large compared to the hop rate. Note that for a fixed symbol rate that the hop rate increases as the number of hops per symbol increases. As a result, the required minimum equivalent noise bandwidth of the bandpass filters in the MFSK demodulator also increases as the number of hops per symbol increases. Hence, as the number of hops increases, the assumption that the channel is frequency-nonselective becomes more restrictive. On the other hand, the assumption that the channel is slowly fading becomes stronger.

II. Analysis

Our analysis concerns the derivation of the bit error probability versus the bit energy-to-interference density ratio for the receiver in Fig. 1 given the description of the channel as Rician. The analysis requires the statistics of the sampled outputs $x_{ik}, i = 1, 2, \cdots, M$ of the quadratic detectors for a given hop k of a symbol as well as the normalized samples $z_{ik}, i = 1, 2, \cdots, M$ that are combined to provide the decision samples $z_i, i = 1, 2, \cdots, M$.

A. Probability Density Function of the Decision Variable Z_i

Let σ_k^2 represent the noise power in a given hop k of a symbol. An accurate measurement of σ_k^2 is a challenging problem in fast frequency-hopped spread spectrum systems. In order to perform a complete evaluation of the noise-normalized receiver, σ_k^2 should be modeled as a random variable. In this paper, σ_k^2 is assumed to be known exactly; hence, the performance obtained for the noise-normalized receiver in this paper is in this sense ideal.

From the noise power spectral density and the equivalent noise bandwidth of the bandpass filters as discussed in the previous section, we have $\sigma_k^2 = N_0B$ with probability $1-\gamma$ and $\sigma_k^2 = (\gamma^{-1}N_I + N_0)B$ with probability γ. We assume that the signal is present in branch 1 of the MFSK demodulator. Then the conditional density of the random variable X_{1k} at the output of the quadratic detector, given a signal amplitude $\sqrt{2}a_k$, is [8]

$$f_{X_{1k}}(x_{1k}|a_k) = \frac{1}{2\sigma_k^2}\exp\left(-\frac{x_{1k}+2a_k^2}{2\sigma_k^2}\right)I_0\left(\frac{a_k\sqrt{2x_{1k}}}{\sigma_k^2}\right) \cdot u(x_{1k}) \quad (1)$$

where $u(\bullet)$ is the unit step function and $I_0(\bullet)$ represents the modified Bessel function of order zero. The average received signal power of hop k is $\overline{a_k^2}$, and the fading of hop k is modeled by assuming a_k to be a Rician random variable. The probability density function of the Rician random variable a_k is [8]

$$f_{A_k}(a_k) = \frac{a_k}{\sigma^2}\exp\left(-\frac{a_k^2+\alpha^2}{2\sigma^2}\right)I_0\left(\frac{a_k\alpha}{\sigma^2}\right)u(a_k) \quad (2)$$

where α^2 is the average power of the nonfaded (direct) component of the signal and $2\sigma^2$ is the average power of the Rayleigh-faded (diffuse) component of the signal. The total average received signal power of hop k is $\overline{a_k^2} = \alpha^2 + 2\sigma^2$ and is assumed to remain constant from hop to hop. Note that if $\alpha^2 = 0$ the channel model is a Rayleigh fading model, and if $2\sigma^2 = 0$ there is no fading.

The conditional probability density function of the normalized random variable $Z_{1k} = X_{1k}/\sigma_k^2$ is given by

$$f_{Z_{1k}}(z_{1k}|a_k) = \frac{1}{2}\exp\left(-\frac{\sigma_k^2 z_{1k}+2a_k^2}{2\sigma_k^2}\right)I_0\left(\frac{a_k\sqrt{2z_{1k}}}{\sigma_k}\right) \cdot u(z_{1k}). \quad (3)$$

The probability density function of Z_{1k} can be found by integrating (3) with respect to a_k

$$f_{Z_{1k}}(z_{1k}) = \int_0^\infty f_{Z_{1k}}(z_{1k}|a_k)f_{A_k}(a_k)da_k. \quad (4)$$

Substituting (2) and (3) into (4), we obtain

$$f_{Z_{1k}}(z_{1k}) = \frac{1}{2(1+\xi_k)}\exp\left[-\frac{1}{2}\left(\frac{z_{1k}+2\rho_k}{1+\xi_k}\right)\right] \cdot I_0\left(\frac{\sqrt{2\rho_k z_{1k}}}{1+\xi_k}\right)u(z_{1k}) \quad (5)$$

where $\rho_k = \alpha^2/\sigma_k^2$ is the signal-to-noise ratio of the nonfaded (direct) component of hop k of a symbol and $\xi_k = 2\sigma^2/\sigma_k^2$ is the signal-to-noise ratio of the Rayleigh faded (diffuse) component of hop k of a symbol.

The probability density function of the noise-normalized random variable $Z_{mk}, m = 2, 3, \cdots, M$ of hop k of a symbol that corresponds to the noise-normalized sampled outputs of the branches m of the demodulator (Fig. 1) that contain no signal is obtained from (5) by letting $\rho_k = \xi_k = 0$ to yield

$$f_{Z_{mk}}(z_{mk}) = \frac{1}{2}\exp\left[-\frac{z_{mk}}{2}\right]u(z_{mk}), \quad m = 2, 3, \cdots, M. \quad (6)$$

Let $Z_{1k}^{(1)}$ and $Z_{1k}^{(2)}$ denote the random variable Z_{1k} when hop k of a symbol has interference and no interference, respectively. Also let i be the number of hops of a symbol that have interference. Then the decision variable Z_1 after L independent hops are combined is given by

$$Z_1 = \sum_{k=1}^{L} Z_{1k} = \sum_{k=1}^{i} Z_{1k}^{(1)} + \sum_{k=i+1}^{L} Z_{1k}^{(2)}. \tag{7}$$

Let $f_{Z_{1k}^{(n)}}\left(z_{1k}^{(n)}\right)$, $n = 1, 2$ denote the probability density function of the random variable $Z_{1k}^{(n)}$, $n = 1, 2$ which is obtained from (5) by replacing ρ_k and ξ_k with $\rho_k^{(n)}$ and $\xi_k^{(n)}$, respectively. Thus, $\rho_k^{(n)}$ is the signal-to-noise ratio of the nonfaded (direct) component of hop k of a symbol with interference ($n = 1$) and without interference ($n = 2$), and $\xi_k^{(n)}$ is the signal-to-noise ratio of the Rayleigh faded (diffuse) component of hop k of a symbol with interference ($n = 1$) and without interference ($n = 2$).

Now the Laplace transform of $f_{Z_{1k}^{(n)}}\left(z_{1k}^{(n)}\right)$, $n = 1, 2$ is obtained from

$$F_{Z_{1k}^{(n)}}(s) = \int_0^\infty f_{Z_{1k}^{(n)}}\left(z_{1k}^{(n)}\right) \exp\left[-s z_{1k}^{(n)}\right] dz_{1k}^{(n)}, \quad n = 1, 2. \tag{8}$$

Substituting (5) into (8) and using the transformation $u = \sqrt{z_{1k}^{(n)}}$, we obtain

$$F_{Z_{1k}^{(n)}}(s) = \frac{1}{1+\xi_k^{(n)}} \exp\left[\frac{-\rho_k^{(n)}}{1+\xi_k^{(n)}}\right] \cdot \int_0^\infty u \exp\left\{-\left[s + \frac{1}{2\left(1+\xi_k^{(n)}\right)}\right] u^2\right\} \cdot I_0\left(\frac{\sqrt{2\rho_k^{(n)}}\, u}{1+\xi_k^{(n)}}\right) du \tag{9}$$

where $n = 1, 2$. The above integral can be evaluated as [9]

$$F_{Z_{1k}^{(n)}}(s) = \frac{\beta_k^{(n)} \exp\left(-2\rho_k^{(n)}\beta_k^{(n)}\right)}{s+\beta_k^{(n)}} \exp\left[\frac{2\rho_k^{(n)}\left(\beta_k^{(n)}\right)^2}{s+\beta_k^{(n)}}\right] \tag{10}$$

where

$$\beta_k^{(n)} = \frac{1}{2\left(1+\xi_k^{(n)}\right)}. \tag{11}$$

Since all L hops are independent, we obtain the conditional probability density function for the decision variable Z_1 given that i hops of a bit have interference from (7) and (10) as

$$f_{Z_1}(z_1|i) = \left[f_{Z_{1k}^{(1)}}\left(z_{1k}^{(1)}\right)\right]^{\otimes i} \otimes \left[f_{Z_{1k}^{(2)}}\left(z_{1k}^{(2)}\right)\right]^{\otimes(L-i)} \tag{12}$$

where $\otimes c_n$ represents a c_n-fold convolution and $\left[f_{Z_{1k}^{(n)}} \cdot \left(z_{1k}^{(n)}\right)\right]^{\otimes c_n}$ is obtained as the inverse Laplace transform of $\left[F_{Z_{1k}^{(n)}}(s)\right]^{c_n}$ which is given by

$$\left[f_{Z_{1k}^{(n)}}\left(z_{1k}^{(n)}\right)\right]^{\otimes c_n} = \frac{\beta_k^{(n)} z_{1k}^{(n)(c_n-1)/2}}{\left(2c_n\rho_k^{(n)}\right)^{(c_n-1)/2}} \cdot \exp\left[-\beta_k^{(n)}\left(z_{1k}^{(n)} + 2c_n\rho_k^{(n)}\right)\right] \cdot I_{c_n-1}\left(2\beta_k^{(n)}\sqrt{2c_n\rho_k^{(n)} z_{1k}^{(n)}}\right) \cdot u\left(z_{1k}^{(n)}\right) \tag{13}$$

where $c_1 = i$ and $c_2 = L - i$.

For the special case of Rayleigh fading, $\rho_k^{(n)} \to 0$, and the Laplace transform of $f_{Z_1}(z_1|i)$ reduces to

$$F_{Z_1}(s|i) = \left(\frac{\beta_k^{(1)}}{s+\beta_k^{(1)}}\right)^i \left(\frac{\beta_k^{(2)}}{s+\beta_k^{(2)}}\right)^{L-i} \tag{14}$$

which can be inverted to yield (15) below where the coefficients $K_{1ji}, j = 1, 2, \cdots, i$ and $K_{2k(L-i)}, k = 1, 2, \cdots, (L-i)$ depend upon L and i. Specific coefficients for several values of L and i are given in Table I.

The probability density function of Z_{mk}, $m = 2, 3, \cdots, M$ of hop k of a symbol is a special case of (13) with $\rho_k^{(n)} = 0$ and $\beta_k^{(n)} = 1/2$ and is given by the chi-squared probability density function with $2L$ degrees of freedom

$$f_{Z_m}(z_m) = \frac{(z_m/2)^{L-1}}{2(L-1)!} \exp\left(\frac{-z_m}{2}\right) u(z_m), \quad m = 2, 3, \cdots, M. \tag{16}$$

B. Probability of Bit Error For Uncoded Signals

The symbol error probability for the receiver in Fig. 1 in the presence of partial-band interference is

$$P_s = \sum_{i=0}^{L} \binom{L}{i} \gamma^i (1-\gamma)^{L-i} P_s(i) \tag{17}$$

$$f_{Z_1}(z_1|i) = \left[K_{21(L-i)} + K_{22(L-i)}z_1 + \cdots + \frac{K_{2(L-i)(L-i)}z_1^{L-i-1}}{(L-i-1)!}\right] \exp\left[-\beta_k^{(2)} z_1\right] u(z_1) + \left[K_{11i} + K_{12i}z_1 + \cdots + \frac{K_{1ii}z_1^{i-1}}{(i-1)!}\right] \exp\left[-\beta_k^{(1)} z_1\right] u(z_1) \tag{15}$$

TABLE I
COEFFICIENTS OF $f_{Z_1}(z_1|i)$ FOR RAYLEIGH FADING FOR $i = 1, 2$ AND $L - i = 1, 2$

$K_{111} = \beta_k^{(1)} \left(\frac{\beta_k^{(2)}}{\beta_k^{(2)} - \beta_k^{(1)}} \right)^{L-1}$	$K_{211} = \beta_k^{(2)} \left(\frac{\beta_k^{(1)}}{\beta_k^{(1)} - \beta_k^{(2)}} \right)^{i}$
$K_{112} = \frac{-(L-2)\left(\beta_k^{(1)}\right)^2 \left(\beta_k^{(2)}\right)^{L-2}}{\left(\beta_k^{(2)} - \beta_k^{(1)}\right)^{L-1}}$	$K_{122} = \left(\beta_k^{(1)}\right)^2 \left(\frac{\beta_k^{(2)}}{\beta_k^{(2)} - \beta_k^{(1)}} \right)^{L-2}$
$K_{212} = \frac{-i\left(\beta_k^{(2)}\right)^2 \left(\beta_k^{(1)}\right)^{i}}{\left(\beta_k^{(1)} - \beta_k^{(2)}\right)^{i+1}}$	$K_{222} = \left(\beta_k^{(2)}\right)^2 \left(\frac{\beta_k^{(1)}}{\beta_k^{(1)} - \beta_k^{(2)}} \right)^{i}$

where $P_s(i)$ is the conditional symbol error probability given that i hops of a symbol have interference. It is well known that $P_s(i)$ can be obtained from

$$P_s(i) = 1 - \int_0^\infty f_{Z_1}(z_1|i) \left[\int_0^{z_1} f_{Z_m}(z_m)\, dz_m \right]^{M-1} dz_1 \quad (18)$$

where the integral in brackets can be evaluated to yield

$$P_s(i) = 1 - \int_0^\infty f_{Z_1}(z_1|i) \cdot \left[1 - \exp\left(\frac{-z_1}{2}\right) \sum_{k=0}^{L-1} \frac{(z_1/2)^{L-1-k}}{(L-1-k)!} \right]^{M-1} \cdot dz_1. \quad (19)$$

For the general case of some hops jammed and others free of interference, (19) must be evaluated numerically. By using first the binomial theorem and then the multinomial theorem, we can express the $[\bullet]^{M-1}$ term in (19) as a double summation of powers of z_1. Then, for the special cases of either all hops jammed or all hops free of interference, (12) reduces to (13), and replacing the modified Bessel function in (13) with a series representation, we can integrate (19) analytically term by term [10]. The result is in the form of a double summation of the product of powers of rational functions, exponentials, and confluent hypergeometric functions with arguments depending on both summation indexes. As a result, it is more straightforward to evaluate (19) numerically for all cases even though an analytic expression is available in two special cases. For a numerical evaluation of (19), it is preferable to use $[\bullet]^{M-1}$ rather than its expansion as a double summation in powers of z_1.

In many cases, the numerical evaluation of (19) can be made substantially easier by noting that

$$\lim_{z_1 \to \infty} \left[1 - \exp\left(\frac{-z_1}{2}\right) \sum_{k=0}^{L-1} \frac{(z_1/2)^{L-1-k}}{(L-1-k)!} \right]^{M-1} = 1. \quad (20)$$

Hence, by adding and subtracting

$$\int_0^\infty f_{Z_1}(z_1|i)\, dz_1 = 1 \quad (21)$$

to the right-hand side of (19), we obtain a computationally efficient expression for the conditional probability of symbol error given that i hops of a symbol have interference as

$$P_s(i) = \int_0^\infty f_{Z_1}(z_1|i) \left\{ 1 - \left[1 - \exp\left(\frac{-z_1}{2}\right) \cdot \sum_{k=0}^{L-1} \frac{(z_1/2)^{L-1-k}}{(L-1-k)!} \right]^{M-1} \right\} dz_1 \quad (22)$$

since the $\{\bullet\}$ term in (22) approaches zero as $z_1 \to \infty$.

For orthogonal MFSK the bit error probability can be related to the symbol error probability by

$$P_b = \frac{M}{2(M-1)} P_s \quad (23)$$

and the energy per bit E_b is related to the energy per symbol E_s by $E_s = (\log_2 M) E_b$.

C. Probability of Bit Error For Coded Signals

The diversity combining discussed above is a form of repetition coding. More complex codes may be used either alone or in conjunction with diversity. For this paper we consider the effects on the noise-normalization combining receiver of both binary convolutional codes with hard-decision Viterbi decoding and Q-ary, linear block codes with hard-decision decoding. For each of the codes considered, the coded energy per bit is related to the uncoded energy per bit and the code rate r by $E_c = rE_b$.

The channel-bit error probability p of the system with convolutional coding and hard-decision Viterbi decoding is given by (23); that is, $p = Mp_s/2(M-1)$ where p_s is the channel M-ary symbol error probability. Using the best rate 1/2, constraint length 7 convolutional code in [11], we obtain the upper bound for the decoded bit error probability as

$$P_b \le \frac{1}{2} \left(36D^{10} + 211D^{12} + 1404D^{14} + 11633D^{16} + \cdots\right) \quad (24)$$

and for the best rate 1/3, constraint length 7 convolutional code [11], we have the upper bound for the decoded bit error probability as

$$P_b \le \frac{1}{2} \left(D^{14} + 20D^{16} + 53D^{18} + 184D^{20} + \cdots\right) \quad (25)$$

where

$$D = 2\sqrt{p(1-p)}. \tag{26}$$

The decoded error probability for a block decoder which accepts an n-tuple of Q-ary coded symbols and outputs a k-tuple of Q-ary information symbols is [12]

$$P_q \cong \frac{d}{n} \sum_{i=t+1}^{d} \binom{n}{i} p_q^i (1-p_q)^{n-i} + \frac{1}{n} \sum_{i=d+1}^{n} i \binom{n}{i} \cdot p_q^i (1-p_q)^{n-i} \tag{27}$$

where $Q = 2^q$, d is the minimum distance between codewords, $t = [(d-1)/2]$ is the maximum number of correctable symbol errors in a code word ($[\bullet]$ = integer part of $\bullet$), and p_q is the channel Q-ary symbol error probability. The channel Q-ary symbol error probability is related to the channel M-ary symbol error probability p_s by [4]

$$p_q = 1 - (1-p_s)^{q/\log_2 M} \quad \text{when } q/\log_2 M = \text{integer} \tag{28}$$

or

$$p_q = \frac{M}{M-1}\left(1 - 2^{-q}\right)p_s \quad \text{when } \log_2 M/q = \text{integer}. \tag{29}$$

The decoded bit error probability is $P_b = QP_q/2(Q-1)$. The linear block codes used in this paper are (n, k) Reed–Solomon codes, for which the minimum distance between code words is $d = n - k + 1$ and the code rate is $r = k/n$.

III. Numerical Results

Computation of the probability of bit error involves a numerical evaluation of (22) for each of the possible combinations of jammed and unjammed hops given L hops per symbol. In addition, except for the special cases of either all hops jammed or all hops free of interference or for Rayleigh fading, the probability density function of Z_1 given that i hops of a bit have interference $f_{Z_1}(z_1|i)$ must be evaluated numerically. Using (10) and (12), we can easily obtain an analytic expression for $F_{Z_1}(s|i)$, the Laplace transform of $f_{Z_1}(z_1|i)$. Hence, the most efficient way to evaluate $f_{Z_1}(z_1|i)$ numerically is to invert $F_{Z_1}(s|i)$ numerically. This is accomplished using a variation of the method detailed in [13] where

$$f_{Z_1}(z_1|i) = \frac{\exp(az_1)}{2z_1}\left[F_{Z_1}(a|i) + 2\sum_{k=1}^{\infty}(-1)^k \operatorname{Re}\{F_{Z_1}(a + jk\pi/z_1|i)\}\right] + \text{Error}. \tag{30}$$

The variable a in (30) is related to the exponential order β of $F_{Z_1}(s|i)$ and the upper bound of the relative error E' by

$$a = \frac{3}{2}\beta - \frac{\ln E'}{2z_1} \tag{31}$$

The magnitude of the error term in (30) is controlled by the choice of the maximum relative error E'.

To obtain worst case partial-band jamming, the jamming fraction γ which maximizes the probability of bit error is found for various values of diversity, fading conditions, signal-to-noise power density ratios, and order of modulation. Both the coded and the uncoded cases are investigated. All results presented in this paper are obtained by assuming that the ratio of direct-to-diffuse signal energy $\alpha^2/2\sigma^2$ is the same for each hop k of a symbol.

A. Uncoded Performance

Receiver performance for specific fractions of partial-band interference are compared to worst case performance for a relatively strong direct signal $\left(\alpha^2/2\sigma^2 = 10\right)$ in Figs. 2–5 for $M = 4$ and diversities of $L = 1$, 2, 3, and 4, respectively. In each of these figures, the ratio of bit energy-to-thermal noise density is $E_b/N_0 = 13.35$ dB. This value of E_b/N_0 corresponds to $P_b = 10^{-5}$ when there is no fading or interference, $M = 2$, and $L = 1$. This corresponds to the signal-to-thermal noise density ratio used in [2] and allows our results to be compared directly to the nonfaded results presented in [2]. As can be seen in Fig. 2, partial-band interference results in a significant degradation in performance over a broad range of bit energy-to-interference noise density ratios as compared with uniform interference when no diversity is used. As a general rule, as the ratio of bit energy-to-interference noise density increases, the degradation due to partial-band interference increases as the fraction of the spread bandwidth being jammed γ decreases; although, for both very large and very small bit energy-to-interference noise density ratios, there is very little degradation due to partial-band interference. As can be seen in Figs. 3–5, the degradation due to partial-band interference is progressively reduced as diversity increases. At this value of E_b/N_0, a diversity of four is sufficient to virtually eliminate any degradation due to partial-band interference. As can be seen in Fig. 5, for $M = 4$ and $L = 4$ worst case performance is obtained for all bit energy-to-interference noise density ratios when the interference is uniform. Hence, for the case of a strong direct component, diversity is capable of completely eliminating the degradation introduced by partial-band interference when no diversity is used. It is also interesting to note that for bit energy-to-interference noise density ratios less than about 15 dB, receiver performance improves dramatically when the interference is partial-band rather than uniform. This is particularly true for $L > 2$. Another interesting result is that for bit energy-to-interference noise density ratios greater than roughly 12 dB there is a distinct improvement when diversity is used. This can be contrasted with the nonfaded results [2], where diversity offers an improvement only for bit energy-to-interference noise density ratios greater than roughly 15 dB and less than roughly 30–35 dB, depending on the order of diversity. Analogous results are obtained when $M = 2$ except overall performance is poorer and there is very little change in performance for $L > 1$.

Receiver performance for specific fractions of partial-band interference are compared to worst case performance for a

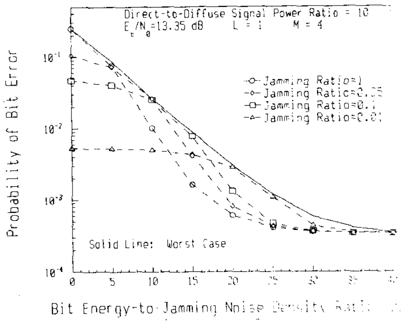

Fig. 2. Performance of the noise-normalization receiver for partial-band jamming fractions of $\gamma = 1.0, 0.25, 0.1$, and 0.01 compared to worst case performance for a strong direct signal with $E_b/N_0 = 13.35$ dB, $M = 4$, and $L = 1$.

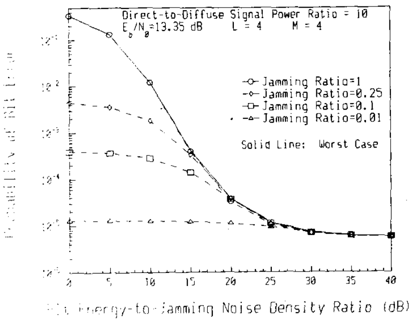

Fig. 5. Performance of the noise-normalization receiver for partial-band jamming fractions of $\gamma = 1.0, 0.25, 0.1$, and 0.01 compared to worst case performance for a strong direct signal with $E_b/N_0 = 13.35$ dB, $M = 4$, and $L = 4$.

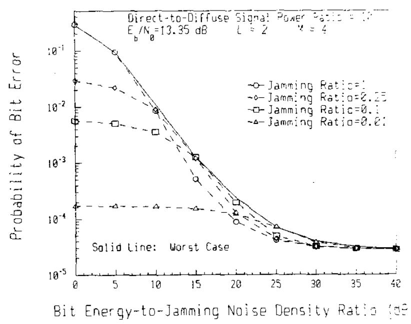

Fig. 3. Performance of the noise-normalization receiver for partial-band jamming fractions of $\gamma = 1.0, 0.25, 0.1$, and 0.01 compared to worst case performance for a strong direct signal with $E_b/N_0 = 13.35$ dB, $M = 4$, and $L = 2$.

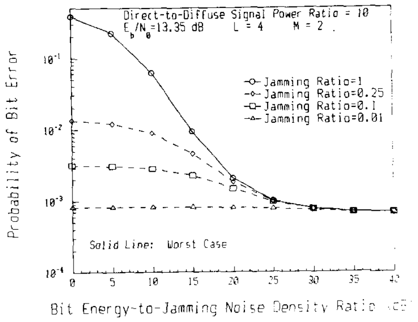

Fig. 6. Performance of the noise-normalization receiver for partial-band jamming fractions of $\gamma = 1.0, 0.25, 0.1$, and 0.01 compared to worst case performance for a strong direct signal with $E_b/N_0 = 13.35$ dB, $L = 4$, and $M = 2$.

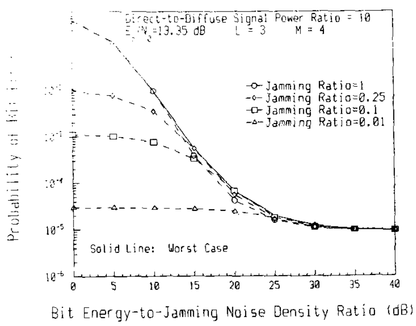

Fig. 4. Performance of the noise-normalization receiver for partial-band jamming fractions of $\gamma = 1.0, 0.25, 0.1$, and 0.01 compared to worst case performance for a strong direct signal with $E_b/N_0 = 13.35$ dB, $M = 4$, and $L = 3$.

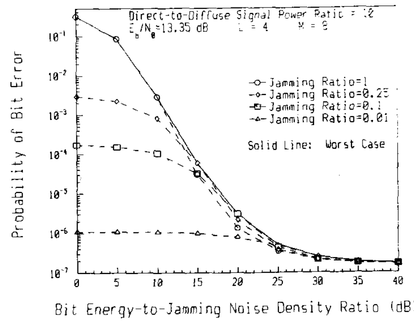

Fig. 7. Performance of the noise-normalization receiver for partial-band jamming fractions of $\gamma = 1.0, 0.25, 0.1$, and 0.01 compared to worst case performance for a strong direct signal with $E_b/N_0 = 13.35$ dB, $L = 4$, and $M = 8$.

relatively strong direct signal $(\alpha^2/2\sigma^2 = 10)$ in Fig. 6, Fig. 5, and Figs. 7–8 for $L = 4$ and modulation orders of $M = 2$, 4, 8, and 16, respectively. The ratio of bit energy-to-thermal noise density is again taken to be $E_b/N_0 = 13.35$ dB. As the order of modulation increases partial-band interference becomes more effective in degrading relative receiver performance; however, for each increase in modulation order there is a significant improvement in receiver performance that far outweighs any degradation that is introduced by partial-band interference.

When there is not a strong direct component to the signal, partial-band interference results in virtually no degradation of receiver performance. In this instance, the worst case performance of the noise-normalization combining receiver is virtually identical to the performance when the interference is uniform. As in the case of a strong direct signal, performance

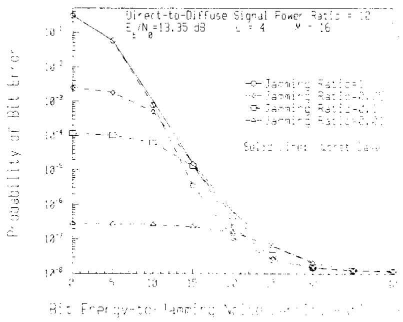

Fig. 8. Performance of the noise-normalization receiver for partial-band jamming fractions of $\gamma = 1.0$, 0.25, 0.1, and 0.01 compared to worst case performance for a strong direct signal with $E_b/N_0 = 13.35$ dB, $L = 4$, and $M = 16$.

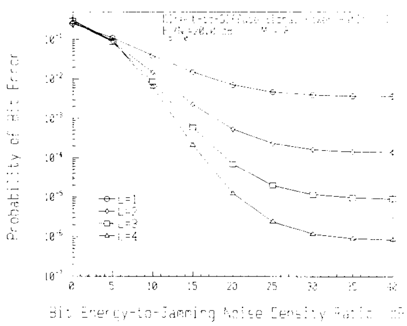

Fig. 9. Performance of the noise-normalization receiver for diversities of $L = 1$, 2, 3, and 4 for a weak direct signal with $E_b/N_0 = 20.0$ dB and $M = 8$.

is dramatically improved for bit energy-to-interference noise density ratios less than about 10 dB when the interference is partial-band in nature. Receiver performance for various values of diversity with modulation order fixed and for various values of modulation order with the level of diversity fixed for a relatively weak direct signal $(\alpha^2/2\sigma^2 = 1)$ is shown in Figs. 9–10, respectively. The ratio of bit energy-to-thermal noise density is taken to be $E_b/N_0 = 20.0$ dB. As can be seen, when the direct signal is weak, significant improvements in performance can be obtained by increasing either the order of diversity or the modulation order.

B. Coded Performance

Coded receiver performance for the case of no diversity with $M = 8$ for a relatively weak direct signal $(\alpha^2/2\sigma^2 = 1)$ is shown in Fig. 11. Since partial-band interference does not significantly affect performance when there is not a strong direct signal component, the interference is taken to be broadband. The ratio of bit energy-to-thermal noise density is taken to be $E_b/N_0 = 20.0$ dB. The performance obtained with the best rate $1/2$, constraint length 7 convolutional code, the best rate $1/3$, constraint length 7 convolutional code, a $(7,3)$ Reed–Solomon code with $Q = 8$, and a $(15,7)$ Reed–Solomon code with $Q = 16$ are all compared with

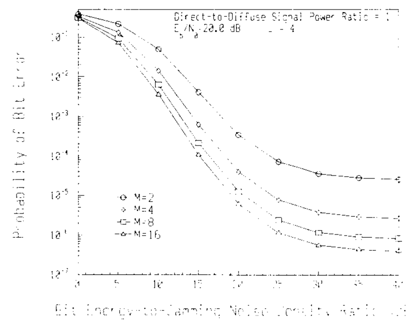

Fig. 10. Performance of the noise-normalization receiver for modulation orders of $M = 2$, 4, 8, and 16 for a weak direct signal with $E_b/N_0 = 20.0$ dB and $L = 4$.

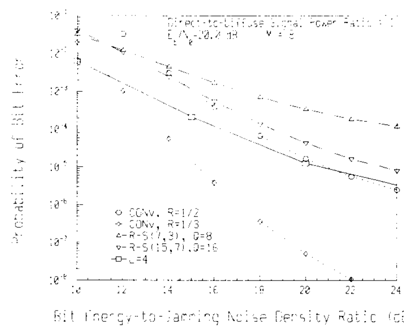

Fig. 11. Performance of the noise-normalization receiver with no diversity and $M = 8$ for various coded signals for a weak direct signal with $E_b/N_0 = 20.0$ dB.

the uncoded receiver performance when a diversity of $L = 4$ is used. As can be seen, for bit energy-to-interference noise density ratios in the 10–20 dB range, the uncoded receiver with diversity performs substantially better than all coded receivers except for the best rate $1/3$, constraint length 7 convolutional code. The best rate $1/3$, constraint length 7 convolutional code performs significantly better than the uncoded receiver with diversity for bit energy-to-interference noise density ratios above about 12 dB. Above bit energy-to-interference noise density ratios of about 21 dB, the best rate $1/2$, constraint length 7 convolutional code also performs better than the uncoded receiver with diversity. The coded receiver performance plotted in Fig. 11 can also be compared to the uncoded performance illustrated in Fig. 9. As can be seen, for bit energy-to-interference noise density ratios above about 14 dB the coded performance is significantly better than the uncoded performance with no diversity. For bit energy-to-interference noise density ratios in the 10 to 24 dB range, the $(15,7)$ Reed–Solomon code with $Q = 16$ is roughly comparable to the uncoded receiver performance with a diversity of $L = 3$. Of course, as the bit energy-to-interference noise density ratio increases, the coded receiver performance becomes extremely good.

Extremely good results can be obtained by combining diversity and coding. Coded receiver performance for a relatively weak direct signal $(\alpha^2/2\sigma^2 = 1)$ when a diversity of $L = 4$

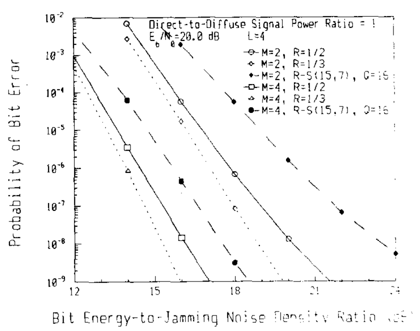

Fig. 12. Performance of the noise-normalization receiver with $L = 4$ and modulation orders $M = 2$ and $M = 4$ for various coded signals for a weak direct signal with $E_b/N_0 = 20.0$ dB.

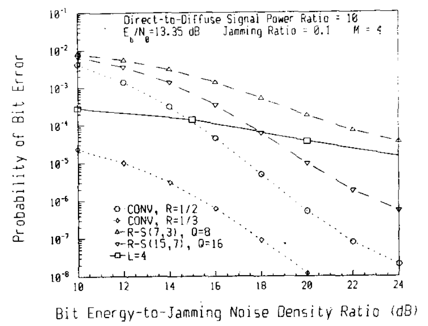

Fig. 13. Performance of the noise-normalization receiver with no diversity and $M = 4$ for various coded signals for a strong direct signal with $E_b/N_0 = 13.35$ dB.

is used is shown in Fig. 12. As in the previous example, the interference is taken to be broadband since there is no strong direct signal component. The ratio of bit energy-to-thermal noise density is again taken to be $E_b/N_0 = 20.0$ dB. Coded receiver performance is obtained for modulation orders of $M = 2$ and $M = 4$ for the best rate $1/2$, constraint length 7 convolutional code, the best rate $1/3$, constraint length 7 convolutional code, and a $(15, 7)$ Reed–Solomon code. As can be seen, as the bit energy-to-interference noise density ratio increases only a few dB, receiver performance improves dramatically for all codes considered. These results can be compared with those in Fig. 10, where the uncoded receiver performance for a modulation order of $M = 16$ is substantially poorer than any of the $M = 2$ coded receiver performances considered for E_b/N_I greater than about 18 dB.

Coded performance for the case of no diversity with $M = 4$ for a relatively strong direct signal $(\alpha^2/2\sigma^2 = 10)$ is shown in Fig. 13. The ratio of bit energy-to-thermal noise density is taken to be $E_b/N_0 = 13.35$ dB so that the results can be compared to Figs. 2–8. A constant partial-band interference ratio of $\gamma = 0.1$ is used since this yields near worst case results in the 10 to 20 dB range of bit energy-to-thermal noise density ratio. The performance obtained with the best rate $1/2$, constraint length 7 convolutional code, the best rate $1/3$, constraint length 7 convolutional code, a $(7, 3)$ Reed–Solomon code with $Q = 8$, and a $(15, 7)$ Reed–Solomon code with $Q = 16$ are all compared to the uncoded receiver performance when a diversity of $L = 4$ is used. As can be seen, for bit energy-to-interference noise density ratios below about 15 dB, the uncoded receiver with diversity performs better than all coded receivers with no diversity except for the best rate $1/3$, constraint length 7 convolutional code. Above bit energy-to-interference noise density ratios of about 18 dB, except for the $(7, 3)$ Reed–Solomon code, the coded performance becomes significantly better than the uncoded performance with diversity. As can be seen by comparings Figs. 8 and 13, in the 14 to 24 dB range of bit energy-to-interference noise density ratio the performance obtained with no diversity and the best rate $1/2$, constraint length 7 convolutional code is roughly comparable to the performance obtained with a diversity of four and a modulation order of 16, while the performance obtained with no diversity and the $(15, 7)$ Reed–Solomon code is between 2 and 3 dB poorer. The performance with no diversity and the best rate $1/3$, constraint length 7 convolutional code provides consistently superior performance for the case of a strong direct signal.

IV. Conclusion

The noise-normalized receiver with diversity is seen to offer dramatic improvement in probabilities of bit error over the noise-normalization receiver with no diversity when fading is present provided the signal-to-interference noise density ratio is 12 dB or more. This is particularly true when the signal does not contain a strong direct component. Even a diversity of only two provides a significant advantage. In addition, diversity is seen to dramatically reduce degradation in receiver performance due to partial-band interference even when the signal contains a strong direct component. For signal-to-interference noise density ratios of less than 10 dB, a superior receiver is obtained when diversity is not used; however, the degradation due to diversity at lower signal-to-interference noise density ratios is slight. For a given diversity, increasing the modulation order to $M > 2$ results in further significant performance improvements regardless of how strong channel fading is.

The use of forward error correction coding significantly improves the performance of the noise-normalization receiver for ratios of bit energy-to-interference noise density above about 12 dB. Below a bit energy-to-interference noise density ratio of about 16 dB, coded receiver performance with no diversity is outperformed by the uncoded receiver performance with a diversity of four. When channel fading is slight, coded receiver performance with no diversity outperforms uncoded receiver performance with a diversity of four for bit energy-to-interference noise density ratios above about 16 dB. The exception to the above generalizations is the coded performance obtained with the best rate $1/3$, constraint length 7 convolutional code when no diversity is used. The best rate $1/3$, constraint length 7 convolutional code provides dramatically superior performance for bit energy-to-interference noise density ratios above about 11 dB when fading is significant and for all bit energy-to-interference noise density ratios when fading is weak.

Significant improvements in receiver performance are obtained by combining coding, diversity, and modulation orders $M > 2$.

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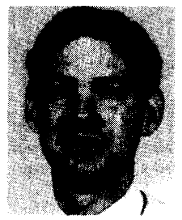

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