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Interleaving Angle Variation Analysis for Variable Frequency PWM Drives

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***Abstract*—Generally, drive manufacturers include input filters and/or output filters to meet system requirements. Depending on damping, these filters have the potential to introduce resonant conditions. Often the switching frequency changes with the rotational speed of the drive machine leading to the possibility that switching harmonics align with the resonant peak of the filters. As the frequency of the drive decreases, the number of pulses per half cycle of the output fundamental can increase to maintain a reasonably constant switching frequency. For drives that utilize interleaving, another possibility investigated in this paper is to dynamically alter the phase angle between interleaved converters. This allows selective elimination of harmonics near the filter resonance while reducing losses due to a lower switching frequency. Typically, the phase shift is based on the number of system inverters and is used to increase the effective switching frequency. The phase-shifting strategy in this paper differs from the traditional method by targeting specific harmonics of interest. This paper[1] describes the details of implementing strategies to dynamically alter the phase shift between interleaved converters to eliminate harmonics.**

Keywords— variable speed drives, interleaving, stacked phase-legs, harmonic elimination.

I. INTRODUCTION

There are a number of methods which may be employed to limit the number of harmonics in the output waveform of an inverter. Some of these methods include: (1) selection of the modulating waveform (i.e. sawtooth versus triangle), (2) locking the output waveform frequency to the modulating waveform, (3) interleaving stacked legs where the modulation frequency is appropriately phase-shifted, (4) pulse-width modulation (PWM) technique (i.e. various forms of space vector modulation (SVM) or analog modulation methods) [1–4].

For instance, sine triangle PWM shows a substantial reduction in both harmonic magnitude and sideband spectrum when compared to sawtooth [5–6]. Intermediate non-symmetrical modulation will also produce results that are generally less desirable than center aligned PWM.

Locking the output frequency of the inverter to switching frequency of the modulator is also desirable. Generally this is done to prevent low frequency standing waves in the output of the inverter. One typically used algorithm defines an odd integer number of pulses per half cycle k as an integer value of the system phase count (i.e. for 3-phase, k would ideally be one of the following: $k = 3, 9, 15, 21, \ldots$).

Further, other modulation characteristics may be desirable such as increasing output-to-input gain via injection or SVM, or dramatically reducing switching events via clamping. One may also want to avoid multiple switching events at any single point in time and/or compensate for various delays (i.e. IGBT turn-on or switching control). Obviously, some modulation techniques will be more favorable to harmonic reduction than others by shifting the spectrum to a more desirable band location.

In this paper, we review two basic PWM strategies for eliminating or controlling harmonics of the switching frequency. This is followed by the introduction of a new technique utilizing nonstandard phase shifting of the modulating waveforms for stacked inverter phase-legs. The technique may be used to target forbidden frequency bands that may cause instability in a filter or ancillary system.

II. ELIMINATION OF HARMONICS BY INDEPENDENT ANGLES AND SYMMETRY

PWM based strategies can eliminate or control harmonic content by creating an output switching waveform with independent angles via a desired template overlaid on a modulating waveform. For instance, a sine template overlaid on a triangle modulation waveform "automatically" creates a quarter wave of independent angles in a straightforward fashion.

A. Harmonics with Sine PWM

Sine PWM is based on the elimination of switching frequency harmonics by creating a quarter-cycle of independent angles via a sine template and a modulating waveform. However with sine PWM, all the pulse transitions in the remaining three-quarters of the cycle are angularly related. A unipolar sine weighted PWM waveform with 9-pulses per half-cycle and a modulation index of 50% is displayed in Fig. 1. The quarter wave symmetry in this waveform is easily identifiable.

[1]This work was supported by the Naval Surface Warfare Center Philadelphia Division in conjunction with the Office of Naval Research.

Typical for 3-phase systems, an odd triplet number of pulses per half cycle is used to minimize harmonics. Low slow-moving beat frequencies between the output and modulating waveforms may be eliminated by locking the sine template to the switching frequency. The spectrum of a sine PWM waveform will vary somewhat based on whether it is bipolar or unipolar [7]. Larger harmonic spectrum disparity will result as pulse count changes (i.e. the switching frequency with respect to the inverter output frequency).

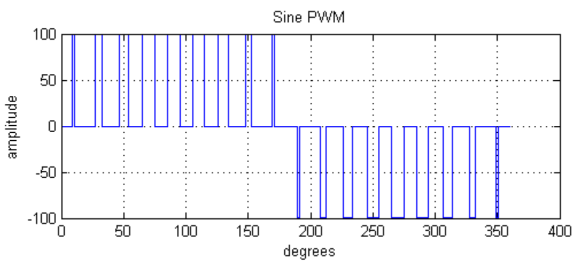

Fig. 1 – Unipolar Sine PWM with 9-Pulses/Half-Cycle.

As a rule of thumb, sine PWM with a locked odd integer number of pulses per half cycle produces upper harmonics with sidebands that are at integer multiples of the switching frequency. However, the modulation index generally affects the sideband spectrum dramatically. Figures 2 and 3 display the spectrum for a typical unipolar sine-triangle (center-aligned) PWM waveform with 9 pulses per half cycle for a modulation index of 5% and 95%, respectively. The magnitudes of the harmonics in the figures have been normalized to the fundamental which has a per unit value of 1.0. When comparing the two figures, it is apparent that a greater modulation index reduces the relative magnitudes of the harmonics while spreading the sidebands. As a note with 9-pulse per half cycle PWM, we would expect the following harmonic pairs at a minimum: $h \in \{17,19,35,37,53,55,...\}$.

Ideally when a harmonic is targeted for elimination, it would be nice to also remove the sidebands associated with that harmonic. The final approach presented in this paper does just that.

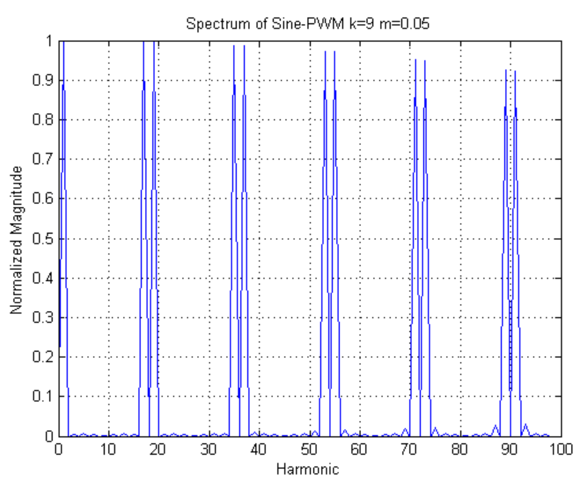

Fig. 2 – Bipolar Sine-PWM with 5% Modulation Index.

B. *Two Independent Angles*

We have seen sine-triangle PWM, but what happens if we want to individually control specific harmonics? In reality, it is possible to control or eliminate any number of individual harmonics given a quarter wave symmetrical PWM waveform where the number of independent control angles matches the number of harmonics one wishes to control or eliminate. With two independent switching times, a simplistic demonstration of harmonic reduction may be shown. Figure 4 shows a symmetrical bipolar PWM waveform with two independent time transition locations in a quarter cycle. Waveform symmetry is maintained such that the function is 'odd' and therefore only odd order harmonics may exist. Since there are two independent times where $t_1 < t_2$ and $t_2 < 0.25T$, two harmonics may be controlled or eliminated. Given more independent times (angles), more harmonics may be controlled. The general equation for the magnitude of the harmonics B_h for multiple independent angles is as follows where h is the harmonic number, T is the period of the output waveform, v_L is the time varying PWM waveform and ω_0 is the frequency in rad/sec of the output waveform [8].

$$B_h = \frac{2}{T}\int_0^T v_L(t)\sin(h\omega_0 t)\,d(\omega_0 t) \qquad (1)$$

The solution for the magnitude of the harmonics B_h for two independent angles is as follows where E is the magnitude of the dc bus voltage of an H-bridge inverter.

$$B_h = \frac{8E}{\omega_0 T}\left\{\int_0^{\alpha_1}\sin(h\omega_0 t)\,d(\omega_0 t) + \int_{\alpha_2}^{\pi/2}\sin(h\omega_0 t)\,d(\omega_0 t)\right\}$$

$$B_h = \frac{4E}{h\pi}\left(1-\cos(h\alpha_1)+\cos(h\alpha_2)\right) \qquad (2)$$

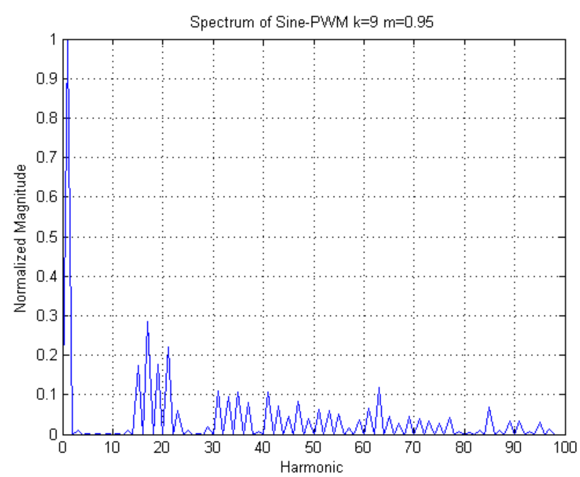

Fig. 3 – Bipolar Sine PWM with 95% Modulation Index.

C. *Example Using Two Independent Angles*

As an example, we will determine the angles α_1 and α_2 such that $B_3 = 0$ and $B_5 = 0$ using (2).

$$B_3 = \frac{4E}{3\pi}\left(1-\cos(3\alpha_1)+\cos(3\alpha_2)\right)=0$$
$$B_5 = \frac{4E}{5\pi}\left(1-\cos(5\alpha_1)+\cos(5\alpha_2)\right)=0 \tag{3}$$

The solution for (3) appears below.

$$\begin{matrix} 1-\cos(3\alpha_1)+\cos(3\alpha_2)=0 \\ 1-\cos(5\alpha_1)+\cos(5\alpha_2)=0 \end{matrix} \Rightarrow \begin{matrix} \alpha_1 = 17.8^\circ \\ \alpha_2 = 38.0^\circ \end{matrix} \tag{4}$$

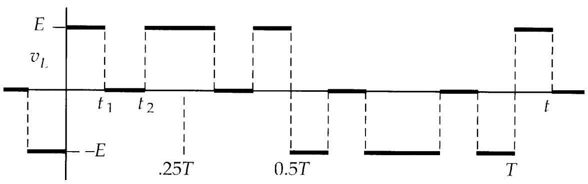

Fig. 4 – PWM Waveform with Two Independent Times [8].

III. Elimination of Harmonics via Phase-Displaced Interleaving

Now that we have explained some of the previous methods of controlling harmonics, we will introduce a new phase-shifting technique that may be used to target harmonics and their sidebands. Therefore, suppose we would like to eliminate specific harmonics of the switching frequency and its associated sidebands (i.e. 5th). Assuming we had two independently adjustable phase legs, it would be possible to phase shift 180 degrees at the 5th harmonic of the switching frequency to remove this component and its sidebands. Similarly with three phase-legs the same could be accomplished at 120 degrees. This process may be extrapolated to any number of phase-legs.

Table I – Effect of phase-leg count on harmonic elimination.

#Phase Legs	5th	10th	15th	20th	25th	30th	35th	40th
2	x	o	x	o	x	o	x	o
3	x	x	o	x	x	o	x	x
4	x	x	x	o	x	x	x	o
5	x	x	x	x	o	x	x	x
6	x	x	x	x	x	o	x	x

Legend: x – harmonic eliminated; o – harmonic present

A. Upper Harmonics and Phase-Leg Count

Consequently, there is a benefit to increasing the number of phase-legs. Table I illustrates upper switching harmonic spectrum elimination based on phase-leg count. It can clearly be seen that increasing phase-leg count for elimination of a single-harmonic, decreases the integer value harmonic content and its sidebands. However, the most electrically compact method would rely on two phase-legs.

In order to independently eliminate two selected harmonics, four independent phase-legs are needed (likewise for 3 harmonics, 8 would be needed or $2\times2\times2$). Rather than using 2^m phase-legs, alternately a composite integer *N* number of phase-legs equal to the product of *m* integers will yield a solution for *m* independently selected harmonics *h*. For example six phase-legs may be used to eliminate two independent harmonics since $6=3\times2$.

B. Illustration with Six Phase-Legs and Two Harmonics

As an example, suppose we wish to eliminate the 5th and 3rd harmonics (and sidebands) of the switching frequency with six independent phase-legs. This could be accomplished by phase shifting (in degrees) the six phase-leg in the pattern shown in Table II where each row indicates the phase shift in degrees with respect to the 1st, 5th and 3rd harmonic of the switching frequency. The phase shifts with respect to the 1st harmonic are designated as θ_i. With respect to the 5th harmonic, the first three phase-legs are separated by 120 degrees and thus cancel; likewise the second group of three also cancel. The phase angle separation with respect to the 3rd harmonic are 180 degrees apart for 1st and 4th, 2nd and 5th, and 3rd and 6th phase-legs; each group of two cancel.

Table II – Phase-angle shift for six phase-leg example.

Phase Leg # / Phase Shift	1	2	3	4	5	6
1st	0°	24°	48°	60°	84°	108°
5th	0°	120°	240°	300°	420°	540°
3rd	0°	72°	144°	180°	252°	324°

C. General Solution

Now that some specific examples have illustrated the process of switching frequency harmonic cancelation, a more general form may be stated. Given *N* phase legs (half-bridges) where $N=\prod_{j=1}^{m} n_j$ and where *m* is the number of factors in a given factorization of *N*, *m* multiples of the switching frequency (f_{sw}) may be chosen for elimination ($f_j = f_{sw}\cdot h_j$). The factors n_j are not necessarily prime, but must be integers. The harmonics associated with each f_j will be eliminated (i.e. f_j and the sidebands around f_j). Additional harmonics associated with multiples of each f_j will also be eliminated. Specifically, the harmonics associated with any p_j^k where *g* and *k* are positive integers.

$$p_j^k = h_j\cdot k \quad \text{and} \quad p_j^k \neq h_j\cdot n_j\cdot g \tag{5}$$

Each half-bridge is switched using sine-triangle (center-aligned) modulation with a delay angle of θ_i where $i\in 1,\ldots,N$ is the half-bridge number. The delay angle can be written as the sum of a series of phase components ϕ_j^k specifically expressed in the following equations.

$$\theta_i = \sum_{j=1}^{m} \phi_j^{(i-1)\bmod n_j} \quad \text{and} \quad \phi_j^k = k\frac{360}{h_j\cdot n_i} \tag{6}$$

The delay angles are given with reference to the switching frequency such that the delay time of each triangle waveform used for modulation is given by the following equation.

$$\tau_i = \frac{\theta_i}{360 f_{sw}} \qquad (7)$$

D. Example with Two Interleaved Phase-Legs

Now that a general form has been defined, we will consider a standard converter with two 180 degree interleaved half-bridges. In this case $N = 2$, $m = 1$ and $n_1 = 2$. Suppose it has a switching frequency of 1kHz and uses standard 180° interleaved PWM, then $f_{sw} = 1$ kHz, $h_1 = 1$ and $f_1 = 1$ kHz. The multiples of the switching frequency whose associated harmonics are eliminated are as follows.

$$p_1^1 = 1 \quad (k = 1 \ \& \ p_1^1 \neq 1 \cdot 2g \ \text{ for any } g)$$
$$p_1^3 = 3 \quad (k = 3 \ \& \ p_1^3 \neq 1 \cdot 2g \ \text{ for any } g)$$
$$p_1^5 = 5 \quad (k = 5 \ \& \ p_1^5 \neq 1 \cdot 2g \ \text{ for any } g)$$
$$\vdots$$

The phase components of the two converters are as follows.

$$\phi_1^0 = 0$$
$$\phi_1^1 = 1 \cdot \frac{360}{1 \cdot 2} = 180°$$

The delay angles and times of the two converters are as follows.

$$\theta_1 = \phi_1^0 \ \text{ and } \ \tau_1 = 0$$
$$\theta_2 = \phi_1^1 \ \text{ and } \ \tau_2 = 0.5\text{ms}$$

E. Example with Six Interleaved Phase-Legs

Let us consider another converter with six interleaved half-bridges. We wish to eliminate the 5th and 6th harmonic components and sidebands. In this case, $N = 6$, $m = 2$, $n_1 = 2$ and $n_2 = 3$. To eliminate the 5th and 6th harmonic components and sidebands, we could choose $h_1 = 6$ and $h_2 = 5$. However, choosing $h_1 = 6$ and $h_2 = 1$ eliminates more harmonics of the switching frequency including the 5th making it a better choice.

Once again, suppose we have a switching frequency of 1kHz, then $f_{sw} = 1$ kHz, $h_1 = 6$ and $f_1 = 6$ kHz, and $h_2 = 1$ and $f_2 = 1$ kHz (where $j \in \{1,2\}$). The multiples of the switching frequency whose associated harmonics are eliminated are as follows.

$$p_1^1 = 6 \quad (k = 1 \ \& \ p_1^1 \neq 6 \cdot 2g \ \text{ for any } g) \quad f = 6 \text{ kHz},$$
$$p_1^3 = 18 \quad (k = 3 \ \& \ p_1^3 \neq 6 \cdot 2g \ \text{ for any } g) \quad f = 18 \text{ kHz},$$
$$p_2^1 = 1 \quad (k = 1 \ \& \ p_2^1 \neq 1 \cdot 3g \ \text{ for any } g) \quad f = 1 \text{ kHz},$$
$$p_2^2 = 2 \quad (k = 2 \ \& \ p_2^2 \neq 1 \cdot 3g \ \text{ for any } g) \quad f = 2 \text{ kHz},$$
$$p_2^4 = 4 \quad (k = 4 \ \& \ p_2^4 \neq 1 \cdot 3g \ \text{ for any } g) \quad f = 4 \text{ kHz},$$
$$p_2^5 = 5 \quad (k = 5 \ \& \ p_2^5 \neq 1 \cdot 3g \ \text{ for any } g) \quad f = 5 \text{ kHz},$$
$$p_2^7 = 7 \quad (k = 7 \ \& \ p_2^7 \neq 1 \cdot 3g \ \text{ for any } g) \quad f = 7 \text{ kHz},$$
$$p_2^8 = 8 \quad (k = 8 \ \& \ p_2^8 \neq 1 \cdot 3g \ \text{ for any } g) \quad f = 8 \text{ kHz},$$
$$p_2^{10} = 10 \quad (k = 10 \ \& \ p_2^{10} \neq 1 \cdot 3g \ \text{ for any } g) \quad f = 10 \text{ kHz},$$
$$p_2^{11} = 11 \quad (k = 11 \ \& \ p_2^{11} \neq 1 \cdot 3g \ \text{ for any } g) \quad f = 11 \text{ kHz},$$
$$p_2^{13} = 13 \quad (k = 13 \ \& \ p_2^{13} \neq 1 \cdot 3g \ \text{ for any } g) \quad f = 13 \text{ kHz},$$
$$p_2^{14} = 14 \quad (k = 14 \ \& \ p_2^{14} \neq 1 \cdot 3g \ \text{ for any } g) \quad f = 14 \text{ kHz},$$
$$\vdots$$

The phase components of the six converters are as follows.

$$\phi_1^0 = 0\,; \qquad \phi_1^1 = 1 \cdot \frac{360}{6 \cdot 2} = 30°$$
$$\phi_2^0 = 0\,; \qquad \phi_2^1 = 1 \cdot \frac{360}{1 \cdot 3} = 120°\,; \qquad \phi_2^2 = 2 \cdot \frac{360}{1 \cdot 3} = 240°$$

The delay angles and times of the six converters are as follows.

$$\theta_1 = \phi_1^0 + \phi_2^0 = 0° \quad \text{and} \quad \tau_1 = 0.0 \ \mu\text{s}$$
$$\theta_2 = \phi_1^1 + \phi_2^1 = 150° \quad \text{and} \quad \tau_2 = 416.7 \ \mu\text{s}$$
$$\theta_3 = \phi_1^0 + \phi_2^2 = 240° \quad \text{and} \quad \tau_3 = 666.7 \ \mu\text{s}$$
$$\theta_4 = \phi_1^1 + \phi_2^0 = 30° \quad \text{and} \quad \tau_4 = 83.3 \ \mu\text{s}$$
$$\theta_5 = \phi_1^0 + \phi_2^1 = 120° \quad \text{and} \quad \tau_5 = 333.3 \ \mu\text{s}$$
$$\theta_6 = \phi_1^1 + \phi_2^2 = 270° \quad \text{and} \quad \tau_6 = 750.0 \ \mu\text{s}$$

IV. Automated Rule Generation

The above elimination process may be utilized to avoid resonant peaks in the converter or ancillary systems (i.e. mechanical drive). A Matlab script file was generated to demonstrate one possible process for the selection of harmonics targeted for elimination. The script operates using a parameterized forbidden region with a specified minimum and maximum frequency ($f_{0\min}$ and $f_{0\max}$) which defines a band. The output of the script produces a table with defined frequency regions and the corresponding harmonics which should be eliminated when the switching frequency is within each region.

We first compute the lowest switching frequency $f_{\min}$ that can successfully cancel all harmonics within the forbidden region. The equation for the minimum switching frequency is listed below.

$$f_{\min} = \frac{f_{0\max} - f_{0\min}}{m}$$

The algorithm to construct the harmonic elimination rules is performed as follows using a Matlab script file.

1) Set $f_k = f_{\min}$.
2) Calculate the highest harmonic of f_k above $f_{0\min}$: $h_{low} = \text{ceil}(f_{0\min}/f_k)$.

3) Calculate the lowest harmonic of f_k below $f_{0\max}$: $h_{high} = \text{floor}(f_{0\max} / f_k)$.
4) Record the set of harmonics to eliminate: $H_k = \{h_{low}, h_{low}+1, ..., h_{high}\}$.
5) Find the next frequency f_{k+1}, this either occurs at the next highest frequency where one of the harmonics enters the forbidden region (f_{enter}) or the next highest frequency where one of the harmonics leaves the forbidden region (f_{leave}). $f_{enter} = f_{0\min} / (h_{low} - 1)$ and $f_{leave} = f_{0\max} / (h_{high})$. Select $f_{k+1} = \min(f_{enter}, f_{leave})$.
6) Record the frequency interval $F_k = [f_k, f_{k+1}]$.
7) Return to step 2 and repeat until $f_k > f_{0\max}$.
8) For any H_k with less than *n* harmonics, add harmonics starting with $h = 1$ and working up until all H_k contain *n* harmonics.

As an example of the above algorithm, a set of rules was generated for a converter with four phase-legs ($m = 4$). The forbidden region in this example is between 6kHz and 8kHz. The resulting set of rules generated is given in Table III. The frequency intervals have defined min and max frequencies and corresponding harmonics that should be eliminated when the switching frequency falls within each interval. With these rules applied, the harmonics of the switching frequency that fall within the forbidden region may be selected for elimination. This table could be applied to a variable speed drive (VSD). As the switching frequency changes, the table above can be used in conjunction with the previously defined equations to dynamically select delay angle for each phase-leg in order to avoid stimulating a system resonance.

Table III – Sample Rules Generated from Script File.

Interval #	$f_{0\min}$ (Hz)	$f_{0\max}$ (Hz)	h_1	h_2
1	1000	1143	6	7
2	1143	1200	1	6
3	1200	1333	5	6
4	1333	1500	1	5
5	1500	1600	4	5
6	1600	2000	1	4
7	2000	2667	1	3
8	2667	3000	1	2
9	3000	4000	1	2
10	4000	6000	1	2

V. CONCLUSION

This paper presented a brief overview of sine triangle PWM control and some basic principles used to limit harmonics in the output voltage waveform. In general, these methods leave an upper frequency spectrum related to the switching frequency with very little ability to choose a specific band of frequencies to eliminate (harmonic and sidebands). When using sine PWM, most designers are only concerned with the noise produced by the switching frequency, but not necessarily its harmonics. Further, locking and/or requiring an integer value of pulses per half cycle requires variability in the switching frequency and half-cycle integer pulse count. This is necessary to maintain a relatively small band of switching frequencies that are compatible with an inverter's output filter cutoff frequency (i.e. as output frequency is reduced, the half-cycle pulse count must be increase).

It was also demonstrated that independently controllable angles in a symmetrical PWM waveform may be used to target specific harmonics for elimination or control. In actuality, sine triangle PWM "automatically" produces the necessary independent angles on the first quarter of the PWM waveform where the remaining three-quarters contains related angles (and thus exhibits quarter wave symmetry).

The new method presented in this paper utilizes non-standard phase-shifting of stacked inverter phase-legs to eliminate targeted harmonics as well as their sidebands. For drives that utilize interleaving, this method allows selective elimination of harmonics near the filter or other connected system resonance while potentially reducing losses due to a lower switching frequency. Typically, the phase shift is based on the number of system inverters and is used to increase the effective switching frequency. The phase-shifting strategy in this paper differs from the traditional method by targeting specific harmonics of interest.

The control strategy has been verified using an angle selection script file as well as brief modeling effort. It will be apparent that such a strategy is very useful on any drive where resonance and switching losses may be a concern.

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