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ELASTIC ELECTRON DEUTERON SCATTERING

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Measurements of the ratio of the deuteron to proton electric form factors were made for low q. The rms radius of the deuteron structure factor was found to be 1.9635 ± 0.0045 fm, yielding an rms charge radius of 2.095 ± 0.006 fm.

Measurements have been made of the ratio of the elastic e-^{2}H to e-^{1}H scattering cross sections in the range of momentum transfers $q^2 = 0.05 - 0.5$ fm^{-2} in order to substantially improve upon the accuracy obtained earlier at the same laboratory by Bumiller et al. [1]. The major improvements were better statistics, the use of H_2 and D_2 gas targets in place of CH_2 and CD_2 polyethylene targets and numerous improvements in accelerator stability and background.

As in the earlier experiment the object was to yield information concerning the deuteron structure and the electric form factor of the neutron. Ratio measurements were made to cancel systematic uncertainties of the experimental apparatus.

The accelerator produces a beam of energy up to 105 MeV, which passes through the two-magnet achromatic deflection system, target chamber and secondary emission beam monitor, located near the target. The scattered electrons are analyzed by a 40 cm radius, 120° double-focussing spectrometer and detected by a 10 channel array of plastic scintillators. For these experiments, the four counters near the center of the ladder were employed, and calculations were made for ratios of D to H scattering for each counter individually. This eliminated the need to know the relative efficiency of each counter. The beam energy was changed for corresponding H and D runs so that the scattered electron had the same energy to assure that the scattering geometry, spectrometer transmission and counter efficiency were the same for both hydrogen and deuterium scattering events.

The gas target cell used in all runs was at a pressure of 13.4 kg cm^{-2} and at liquid nitrogen temperature. Approximately the same beam current was used in corresponding H and D runs to insure that target heating effects would be the same. The number of molecules per cm^3 in the ratio of densities of H and D was calculated using the virial coefficients for these gases [2]. The total variation in target properties contributed less than 0.1% error to the ratio determinations. Thirty-three separate ratio experiments [3] were performed at nine values of q^2, with several different values of the energy and angle for a given q^2.

The ratio of the D to H elastic scattering yields the ratio of the corresponding cross sections, after appropriate radiative, bremsstrahlung and ionization corrections are applied (see refs. [4–6], respectively). The ratio of such corrections differed from unity by less than 0.05%. From the ratio of the cross sections, the ratio G_{ED}/G_{EP} was calculated, using a scaling relation to correct for the small (< 1%) magnetic contribution to the D cross section, neglecting the quadrupole contribution (< 0.05%), and assuming that the deVries [7] b' fit describes the proton scattering. Finally, the equation relating the experimental results to the important physical quantities is

$$G_{ED}/G_{EP} = (G_{EN}+G_{EP})(G_{EP})^{-1}\, C_E(1-q^2/8M_p^2), \tag{1}$$

where $C_E(q^2)$ is the deuteron charge structure factor determined from the deuteron wave function, and the term in the square bracket is the relativistic correction of Casper and Gross [8] in the form quoted in ref. [9]; M_P is the proton mass (C_E is equivalent to D_C in ref. [1]).

If C_E is assumed to be known, then the experimen-

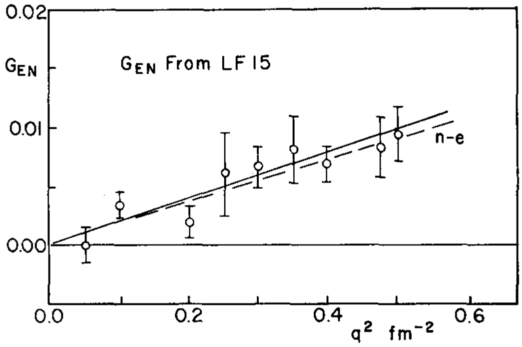

Fig. 1. G_{EN} versus q^2 compared with the neutron-electron interaction slope of ref. [13]. Deuteron model LF15 has been assumed in this plot. The solid line is the slope of G_{EN} obtained from this experiment and the dashed line is that predicted by the thermal neutron-electron interaction. For this graph we have assumed $a = 0$ and $c = -0.0036$ as discussed in the text.

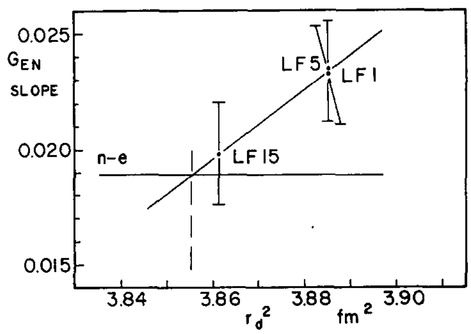

Fig. 2. Extrapolation of the deuteron charge structure radius, r_d, using the prescription of Schumacher and Bethe [9].

tally obtained ratio G_{ED}/G_{EP} may be used to calculate the neutron electric form factor, G_{EN}. This was done for three boundary condition deuteron wave functions of Lomon and Feshbach [10], designated LF1, LF5 and LF15 which correspond to D state probabilities of 4.57%, 5.20% and 7.53%, respectively. The values of C_E were furnished by Lomon [11]. For each wave function, the values of G_{EN} were fitted to a polynomial of the form

$$G_{EN} = a + bq^2 + cq^4 . \qquad (2)$$

This equation was fitted with various assumptions in order to test the data. Table 2 contains the results obtained by setting a and/or $c = 0$. With $c = 0$, the small values of the intercept, a, give credibility to the consistency of the data with the physical requirement that $a = 0$. Our data do not determine c because of the small values of q^2.

In our subsequent analysis the value $c = -0.0036$ fm^4 was adopted, according to the results of the high-q experiment of Galster et al. [12], with $a = 0$. It can be seen that there is an effect on the slope of G_{EN}, but these changes caused in b are well within the uncertainties of the data. For all fits, the value of b associated with deuteron model LF15 most closely approximates the value 0.0189 ± 0.0004 fm^2 obtained from thermal neutron-electron scattering by Krohn and Ringo [13].

A further analysis of the data, proposed by Schumacher and Bethe [9], uses the same experimental data to obtain the rms structure radius, r_d, defined by

$$C_E(q^2) = 1 - r_d^2 q^2/6 + \ldots ; \qquad (3)$$

Table 1
Average G_{ED}/G_{EP} and G_{EN} obtained using various deuteron models.

q^2(fm^{-2})	G_{ED}/G_{EP}	G_{EN}(LF1)	G_{EN}(LF5)	G_{EN}(LF15)
0.05	0.9690 ± 0.0015	0.0002 ± 0.0015	0.0002 ± 0.0015	0.0000 ± 0.0015
0.10	0.9431 ± 0.0010	0.0037 ± 0.0011	0.0037 ± 0.0011	0.0033 ± 0.0011
0.20	0.8892 ± 0.0012	0.0027 ± 0.0013	0.0028 ± 0.0013	0.0020 ± 0.0013
0.25	0.8687 ± 0.0030	0.0070 ± 0.0034	0.0070 ± 0.0034	0.0061 ± 0.0034
0.30	0.8463 ± 0.0016	0.0076 ± 0.0019	0.0076 ± 0.0019	0.0066 ± 0.0019
0.35	0.8259 ± 0.0023	0.0093 ± 0.0027	0.0094 ± 0.0027	0.0082 ± 0.0027
0.40	0.8044 ± 0.0016	0.0084 ± 0.0020	0.0084 ± 0.0020	0.0070 ± 0.0020
0.477	0.7756 ± 0.0020	0.0098 ± 0.0025	0.0099 ± 0.0025	0.0083 ± 0.0025
0.50	0.7678 ± 0.0018	0.0106 ± 0.0023	0.0107 ± 0.0023	0.0090 ± 0.0023

Table 2
Slope of G_{EN} as determined by fitting G_{EN} to the function $G_{EN} = a + bq^2 + cq^4$.

Type of fit	Model	a	$b(\text{fm}^2)$	$c(\text{fm}^4)$
Linear, intercept fitted	LF1	0.0003 ± 0.0010	0.0208 ± 0.0040	0
	LF5	0.0003 ± 0.0010	0.0210 ± 0.0040	0
	LF15	0.0002 ± 0.0010	0.0177 ± 0.0040	0
Linear, intercept zero	LF1	0	0.0218 ± 0.0022	0
	LF5	0	0.0220 ± 0.0022	0
	LF15	0	0.0185 ± 0.0022	0
Quadratic, c assumed	LF1	0	0.0232 ± 0.0022	−0.0036
	LF5	0	0.0232 ± 0.0022	−0.0036
	LF15	0	0.0198 ± 0.0023	−0.0036
n-e interaction [13]			0.0189 ± 0.0004	

Schumacher and Bethe noted that different wave functions yield different values of b using the same experimental data set. A value of r_d is also associated with each wave function. If these values of b are plotted as a function of the corresponding r_d, the points obtained from various wave functions lie on a straight line. Then, if the value of b is required to agree with the thermal n-e scattering experiment, the value of r_d is determined. By this procedure, $r_d = 1.9635 \pm 0.0045$ fm is obtained, and a similar interpolation procedure for the effective range yields $\rho = 1.751 \pm 0.014$ fm. The accuracy of our data is not sufficient to determine the sign of the shape term in the effective range expansion [14]. Our result is between the two possibilities and the error flags overlap them.

From the value of r_d, the rms charge radius may be found from

$$r_{ch}^2 = r_d^2 + r_p^2 - 6b. \tag{4}$$

This yields $r_{ch} = 2.0952 \pm 0.0060$ fm using the values $r_d = 1.9635$ fm and $r_p = 0.805 \pm 0.011$ fm from Hand, Miller and Wilson [15]. This changes the theoretical value of the Lamb shift, as quoted by Erickson [16] to 1059.283 ± 0.024 MHz, compared with the experimental value 1059.28 ± 0.06 MHz. The change produced by the new value for r_{ch} is not very large, but more significant is the fact that the contribution to the error in the frequency due to the error in r_{ch} is only 0.0026 MHz, which is a significant reduction from the corresponding value 0.050 MHz quoted earlier by Erickson and Yennie [17].

Finally, it can be noted that a different calculation of r_{ch} may be made if one of the wave functions is assumed to be correct, and the interpolation procedure is not used; LF1, 5 and 15 yield 2.1036 ± 0.0052 fm, 2.0960 ± 0.0052 fm and 2.0954 ± 0.0052 fm, respectively. These, as well as the value quoted earlier from the interpolation calculation, are so close that, in a limited sense, the quoted value of r_{ch} may be considered to be model independent.

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