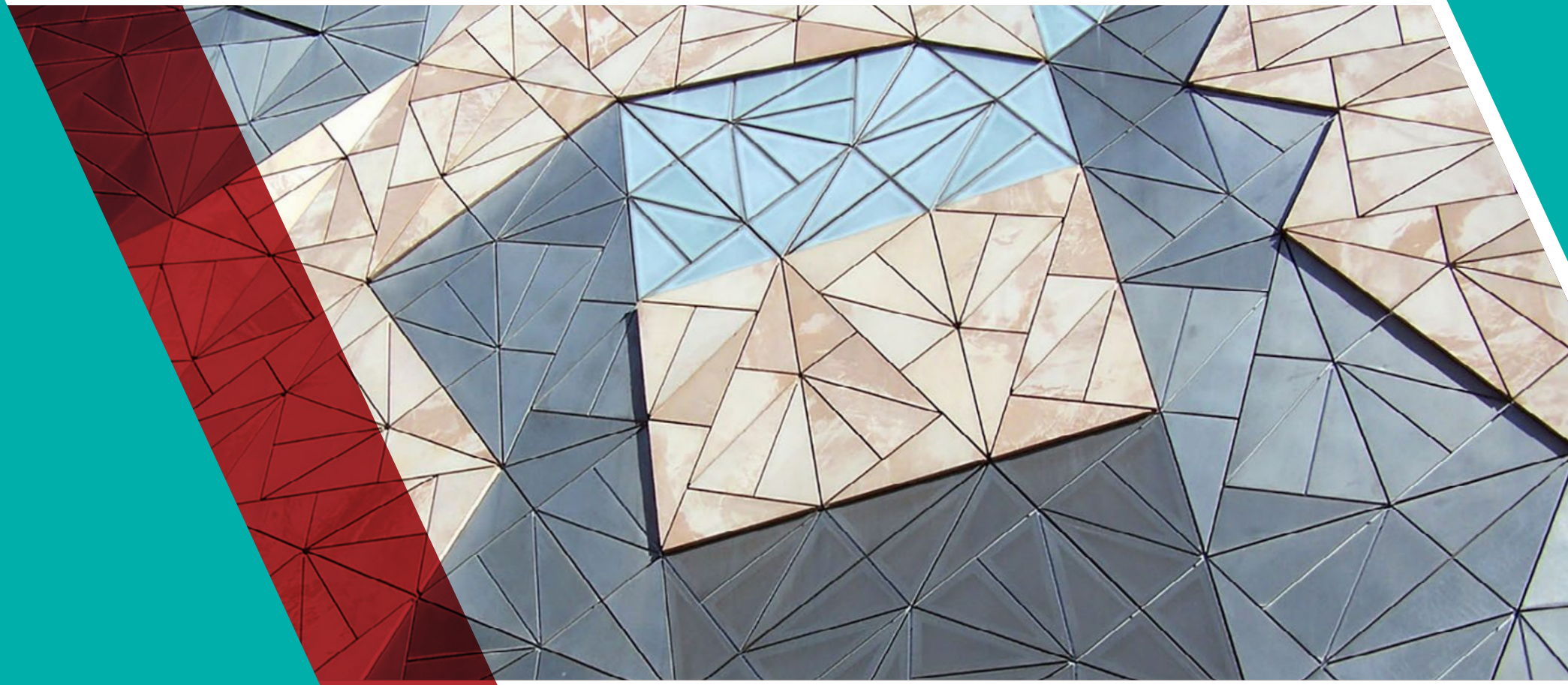
MATHEMATICS TEACHING toolkit

ISSUES IN THE TEACHING OF MATHEMATICS

Critical Connections Between Numeracy and Mathematics



Dave Tout

Senior Research Fellow, Numeracy and Mathematics, Tertiary and Vocational Assessment Services, Australian Council for Educational Research (ACER)

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INTRODUCTION

Why connections between numeracy and mathematics is an important issue in mathematics teaching and learning in the 21st Century?

Increasingly research is showing that life and work in the 21st century is requiring higher levels of mathematics and numeracy of its citizens. Numeracy and mathematics are intrinsically connected and BOTH are needed in our ever changing, globalised and technological world.

This paper looks at the implications of this for the skills we want our students to develop and leave school with, and how we can better address these in our teaching and learning.

RESEARCH ABOUT 21ST CENTURY SKILLS

Research is showing that the skills and knowledge now needed to succeed in work, life and citizenship have significantly changed in the 21st century, often driven by technological advances and an ever-increasing use of numerical and quantitative information and data. This also connects with the transforming nature of the workforce associated with Industry 4.0 and the Gig economy, with increasing demands for science, technology, engineering and mathematics (STEM) skills *(e.g. see AAMT and AiGroup 2014; Binkley et al. 2012; FYA 2017; Gravemeijer et al 2017; Griffin et al. 2012; Hoyles et al. 2010; NCTM, 2017; P21 2016; Pellegrino et al. 2012; Wake 2015)*.

In their 2017 review of mathematics education for the 21st century, the National Council of Teachers of Mathematics (NCTM) in the USA (2017) argued that mathematics is at the heart of most innovations in the information economy. They saw mathematical and statistical literacy as needed more than ever to filter, understand, and act on the enormous amount of data and information that we encounter every day.

WHAT NUMERACY AND MATHS SKILLS?

One of the key outcomes of such research is that the mathematics, or numeracy, related tasks that people undertake involve much more than basic arithmetic skills and straightforward procedural competence. For example, in an Australian project undertaken by practising maths teachers for the Australian Association of Mathematics Teachers (AAMT) and funded by the Office of the Chief Scientist, Quantitative Skills in 21st Century Workplaces, the research identified and analysed the gaps between young peoples' numeracy skills and the expectations of 21st century workplaces.

Mathematics was considered extremely important in all of the companies involved, and changing work practices were found to be generating new demands for mathematical skills, particularly in areas such as efficiency, innovation and Quality & Continuous Improvement.

The project found that workers needed a blend of the following key skills:

- Ability to recognise and identify how and when mathematics is used in the workplace.
- An understanding of mathematical concepts, procedures and skills.
- An understanding of the kinds of practical tasks they need to perform.
- The strategic processes they should be able to use in using and applying mathematics.

Although the skills observed appeared to be fundamental, it is their use and application in work contexts that is not straightforward *(AAMT & AiGroup, 2014)*.

> ***The application of mathematics in the workplace is not straightforward and goes well beyond a command of 'core' mathematical content. Workers perform sophisticated functions which require them to be confident to use mathematical skills in problem-solving situations and to see the consequences of the mathematics related procedures. (AAMT & AiGroup, 2014)***

THE CRITICAL CONNECTIONS BETWEEN NUMERACY AND MATHEMATICS

What is meant by the term numeracy, and what is its relationship to terms such as quantitative or mathematical literacy, and how does it connect to the world of (school) mathematics?

The term numeracy is used in some countries, like in Australia, however, other expressions are used as well, for example, mathematical literacy or quantitative literacy.

This is further complicated by the lack of an equivalent term in some languages. Moreover, what is meant by numeracy also varies between countries, and can vary between how it is understood when applied to school education compared to within adult education. However, increasingly numeracy now refers to the capability to use a range of mathematical and statistical knowledge and skills to solve problems in the real world for a purpose.

One underpinning connection and difference that needs to be acknowledged and highlighted is between numeracy and mathematics as a domain of knowledge.

The meaning of numeracy used here encompasses the need for individuals to be able to understand, use and apply mathematical (and statistical) skills and knowledge.

Thus, to be considered numerate, it is expected that people will need to know some mathematics, and be able to apply that mathematics within a real-world context.

> ***... numeracy is not the same as mathematics, nor is it an alternative to mathematics. Today's students need both mathematics and numeracy. Whereas mathematics asks students to rise above context, quantitative literacy is anchored in real data that reflect engagement with life's diverse contexts and situations. (Steen, 2001, p.10)***

CONCLUSION

With 21st century life and workplaces requiring more critical, reflective mathematical reasoning skills and the ability to interpret and understand a broader range of data and processes, our school leavers need better numeracy and maths skills than ever before.

Hence schools need to teach both numeracy and mathematics well – within maths classes by maths teachers and also as part of numeracy across the curriculum.

KEY TERMS & DEFINITIONS

The need for numeracy and maths skills is recognised in Australian curriculum. For example, the Australian Curriculum: Mathematics (V 8.4) includes numeracy as one of the General Capabilities that must be observed while teaching and learning mathematics stating:

... students become numerate as they develop the knowledge and skills to use mathematics confidently across other learning areas at school and in their lives more broadly.

Numeracy encompasses the knowledge, skills, behaviours and dispositions that students need to use mathematics in a wide range of situations. It involves students recognising and understanding the role of mathematics in the world and having the dispositions and capacities to use mathematical knowledge and skills purposefully.

Similarly, the Victorian Curriculum and Assessment Authority defines numeracy as:

Numeracy is the knowledge, skills, behaviours and dispositions that students need in order to use mathematics in a wide range of situations.

Internationally, the related term mathematical literacy is used in the OECD's Programme for International Student Assessment (PISA). It is defined as:

... an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens. *(OECD, 2019, p. 75)*

It should be noted that at the conceptual and implementation level, numeracy and mathematical literacy are very closely related constructs in terms of their core, underlying ideas *(e.g., see Gal & Tout, 2014)*.

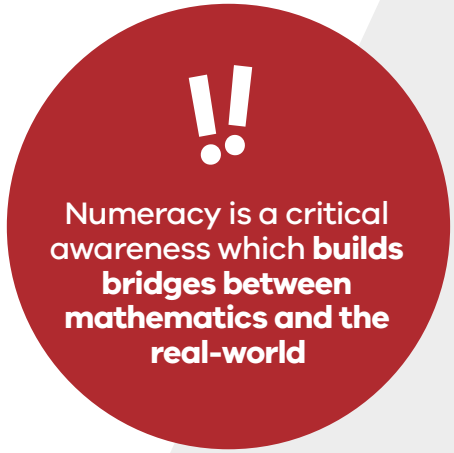
As with the above definitions, most 21st century descriptions and definitions of numeracy now refer to the need to use and apply a range of mathematical knowledge.

It is important to note therefore that numeracy is not seen as synonymous with a minimal or low level of mathematical knowledge and skills – but covers a broad spectrum of skills and knowledge. Sometimes numeracy can be viewed as a lower level skill, simply being about numbers and arithmetical operations.

CRITICAL ELEMENT TO NUMERACY

The issue of the need to be critical, and use reflective mathematical reasoning skills is now identified as a key skill and is routinely included in many numeracy frameworks as will be shown below. An emphasis on critical reflection or interpretation has mainly arisen out of adult numeracy research and development, where, for example, Johnston (1994) argued that

To be numerate is more than being able to manipulate numbers, or even being able to 'succeed' in school or university mathematics. Numeracy is a critical awareness which builds bridges between mathematics and the real-world, with all its diversity (p. 34).



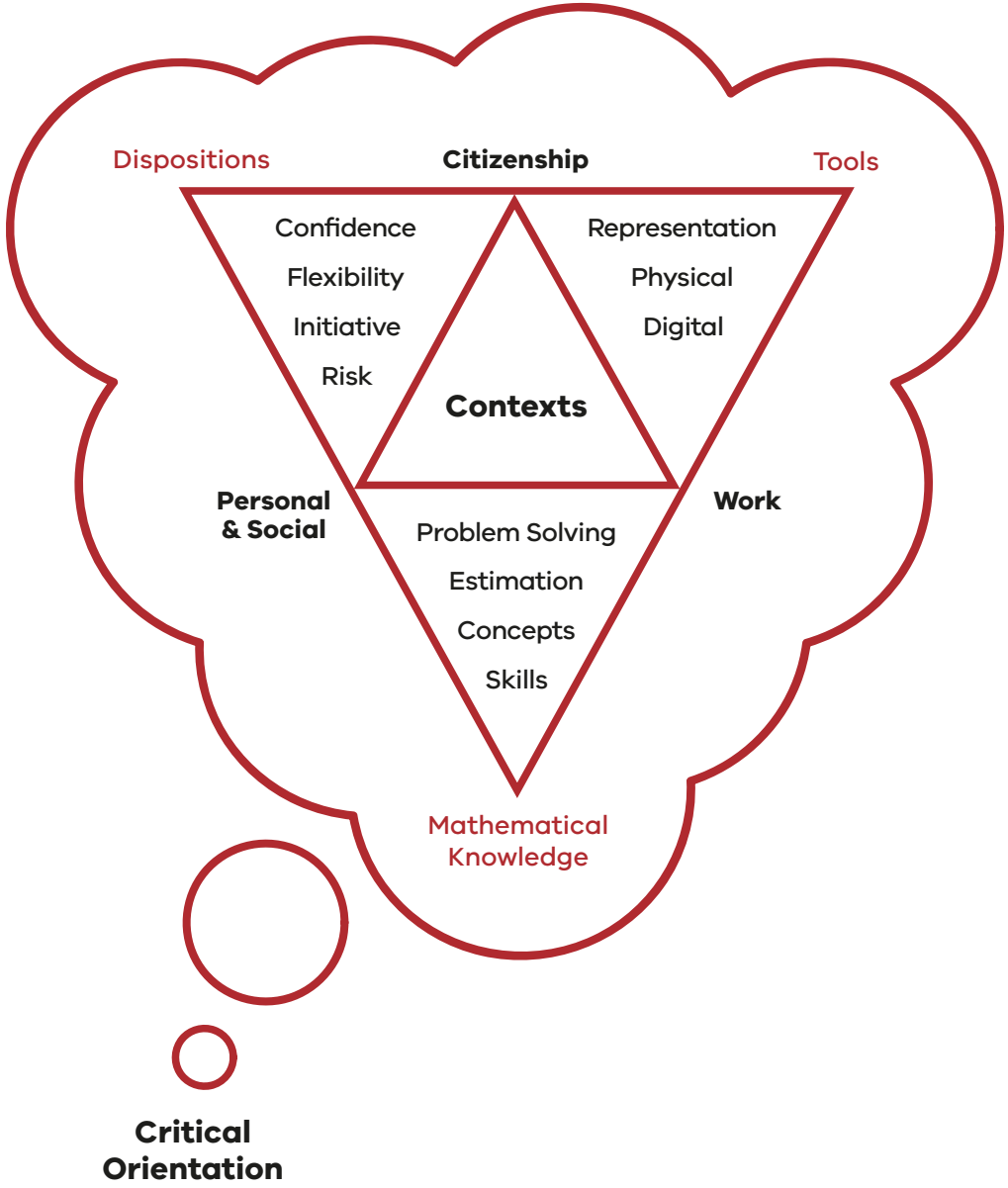
EVIDENCE BASE

Based on, and as a part of, the research into numeracy and mathematical literacy and the increasing demands for higher levels of numeracy and mathematics in the 21st century summarised above, there are now a number of existing frameworks or models for numeracy that can be utilised to frame what we need to do in order to improve and enhance our teaching and learning of numeracy and maths.

For example, a significant Australian model is illustrated in Figure 1. This model incorporates four dimensions of settings/contexts, mathematical knowledge, tools, and dispositions that are embedded in a critical orientation to using mathematics. These dimensions are described more fully in other publications *(e.g., Geiger et al. 2015; Goos et al, 2014)*.

The model views numeracy as having the capacity to use mathematical knowledge in a range of contexts, both within schools and beyond school settings; to have positive dispositions to maths and the confidence, willingness and preparedness to flexibly use mathematical approaches and knowledge to engage with life-related tasks; to use physical materials and tools (models, measuring instruments), representations (symbol systems, graphs, maps, diagrams, drawings, tables) and digital tools (computers, software, calculators, internet) to mediate and shape the mathematical actions and thinking. And the model suggests there is a need for a critical orientation that enables the use of mathematical information to make decisions and judgements, and to argue and challenge.

Figure 1. Numeracy in the 21st century (Goos et al, 2014; Geiger et al. 2015)



PISA mathematical literacy frames its model along similar lines, but is based more explicitly on a real-world, problem solving cycle. One core idea behind the PISA description is that when individuals use mathematics and mathematical tools to solve problems set in a real world situation, they work their way through a series of stages as depicted in Figure 2. The formulating, employing, interpreting, and evaluating processes are key components of the cycle. These processes are shown as drawing on the problem solver's fundamental mathematical capabilities and knowledge across four maths content areas. The PISA definition and model covers many of the same elements as shown in the Goos et al model above, including the use of tools and different representations.

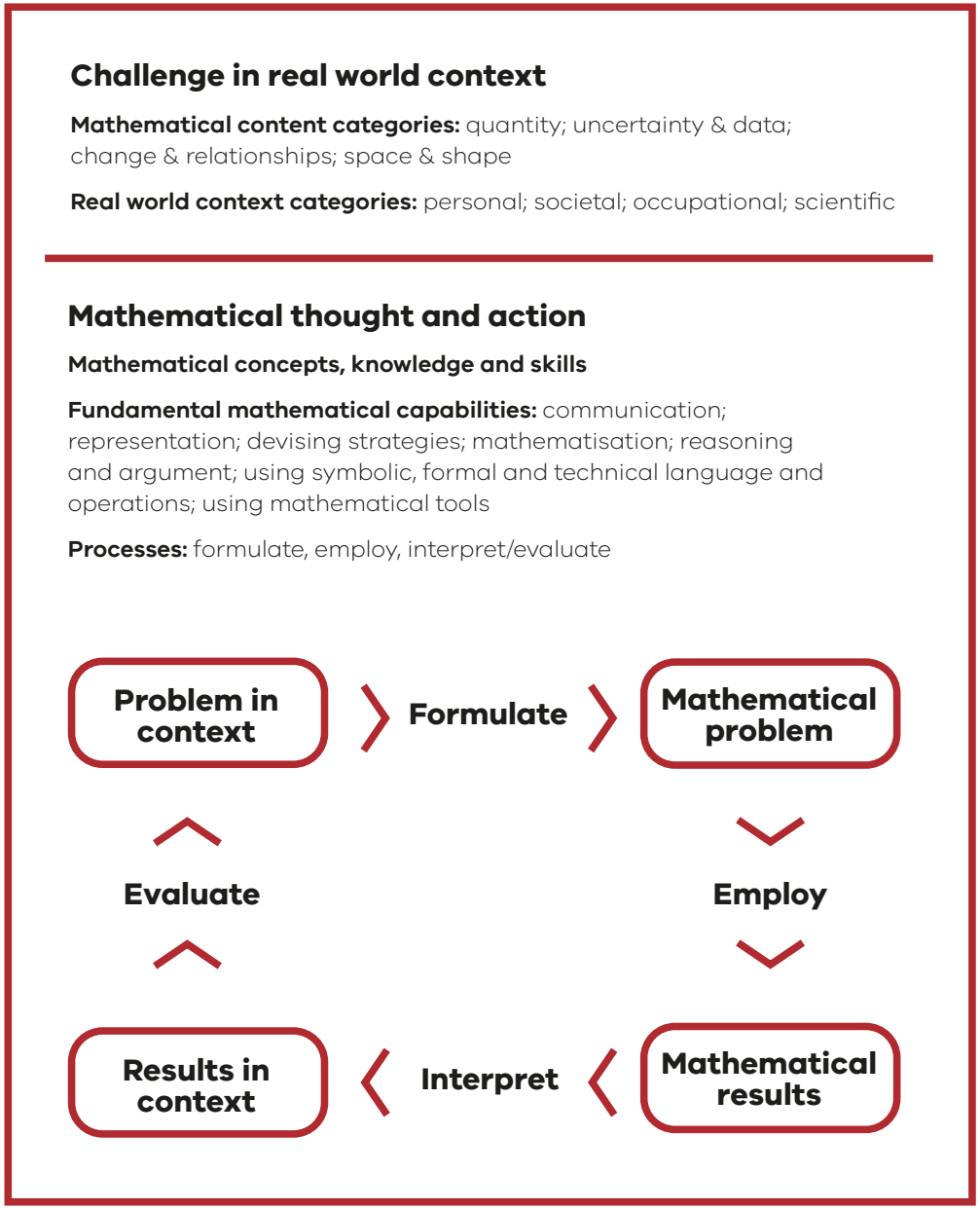
These two models serve to highlight the complex nature of numeracy, and the broad range of factors and skills that need to be addressed and taken on board when supporting students to become numerate.

They are also consistent with the earlier AAMT list of maths skills required of workers, which required workers to have the ability to recognise and identify how and when mathematics is used in the workplace; have an understanding of mathematical concepts, procedures and skills, and have an understanding of the strategic processes they should be able to use in using and applying mathematics.

Being numerate therefore involves more than just mastering basic mathematics in a school classroom, because numeracy connects the mathematics learned at school with out of-school situations that additionally require problem solving, critical judgement, and making sense of the non-mathematical context.

But why do we need to address this issue of numeracy and the demands for higher levels of numeracy and maths? Aren't Australian school children sufficiently skilled in numeracy and mathematics?

Figure 2. Representation of key elements of the PISA mathematics framework (from OECD, 2019)



WHAT IS RESEARCH SHOWING ABOUT AUSTRALIAN'S SKILLS AND ABILITIES IN NUMERACY?

Australian 15 year-old students' performance in the Programme for International Student Assessment (PISA) of mathematical literacy has declined since 2003, and there are persistent equity gaps in the performance of students from disadvantaged groups *(e.g., see Thomson et al, 2019)*.

The PISA results also show clearly that the same students are relatively much stronger in reading (literacy) than in numeracy, and this is reinforced by analysing Australia's performance in the adult equivalent to PISA, the Programme for International Assessment of Adult Competencies (PIAAC), which was most recently conducted in Australia in 2011–2012 *(e.g., see Turner & Tout, 2016)*. The data from both PISA and PIAAC indicate clearly that Australian students and adults perform significantly better in reading literacy than in numeracy.

This evidence indicates we need to do more about improving the ability of our students to be able to understand, use and apply mathematics in their lives – to become more numerate.

The OECD released a targeted country report on Australia's performance in PIAAC, Building Skills for All in Australia: Policy Insights from the Survey of Adult Skills *(OECD, 2017)*. This closer examination of Australia's performance revealed six key challenges, with the first three explicitly focused on numeracy:

- Numeracy represents a particular challenge in australia.
- Signs of poor numeracy performance can be traced back to initial schooling.
- Women have weaker numeracy skills than men. *(OECD, 2017, p. 9)*

The other three challenges named referred to both literacy and numeracy issues that are seen as challenges for Australia. This report adds to the evidence that in Australia we need to better address the numeracy competence of our young people (and our adults).

> ***Taken together, although Australia's average results are not poor, the challenges presented by adults with low basic skills may lead to Australia being left behind in terms of innovation and economic growth by countries that have been more successfully investing in the skills of all their people. (OECD, 2017, p. 9)***

DOES IT MATTER?

Investing in the mathematical literacy/numeracy skills of young people and adults has significant benefits – for the individual, for society and for the economy and in some areas this is more significant for numeracy (*e.g., see Bynner & Parsons, 2005; OECD 2017 & 2019; Turner & Tout, 2016*).

Longitudinal research from the UK indicates that, especially for women, low numeracy has a greater negative effect even than low literacy.

> ***For women, while the impact of low literacy and low numeracy is substantial, low numeracy has the greatest negative effect, even when it is combined with competent literacy. … Poor numeracy skills make it difficult to function effectively in all areas of modern life, particularly for women. (Bynner & Parsons, 2005, p. 7)***

ISSUES AND CHALLENGES

In addressing connections between numeracy and mathematics in schools

So how do we go about addressing this challenge? Whilst there are moves to highlight the importance of numeracy across the curriculum, as with the numeracy general capability in the Australian Curriculum, and in related work and research about the training of all teachers in numeracy *(e.g., see Forgasz, 2019; Goos et al, 2014; Goos 2016)*, there is the need for more work and to extend this approach.

There are a number of issues and challenges to face in order to move forward. These challenges include, amongst others:

- The disconnect between school maths and the real world, including the world of work.
- A lack of awareness and knowledge about the way to teach numeracy.
- The related issue of word problems in maths.

Following is a short introduction to each of these issues. The final section has some suggested professional learning activities for schools and teachers to undertake to discuss and address some of these issues and challenges.

THE DISCONNECT BETWEEN SECONDARY SCHOOL MATHS AND THE REAL WORLD

As the presented research is indicating, Australian schools generally do NOT prepare students particularly well for mathematics in the real world.

If students have little experience grappling with messy real-world situations and problems, and if they can only apply mathematical procedures when problems are packaged in very familiar ways like in a classroom, then why would we expect them to be numerate and be able to see, use and apply maths in the world outside the classroom? The AAMT research project mentioned earlier observed this, with one of the teachers involved commenting on this disconnect:

This is one of the most interesting aspects/ concepts of this project. The relationship between workplace mathematical skills and school mathematics could be described as 'distant' at best – teacher observation (AAMT & AiGroup, 2014)

So a challenge is to use ways and approaches whereby we can engage our students in real world and authentic numeracy tasks and activities.

A key to being numerate is to be able to start from a real world problem or situation and excavate the maths and formulate it as a mathematical problem to solve, as highlighted above. This implies we need to take the students out into the real world to do some mathematical investigations, or alternatively bring the real world into the maths classroom.

How this can be done is addressed in a number of the activities in the next section.

> *Despite calls from many reports, research, and mathematics organizations to contextualize mathematics, many schools continue to present mathematics as abstract, decontextualized problems. (Gravemeijer et al 2017, p. 109)*

LACK OF AWARENESS AND KNOWLEDGE ABOUT THE WAY TO TEACH NUMERACY

Because of this disconnect between school maths and the real world, including the world of work, the challenge for school teachers of maths is about how to see and incorporate numeracy as an integral part of their teaching.

A current common approach is to teach some maths content, and then maybe find an application where that maths can be used and applied. Unfortunately there are a couple of issues with that approach:

- The maths content is already specified – so the student does not need to formulate the problem or decide on what maths to use, not like in the real world.
- Often this approach uses an unauthentic context or word problem, which can have a negative impact (see below).

As in the models above, and in the AAMT research, the ability to recognise, access and identify how and when mathematics is used in the workplace and the real world is critical.

Numeracy needs to be explicitly taught – leaving it to providence will not guarantee success. We need:

- To use problems set in real world contexts.
- An explicit focus on using different mathematical processes (such as communication, modelling, devising strategies, representation, and reasoning).
- An explicit focus on all the stages of mathematical modelling (formulating, employing, interpreting/evaluating) and how to see and excavate the maths out of a context.

Teachers need to teach students how to "read" and interpret the context in which the maths is embedded, often in a physical object or in a printed or digital format. Moreover, in teaching numeracy you need to explicitly cover a range of different cognitive processes and mathematical content areas, the use of appropriate tools and representations, and the skills required to reason, argue, reflect and evaluate and then communicate the results as in the Goos et al model of numeracy described earlier.

CHALLENGES REGARDING THE VALUE OF "WORD" PROBLEMS IN MATHS

Many traditional school-based mathematical word problems often disregard and challenge students' sense making and only help serve to distance students from the real world, and the usefulness and value of mathematics. It is critical therefore in the teaching of numeracy and mathematics that a key value in mathematics is about its relationship with real world things – whereas word problems often do the opposite.

Using authentic situations in numeracy activities and tasks helps encourage a more positive disposition towards solving relevant and engaging maths problems, not irrelevant, word problems as can sometimes be met in school maths classrooms. This issue of authenticity and word problems in mathematics education has been researched and documented *(e.g., see Palm 2006, 2008; Verschaffel et al., 2009; Stacey, 2015)*.

Instead of using traditional word problems of the kind shown above, we encourage greater use of short realistic mathematics tasks more like those used in PISA. See the activity below, Engagement activity 3: the challenge of word problems, for more information.

Four horses cost as much as three cows, four sheep as much as two horses, and three lambs as much as one sheep. How many cows could I exchange for 40 lambs?

A farmer has cows and chickens. He only sees 50 legs and 18 heads. How many are cows and how many are chickens?

SO, WHAT CAN WE DO?

The approach recommended is to integrate the actions below into your classroom teaching and learning:

- Use a problem solving, investigative, open-ended approach – use real texts and real situations – make connections between maths and the real world.
- Start from the real world – teach students how to identify and extract the maths from the messy, real-life situations that they are likely to face (what can be called "excavating the maths").
- A key part of this is to make the maths explicit and then, when the need arises, or gaps in knowledge appear, teach the maths that is required: "just in time, not just in case".

For the engaged and the disengaged student, and the competent and less competent student, the context can provide the challenge, the motivation and the purpose for understanding and using mathematics. However, in order for this to work in the classroom, we need to explicitly integrate the Goos et al or PISA cycle of skills into our teaching and learning.

Two activities in the next section attempt to demonstrate how to do this:

Engagement activity 3:

The challenge of word problems.

Engagement activity 4:

Connecting to the real world – how to do it?

Note. See the Resources section at the end for some links, ideas and resources about how to do this. But one key and simple way to get involved is to participate in the International Mathematical Modeling Challenge (IM^{2}C): https://www.immchallenge.org.au/