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MASTER'S THESIS

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*Ljubljana, 2020*

**THE ROLE OF MANIPULATIVE TOOLS ON TEACHING AND  
LEARNING MULTIPLICATION AND DIVISION IN SECOND GRADES  
IN KOSOVO**

**Vloga didaktičnih pripomočkov pri učenju in poučevanju množenja in deljenja v 2.  
razredu osnovne šole na Kosovem**

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## ACKNOWLEDGEMENT

Conducting this research required hard work, dedication, as well as assistance from others. The greatest supporters while doing this research were my family, my mentor professor, the principal and teachers of "Vaso Pash Shkodrani" school and the students with whom the research was conducted and their parents.

My work and success to date would never have been possible without the support of my family, who have supported me at every step since the beginning of my studies.

Special thanks go to Professor Mentor, Dr. Vida Manfreda Kolar, who constantly offered me help, cooperation and support regarding the research, not only professionally but in a friendly way as well. Thanks to her motivating me, I consider that this paper has all the elements it should have and has been implemented as planned.

I also thank the school management for making this research possible and the teachers who supported, helped, and collaborated with me.

I also have to specially thank the students who participated in this research and did an amazing job while conducting the research.

*Thank you!*

## LIST OF FIGURES

<i>Figure number</i>	<i>Scheme</i>	<i>Page</i>
<b>Fig.1</b>	a) Introducing multiplication by equal groups b) Introducing multiplication by commutative property	8-9
<b>Fig.3</b>	a) Dividing apples in a partitive way b) Dividing apples in a measurement way	10
<b>Fig.3</b>	An example of what CRA would look like	12
<b>Fig.4</b>	Representing multiplication and division using sets	22
<b>Fig.5</b>	Successive clarification of word problems	22
<b>Fig.6</b>	Division separated into equal parts	23
<b>Fig.7</b>	Multiplication in relation to addition	23
<b>Fig.8</b>	The commutative property	23
<b>Fig.9</b>	The distribution property	23
<b>Fig.10</b>	The associative property	24
<b>Fig.11</b>	Applying multiplication, addition and division in a task	24
<b>Fig.12</b>	Various word problems with multiplication and division	24
<b>Fig.13</b>	The students completing the pre-test	30
<b>Fig.14</b>	Cubes	32
<b>Fig.15</b>	Clothespins and plastic plates	32
<b>Fig.16</b>	Popsicle sticks with paper cups	32

<b>Fig.17</b>	Beads with muffin liners	32
<b>Fig.18</b>	a) Solving the task $3 \times 6$ using sticky figures b) Solving the task $3 \times 6$ using beads	33
<b>Fig.19</b>	a) A student solving the task $5 \times 6$ b) Solving the task $5 \times 6$ task layout with sticks	33
<b>Fig.20</b>	Presenting the multiplication task of $7 \times 6$ through pictures	34
<b>Fig.21</b>	a) Presenting the multiplication task $4 \times 6$ through pictures b) The layout of the task $4 \times 6$ by using clothespins	35
<b>Fig.22</b>	The layout of the task $5 \times 6$ in the control class	36
<b>Fig.23</b>	The layout of the task $8 \times 6$ in the control	36
<b>Fig.24</b>	The layout of the first word problem	37
<b>Fig.25</b>	The layout of the second word problem	37
<b>Fig.26</b>	The activity using hands	39
<b>Fig.27</b>	a) Presenting the task $18:8$ through pictures b) The layout of task $18:6$ using cubes c) The layout of task $18:6$ using clothespins and plates	40
<b>Fig. 28</b>	a) Dividing number 42 into 6 equal parts b) The layout of task $42:6$ using cubes	41
<b>Fig. 29</b>	Division tasks with number 6	42
<b>Fig. 30</b>	Word problem drawn on student notebook	43
<b>Fig. 31</b>	a) Drawing groups to divide elements equally b) Drawing elements one by one to each group c) Finding the result of the task	44
<b>Fig. 32</b>	Solving tasks in the notebook	44
<b>Fig. 33</b>	Multiplication tasks and the manipulative tools placed near the blackboard	46

<b>Fig. 34</b>	a) The student solving the task by using clothespins and plates b) Writing the task down on the board c) Putting the result at the task	46
<b>Fig. 35</b>	Presenting the second task on the board	47
<b>Fig. 36</b>	Presenting the third task on the board	48
<b>Fig. 37</b>	Solving tasks using cubes	49
<b>Fig. 38</b>	Solving tasks using popsicle sticks	49
<b>Fig. 39</b>	Solving tasks using beads	49
<b>Fig. 40</b>	A student solving the task $3 \times 6$	50
<b>Fig. 41</b>	Writing the tasks down in the notebook	51
<b>Fig. 42</b>	A student finding the results by drawing the elements	51
<b>Fig.43</b>	a) Completing the post-test using the clothespins and plates b) Completing the post-test using popsicle sticks c) Completing the post-test using cubes d) Completing the post-test using beads and paper plates	52
<b>Fig. 44</b>	The students of the control class completing the post-test.	53

## LIST OF TABLES

<i>Table number</i>	<i>Scheme</i>	<i>Page</i>
<b>Table 1</b>	Comparison of control and experimental group on pre and post test	54
<b>Table 2</b>	Pre-Test and Post-Test significant comparison	55
<b>Table 3</b>	Use of manipulative tools according to a living place	57

<b>Table 4</b>	Use of manipulative tools by different ages	58
<b>Table 5</b>	The reason why teachers do not use manipulative tools	59
<b>Table 6</b>	Manipulative tools that teachers use the most	61
<b>Table 7</b>	Other materials that teachers use instead of manipulative tools	63
<b>Table 8</b>	The method by which teachers use manipulatives	64
<b>Table 9</b>	Students' attitude towards the use of manipulatives	65
<b>Table 10</b>	The role of manipulatives in teaching multiplication and division	66
<b>Table 11</b>	The difficulty of managing the lesson while using manipulative tools	67
<b>Table 12</b>	Teachers' beliefs about the role of manipulatives on students' achievement	69
<b>Table 13</b>	Teachers' beliefs about the students' confusion while using manipulative tools	71

#### LIST OF HISTOGRAMS

<i>Histogram number</i>	<i>Skema</i>	<i>Page</i>
<b>Histogram 1</b>	Use of manipulative tools according to a living place	57
<b>Histogram 2</b>	Use of manipulative tools by different ages	58
<b>Histogram 3</b>	The reason why teachers do not use manipulative tools	59
<b>Histogram 4</b>	Manipulative tools that teachers use the most	61
<b>Histogram 5</b>	Other materials that teachers use instead of manipulative tools	63



<b>Histogram 6</b>	The method by which teachers use manipulatives	64
<b>Histogram 7</b>	Students' attitude towards the use of manipulatives	65
<b>Histogram 8</b>	The role of manipulatives in teaching multiplication and division	66
<b>Histogram 9</b>	The difficulty of managing the lesson while using manipulative tools	67
<b>Histogram 10</b>	Teachers' beliefs about the role of manipulatives on students 'achievement	69
<b>Histogram 11</b>	Teachers' beliefs about the students' confusion while using manipulative tools	71

## ABSTRACT

The purpose of this research is to examine the role of manipulative tools in learning and teaching multiplication and division in second grades in Kosovo. In Kosovo there are a lot of teachers that still use the traditional method for teaching mathematics to children. Traditional method consists of learning a concept by observing the teacher “do” problems at the board and copying down the steps during note taking. Use of manipulative tools presents a more effective approach towards developing students' understanding of mathematical concepts. The usage of manipulative tools also has positive affects on students' motivation while teaching and learning mathematics. Concrete nature of manipulatives makes them particularly appropriate for kindergartners and young elementary school children. The purpose of our research is to investigate the influence of the teaching approach on pupils' understanding of multiplication and division: two different methods were compared, a traditional method which is used in most Kosovo schools and method of teaching and learning through use of manipulative tools.

This research was conducted in a selected primary school and two second-grade classes were included. The first one was the experimental group and the other one was the control group. While working with pupils, manipulative tools were used with the experimental group whereas only school textbooks were used with the control group.

The data has been derived from the assessment of the knowledge test for multiplication and division that was conducted before and after the experiment. Additionally, data were also extracted from a questionnaire for second grade teachers. The results show that the use of manipulative tools positively impacts the students' results in solving multiplication and division tasks. From the analysis of the questionnaires distributed to the teachers, it has been concluded that the place of residence does not affect the extent to which the teachers use manipulative means, but on the other hand the age of teachers influences the use of manipulative tools during the teaching of multiplication and division in the second grade – younger teachers tend to use more manipulatives the elder teachers'.

**Key words:** mathematics, teaching approach, manipulative tools, multiplication, division.

## POVZETEK

Namen te raziskave je preučiti vlogo didaktičnih sredstev pri učenju in poučevanju množenja in deljenja v drugem razredu osnovne šole na Kosovu. Na Kosovu veliko učiteljev še vedno uporablja tradicionalno metodo poučevanja matematike, kar pomeni, brez uporabe didaktičnih sredstev, ampak le knjige, zvezke, mize in krede. Če bi učitelji med poučevanjem matematike uporabljali različna manipulativna orodja, bi to otrokom pomagalo do boljšega in lažjega razumevanja. Uporaba didaktičnih sredstev pozitivno vpliva tudi na motivacijo učencev med poučevanjem in učenjem matematike. Zaradi konkretne narave ponazoril so le-ta še posebej primerna v vrtcih in nižjih razredih osnovne šole. Namen naše raziskave je raziskati vpliv učnega pristopa na razumevanje množenja in delitve učencev: primerjali smo dva različna pristopa: tradicionalnega, ki se uporablja v večini kosovskih osnovnih šol in pristop poučevanja in učenja z uporabo didaktičnih sredstev.

Raziskava je bila izvedena v izbrani osnovni šoli, vključena sta bila dva oddelka drugega razreda. Prvi oddelke je predstavljal kontrolno skupino, drugi pa eksperimentalno skupino. Pri delu z učenci so bila v eksperimentalni skupini uporabljana didaktična sredstva, pri kontrolni skupini pa le šolski učbeniki.

Podatki so bili pridobljeni iz testov znanja za množenje in deljenje, ki so bili ivedeni pred in po eksperimentu. Poleg tega so bili podatki pridobljeni tudi iz vprašalnika za kosovske učitelje drugega razreda osnovnih šol. Rezultati so pokazali, da uporaba didaktičnih sredstev pozitivno vpliva na dosežke učencev pri reševanju nalog množenja in deljenja. Na osnovi analize vprašalnikov, razdeljenih učiteljem, ugotovljamo, da lokacija prebivališče ne vpliva na to, v kolikšni meri učitelji uporabljajo didaktična sredstva, po drugi strani pa starost učiteljev vpliva na njihovo uporabo med poučevanjem množenja in deljenja v drugem razredu - mlajši učitelji ponavadi uporabljajo več didaktičnih sredstev kot starejši učitelji.

**Ključne besede:** matematika, učni pristop, didaktična sredstva, množenje, deljenje

## Table of Contents

<b>I. INTRODUCTION</b> .....	3
<b>II. THEORETICAL PART</b> .....	6
2.1 Introduction of key concepts .....	6
2.1.1 Teaching approach .....	6
2.1.2 Manipulative tools.....	6
2.1.3 Multiplication.....	8
2.1.4 Division.....	9
2.2 Concrete-Representational-Abstract Approach.....	11
2.3 Advantages of using manipulatives while teaching math.....	14
2.4 Disadvantages of using manipulatives while teaching math.....	16
2.5 The use of manipulative tools in multiplication and division .....	18
2.6 Teaching multiplication and division in Kosovo schools .....	20
<b>III. THE EMPIRICAL PART</b> .....	25
3.1 The research problem.....	25
3.2 The research questions and hypothesis .....	26
3.3 Research methodology.....	26
3.4 The school context and participants .....	27
3.5 Data collection instruments.....	27
3.6 Teaching program.....	29
3.6.1 Day one .....	29
3.6.2 Day two .....	31
3.6.3 Day three .....	38
3.6.4 Day four.....	45
3.6.5 Day five .....	51
<b>3.7 Data analyses and results</b> .....	54
3.7.1 Pre and post test result.....	54

3.7.2 Results of teachers' questionnaire .....	56
<b>IV. DISCUSSION</b> .....	73
<b>V. CONCLUSION</b> .....	75
REFERENCES.....	79
APPENDIXIS .....	85
Appendix A.....	86
Appendix B.....	87
Appendix C.....	88
Appendix D.....	90

## I. INTRODUCTION

Every branch of mathematics and every mathematical concept has its own importance and it is applied in everyday life. Math is everywhere around us, it helps us in our daily lives because math is present in a type of a clock that wakes us up in the morning at the set time, the number of chairs one has at home, the amount of milk a child needs to drink during the day, the number of buttons on a shirt, the wooden cubes the child plays with every day, the grouping of toys according to certain criteria, etc. It follows us through every step of life; therefore, it is important to have knowledge of it from childhood, and this possession of knowledge for children should be seen by teachers as the main objective in their profession.

The Kosovo Curriculum Framework presents mathematics as an important subject area for the development of thinking, learning and working competencies. However, in addition to these competences, mathematics also helps to develop communication competence through the use of different codes and symbols, while through the development of logical reasoning which is characteristic of this subject, it also contributes to the development of personal and civic competence (Korniza e Kurrikulit të Kosovës,2016).

Multiplication and division as mathematical concepts are taught in schools starting from the lower grades and continuously, while in real life one faces them every day. The notion of multiplication and division is used for the first time in the second grade in Kosovo (at the age of 7) but this age may not be appropriate to understand multiplication and division logically.

The teaching of multiplication and division in this period should be seen as a brief introduction to students with the notion of multiplication and the notion of division or division as a mutual multiplication operation. It is important for students to learn multiplication and division because they are indispensable in every job and occupation a person deals with in his or her life, and thus, it is best to acquire them as a child as they create opportunities for elaboration and absorption of knowledge regarding various natural phenomena. Learning multiplication and division is of great importance to everyone, and especially to children, because higher level of thinking and reasoning is achieved concerning various problems of daily life.

Having this issue into consideration, the task of a teacher is to deal more with the learning of multiplication and division, and to combine the provision of new problems from daily life, which requires the active participation of all students. This implies that one should not only rely on the relevant student literature, but to also take self-initiative measures and assist students to the maximum extent in their understanding of multiplication and division, use different tools so that students understand at best and in the simplest way the notions of multiplication and division.

According to Rabano and Torres (2012), a combination of creative development and educational methodology can help to support the development of creative thinking skills. Lack of literature should not be seen as an obstacle to learning concepts in general. It should not be a barrier to student's success because the teacher is the one who pilots his/her classes and can use the tools he/she deems necessary and reasonable to achieve the goal of highlighting students' knowledge and creativity.

Besides being an educator and a pedagogue, the teacher must also be an artist and use creativity in the classroom and in teaching in order to discover students' creativity and transform the class into an entertaining environment for students, by always preparing new things and concrete materials about the lesson where the students are active participants, and not to prepare the lesson in the form of a lecture where the student comes into the listener's position and bring passivity to them, as it creates an inappropriate and monotonous class climate.

Despite the importance of the subject of mathematics, many teachers in Kosovo continue to apply traditional teaching methods relying exclusively on mathematics texts and the requirement to "fill" pages with calculations and to complete arithmetic operations (Vula & Berdynaj, 2011) Thus, a more effective approach towards teaching is needed in order to develop students' understanding of multiplication and division in the process of problem solving in mathematics (Jonassen, 2003).

Therefore, it is important for teachers as facilitators to assist students in building their mindset about the problems of multiplication and division of numbers, as well as the relationships between different factors (Griffin & Jitendra, 2009). In this research, manipulative tools were used in teaching multiplication and division in second grade. The theory that underpins manipulative tools is the cognitive theory (Musai, 1999). The manipulative tools often found in classrooms such as: tables, plates and cubes, geo charts and tangram collections have been used as a better and easier

way to help students learn math. But, according to Burns (1996) many teachers still ask: “Are manipulative tools functional? How to adapt them to my instructions? How to convince students that they are useful for learning and they are not toys? How can I communicate their value to the parents?”

Taking into account everything that was stated above, this research will examine the impact of manipulative tools on the development and understanding of multiplication and division among students. Thus, this impact on students' success in completing multiplication and division tasks among second graders will be assessed.



## II. THEORETICAL PART

In this chapter the basic notions will be explained. Additionally, some of the many researches done by various scholars on manipulative tools will be discussed; their role, advantages and disadvantages in teaching mathematical concepts, namely teaching multiplication and division in the second grade. Furthermore, the physical environment of schools in Kosovo will be discussed, as well as the goals and objectives of second-grade education in mathematics in Kosovo schools.

We are first going to present the key concepts on which our research work is based.

### 2.1 Introduction of key concepts

**2.1.1 Teaching approach** - “The arrangement is hierarchical. The organizational key is that technique carry out a method which is consistent with an approach. An approach is a set of correlative assumptions dealing with the nature of language teaching and learning. An approach is axiomatic. It describes the nature of the subject matter to be taught. Method is an overall plan for the orderly presentation of language material, no part of which contradicts, and all of which is based upon, the selected approach. An approach is axiomatic, a method is procedural. Within one approach, there can be many methods. A technique is implementational - that which actually takes place in a classroom. It is a particular trick, stratagem, or contrivance used to accomplish an immediate objective. Techniques must be consistent with a method, and therefore in harmony with an approach as well” (Anthony 1963, p. 63-7; Richards and Rodgers, 1999, p. 15).

**2.1.2 Manipulative tools** – Manipulative tools are objects that must be used and organized by students and teachers, who use them to convey abstract ideas or concepts by modeling or presenting their ideas (NCTM, 2000).

Manipulatives can be used in teaching a wide variety of topics in mathematics, including the objectives from the five NCTM standards: problem solving, communicating, reasoning, connections, and estimation. The materials should “foster children’s concepts of numbers and operations, patterns, geometry, measurement, data analysis, problem solving, reasoning, connections, and representations” (Seefeldt & Wasik, 2006, p.93). Mathematical manipulatives

can be structured or unstructured by the teacher. Teachers could use counters, place-value mats, base-ten blocks, and fraction strips while teaching from the numbers and operations standard. The counters could be used to teach one-on-one correspondence, ordinal numbers, and basic addition and subtraction. The fraction strips could be used to add and subtract fractions or to show equivalent fractions. Pattern blocks, attribute blocks and scales could be used to assist students in the learning basic algebra. Student could use geoboards when trying to identify simple geometric shapes. They could also use geometric solid models when learning about spatial reasoning. The manipulative materials should relate to the students' real world (Heddens, 1997).

Manipulative tools have been around us since the beginning of time. Manipulative tools are objects that can be used to stimulate the understanding of abstract mathematical ideas, and understanding of other subjects. These tools link theory to practice, which enables students to better and quickly understand lesson units, in particular new concepts. Children can perceive a type of didactic material in a desired way – mathematically. In such a case, didactic material functions as a representation of an abstract mathematical concept. However, didactic material can also be perceived non-mathematically, i.e. children may only see it as a physical object and may not see the mathematical relations in the background (Gravemeijer 1991).

When selecting manipulatives, Zbiek, Heid, Blume & Dick (2007) recommended that the following aspects be considered:

- Mathematical fidelity: the degree to which the mathematical object is faithful to the underlying mathematical properties of that object in the virtual environment;
- Cognitive fidelity: how well the virtual tool reflects the user's cognitive actions and possible choices while using the tool in the virtual environment;
- Pedagogical fidelity: the extent to which teachers and students believe that a tool allows students to act mathematically in ways that correspond to the nature of mathematical learning that underlies a teacher's practice.

**2.1.3 Multiplication** - Many authors have suggested that multiplication is significantly more difficult than addition and subtraction (Anghileri (2000); Davzdov (1991); Greer (1992)). Nunes and Bryant (1996) stated that a common view of multiplication and division is simply that they are

"different arithmetic operations ... learned after having learned the addition and subtraction". However, they stated that this is very limited and that in fact "multiplication and division represent an important qualitative change in children's thinking".

Multiplication is used when you count things that are divided into groups or rows, when you need to know the room space or other spaces, and when you need to know how much money it will cost to buy more than one thing (Wingard-Nelson, 2005). Geary (2008) is a mathematics teacher, he recommends to teach multiplication through an approach that combines conceptual understanding and procedural knowledge (Geary, Berch, Embretson, Rezna, & Siegler, 2008). Meanwhile Lampert (1986), a researcher of the multiplication concept, recommends to teach multiplication by combining knowledge with practice using modern manipulative tools, techniques and methods.

The multiplication of whole numbers may be thought as a repeated addition; that is, the multiplication of two numbers is equivalent to adding as many copies of one of them, the multiplicand, as the value of the other one, the multiplier.

For example, 3 multiplied by 2 (often written as  $3 \times 2$  and spoken as "3 times 2 ") can be calculated by adding 3 copies of 2 together.

$$3 \cdot 2 = 2 + 2 + 2 = 6$$

Here 3 and 2 are the factors and 6 is the product.

We imagine that we have 3 groups of 2 pears. As a result, we have:

$$3 \cdot 2 = 6 \text{ - Therefore, we have 6 pears.}$$

(the first factor)  $\cdot$  (the second factor) = product (Sometimes instead of the symbol  $\cdot$ , one may use  $\times$  as well.



Fig. 1. a.  
Introducing  
multiplication  
by equal groups

One of the main properties of multiplication is the commutative property: adding 3 copies of 2 gives the same result as adding 2 copies of 3:

$$2 \cdot 3 = 3 + 3 = 6$$

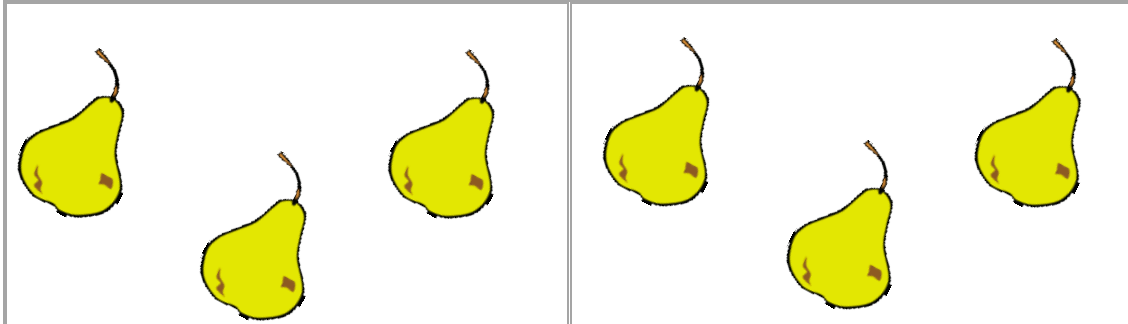


Fig. 1. b. Introducing multiplication by commutative property

Thus, the designation of multiplier and multiplicand does not affect the result of the multiplication (Devlin, Keith 2011)

Another way to approach whole-number multiplication is to use the Cartesian product of two sets.

If  $a$  and  $b$  are any whole numbers where  $a = n\{A\}$  and  $b = n\{B\}$ , then  $a \times b = n\{A \times B\}$

Solution.: We represent each number with a set, form the Cartesian product, and then count the number of ordered pairs. For instance, if we let  $A = \{a, b, c\}$  and  $B = \{1, 2, 3, 4, 5\}$  then  $n\{A\} = 3$ ,  $n\{B\} = 5$ , and

$$\begin{aligned} 3 \cdot 5 &= n\{A \times B\} \\ &= n\{a, b, c\} \cdot n\{1, 2, 3, 4, 5\} \\ &= n\{(a,1), (a,2), (a,3), (a,4), (a,5), (b,1), (b,2), (b,3), (b,4), (b,5), (c,1), (c,2), (c,3), (c,4), (c,5)\} \\ &= 15 \end{aligned}$$

**2.1.4 Division** – is the inverse operation to multiplication, and enters the set of the most difficult operations along with multiplication. The meaning of division is closely related to the common use of the verb "to divide". In fact, children easily understand that to divide a certain amount of money (or a certain number of objects) into equal parts among a certain number of people, it is necessary to divide the "amount of money (or number of objects) in as many equal parts as there are people (Boero, Ferrari, & Ferrero, 1989).

There are essentially two ways of thinking of division:

- ❖ Partition division (also known as partitive, sharing and grouping division) is a way of understanding division in which you divide an amount into a given number of groups. Dividing 10 by 2 in a partitive way involves taking 10 objects and ‘dealing’ them out evenly into two groups and we wish to know how many groups we can make.

Imagine we have 10 apples. We want to put them in two boxes so that each box has the same number of apples. How will you share them?

Each counter is moved separately and this takes 10 steps.

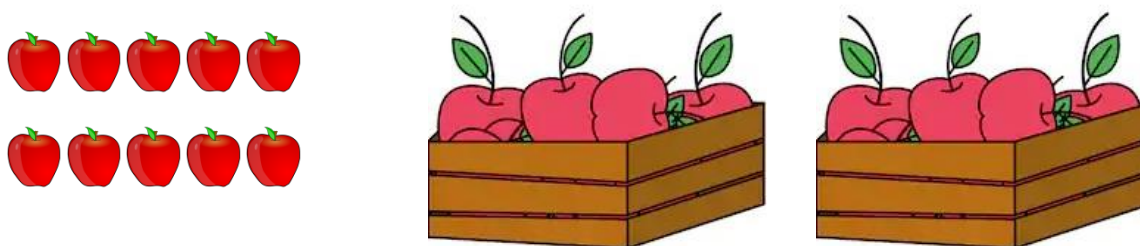


Fig. 2. a. Dividing apples in a partitive way

- ❖ Measurement division (also called repeated subtraction division) is a way of understanding division in which you divide an amount into groups of a given size. Dividing 10 by 2 in a measurement way involves taking 10 objects and making groups of 2 from them. With a small number two in each group, this is typically a faster process because 2 counters are removed at a time, so this might take only 5 steps.

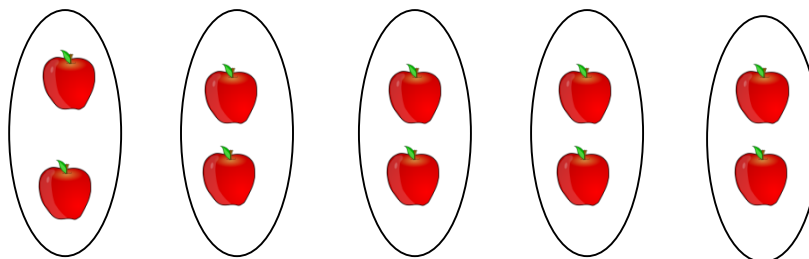


Fig. 2. b. Dividing apples in a measurement way

While the answer in both cases is the same, it represents different things.

In the partitive interpretation, 5 is the number of objects in each group  $10:2 = 5$ .

In the measurement interpretation, 5 is the number of groups.

A division is made of different numbers, and each of these numbers has a particular name.

For example the task  $10 : 2 = 5$ :

**10** is the total number of elements we have to divide and is called "divisor".

**2** is the number we want to divide with and is called "dividend".

**5** is the result of division and is called "quotient".

## 2.2 Concrete-Representational-Abstract Approach

The CRA Instructional Approach is “an intervention for mathematics instruction that research suggests can enhance the mathematics performance of students.” (Hauser,2009). The Approach is a “three-part instructional strategy, with each part building on the previous instruction to promote student’s learning and retention and to address conceptual knowledge.” The three parts are as follows:

- **Concrete:** In this stage, the teacher begins instruction by modeling each mathematical concept with concrete materials. In other words, this stage is the “doing” stage, using concrete objects to model problems.
- **Representational:** In this stage, the teacher transforms the concrete model into a representational (semi-concrete) level, which may involve drawing pictures; using circles, dots, and tallies; or using stamps to imprint pictures for counting. In other words, this is the “seeing” stage, using representations of the objects to model problems.
- **Abstract:** In this stage, the teacher models the mathematics concept at a symbolic level, using only numbers, notation, and mathematical symbols to represent the number of circles or groups of circles. The teacher uses operation symbols (+, −, x, :) to indicate addition, multiplication, or division. This is the “symbolic” stage, where students are able to use abstract symbols to model problems (Hauser,2009).

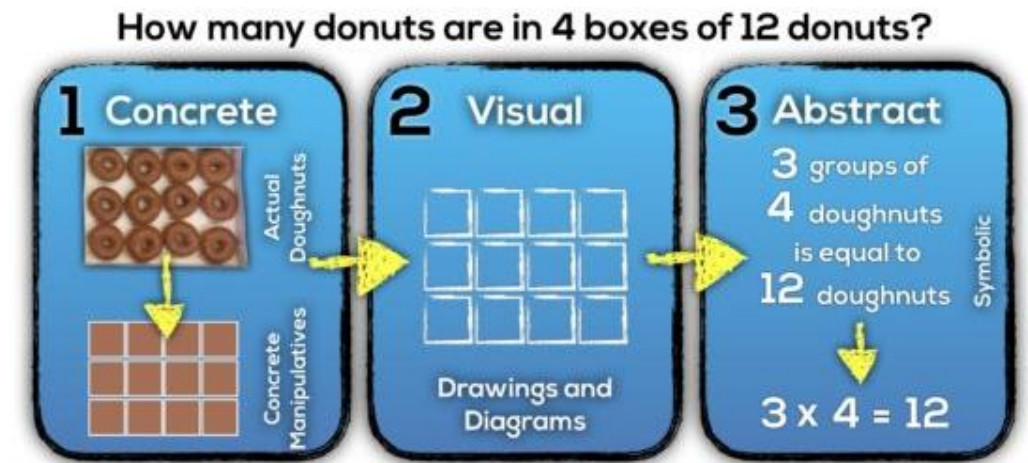


Fig. 3. An example of what CRA would look like

This instructional approach benefits all students but has been shown to be particularly effective with students who have mathematical difficulties, mainly because it moves gradually from actual objects through pictures and then to symbols (Sousa, 2007).

The CRA sequence has been shown to be effective for remediating deficits in basic mathematics computation (Harris, Miller, & Mercer, 1995; Mercer & Miller, 1992; Miller & Mercer, 1993; Morin & Miller, 1998), in placing value (Peterson, Mercer, & O’Shea, 1988), fractions (Butler, Miller, Crehan, Babbitt, & Pierce, 2003), and algebra (Maccini & Ruhl, 2000; Witzel, 2005; Witzel, Mercer, & Miller, 2003).

In the classroom, this approach is a facilitating framework for students to create meaningful connections between concrete, representational, and abstract levels of thinking and understanding. Students’ learning starts out with visual, tangible, and kinesthetic experiences to establish basic understanding, and then students are able to extend their knowledge through pictorial representations (drawings, diagrams, or sketches) and then finally are able to move to the abstract level of thinking, where students are exclusively using mathematical symbols to represent and model problems (Hauser, 2009).

Studies have shown that “students who use concrete materials develop more precise and more comprehensive mental representations, often show more motivation and on-task behavior, understand mathematical ideas, and better apply these ideas to life situations,” (Hauser, 2009). The overarching purpose of the CRA instructional approach is to “ensure students develop a tangible

understanding of the math concepts/skills they learn.” (Special Connections, 2005) Using their concrete level of understanding of mathematical concepts and skills, students are able to later use this foundation and add/link their conceptual understanding to abstract problems and learning. Making the students to go through these three steps, it provides them with a deeper understanding of mathematical concepts and ideas and provides an excellent foundational strategy for problem solving in other areas in the future (Special Connections, 2005).

Mercer and Miller (1992) taught basic addition, subtraction, multiplication, and division to students with high-incidence disabilities using either the CRA sequence or a traditional curriculum. Mercer and Miller found that students performed significantly better when instruction involved the CRA sequence and the mnemonic strategy. After CRA instruction, students demonstrated generalization by transferring basic computation knowledge to completion of one-step word problems. Miller and Mercer (1993) replicated their findings with regard to addition fact fluency and generalization to word problems. For example, when teaching a multiplication fact such as

$3 \cdot 2$ , a number sentence was translated to these words, “Three groups of two objects equals how many?”

At the concrete level, three groups of plates were arranged; two objects were placed on each plate; and the total number of objects was counted.

At the representational level, three lines were drawn to represent the groups; two tallies were drawn on each line; and the total number of tallies was counted.

The view that learning takes place when students create (construct) new mathematical knowledge by reflecting on their physical and mental actions is most derived from Piaget’s descriptive theory of development stages and Bruner’s (Bruner, 1966) prescriptive theory of modes of representations thought (Watson, J. M., Campbell & Collis, K. 1993). While Piaget suggests that intellectual development progresses through different stages in which constructions precedes analysis, Bruner thought that learning by discovery involved an internal reorganization of previously known ideas and stipulated that children move through three modes or levels of representations as they learn. In the first or enactive level, the child needs actions on materials to understand a concept. In the second or iconic level, the child creates mental representations of the objects but does not



manipulate them directly; rather, the concept is represented pictorially. Finally, in the third or symbolic level, the child is strictly manipulating symbols and does not need to manipulate objects.

### 2.3 Advantages of using manipulatives while teaching math

Based on some researches (Canny, 1984; Clements and Battista, 1990; Clements, 1999; Dienes, 1960; Driscoll, 1981; Fennema, 1972, 1973; Skemp, 1987; Sugiyama, 1987; Suydam, 1984) it is proven that students learn better by doing thus, they should be involved in different activities using manipulative tools. When conducting various educational manipulative tools or even competing with each other then they learn without noticing that they are doing math.

Browsing the different literature connected to our research, evidences show that the lack of use of manipulative tools during math classes can be noticed not only among the Kosovan society but it is also present in other developed societies as well. Therefore, many researchers have explored on finding ways to overcome this problem.

Including activities that use manipulative tools in the teaching process is an essential component for a normal developed education for every child. Teachers while using manipulative activities engage students in the teaching process increasing motivation and self-confidence, and in the meantime, students have fun. It is important for children to have a variety of materials to manipulate and the opportunity to sort, classify, weigh, stack and explore while they are constructing mathematical knowledge. "In order to have opportunities to learn math, children need firsthand experiences related to math, interaction with other children and adults concerning these experiences and time to reflect on the experiences" (Seefeldt & Wasik, 2006).

Since the early 1900s, manipulative tools have come to be considered essential in teaching mathematics at the elementary-school level. The National Council of Teachers of Mathematics (NCTM) for decades has recommended the usage of manipulative tools in teaching mathematical concepts at all grade levels. NCTM emphasizes that the curriculum should provide ways for teachers to use as many activities in mathematics.

Teachers have supported the use of manipulative tools in the subject of mathematics (Burns, 1996), based on theories claiming that children need physical referents to develop abstract concepts of mathematics (Piaget, 1952) and many researches that demonstrate advantage from the use of

manipulative tools in the subject of mathematics (Sowell, 1989). Teaching with manipulatives supports the constructivist theory that learning occurs when students construct personal meaning through hands on experiences. Piaget (1954a) considered the concrete stage a major turning point in the child's cognitive development, because it marks the beginning of logical or operational thought. The child is now mature enough to use logical thought or operations (i.e. rules) but can only apply logic to physical objects (hence concrete operational). Hiebert (1997) pointed out: "Mathematical tools should be seen as supports for learning. But using tools as supports does not happen automatically. Students must construct meaning for them. This requires more than watching demonstrations; it requires working with tools over extended periods of time, trying them out, and watching what happens. Meaning does not reside in tools; it is constructed by students as they use tools (p. 10)".

In the subject of mathematics, a manipulator is an object, which is designed in that way that a learner can understand certain mathematical concepts by manipulating with it, namely its name (Kellner, 1989). The use of manipulative tools provides a way for children to learn concepts through hands-on experience while adapting to their development.

The use of manipulative tools in mathematics classes increased significantly around the world during the second half of the 20th century (Jitendra, Griffin, Haria Leh, Adams, & Kaduvettor, 2007).

Mathematical manipulative tools provide students with a better way of understanding abstract mathematical concepts by enabling them to relate concepts more to concrete informal ideas (Uribe-Florefery & Wilkins, 2010). The use of manipulative tools to promote student learning is considered a practical pedagogical technique (Moch, 2001). As stated by Ball (1992), teaching while using manipulative means is not simply a matter of pedagogical and technical strategy. Using a manipulative model while teaching mathematics requires knowledge, skills, and experience needed to respond to students learning mathematics in this setting (Hatfield, 1994). Mathematical manipulative tools can be structured or non-structured. Structured manipulative tools used include: cubes, tangrams, rods, number patterns, color plates, blocks, colored sticks geometric figures, 3D figures, dice, domains, etc. Whereas, the manipulative tools made by the teachers themselves include: beans, bean sticks, cardboard, various relevant types of food to fit the lesson, threads and various recyclables that can be used as manipulative tools. Multiple manipulative experiences

provide children with basic conceptual understanding of mathematics at a conceptual level and are recommended by NCTM (2000).

Mathematical manipulatives play a key role in the development and understanding of mathematics among young children. These concrete objects facilitate students' understanding of important mathematical concepts, and later help them to relate these ideas to abstract representations and ideas.

Sowell (1989) conducted a comprehensive analysis of 60 students from 1967 to 1987 on the effect of using manipulative tools in mathematics, from kindergarten to college. The results showed that math achievement increased through the long-term use of manipulative tools, and that students' attitudes toward math improved positively when they were taught using concrete materials provided by their teachers.

Balka (1993) described the benefit of using manipulative tools by saying: "The use of manipulative tools allows students to make the important connection between conceptual and procedural knowledge, to know the relationships between different areas of mathematics, to see mathematics as an integrated whole using physical models and linking procedures to an equivalent representation" (p. 22). Despite the existence of many researches and the positive support regarding the use of manipulative tools in classrooms, they are not used regularly and sometimes they are not used by teachers at all.

#### 2.4 Disadvantages of using manipulatives for teaching math

However, there are also studies that do not support the use of manipulatives. These studies focus on students not making connections between multiple representations, learning procedures rather than understanding concepts, inability to transfer knowledge to new applications, and learners having fun with the manipulative and essentially utilizing it as a toy.

Ball points out that manipulatives are not magical transmitters of meaning or insight (Ball, 1992). According to Ball (1992), "although kinesthetic experience can enhance perception and thinking, understanding does not travel through the fingertips and up the arm".

Boulton-Lewis (1998) found that conceptual processing could not occur unless the student has reached a point of automaticity with the manipulative. A manipulative is an artifact not a tool if the student is constantly aware of it. In this situation, the manipulative is not helping the student move toward a goal state of understanding (Winograd and Flores, 1986).

Learners often use manipulatives in a mechanical manner, with little or no learning of the mathematical concepts behind the procedures (Hiebert and Wearne, 1992). As a result, students are often unable to make connections between their actions with manipulatives and abstract symbols (Thompson & Thompson, 1990). For teaching and learning multiplication and division up to number 10 in Kosovo schools' teachers can use 100 blocks as a manipulative tool since in most cases learners have them at home and teachers can ask to bring them to school. Learners can group or divide cubes based on the tasks with multiplication and division presented from the teacher or in their books. But they can also manipulate with them in a form of game or fun with no relation to mathematical goals on multiplication and division content.

Meira (1998) defines the concept of transparency of instructional devices as “an index of access to knowledge and activities rather than as an inherent feature of objects . . . a process mediated by unfolding activities and users' participation in ongoing sociocultural practices”.

Kim (1993), mentioned some of the challenges of applying manipulative tools:

- Time management
- Structuring, monitoring and evaluating the use of manipulative tools
- Manipulative connection to mathematical symbols and procedures
- Lack of financial resources
- Lack of professional development

A teacher who uses manipulatives in classrooms, especially in mathematics, must be a good manager, have control over the class and his or her students, because they can be noisy when using manipulatives and the situation can get out of control, and the same can happen with the lesson as well. Afterwards, a good teacher should have a plan, know how to use these tools, and when to use them, adapt them to the lesson and to the age of the students so that the lesson can be successful and effective. Moyer (2001), for example, reported that ten high school teachers use manipulative tools to teach math concepts in a one-year project. Through interviews and observations, Moyer

revealed how and why teachers use manipulatives in their classes, and showed that teachers did not always understand the purpose of using manipulatives and did not use them effectively. Consequently, a teacher must first master his or her subject matter, be a good acquaintance of the subject and then know how to use manipulatives in that subject rather than saying that manipulatives are not useful or do not have positive effects while teaching a particular topic.

Some of the objections to using manipulative tools come from the fact that these tools can make the hard work seem easy, but at the same time can mask the lack of understanding.

### 2.5 The use of manipulative tools in multiplication and division

One of the key domains in primary mathematics educational schools is the domain of multiplication and division, or multiplicative reasoning.

Several researchers have studied how young students multiply and divide. Nunes and Bryant (1996) indicated that a general point of view about multiplication and division is that they simply “are inverse arithmetical operations ... that are taught after addition and subtraction”. However, they stress that such a viewpoint is incomplete knowing the fact that “multiplication and division represent a significant qualitative change in children’s thinking.”

The most common and traditional way to learn multiplication and division is to train and practice them in everyday life (Wingard, 2005). Geary (2011) is a teacher of mathematics; he recommends that multiplication should be taught through an approach that combines conceptual understanding and procedural knowledge (Geary, Berch, Embretson, Reyna, & Siegler, 2008), while Lampert (1986), researcher of the concept of multiplication, recommends that multiplication should be taught by combining knowledge with practice using modern manipulative tools, techniques and methods.

Children can model multiplication and division using manipulative tools, pictures, diagrams, counters, blocks, shells or any materials that are available in the classroom. This will make students more creative and will bring them close to the real life by using reality. Students’ abstract thinking is closely anchored in their concrete perceptions of the world (Thompson, 1992), actively manipulating these materials allows learners to develop a repertoire of images that can be used in the mental manipulation of abstract concepts.

Most of the research points that it has been proven that students who use physical and visual manipulatives properly, to learn the concepts of multiplication and division, outperform those students who do not use these manipulatives " (Raphael and Wahlstrom, 1989, Clements,1999). Clements reports that these benefits are true no matter the level, ability or topic of knowledge. He also adds that students' attitudes towards mathematics are improved when teachers teach them using concrete materials.

According to Naiser et al., (2004), teachers need to find a variety of strategies, including the use of physical and visual manipulative tools, in order to teach multiplication and division.

“Teachers should help their students to generalize multiplication and division from their experiences with the contexts of real-world problems, with physical manipulative, visual tools, and pictures” (Bezuk and Armstrong, 1992, p.729). This will enable students to build their mathematical understanding, which is essential for a satisfactory basis for understanding the real world in the future (NCTM, 1989).

Neiser et al., (2004), found out through their research that the use of manipulative tools to teach multiplication and division made the lessons more active and provided an effective way for students to represent their thinking. They also found that teachers were able to understand students 'thinking by observing them while using manipulative tools, and on the other hand while using paper and pencils, students' 'real thinking' was not possible to be observed.

Krech (2000) through his research found that hands-on activities are the best way for students to learn about multiplication and division. He also claims that manipulative learning allows students to experience multiplication and division on a concrete level. May (1994) said that the best way for students to learn multiplication and division is to create those models that make sense of the operation. She believes that the best way to do this is to provide them with good activities and the right kind of manipulative tools. When manipulatives are not used and students simply learn symbolic patterns, they do not advance beyond simple computation and do not learn how to use patterns to solve real-life problems (Moyer and Jones, 2004). Many textbook publishers provide a kit with manipulative tools for each elementary math series. The National Council of Teachers of Mathematics (NCTM) promotes the use of manipulative tools to teach the concepts of multiplication and division and to concretize abstract ideas. Hiebert et al. (1997) point out that:

"Manipulative tools in learning multiplication and division must be seen as support while teaching them. But using them as tools does not happen automatically, students need to make sense of them and this requires more than just watching their demonstration; it requires working longer with these manipulative tools over a long period of time, by trying them out and seeing what happens. Meaning does not live on tools but it is built by students when they use those tools (p. 10)". Some of the manipulative tools that can be used precisely to teach multiplication and division are: buttons, cubes, sticks and beans. All of these can be easily used during multiplication and division instruction where students can divide them into groups based on the task requirement. For instance, to solve the task of  $3 \times 4$ , students can use buttons and present 3 sets of 4 buttons (elements) and finally count all the buttons to find the product of these two factors.

## 2.6 Teaching multiplication and division in Kosovo schools

Kosovo's schools are still not built in compliance with European standards. Most of them were built before the war and they are enormously outdated and need to be repaired or rebuild. Some schools in Kosovo also lack basic needs for students, such as water, and obviously lack of learning tools. Even 20 years after the war, our schools still do not have sufficient and appropriate learning tools such as colored chalks, the colored paper and sometimes even copy paper needed for printing (tests, texts, exercises). A worrying truth is the large number of students in classes where each class has 35-40 students and sometimes even more. It is very difficult for a single teacher to manage the classroom with such a large number of students. Many schools do not even have separate cabinets for special subjects such as math, biology, physics, etc. As long as there are no cabinets, it means that there are no concrete materials to teach students. Poor facilities conditions in schools make it impossible for schools to be provided with additional materials such as various manipulative tools, namely in mathematics. Teachers may choose not to spend money, they may use different things instead of manipulative tools such as beans, pasta, stamps, pencils, paints, but the large number of students in classrooms still makes it difficult to work with them because the involvement of all students in activities and time management becomes increasingly challenging. After the student in the first grade has passed the level of acquisition of basic meanings of mathematics (such as the notion of sets, relations, number and different geometric meanings), in the second grade we would have to expand the demands for the advancement of his/her knowledge.

Mathematics in the second grade is broadly related to the same concepts as in the first grade, but in a substantially expanded form. As in the first grade, there should still exist the intention of teachers to develop students' abilities to notice similarities and differences in shares and relations as well as the ability to make comparisons. To achieve this objective, new teaching methodologies should be put into operation during teaching.

Since our research is based on working with second grades, we will describe the curriculum of second grade more in details.

The second-grade mathematics course aims (MASHT, 2016):

- To develop students' imagination, curiosity, attention, memory and intuition;
- To enable students to express themselves correctly, clearly and accurately;
- To cultivate self-initiative, independent work and collaboration;
- To enable students to apply their acquired knowledge to daily life situations;
- To enable students to use their acquired knowledge when learning mathematics and other subjects at the highest levels of education.

From the second-grade program content the students should be able to (MASHT, 2016):

- Perform the operations of addition, subtraction, multiplication and division of numbers from 1 to 100;
- Apply the acquired knowledge in solving different daily life problems;
- Solve simple numerical equations and inequalities and apply them in solving concrete problems, etc.

Within the curriculum of Kosovo's primary education (MASHT, 2016), multiplication and division are introduced for the first time in the second grade. According to this curriculum, second graders learn the meaning of multiplication as repeated addition, and division as an inverse operation of multiplication (finding a factor, when the product and the other factor are known). As in most traditional lesson plans, these concepts are taught separately with multiplication preceding division. The teaching is very similar in most of the classes. Each teacher quite rigorously relies on school math textbooks. They use them for preparing the lesson, class organization and as



resource for students' work. Traditionally, for the first 10 weeks of the second grade, in all schools, students learn the „multiplication table“ - 10 x 10, and after that they start with division (as inverse of multiplication). But there are schools that teach multiplication and division simultaneously, in which cases they teach multiplication of a certain number and then division by the same number follows.

Very few teachers use their own strategies and methods by including various manipulatives while teaching multiplication and division. Since classroom manipulatives are lacking most teachers explain multiplication and division in the traditional method using only textbooks and chalks which makes it even more difficult for students to understand and learn them. It can be claimed that the multiplication table is learned by heart mostly and very few pupils can represent the multiplication and division graphically or through manipulative tools since they are not even given the opportunity to work through them. This is due to the lack of conditions at school, the lack of interest and creativity of the teacher as well as the large number of students in the classrooms, often preventing the teacher from doing many activities in the classroom.

The following pictures show some multiplication and division tasks that have been taken from mathematics books of the second grade:

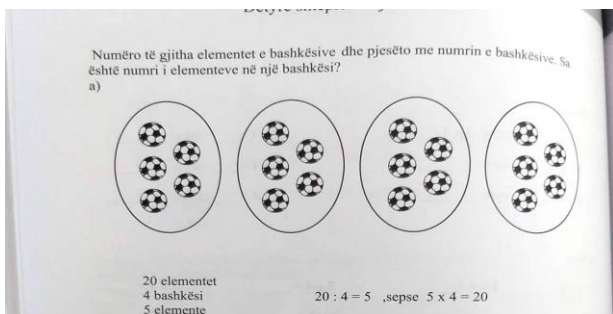


Fig. 4. Representing multiplication and division using sets

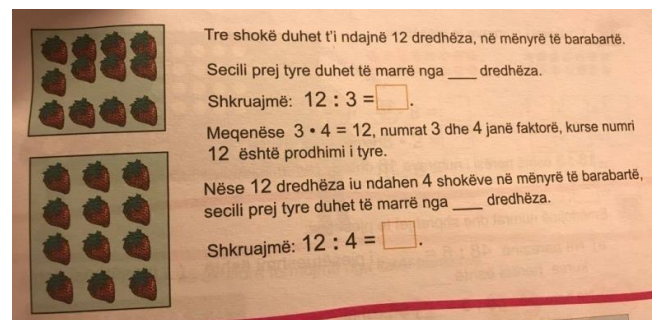


Fig. 5. Successive clarification of word problems

The translation of fig.5:

Count all the set elements and divide them by the number of sets. How many elements are in one set?

The translation of the second picture:

Three friends have to divide 12 strawberries equally. Each of them has to take \_\_\_ strawberries. This is written as  $12:3=$ \_\_

Since  $3 \cdot 4 = 12$ , then the numbers 3 and 4 are factors, while number 12 is their product.

If 12 strawberries are equally divided among 4 friends, then each of them has to take \_\_\_ strawberries. This is written as  $12:4=$ \_\_

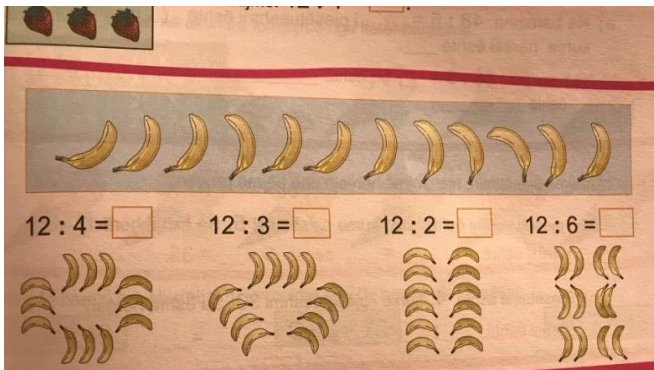


Fig. 6. Division separated into equal parts

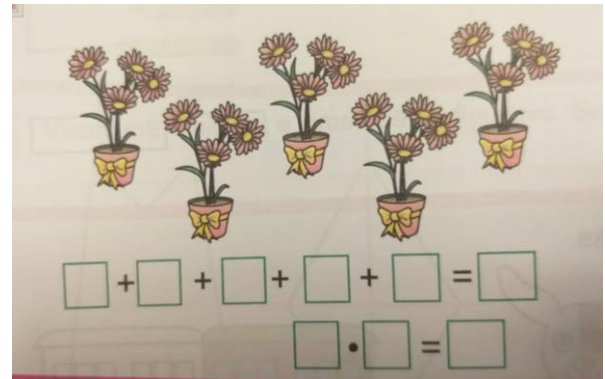


Fig. 7. Multiplication in relation to addition

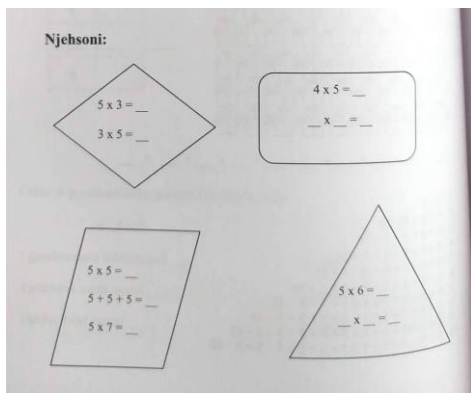


Fig. 8. The commutative property

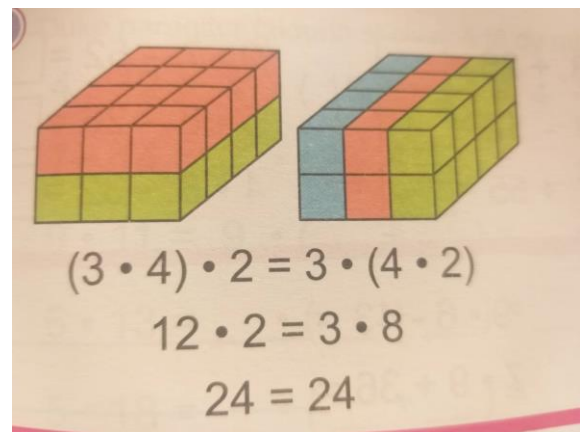


Fig. 9. The associative property

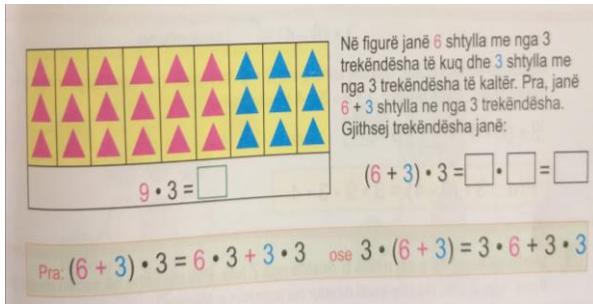


Fig. 10. The distribution property

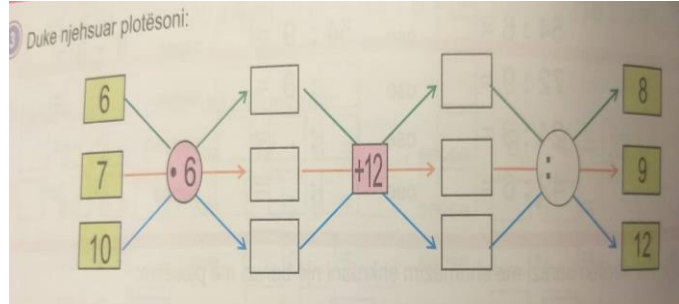


Fig. 11. Applying multiplication, addition and division in a task

The translation of fig. 10:

There are 6 columns with 3 red triangles and 3 columns with 3 blue triangles. Thus, there are 6+3 columns having 3 triangles within. In total, how many triangles are there?

The translation of Fig. 12:

2) Find X which is six times smaller than 54:

The required number is  $X = \_ : \_ = \_$

Find X which is smaller than 54 for six numbers:

The required number is  $X = \_ - \_ = \_$

3) The father is 42 years old, his son is six times younger than him, whereas the mother is 7 years younger than the father. How many years do they all have in total?

Therefore:  $42 + (\_ : \_) + (\_ - 7) = \_$

Thus, in total all of them together have  $\_$  years.

4) Divide the sum of numbers 42 and 14 by 7.

Therefore:  $(42 + \_) : \_ = \_$

Multiply the difference of 42 and 35 by 6.

Therefore:  $(42 - \_) \cdot \_ = \_$

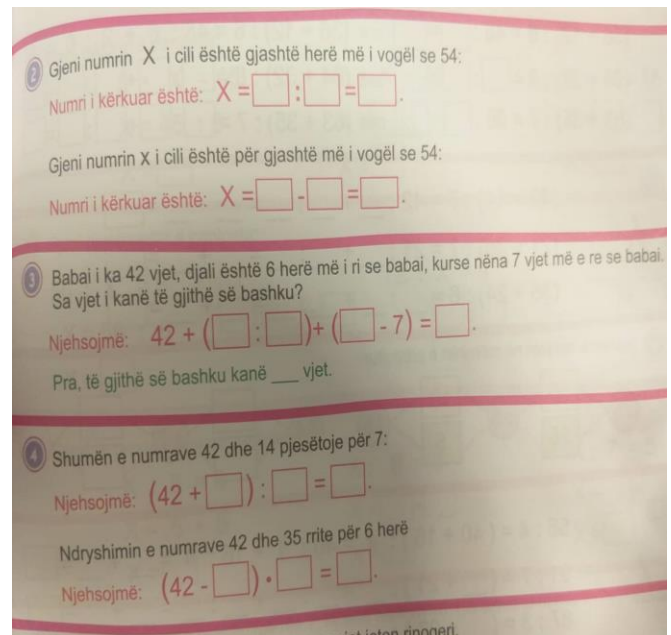


Fig. 12. Various word problems with multiplication and division

### III. THE EMPIRICAL PART

In this chapter are presented: the research problem, the research questions and hypothesis, the school context and participant, and the instrument we used to collect data. Also, it describes the teaching process over a full week in both classes, the experimental group and the control one. Each day of the lessons held has a separate description, meaning that the days are described separately.

#### 3.1 The research problem

While working with second graders in Kosovo, one must try to take into account the difficulties that second grade students face every day. Going through the mathematical lesson plan of the second-grade it is obvious that students have to learn multiplication and division of numbers almost every day.

The problem that the research aims to solve is that in our country there are only a few teachers that use activities in general, especially the activities that include work with manipulatives on multiplication and division. The purpose of the study is to examine the role of the manipulative tools in mathematics, particularly for teaching and learning multiplication and division and also to inform the teachers about the importance and the role of manipulative tools in order for them to be able to make the most of their use of mathematics, especially during the teaching of multiplication and division. Regarding the importance of manipulative tools teaching math concepts has not been researched as much in our country as in other countries. Many foreign researchers have addressed this topic as a very important part of mathematics class.

The objectives of this study are:

- To examine the role of manipulative tools on teaching and learning multiplication and division in the second grade in Kosovo.
- To examine whether the teachers' age affects the usage of manipulative tools when teaching multiplication and division.
- To examine whether there are any differences between teachers in rural and urban areas on teaching and learning multiplication and division in the second grade.

### 3.2 The research questions and hypothesis

The main question of the research is:

- Do manipulative tools affect on a better acquisition of multiplication and division in the second-grade?

Other questions include:

- Does teachers' age affect the usage of manipulative tools when teaching multiplication and division?
- Are there any differences between teachers in rural and urban areas in the usage of manipulative tools when teaching multiplication and division?

#### **The hypotheses of the research**

- Hypothesis 1: The students that have learned multiplication and division by using manipulative tools will score higher on the Post-test than the controlled group of students that have learned multiplication and division through traditional method without using manipulative tools.
- Hypothesis 2: The teacher's age has an impact on the usage of manipulative tools when teaching multiplication and division: younger teachers' use more manipulative tools than the older teachers)
- Hypothesis 3: There won't be any differences between teachers in rural or urban areas in the usage of manipulative tools when teaching multiplication and division

### 3.3 Research methodology

The research was conducted using quasi-experimental method. This methodology was chosen because quasi-experimental research can be used in situations where two or more groups of participants or data that are naturally different from one another can be identified, and where one can be used as a control group and the other as an experimental.

The design of the research is empirical. Intended steps of conducting the methodology of this research include quantitative approach by using:

- A Non-pedagogical experiment: questionnaire for teachers and
- A pedagogical experiment with students

The pedagogical experiment is held with two second grade classrooms and it consists of three phases:

- The pre-test phase
- The experimental part (teaching part) and
- The post-test phase.

. The results were compared by measuring the level of multiplication and division task solving skills, before and after teaching in these two classes.

### 3.4 The school context and participants

The study was carried out in a public school in Kosovo. Two groups of second-grade students were part of this research where one class is taken as the control group in which the traditional method (without using manipulative tools) was used and the other class as the experimental group where manipulative tools were used. The classes participating in this research were: the experimental group with a total of 19 students; 11 girls and 8 boys and the control group with a total of 17 students; 8 girls and 9 boys. The sample is intentionally homogeneous (students have the same age, experience, etc.) Furthermore, one hundred and fifty teachers who teach second-grade students participated in the process via questionnaires.

### 3.5 Data collection instruments

Questionnaires and tests were used with the intention of conducting methodology and covering the quantitative side of the research. One hundred and five teachers that teach second graders of Kosovo filled in the questionnaires. The purpose of questionnaires, was to find out whether teachers' age affects the inclusion of extra activities using manipulative tools when teaching

multiplication and division. And to examine whether there are any differences between teachers in rural and urban areas regarding the usage of activities while teaching multiplication and division

Pre-test and Post-test was conducted within two phases: before the action plan, and after the action plan in order to compare the results and assess the impact of manipulative tools in teaching multiplication and division. Before introducing the topic, both groups took the pre-test in order to get information about the similarities and differences between the groups and to plan the follow-up work. Afterwards the experimental teaching program was done. By using the traditional methods, the multiplication and division by number 6 was taught to the controlled group. Indicating interesting activities using manipulative tools such as pictures, diagrams, counters, blocks, shells or any materials that were available in the classroom, multiplication and division by number 6 was taught to the experimental group.

In order to find out which group was more successful and has acquired the operations of multiplication and division in a better way the Post-Test was held with each group. The experimental teaching with students, the Pre-Test and Post-Test were done during May. The whole process was finished within a school week. Questionnaires for teachers of second grades were delivered during May and June.

The following chapter describes the teaching process over a full week in both classes, the experimental group and the control one. Each day of the lessons held has a separate description, meaning that the days are described separately.

### 3.6 Teaching program

Conducting this research required the permission of the school principal, who approved the research, and the school staff who were willing to assist if needed. Additionally, a written announcement was sent home to the parents of the students in order to let them know about my research. At the end of the announcement, the contact number and the email address of the researcher had been written in case the parents would be interested in getting additional information about research and they would be free to contact at any time. The parents were willing to help as much as possible if needed.

The research was conducted during the school year of 2018/2019 and lasted one school week. The total number of students who participated in this research is 36. Two second grades were included in the study where one of them was the control group with 17 students and the other one was the experimental group with 19 students. A common agreement was reached with the teachers of these grades that during the research week, the math classes would be taught by the researcher in both classes. Firstly, one class would be taught and then the teaching process would continue with the other grade on the same day.

The following paragraphs describe the teaching process for each day. Firstly, the teaching and the learning process is described in one class and then in the other one.

**3.6.1 Day one** - The first teaching day was identical for both groups, the actions that were done with the experimental group were done with the control group as well. Firstly, the new teacher was introduced to the students and they were informed that during this week, they would be learning math together. The students were asked what was the last unit they had learned and before handing out the pre-test to them, they answered some questions about multiplication and division simply to make the students feel comfortable with their new math teacher. The students were curious to know what was the reason that the new math teacher would be part of the lesson for a week. Therefore, they were also told that this was related to a research needed for the researcher's master studies. They were asked whether they were willing to help with what was needed and they confirmed that they were more than willing to do so. Nevertheless, they were told that in order to help they needed to listen attentively and be polite during classes. They were also asked about the



last unit they had learned, and then some other questions about multiplication and division. Afterwards, they were informed that some exercise sheets would be given to them and they had to fill them in but they should not be scared of it because it was just an exercise which would not be graded. The pre-test included multiplication and division tasks up to number 5, and as a result know the students' knowledge up to this level would get to be known. After all the tasks were explained in sequence, the students began to complete the tasks individually and they had 40 minutes to complete the tasks. After the class ended, before continuing to teach the other group, a written announcement regarding this research was handed out to students, which should be given to their parents to read it and get informed about the work that would be done with their children and the reason why this research was being done at that school.



Fig. 13. The students completing the pre-test

### 3.6.2 Day two

#### The experimental group

##### **Objectives:**

- Learn multiplication by number 6
- To present multiplication by number 6 through manipulative tools
- To solve simple multiplication word problems by using manipulative tools

##### **Materials:**

- Notebooks
- Pencil
- Blackboard
- Chalk
- Manipulative tools

##### **The manipulative tools that were used included:**

- Cubes – Students would manipulate cubes by grabbing them into as many groups as needed and from as many cubes as needed (fig. 14).
- Clothespins and plastic plates - Students would cling the clothespins to plastic plates where the clamps represented the number of elements while the plastic plates represented the sets or groups of elements to be clasped based on the task request (fig. 15).
- Popsicle sticks and paper cups - Students would put the popsicle sticks in plastic cups where the sticks represented the number of elements while the paper cups were the sets or groups of elements to be inserted according to the task requirement (fig. 16).
- Beads and muffin liners - Students would place the beads on the muffin liners where the beads represented the number of elements while the muffin liners were the sets or groups of elements to be inserted as required by the task (fig. 17)

Description of the lesson: The teaching process was started at the experimental group where all the students (19 students) were present. They had to be divided into three groups of five students and one group of four remaining students. In front of all the students, the manipulative tools, which

would be used to teach them the multiplication by number 6, were introduced. During the presentation it was also explained how to use these manipulatives and they were also given an example so that they could more clearly understand how to work with them.



Fig.14. Cubes



Fig. 15. Clothespins and plastic plates



Fig.16. Popsicle sticks with paper cups



Fig.17. Beads with muffin liners

For each task, students would have the opportunity to manipulate with those tools in order to understand the course of tasks. After the manipulatives were introduced to the students they were placed in each group and the lesson began to learn multiplying by number 6, from  $1 \times 6$  to  $10 \times 6$ . When explaining or working on the tasks, some sticky figures were used to show the elements on the board so that they were more attractive to students. By grouping the figures on the board, students would have to follow the teacher and then manipulate them into groups by laying out the

elements in as many pieces as needed, i.e. in fig. 18.a, students represented the assignment  $3 \times 6$  whereby students have placed 6 elements into 3 places as seen below:

$3 \times 6 = 18$  - 3 plates of 6 beads with a total of 18 beads.

Whereas in figure 19. b. the students have put the popsicle sticks into cups:

$5 \times 6 = 30$  - 5 cups with 6 sticks and a total of 30 sticks.

In addition to working in groups, students also expressed a desire to come to the blackboard and present the elements based on the given assignment (fig. 19. a).



Fig. 18. a. Solving the task  $3 \times 6$  using sticky figures



Fig. 18. b. Solving the task  $3 \times 6$  using beads



Fig. 19. a. A student solving the task  $5 \times 6$

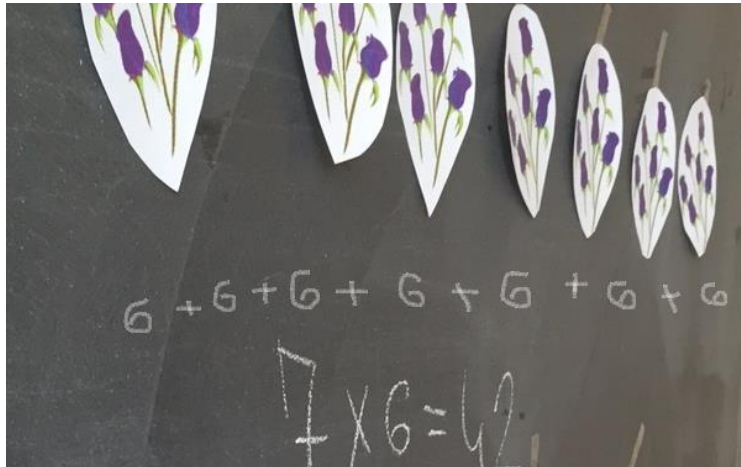


Fig. 19. b. Solving the task  $5 \times 6$  task using sticks

In the same way the students continued solving other tasks. As students manipulated the tools at their disposal then they also wrote the tasks in their notebooks. Consequently, the tasks would be further enhanced in their memory. In the photo (fig. 20) it can be seen that during the lesson on

multiplication by number 6, the students were reminded that multiplication is otherwise called fast addition and tasks were also presented as sums of numbers. In this case the task  $7 \times 6$  can be seen which is also presented as a fast addition done by pupils.

In the exercise phase two tasks with word problems were solved. Some pictures were put on the board and then the tasks were read.



Word problem 1: Klea prepared 7 bunches with 6 flowers. How many flowers did Klea have?

As a result, along with students it was found out that Klea had 42 flowers in total.

Fig. 20. Presenting the multiplication task of  $7 \times 6$  through pictures

Word problem 2 (fig. 21.a):

Ana has 4 boxes with 6 pieces of chocolate.

How many pieces of chocolate does Ana have?

As a result, we found out that the result was 24, so Ana had 24 pieces of chocolate in total.

By seeing the pictures on the board, students would have to follow the teacher and then manipulate with the tools to present the task  $4 \times 6$ . In fig. 21. b can be seen how students have presented the assignment  $4 \times 6$  whereby students have placed 6 clothespins into each of 4 plates.



Fig. 21. a. Presenting the multiplication task  $4 \times 6$  through pictures



Fig. 21. b. The layout of the task  $4 \times 6$  by using clothespins

The students were constantly attentive, worked tirelessly, and enjoyed themselves immensely. Many times, during the teaching process students even declared that they were enjoying this kind of activity and said: How nice teacher, we are playing and learning at the same time!

These activities have had a good impact on the students. Their concentration has been at a maximum level and their constant interest was shown by continuously raising their hand and solving tasks both on the board and while sitting down on their seats as well.

### The control group

#### Objectives:

- To learn multiplication by number 6
- To present multiplication tasks through drawing
- To solve simple multiplication word problems

#### Materials:

- Notebooks,
- pencil,
- blackboard,
- chalk ,
- pictures of flowers and bars of chocolate

Description of the lesson: After teaching the experimental group, then the teaching process continued with the control group. There the lesson was taught in the traditional way using only the board and the chalk while the students only used the pencil and the notebook. This was done contrary to the experimental group where students used manipulative tools. Fig. 22 shows a student who voluntarily came to the board to draw elements (circles) in groups according to the task requirement, i.e.:

$5 \times 6$  - 5 groups with 6 elements (6 circles).

Whereas in fig. 23, another student can also be seen presenting the task  $8 \times 6$ , namely:

$8 \times 6$  - 8 sets of 6 elements (circles).

After drawing the elements in groups, they then counted the number of elements and in this way, they also found the result of the task.

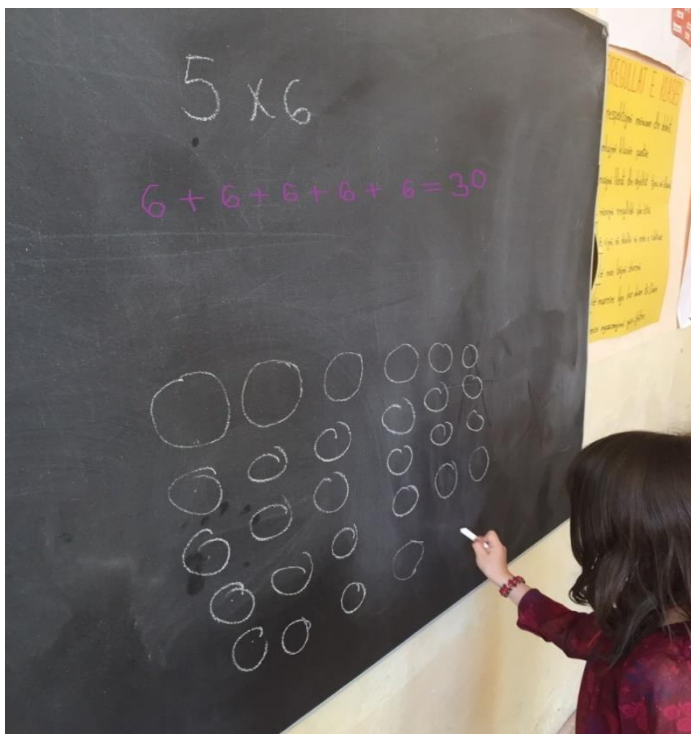


Fig. 22. The layout of the task  $5 \times 6$  in the control class



Fig. 23. The layout of the task  $8 \times 6$  in the control class

Each task that was solved on the board was also written by the students in their notebooks, showing the grouped elements as shown in the table. The word problems were solved with the control group similarly as it was done with the experimental group. However, the only difference here was that the pictures were not used there but the elements were simply drawn on the board and the students wrote them in their notebooks. (fig. 24 and fig. 25).

In the exercise phase two tasks with word problems were solved.



Word problem 1:

Ana has 8 boxes of 6 chocolates. How many chocolates does Ana have in total?

$8 \times 6$  - so Anna had 48 chocolates in total.

Fig. 24. The layout of the first word problem

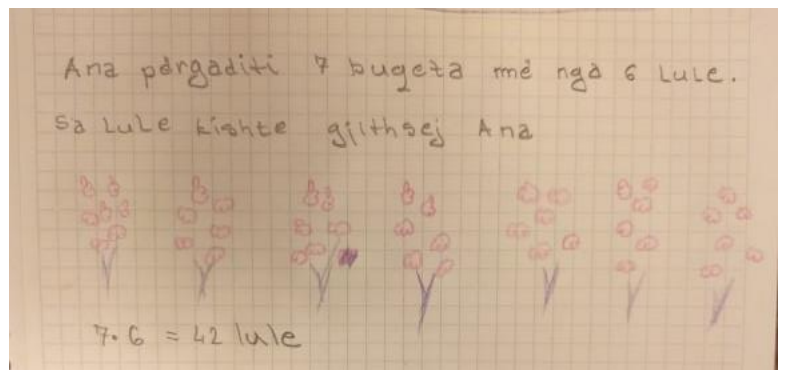


Fig. 25. The layout of the second word problem

Word problem 2:

Ana had prepared 7 bouquets of 6 flowers.

How many flowers did Ana have in total?

$7 \times 6$  - so Anna had a total of 42 flowers.

While teaching multiplication by number 6, ( $1 \times 6$  to  $10 \times 6$ ) the students were always reminded that multiplication is a kind of rapid addition and multiplication was also presented as a rapid addition of numbers.



For the students, the lesson was as common as any other day. The only difference being was that they were being taught by the researcher and not the form teacher. During the lesson the pupils were not so attentive. They also lacked concentration and interest in the lesson they were learning because it seemed to them that they had already learned the multiplication of numbers. Most students simply tried to copy in their notebook what the teacher or any of their classmates were writing on the board.

### 3.6.3 Day three

#### **The experimental group**

##### **Objectives:**

- To learn division by number 6
- To present division tasks through manipulative tools
- To solve simple division tasks through manipulative tools

##### **Materials:**

- Notebooks
- Pencil
- Blackboard,
- Chalk,
- Manipulative tools

##### **The manipulative tools that used included:**

- Cubes
- Clothespins and plastic plates
- Popsicle sticks and paper cups
- Beads and muffin liners

Description of the lesson: On the third day, the students learned the division by number 6,  $6 : 6$  to  $60 : 6$ . Teaching started in the experimental class, and the students were eagerly awaiting because they knew they were going to work again with manipulative tools and in a way, it seemed to them like they were playing while working with them. Again, the students were divided into groups;

three groups of five students and one group of four remaining students. The manipulative tools were also distributed to each group and while solving the tasks on the board the students would be able to divide or group them in their own places.

Up to now, students had learned that division means dividing a whole into several equal parts. However, before starting to explain the unit a revision on what division was done. To explain this lesson, the partition division method was used, each task would always ask students to divide a number into 6 groups and find out how many elements each group contained.

The lesson began by inviting 3 students to come to the board (fig. 26). The students were asked to open their fingers and form a circle with their hands, where first it was started with multiplying and then moved on to division.

The students were asked by the teacher:

How many fingers do you have in total?

30 fingers.

How many hands do the three of you have in total?

6 hands.

How many fingers does each hand have?

5 fingers.

So, how many fives are there in number 30?

6 fives or 6 hands

If we divide number 30 (30 fingers) into 6 groups (6 hands) we will have 5 elements for each group (5 fingers).

$30: 6 = 5$



Fig. 26. The activity using hands

In this way, the teaching process was started with a simple hands-on activity where a clear distinction could be done between multiplying and dividing so students would find it easier to understand.

On the board some pictures with bananas on them were put, totaling 18 bananas.

How many bananas do we have?

18 bananas.

We will divide these 18 bananas equally to 6 students.

So, how many bananas will each student get?

$18 : 6 = ?$

Because:

$$X \cdot 6 = 18$$

How many sixes are there in 18?

3 sixes.

So, number 18 is divided into 6 equal parts it means

3 elements for each.

$$18 : 6 = 3$$

The same was done by the other students in their groups who also presented the task using

manipulative tools (fig. 27.b and 27.c)

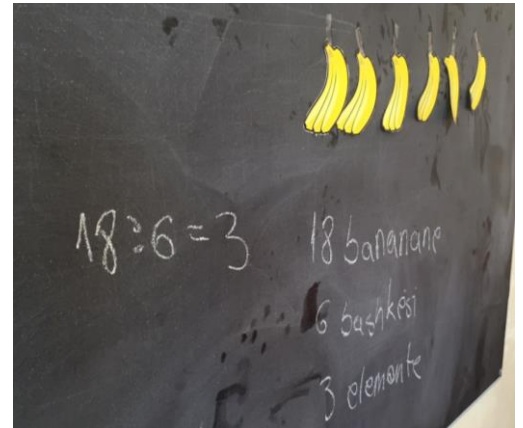


Fig. 27. a. Presenting the task 18:6 through pictures



Fig. 27. b. The layout of task 18:6 using cubes



Fig. 27. c. The layout of task 18:6 using clothespins and plates

Then the lesson continued by solving other tasks. For each task, different stickers would be put on the board, as many elements as needed, and a volunteer student would come to the board to divide the elements into equal parts.

In fig. 28. a, a student who has divided the number 42 into 6 equal parts can be seen. He counted the elements to make sure he had 42, then he put 1 element into each group, meaning 6 groups of 1, then continued adding an element to each one until no more elements remained. In this way the equal division of elements was done and he got 6 groups of 7 elements. The same was done by the other students in their groups who also presented each task using manipulative tools (Fig. 28. b).



Fig. 28. a. Dividing number 42 into 6 equal parts



Fig. 28. b. The layout of task  $42:6$  using cubes

It was a pleasure to see students gaining knowledge of division by manipulating the manipulative tools, which provided a lot of pleasure to them.

At the end of the lesson, the students were instructed to fill out a page in the book (fig.29) containing division tasks by number 6. After giving the necessary explanations about the assignments, the students started working. Students were free to use manipulative tools if they were handy.

Upon completing this page, we could also notice students who had mastered the lesson well but there also were the others who still needed to practice more. Thus, they used the manipulative tools to find the results.

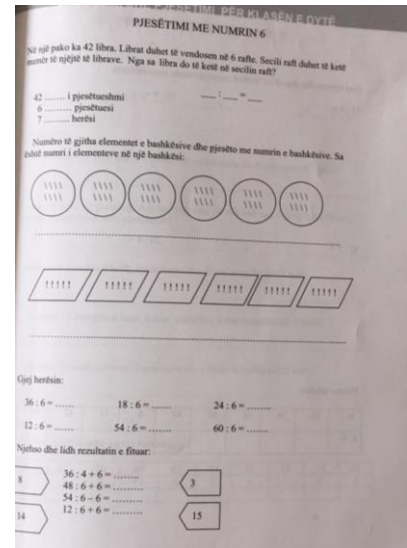


Fig. 29. Division tasks by number 6

**The control group:**

**Objectives:**

- To learn division by number 6
- To present division tasks through drawing
- To solve simple division tasks

**Materials:**

- Notebooks
- Pencil
- Blackboard,
- Chalk,
- Pictures of bananas

Description of the lesson: The teaching process continued at the control group where the teaching would with these students would be done again by the traditional method of using only the pencil, notebook and the chalk along with the board as it was done during the lesson on multiplication. Firstly, a short revision was done regarding the previous unit, thus, multiplying by number 6.

To begin explaining division, the teaching process was started in the same way as with the experimental group using the activity with hands. The difference was that after doing this activity, the students did not have the manipulative tools to practice element division but would only draw the elements in their notebooks using circles and the same would be done on the board by presenting tasks through circles rather than sticky figures.

Word problems were solved with the control group similarly as it was done with the experimental group. However, the only difference here was that the pictures of bananas were not used there but the elements were simply drawn on the board and the students draw them in their notebooks too (Fig. 30).

Word problem:

Ana has 18 bananas and will share them to 6 friends equally.

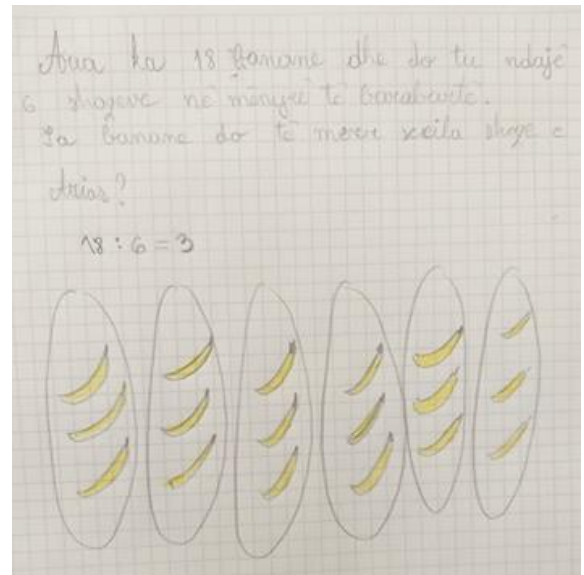


Fig. 30. Word problem drawn on student notebook

How many bananas will get each of Arias friends?  $18 : 6 = 3$

The lesson continued by explaining and practicing other tasks divided by number 6.

After writing the task on the blackboard i.e.  $42 : 6$ , the pupils would be asked:

How many elements do we have in total?

42 elements.

In how many equal parts should we divide the number 42?

In 6 equal parts.

A volunteer student would come to the blackboard to draw the elements and then divide them into 6 groups. Thus, by dividing it into 6 equal groups the students would find out how many elements each group contained. In this case  $42 : 6$ , each group contained 7 elements (Fig. 31.c).

The student first drew 6 cycles (fig.31.a) as it was required to divide them into 6 groups and then he drew elements one by one to each group (fig.31.b) until a total of 42 elements were drawn. After dividing elements, it resulted that 7 elements belonged to each group.

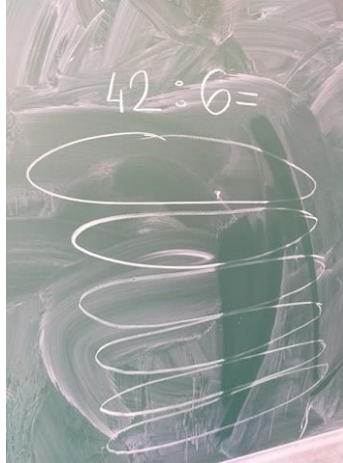


Fig. 31. a. Drawing groups to divide elements equally

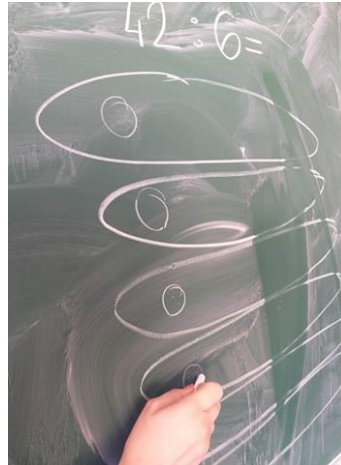


Fig. 31. b. Drawing elements one by one to each group

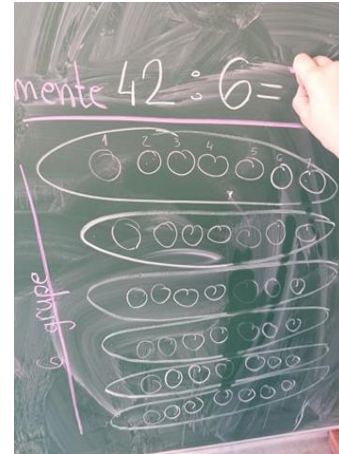


Fig. 31. c. Finding the result of the task

As with these assignments, the same was done with the other division tasks with number 6, up to  $60:6$ . The students wrote down each assignment in their notebooks as well and some of the tasks can be seen on fig 32.

At the end of the lesson, these students were also instructed to complete the same book

page as the experimental group did. Upon completion,

we could notice that students were having difficulties as they needed a lot of exercise in order to properly understand this lesson.

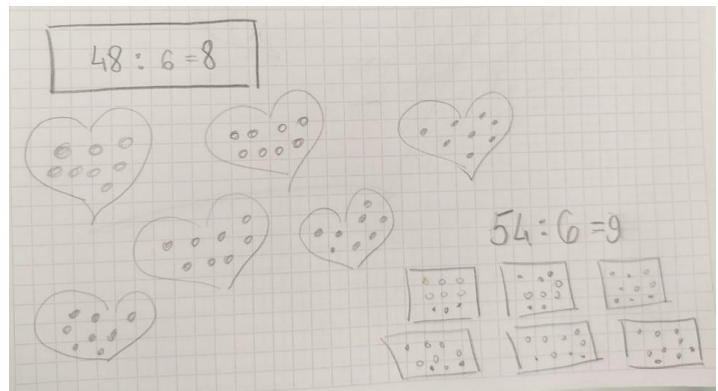


Fig. 32) Solving tasks in the notebook

### 3.6.4 Day four

#### The experimental group

##### **Objectives:**

- To solve multiplication and division tasks by number 6
- To present multiplication division tasks through manipulative tools
- To solve word problems by using manipulative tools
- To reinforce their knowledge about multiplication and division by number 6

##### **Materials:**

- Notebooks
- Pencil
- Blackboard
- Chalk
- Flip charts
- Manipulative tools

##### **The manipulative tools that used included:**

- Cubes
- Clothespins and plastic plates
- Popsicle sticks and paper cups
- Beads and muffin liners

Description of the lesson: In the previous days the students worked on multiplication and division by number 6, while on this day a revision on these two units was done. Thus, the students had the opportunity to recall and reinforce these tasks even better. The teaching process started at the experimental group. The tasks that had been done beforehand were written in A3 kraft paper so that no time would be wasted in writing or drawing them on the board and would also look more attractive to the students. Near the blackboard a desk was placed on which the manipulative tools were put (fig. 33) and a student would come to the blackboard to find the result of a task. He/she



would have the chance to choose one of the manipulative tools and to group the elements in order to find the result

The description of the first task:

The task included multiplication where the number 6 was placed in the middle while around it was written the numbers from 1 to 9. The volunteer students would come to the blackboard and solve one of the tasks, except that the student would have to



Fig.33. Multiplication tasks and the manipulative tools placed near the blackboard

choose which task to solve, he/she would also have to choose one of the manipulative tools to find the result.

Figure 33 shows the first task set out in the table where it can also be seen that some of the results are solved. Under the kraft paper one can see that the task is written down as  $4 \times 6 = 24$  and the student who came to the blackboard decided to find the result of this task and on the desk one can also see the cubes using which the student found the result, i.e. 4 places with 6 cubes totaling 24.

The following figures show a student who has decided to solve the task  $8 \times 6$ .



Fig. 34. a. The student solving the task by using clothespins and plates

Fig. 34.b. Writing the task down on the board

Fig. 34.c. Putting the result at the task

The student in question has decided to use plastic plates along with the clothespins where he can be seen dividing 6 elements into 8 groups or putting 6 clothespins into 8 plates (fig. 34.a). After finding that the result is 48, he writes it on the board first (fig. 34.b) and then on the sticker he writes the number 48 and places it on the kraft paper at its belonging place where numbers 8 and 6 are located (fig. 34.b).

That way all the results are found and then the students can move on to the other two tasks. The paper was put on the board and the students were instructed to write assignments in their notebooks and with the help of manipulative tools to find the results and after solving them, to write the results together on the board. By writing the results on the board, the students would check if they had solved the tasks correctly or if they made any mistakes to correct them.

The description of the second task (fig. 35):

The second task contained both multiplication and division tasks. Firstly, the students had to find the results of the assignments compiled of the factor and the product and then to compare the results by putting one of the comparison symbols:  $<$ ,  $>$ ,  $=$ .

The description of the third task (fig. 36).

The third task consisted of word problems of which the first two ones included multiplication and division while the other two contained both multiplication and

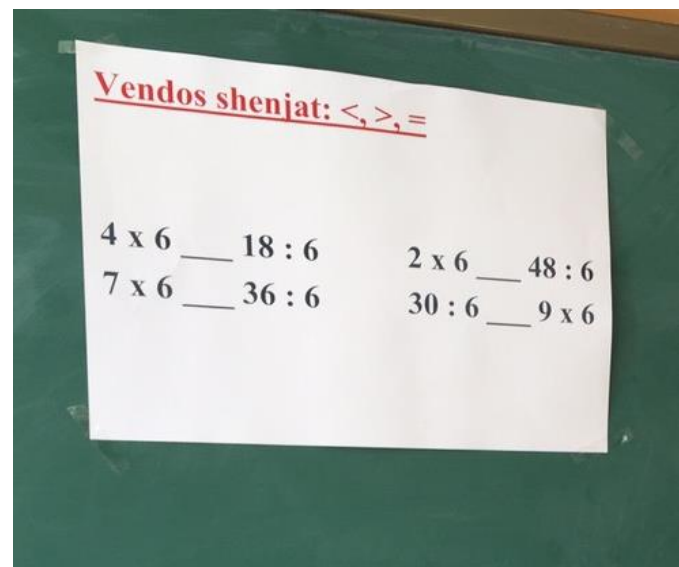


Fig. 35. Presenting the second task on the board

division within the same task. The students had to read the tasks carefully and with the help of manipulative tools to present the elements as required and thus, to find the result.

Word problems:

- ❖ If one notebook costs 7 euros, how much do 6 notebooks cost?

The task included multiplication where they would multiply:

$6 \times 7 = 42$ , so 6 notebooks cost 42 euros

- ❖ 24 flowers should be evenly divided into 6 boxes. How many flowers will be in each box?

Dividing the elements evenly means that we are dealing with division:

$24 : 6 = 4$ , therefore 4 flowers will be placed into each box.

- ❖ Reduce the product of numbers 9 and 6 for the dividend of 30 and 6.

We know that when we say production we are dealing with multiplication and when we say dividend we are dealing with division, and when we say reduce, we must subtract, therefore:  $(9 \times 6) - (30 : 6) = 54 - 5 = 49$

- ❖ Add the dividend of the numbers 18 and 6 to the production of numbers 8 and 6.

The above task had to deal with subtraction and now we have addition since we are mentioning addition then we know we need to add:  $(18 : 6) + (8 \times 6) = 3 + 48 = 51$

Students were free to use the manipulative tools to solve tasks. As the manipulative tools were sufficient, the students also had the opportunity to choose which of them they wanted to work with. As the lesson continued by moving on to another task, they could exchange their manipulative tools with each other. Some students wrote the results without even using the manipulative tools at all, but as the 3rd and 4th tasks were more difficult, they obviously used them to find the result.

The following figures show students who have written some assignments down in their notebooks and are using manipulative tools to find the results of the tasks presented before.

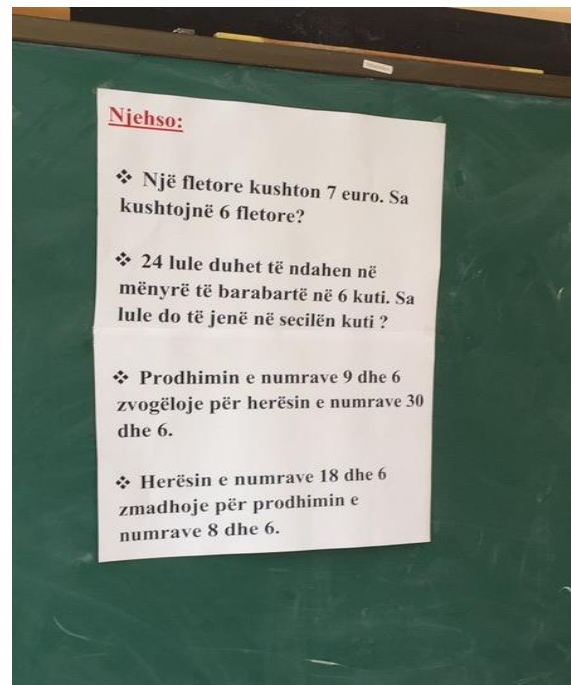


Fig. 36. Presenting the third task on the board



Fig. 37. Solving tasks using cubes

Fig. 38. Solving tasks using popsicle sticks

Fig. 39. Solving tasks using beads

After the students solved the assignments in their notebooks, a discussion was done by solving the assignments and bringing the volunteer students to the board to solve them. In this way all the students checked the results of the assignments in order to see if they had completed them correctly or not.

### The control group

#### Objectives:

- To solve multiplication and division tasks by number 6
- To present multiplication division tasks through drawing
- To solve word problems
- To reinforce their knowledge about multiplication and division by number 6

#### Materials:

- notebooks,
- pencil,
- blackboard,
- chalk,
- flip charts

Description of the lesson: The teaching process continued at the control group, and in this class, the same tasks which had been written on the kraft paper would be solved. Nevertheless, unlike the experimental group, these students did not have the help of manipulative tools in order to find the results. All the students could do was the opportunity to draw the elements in their notebooks or on the board and group them in groups just like the previous days.

In fig. 40, a student can be seen solving the task  $3 \times 6$  by drawing the elements, where in three groups he has drawn 6 elements and there are 18 elements in total.

In this way, all the tasks were solved one by one by having a volunteer student coming to the board to solve them. After solving these tasks, then it was proceeded with the other two tasks: one

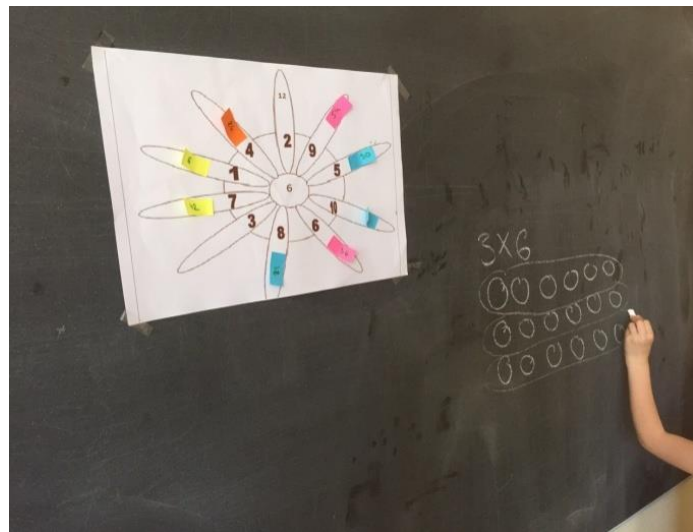


Fig. 40. A student solving the task  $3 \times 6$

included comparing the factor and product

while the other task contained the word problems, similar to the ones as in the experimental class. After giving the necessary explanations for these assignments, the students were instructed to write them down in their notebooks and to complete them. Some of the students drew the elements and grouped them to find the results and while some others did not draw at all but only wrote the results either correctly or incorrectly as it was difficult for them to memorize the multiplication and division by heart.

In the figures 41 and 42, two different notebooks of the students of the control group can be seen, where in one a student can be seen solving the tasks without drawing the elements while the other student was solving the tasks by drawing the elements and grouping or dividing them.

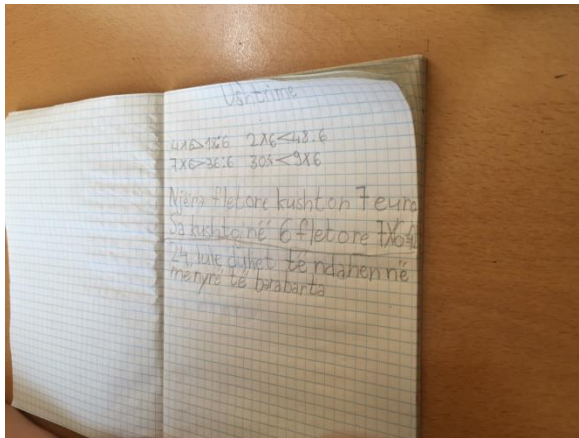


Fig. 41. Writing the tasks down in the notebook

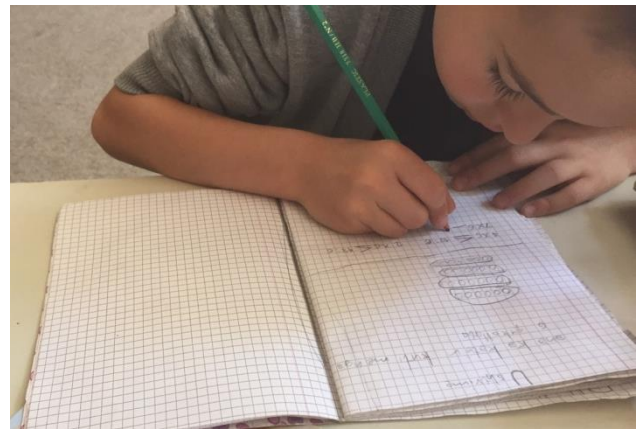


Fig. 42. A student finding the results by drawing the elements

As the students solved their tasks in their notebooks, each task was discussed and the students would come to the board and solve the tasks. The students were kindly asked to write them on the board by drawing the elements no matter how they had solved them in their notebooks as it would probably be easier and simpler to find the result.

The students were asked to practice these tasks at home as they would have a test including multiplication and division by number 6 the next day.

**3.6.5 Day five** - On this day the students would complete the post-test which included multiplication and division tasks only with number 6 in order to evaluate the knowledge gained on multiplication and division by number 6. Through the post-test, a comparison would be done between the knowledge gained by the students of the experimental group to those of the control group. Additionally, in case that the students of the experimental group will have better achievements then one could become aware of the impact of manipulative tools on learning multiplication and division.

The lesson with the experimental group started and the students were already prepared for the test that they were going to take after being informed about it in advance. The manipulative tools were placed on each desk. Hence, the students would not have to get out of their places and in order to get the manipulatives, as this would create a mess and enable the students to find a way to communicate with each other. Even though all the students were sitting in pairs, meaning that two

students would be sharing the same desk, they were always under supervision and usually the school pupils in Kosovo are aware of the fact that if they turn their heads to talk to a friend or look at a friend, then their tests would be taken and they would be punished with a poor grade. Students were free to use manipulative tools while completing the test if they were handy to help them complete the task.

Firstly, all the tasks were explained and then the students started completing the test. While all the students were completing it, one could notice that some of them used the manipulative tools and some other had clearly understood the tasks, therefore they did not even need them to find results.

In the pictures below we can see students using 4 types of manipulative tools while completing the post-test.



Fig. 43. a. Completing the post-test using the clothespins and plates

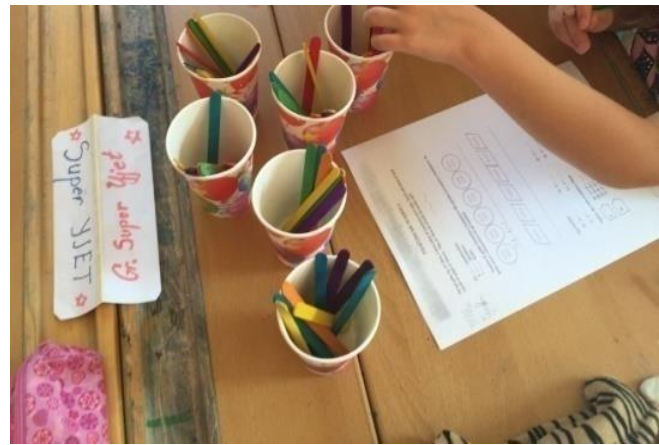


Fig. 43. b. Completing the post-test using popsicle sticks



Fig. 43. c. Completing the post-test using cubes



Fig. 43. d. Completing the post-test using beads and paper plates

As the class ended, the tests were collected and the students were thanked for the cooperation, commitment, and hard work they had done throughout the week, for their discipline, attention, and interest in working with manipulative tools.

Afterwards, the lesson in the control class continued. These students were also prepared to take the test after they had been informed the day before. After the tests were distributed, all the tasks were explained one by one. Consequently, the students began to work individually. Students were free to keep a sheet of paper if they needed to present a task, or to group and divide items in order to find the result.

When the class ended, the tests were collected, and these students were also thanked for their cooperation and work that was done together for a whole week.

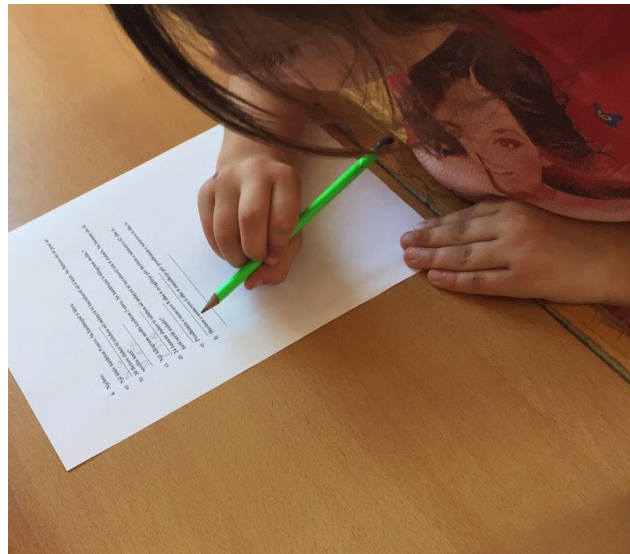


Fig. 44. A student of the control group completing the post-test.



### 3.7 Data analysis and results

This chapter presents the research results, including pre and post-test results held with the second-grade pupils and the results of the questionnaires filled in by second-grade teachers.

Based on the results obtained before and after the test we have managed to answer the main question of this research:

- Do manipulative tools affect on a better acquisition of multiplication and division in the second grade?

On the other hand, the results obtained from the questionnaire filled in by the second-grade teachers we have managed to answer two other questions of this research:

- Are there any differences between teachers in rural and urban areas in the usage of manipulative tools when teaching multiplication and division?
- Does teachers' age affect the usage of more manipulative tools when teaching multiplication and division?

**3.7.1 Pre and post test result** - The following tables show the pre and post-test results, which have been held in order to answer the main question of this research “Do manipulative tools affect on a better acquisition of multiplication and division in the second grade?”

**Table 1: Comparison of control and experimental group on pre and post test**

	Class	N	Mean	Std. Deviation	Std. Error Mean
Pre-Test	Control	17	36.06	2.817	.683
	Experimental	19	37.95	2.460	.564
Post-Test	Control	17	43.47	3.105	.753
	Experimental	19	48.26	1.558	.357

**Table 2: Pre-Test and Post-Test significant comparison**

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	T	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Pre_Test	Equal variances assumed	.065	.800	-2.148	34	.039	-1.889	.879	-3.676	-.101
	Equal variances not assumed			-2.131	32.032	.041	-1.889	.886	-3.693	-.084
Post_Test	Equal variances assumed	7.324	.011	-5.950	34	.000	-4.793	.805	-6.430	-3.156
	Equal variances not assumed			-5.750	22.984	.000	-4.793	.834	-6.517	-3.068

Levene's test is an inferential statistic used to assess the equality of variances for a variable calculated for two or more groups (Howard,1960), so, we have used it to compare the results of pre and post- tests done with two groups of students.

Based on the results of the pre-test, we conclude that the control group and the experimental group have no major difference before starting the teaching program. The first tester sign is 0.8.

These results show that the experimental and control group started with no statistical important differences in knowledge about multiplication and division. While two groups of students had no major difference we could continue with the teaching program in order to follow its impact on progressing knowledge between two groups of students.

Based on the results of the post-test we can conclude that the experimental classroom pupils who have learned multiplication and sharing using manipulative tools, have had better outcomes than the pupils of the control group. The post-test signal is equal to 0.01, a value which is less than 0.05.

According to the results, the statistical Table 2 shows that the hypothesis 1 has been verified: There are statistically significant differences in achievement between students who use manipulative tools while learning multiplication and division on second-grade and those who learn multiplication and division without manipulative tools.

3.7.2 Results of the teachers' questionnaire - The following tables show the results obtained from the questionnaire filled in by the teachers. The questionnaire was filled in by the teachers in order to answer the research questions:

- Are there any differences between teachers in rural and urban areas in the usage of manipulative tools when teaching multiplication and division?
- Does teachers' age affect the usage of more manipulative tools when teaching multiplication and division?

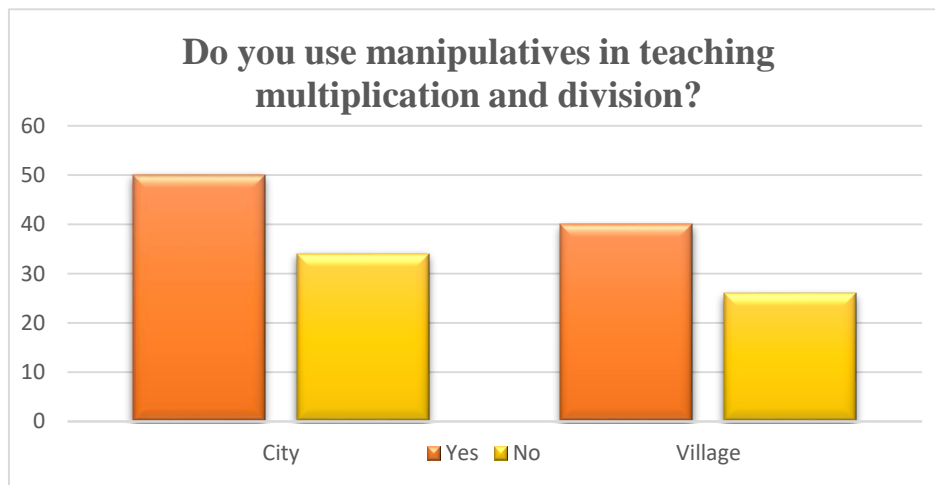
Teachers' answers are presented in percentages. After each table the results are also presented by histograms.

Table 3 shows the results of the question: “Do you use manipulatives in teaching multiplication and division?”. The answers were drawn based on the teachers' residence (village-town).

**Table 3: Use of manipulative tools according to the living place**

Do you use manipulatives in teaching multiplication and division?			
Answers	Living place		Total
	City	Village	
Yes	59.5%	60.6 %	56%
No	40.5%	39.4%	44%
Total	100 %	100 %	100 %

**Histogram 1: Use of manipulative tools according to the living place**



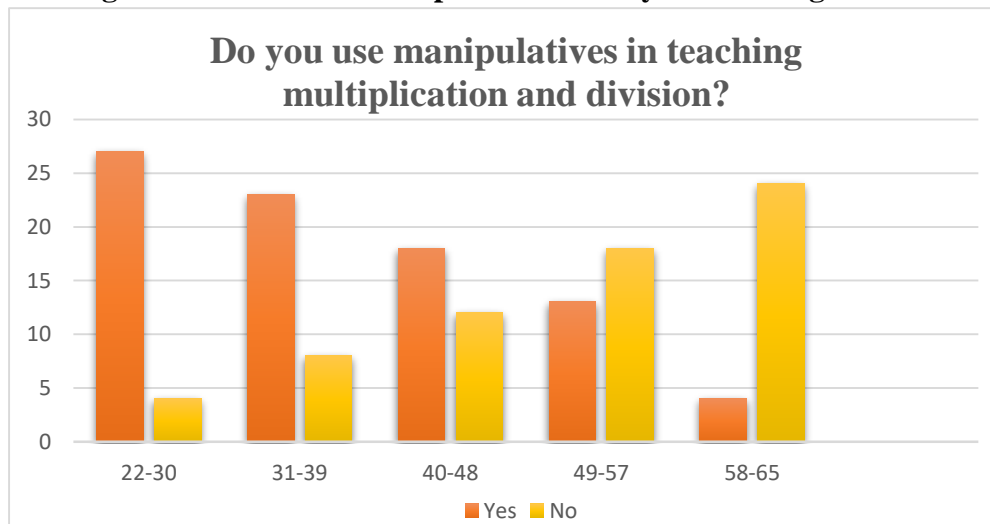
Based on the result of Table 3 we can see that 59.6% of teachers in the city answered with “Yes” and 60.6% of teachers in villages answered with “Yes”. While the percentages are almost equal we conclude that teacher’s residence place does not affect the use of manipulatives during multiplication and division teaching. This confirms the second research hypothesis: “There aren’t any differences between teachers in rural or urban areas in the usage of manipulative tools when teaching multiplication and division”.

Furthermore, we analyzed the influence of teachers' age on the use of manipulatives for teaching and learning multiplication and division. Table 4 presents the results to the question: "Does teachers' age affect the usage of more manipulative tools when teaching multiplication and division?". As we can see in Table 4, we have divided teachers into five intervals of age, each of the groups having approximately 30 teachers.

**Table 4: The use of manipulative tools by different ages**

Do you use manipulatives in teaching multiplication and division?					
Answers	Age				
	22-30	31-39	40-48	49-57	58-65
Yes	87.1%	73.3%	60%	41.9%	14.2%
No	12.9%	26.7%	40%	58.1%	85.8%
Total	100%	100%	100%	100%	100%

**Histogram 2: The use of manipulative tools by different ages**



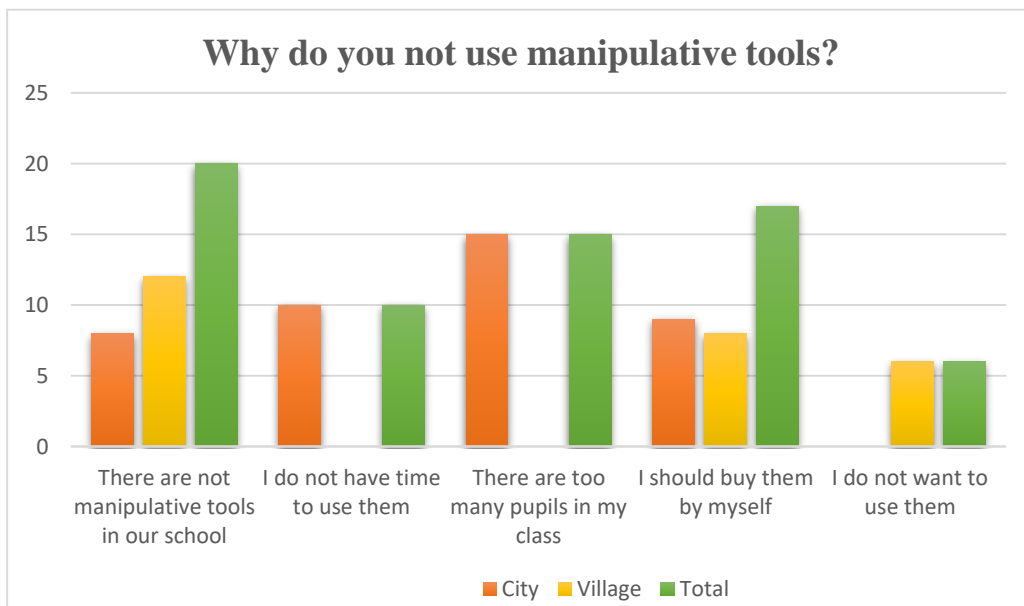
Based on the obtained results, it can be concluded that younger teachers, aged "22-30", as well as the second age group aged "31-39", use more manipulative tools in their multiplication and division teaching. Meanwhile older ones use less manipulative tools or do not use them at all. In this case, the third hypothesis of this research is confirmed: "The teachers' age has an impact in the use of manipulative tools when teaching multiplication and division".

As we were conducting this research, we were also interested in knowing why certain teachers did not use manipulative tools. Due to this fact, the teachers who answered no to the previous question would also have to answer the question: Why do they not use manipulatives when teaching multiplication and division? Within this question, teachers could select one of the 5 given options.

**Table 5: The reasons why teachers do not use manipulative tools**

Why do you not use manipulative tools?			
Answers	Living place		Total
	City	Village	
There are not any manipulative tools in our school	12.1%	17.9%	30%
I do not have time to use them	14.3%	0%	14.3%
There are too many students in my class	22%	0%	22%
I should buy them by myself	12.6%	12.1%	24.7%
I do not want to use them	0%	9%	9%
Total	61%	39%	100%

**Histogram 3: The reason why teachers do not use manipulative tools**



Based on the results, we can see that the main reasons why manipulative tools are not used in Kosovo schools are: lack of manipulative tools in schools, large number of students in the classroom and if teachers want to use manipulative tools, they should buy them with their own money. Thus, they prefer neither buying nor working with them. From the results we can see that most teachers have responded as following: “There are not manipulative tools in our school” with 30 % and “I should buy them by myself” with 24.7 %.

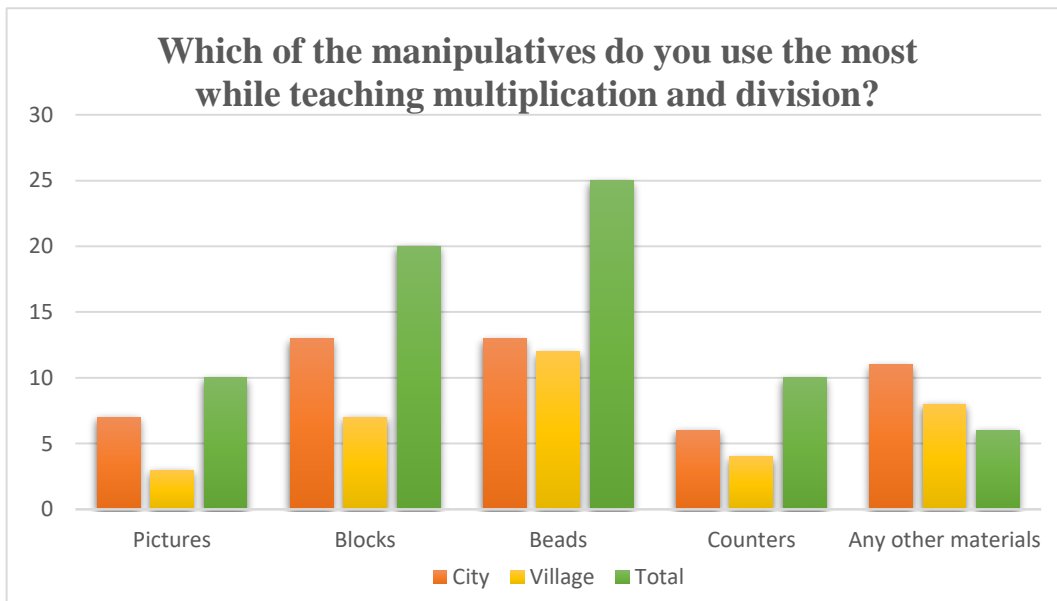
Also, from the results of teachers in the city schools we can see that they have also answered with “I do not have time to use them” and “There are too many students in my class” while on the other hand, none of the teachers in village schools have chosen those answers. Based on the answers we can conclude that one of the reasons why teachers who work in the city schools do not use manipulative tools, are crowded classrooms.

The following questions were answered only by the 56% of teachers who claimed that they use manipulative tools when teaching multiplication and division. Table 6 shows the results of the question: “Which of the manipulative tools do you use the most while teaching multiplication and division?”. The teachers could choose between 5 given options and one of them included "Any Other materials".

**Table 6: Manipulative tools that teachers use the most**

Which of the manipulatives do you use the most while teaching multiplication and division ?			
	Living place		Total
	City	Village	
Pictures	8.4%	3.5%	11.9%
Blocks	15.5%	8.3%	23.8%
Beads	15.4%	14.3%	29.7%
Counters	7.2%	4.7%	11.9%
Any other materials	13.2%	9.5%	22.7%
Total	59.7%	40.3%	100%

**Histogram 4: Manipulative tools that teachers use the most**





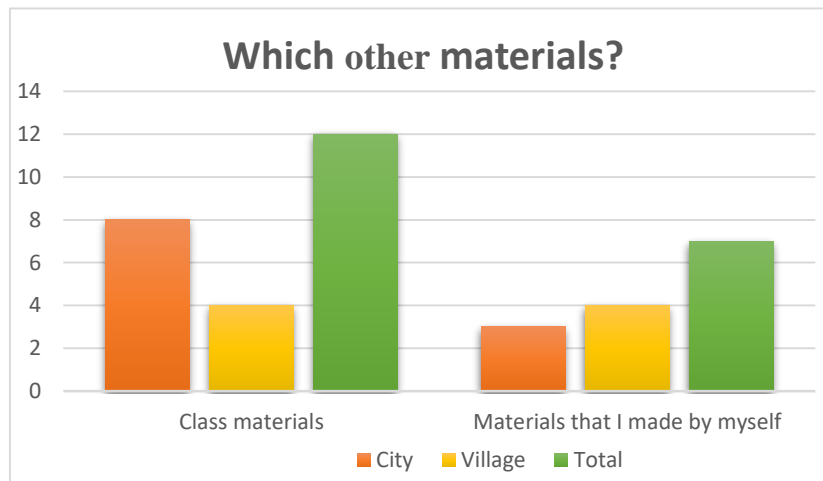
Based on the given answers we can see that the most selected options by the teachers are: "Beads" with 29.7 % and "Blocks" with 23.8%. If we compare the results according to the working place we can see that there aren't big differences in percentages, except to the answer "Blocks", teachers' in the city have chosen this option more than the teachers in the village.

Manipulative tools include either self-made manipulatives or tools that can be found within the classroom and they are used as such. These improvised tools include: pencils, colors pencils, erasers, etc. Teachers who responded with "Other materials" would have to answer the question "Which other materials do you use instead of manipulative tools?" where only two possible answers were given.

**Table 7: Other materials that teachers use instead of manipulative tools**

Which other materials?			
Answers	Living place		Total
	City	Village	
Class materials	42.1%	21.1%	63.2%
Materials that I made by myself	15.7%	21.1%	36.8%
Total	57.8%	42.2%	100%

**Histogram 5: Other materials that teachers use instead of manipulative tools**



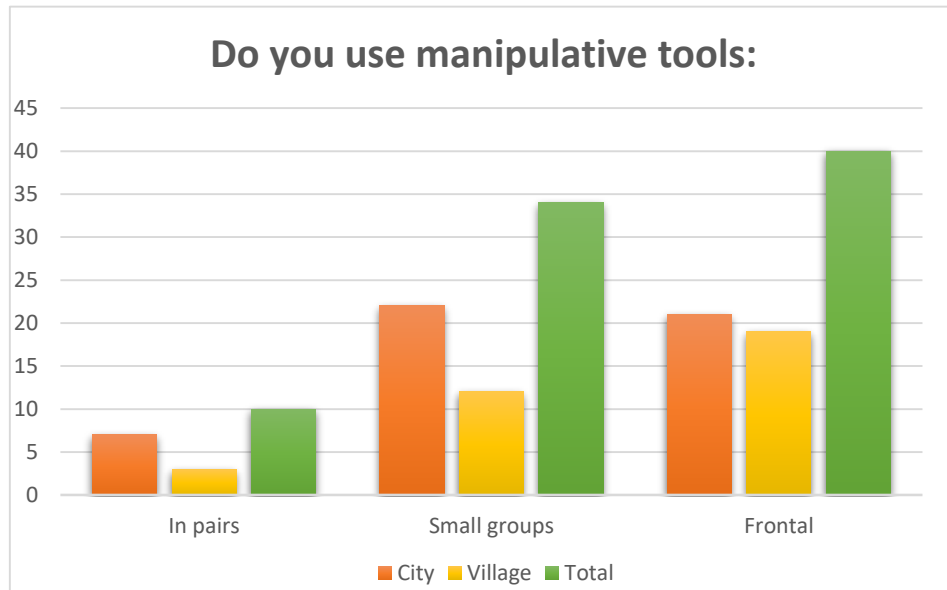
Based on the result of Table 3 we can conclude that teachers use “Class materials” more, since 63.1% of teachers responded that they use classroom tools (pencils, colored pencils, erasers) instead of manipulative tools.

We were also interested in knowing the method how the teachers use manipulative tools: are there sufficient tools to use them individually for each student, to use them to small groups or to all class?

**Table 8: The method by which teachers use manipulatives**

Do you use manipulative tools:			
	Living place		Total
	City	Village	
In pairs	8.4%	3.5%	11.9%
Small groups	26.2%	14.3%	40.5%
Frontal	25%	22.6%	47.6%
Total	59.6%	40.4%	100%

**Histogram 6: The method by which teachers use manipulatives**



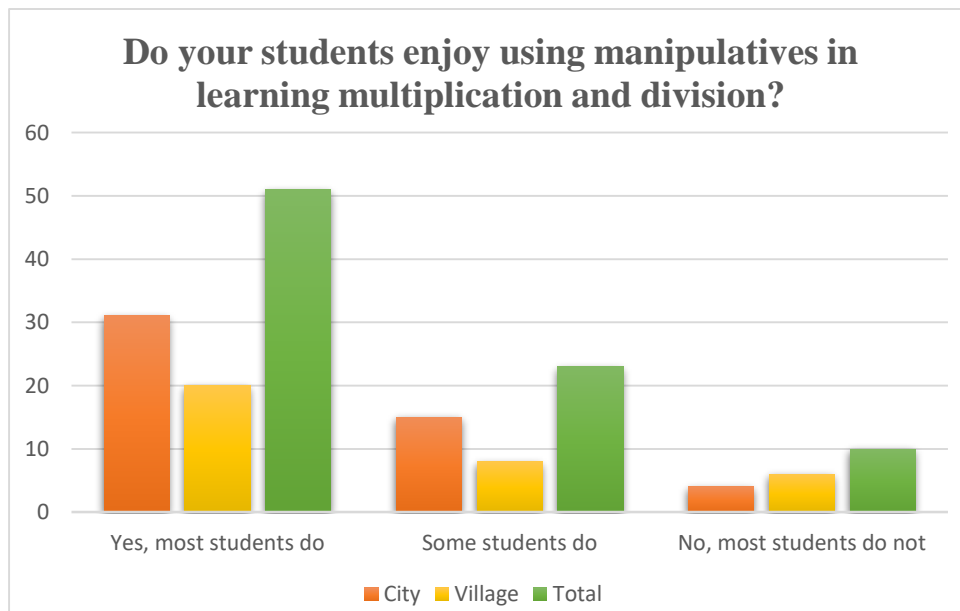
Most of the teachers responded with "Frontal" as well as "Small groups". I believe that the teachers have given such answers because of the lack of manipulative tools available to each student or to a pair of students so they could learn with them. If we compare the results based on the working place, we can see that 26.2% of teachers in the city have answered "Small groups" and on the other hand 22.6 of teachers in the village have answered "Frontal".

In the following table we have presented the results of the question “Do your students enjoy using manipulatives to learn multiplication and division?”.

**Table 9: Students’ attitude towards the use of manipulatives**

Do your students enjoy using manipulatives to learn multiplication and division?			
	Living place		Total
	City	Village	
Yes, most students do	36.9%	23.8%	60.7%
Some students do	17.9%	9.5%	27.4%
No, most students do not	4.8%	7.1%	11.9%
Total	59.6%	40.4%	100%

**Histogram 7: Students’ attitude towards the use of manipulatives**



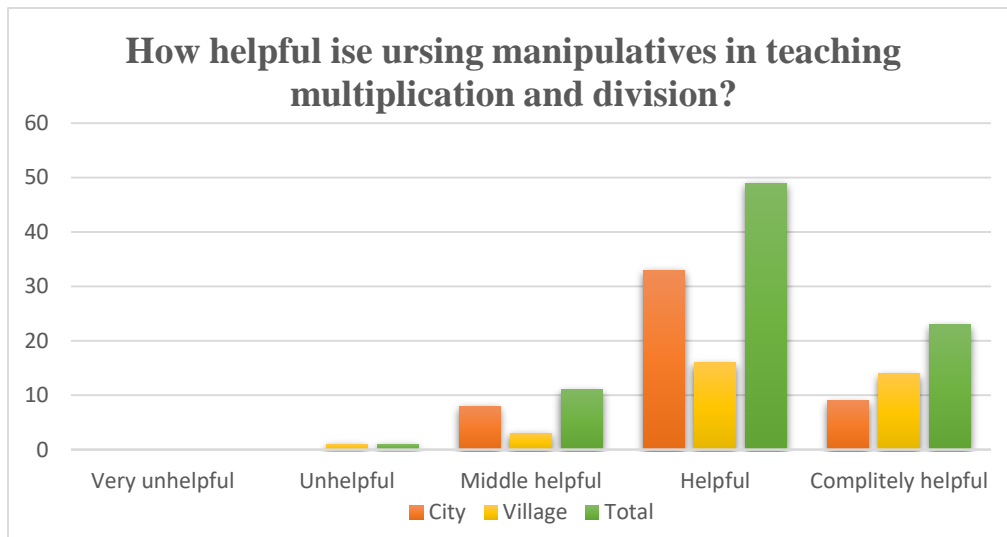
When working with students, teachers undoubtedly notice whether their students are enjoying a particular activity or if it is a boring and non-motivating activity to learn. 60.7% of teachers responded by circling that "Yes, they do". Furthermore, while conversing with some of the teachers, they said that children enjoy working with manipulative tools because they consider them to be a game by which they both play and learn.

Based on the teachers' answers to the question "How helpful is using manipulative tools in teaching multiplication and division?", we can further reinforce the answer to the key question of this research whether manipulative tools affect the acquisition of multiplication and division in the second-grade.

**Table 10: The role of manipulative tools in teaching multiplication and division**

How helpful is using manipulatives in teaching multiplication and division?			
	Living place		Total
	City	Village	
Very unhelpful	0%	0%	0%
Unhelpful	0%	1.2%	1.2%
Middle helpful	9.5%	3.5%	13%
Helpful	39.3%	19%	58.3%
Completely helpful	10.9%	16.6%	27.5%
Total	59.7%	40.3%	100%

**Histogram 8: The role of manipulative tools in teaching multiplication and division**



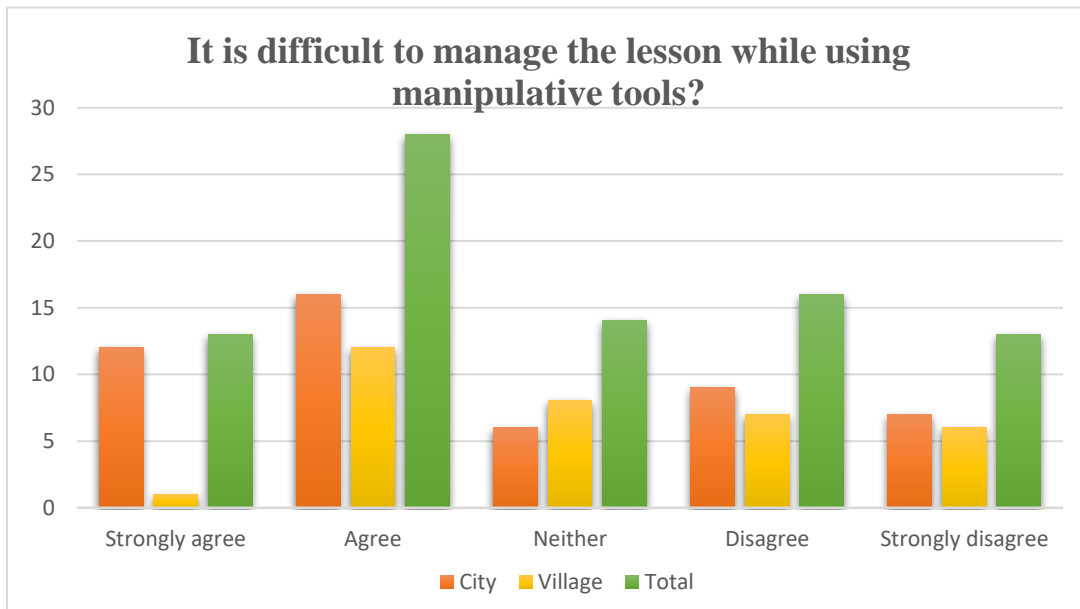
Due to the fact that 85 % of teachers have responded with "Helpful" and "Completely Helpful", it can be concluded that manipulative tools help or have an impact on better acquisition of multiplication and division.

Many teachers think that using a manipulative tool creates a mess in the classroom and the lesson becomes difficult to be managed. Due to this reason, for the teachers who use manipulative tools we compiled the question "Is it difficult to manage the lesson while using manipulative tools?".

**Table 11: The difficulty of managing the lesson while using manipulative tools**

It is difficult to manage the lesson while using manipulative tools?			
	Living place		Total
	City	Village	
Strongly agree	14.2%	1.2%	15.4%
Agree	19.2%	14.3%	33.5%
Neither	7.1%	9.5%	16.6%
Disagree	10.8%	8.2%	19%
Strongly disagree	8.3%	7.2%	15.5%
Total	59.6%	40.4%	100%

**Histogram 9: The difficulty of managing the lesson while using manipulative tools**



48% of teachers responded with "Agree" and "Strongly Agree". But even though these teachers know that it is difficult to manage the class when using manipulative tools, they still use the tools with great care and by planning everything correctly so that there is no mess in the classroom.

Based on the working place we can see that there is a difference in the percentages of the answer "Strongly Agree". 14.2 % of teachers in the city have answered "Strongly Agree" while on the other hand only 1.1% of teachers in the village have answered to this option. Maybe this difference is due to the fact that in the city schools there is a larger number of students in a classroom and when the number is larger of course that it is harder to manage the lesson while using manipulative tools.

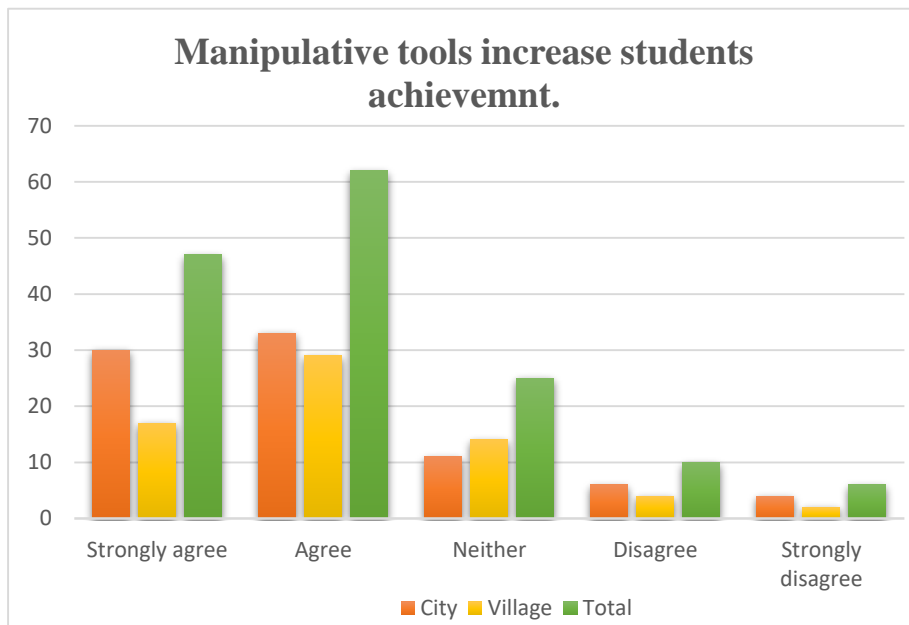
In the following two statements all the teachers had the opportunity to answer and not only those who said that they use manipulative tools during the teaching of multiplication and division.

In the following table we have presented the results of the statement “Manipulative tools increase students’ achievement.”

**Table 12: Teachers’ beliefs about the role of manipulative tools on students’ achievement**

Manipulative tools increase students’ achievement			
	Living place		Total
	City	Village	
Strongly agree	20%	11.3%	31.3%
Agree	22%	19.3%	41.3%
Neither	7.3%	9.3%	16.6%
Disagree	4%	2.7%	6.7%
Strongly disagree	2.6%	1.5%	4.1%
Total	55.9%	44.1%	100%

**Histogram 10: Teachers’ beliefs about the role of manipulative tools on students’ achievement**





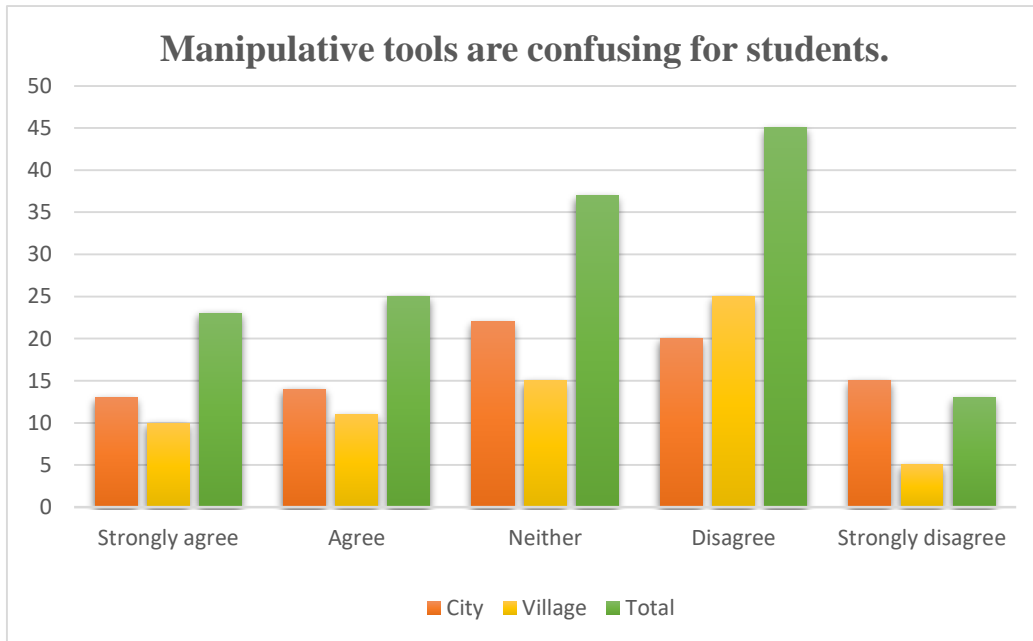
In the statement "Manipulative tools increase students' achievement", 72.6% of teachers responded with "Strongly agree" and "Agree". This draws us to the conclusion that many teachers think that manipulatives have an impact on students' achievements in mathematics, namely on teaching different mathematical concepts.

The statement "Manipulative tools are confusing for students" was part of the teachers' questionnaire as well and in the following table we can see the result.

**Table 13: Teachers' beliefs about the students' confusion while using manipulative tools**

Manipulative tools are confusing for students.			
	Living place		Total
	City	Village	
Strongly agree	8.9%	6.6%	15.5%
Agree	9.3%	7.3%	16.6%
Neither	14.6%	10%	24.6%
Disagree	13.4%	16.6%	30%
Strongly disagree	10%	3.3%	13.3%
Total	56.2%	43.8%	100%

**Histogram 11: Teachers' beliefs about the students' confusion while using manipulative tools**



The results show that 32% teachers responded with "Agree" and "Strongly Agree", while the rest disagree with this statement, since they probably think that manipulatives simplify a lesson and do not complicate it. According to the working place, we can see that between the answers of two groups there is not any differences, both groups gave almost equal answers.

#### IV. DISCUSSION

The purpose of this research was to investigate the impact of manipulative tools in teaching multiplication and division in the second grade. To come to the results of this research two second-grade students were taken as participants, one being the control group and the other one the experimental one. At the control group for an entire week, the multiplication and division by number 6 were taught using the radical method, i.e. using only the book, notebook, chalk and the blackboard while with the experimental group during the entire week multiplication and division by number 6 was learned using manipulative tools to solve each task. By comparing the results obtained in the pre-test and the post-test it can be concluded that the purpose of this research was achieved. Consequently, from the data obtained, it has been found that the results in the post-test at the experimental group were higher than the ones at the control group. With the experimental group the manipulative tools were used all the time. This has also increased the knowledge of second-grade students when it comes to the learned multiplication and division tasks.

From the obtained results, it became possible to validate the hypothesis stated above: students using manipulative tools in learning multiplication and division achieve higher results in comparison to the students using math textbooks only.

The question of this research was: Do manipulative tools affect on a better acquisition of multiplication and division in the second grade? From the data obtained, as shown in Tables 1 and 2 we can see that manipulative tools have a positive impact on the acquisition and solving of multiplication and division tasks among the second-grade students. The manipulative tools used in the experimental group made it possible to develop a new spirit of teaching and learning. This made the students very happy and also aware of the importance of manipulative tools and the opportunity to teach them things in a simpler, faster and clearer way. The classes held in this form were also very motivating for the students who have difficulties with math and made it possible for them to be an active part of the class. " In most cases it has been proven that students who use manipulative tools properly to learn the concepts of multiplication and division outperform those students who do not use these tools" (Raphael and Wahlstrom, 1989, p.46). Also, based on the results of Table 10 and Table 12, we can conclude that teachers also agree that manipulative tools help second grade students on a better acquisition of multiplication and division.

Other research questions included:

- Does teachers' age affect the usage of more manipulative tools when teaching multiplication and division?
- Are there any differences between teachers in rural and urban areas in the usage of manipulative tools when teaching multiplication and division?

Based on the questionnaire we conducted with 150 second-grade teachers, the answers to the above questions were also validated and the other 2 hypotheses put forward in the research were also confirmed. Based on the results of Table 4, we can conclude that teachers' age affects the frequency of how often manipulative tools are used. This is probably due to the fact that the older teachers do not have as much energy as at the beginning of their career which also requires more commitment and attention while using them. On the other hand, younger teachers have more energy and patience and cannot wait for their students to have fun in different ways. Thus, they find easier methods to use them.

Regarding the other research question which refers to the extent that the town and village have on the use of manipulative tools, based on the obtained results on Table 3, we come to the conclusion that it does not affect the place in which they teach. Teachers, be it in towns and villages as well are willing to work with manipulative tools. Based on the results from Table 5, teachers would work with manipulatives but there are not any manipulative tools in their schools and the financial conditions to buy them would in many cases make it impossible for them to work with them. Based on the results of table 7, instead of manipulative tools they use various tools within the classroom such as pencils, chalks, colors, erasers, etc.

## V. CONCLUSION

Possibly this study has answered the questions and concerns that the researcher had about the problem and also provided the expected results. It can also be noted that it has largely met the objectives. From the results of the pre-test obtained with the experimental group and the control one as well, it has been concluded that manipulative tools positively impact on the acquisition and results while solving multiplication and division tasks in second-grade students. The students of the experimental class did not find it difficult to work with manipulative tools. They became so familiar with this form of learning that they considered it more of a game and often forgot that they were dealing with multiplication and division. Thus, they learned in a very natural way and with great desire and will. It is widely believed that teachers should be the ones who understand the interest of children, and should know how to motivate children to learn. Always bear in mind that students need constant motivation, and who better than a teacher understands their needs. Teachers are always looking for more advanced practices that bring results when explaining things. It is preferable to use a variety of methods and strategies and not to become completely dependent on the explanations they encounter only in the basic textbook.

From the work done with the students of the experimental group, from the observation, regarding the use of manipulative tools in the classroom it can be concluded that:

- Students were more active during the lesson;
- They better understood the operations of multiplication and division;
- They were more cooperative and helped each other;
- They learned about recycling things and realized that many things they had at their houses could be used to learn mathematics;
- They developed their psychomotor skills;
- They developed critical and creative thinking;
- They achieved better results in solving multiplication and division tasks;
- Students started to love math more and they considered it much more entertaining.

On the other hand, regarding the control group students who had the mathematics textbook as their only source of learning, some characteristics were observed, such as:

- Not all the students were active during lesson;
- There was a lack of students' attention to follow the class;
- They did not manage to absorb the multiplication and division operations quite well;
- They were quieter during class;
- Some students were not at all interested in understanding what the teacher was explaining;
- They did not develop their psychomotor skills;
- They did not develop critical thinking;
- The students did not achieve good results in solving multiplication and division tasks;
- They did not enjoy the math class;
- They tried to memorize the results of multiplication and division and not to learn them logically;
- There was a lack of collaboration between students since every student wanted to dominate.

Therefore, it can be stated that the best way for students to come to school very happy, cheerful and to like all subjects similarly is to use as many concrete and manipulative tools as possible during a lesson. Teachers have to choose the manipulatives very carefully. It is important to develop mathematical concepts through use of concrete material and follow the steps: from concrete to abstract, but it doesn't mean that we should exaggerate with the use of them. Too much dependence on work with manipulatives can also prohibit a student from progressing to a more abstract level of thinking. The problem is that "a manipulatives' physical nature does not carry the meaning of a mathematical idea" (Clements, 1999, p.56) meaning that when using manipulatives teachers need to carefully plan how they are going to use them and what mathematical concept they are trying to teach. It is easy for students to "overlook the consequences of their actions" (p.55) when working with manipulatives and "even when manipulatives are planned as an integral part of the lesson, student thinking and reasoning can become routine and mechanical" (Stein & Bovalino, 2001, p.356), causing manipulatives to lose their value.

Having an experience in teaching at primary school and as an observer of the use of manipulative tools when explaining multiplication and division in the second-grade, we can feel competent and willing to outline some of the recommendations that are considered as reasonable while teaching multiplication and division:

- Teachers should integrate manipulative tools into the classroom as they really bring efficiency to student learning;
- Always integrate manipulative tools in the teaching of multiplication and division, relying on different activities, because this way the students' creativity is highlighted and helps them to express their opinions;
- While using manipulative tools learners can make multiple presentations showing how they have obtained the result;
- Inclusion is one of the most important issues in the education of children. Thus, the students with learning difficulties and special needs can simultaneously learn more and be entertained while using manipulative tools;
- Before using manipulative tools make sure that they are appropriate for the age and psycho-physical ability of the students;
- Large quantities of classroom manipulative tools can deconcentrate pupils and therefore bring to the classroom a limited number of manipulative tools;
- Set a goal in advance on what kind of manipulative tools you will use and how long you will use them.

Based on the data of this research, the obtained results from the work done with pupils as well as from the questionnaire filled in by the teachers, it is reasonable to think that many teachers in Kosovo should be aware of the importance of manipulative tools in teaching mathematical concepts, respectively, teaching multiplication and division to the second-grade. This research provides suggestions on how elementary school teachers can use manipulative tools in various activities in order to encourage the development of students' thinking, and to facilitate the learning of multiplication and division among second graders. Although Kosovo's schools lack the physical conditions to purchase various manipulative tools, this is not a reason not to use them. It is not necessary, however, to spend money on purchasing manipulative tools when instead they can use tools such as beans, rice, pasta, stamps, etc., which can be obtained by each student from their home, and then other tools can be used in the classroom such as pencils, colored pencils, erasers, etc. It is important for teachers to understand the importance of manipulative tools in teaching and



then when there is a desire to use them, there is no doubt that there are ways to use them or what can be used instead of them.

While working on this research certain limitations were faced and which included:

- Lack of literature and articles in the Albanian language regarding the relevant topic;
- There was no individual desk for each student in the classroom, since the students were seated in pairs and this allowed them to look at each other during the pre and post-test, or, in other words, to cheat from the friend they were sitting with.

Through this research and based on the obtained results we will try to convince other teachers of the importance of manipulative tools while teaching different concepts of math and especially while teaching and learning multiplication and division in second grade. Starting from the place we work, we can discuss with the teachers about the topics such as: What are manipulative tools? What we can use as manipulative tools to teach different math concepts?; How can we use manipulative tools?; Advantages of using manipulatives while teaching and learning math?; We could also have a lesson with teachers and integrate the manipulatives in some different activities. If this works with a school than we can present the idea to the relevant persons of education and continue the presentation with other teachers. While this is a previously undisputed topic in our country this could be a way the teachers of our country could raise their knowledge about manipulative tools and the importance of using manipulative tools while teaching and learning.

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**APPENDIX:**

- Appendix A - Announcement
- Appendix B – Pre-Test
- Appendix C – Post- Test
- Appendix D – Teachers questionnaire



## **Appendix A - Announcement**

### ANNOUNCEMENT

Dear parent,

I am conducting a master's degree research study, where as a study topic I am dealing with "THE ROLE OF MANIPULATIVE TOOLS ON TEACHING AND LEARNING MULTIPLICATION AND DIVISION IN SECOND GRADES IN KOSOVO". My study will focus on second grade students and second grade teachers.

Initially, the students will take a pre-test, which will help me know how much the students know about multiplication and division. Then for a week we will have the math class together and teach the lessons using manipulative tools in activities including multiplication and division. In the end, students will take a post-test, which will help me understand how manipulative tools have impacted the teaching of multiplication and division.

These data will help me complete my research but at the same time will help all teachers in our country to use manipulative tools while working with students.

For any additional information, do not hesitate to contact me!

Email: [a.abdullahi92@hotmail.com](mailto:a.abdullahi92@hotmail.com)

*Thank you for your cooperation!*

*Sincerely,*

*Albulena Gashi*

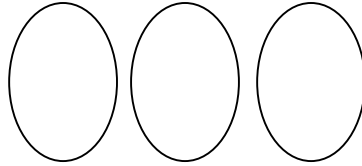
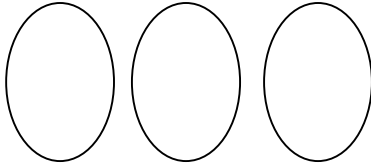
**Appendix B - Pre-test**

**MULTIPLICATION AND DIVISION**

**1. Fill communities optionally with elements based on the request of the task.**

$3 \times 5 = \underline{\quad}$

$27 : 3 = \underline{\quad}$



**2. Calculate:**

$2 \times 4 = \underline{\quad}$        $7 \times 3 = \underline{\quad}$        $4 \times 5 = \underline{\quad}$        $3 \times 3 = \underline{\quad}$        $9 \times 2 = \underline{\quad}$

$4 : 2 = \underline{\quad}$        $27 : 3 = \underline{\quad}$        $35 : 5 = \underline{\quad}$        $18 : 3 = \underline{\quad}$        $14 : 2 = \underline{\quad}$

**3. Set the signs  $<$ ,  $>$ ,  $=$**

$6 \times 4 \underline{\quad} 9 : 3$        $25 : 5 \underline{\quad} 2 \times 4$        $3 \times 3 \underline{\quad} 9 : 1$        $40 : 5 \underline{\quad} 5 \times 2$

**4. Find the unknown factor.**

$X \times 5 = 25$        $X : 4 = 7$        $X \times 3 = 21$        $X : 2 = 8$

$25 : 5 = \underline{\quad}$        $\underline{\quad}$        $\underline{\quad}$        $\underline{\quad}$

$X = \underline{\quad}$        $\underline{\quad}$        $\underline{\quad}$        $\underline{\quad}$

**5. Calculate:**

A book costs 9 euros. How much do cost 5 books ?  $\underline{\hspace{2cm}}$

32 notebooks should be equally divided into 4 boxes. How many notebooks would be in each box?  $\underline{\hspace{2cm}}$

The production of numbers 3 and 5 reduce for the quotient of numbers 40 and 4.

$\underline{\hspace{10cm}}$

The quotient of numbers 12 and 2 sum with the production of numbers 18 and 3.

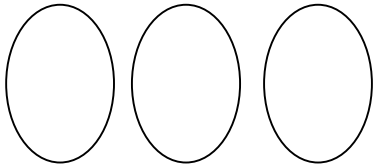
$\underline{\hspace{10cm}}$

**Appendix C - Post test**

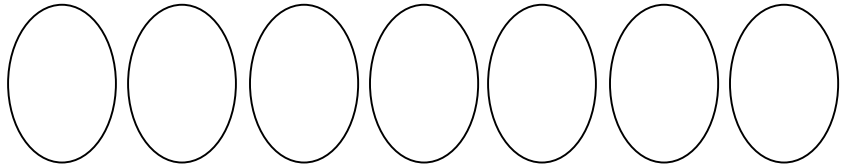
**MULTIPLICATION AND DIVISION WITH NUMBER 6**

**1. Fill communities optionally with elements based on the request of the task.**

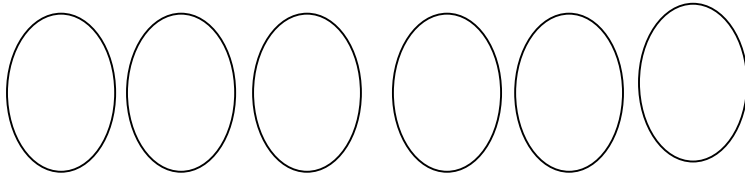
**3 x 6 = \_\_\_\_**



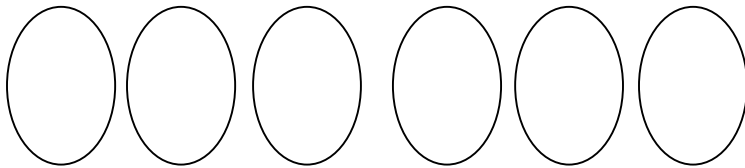
**7 x 6 = \_\_\_\_**



**12 : 6 = \_\_\_\_**



**54 : 6 = \_\_\_\_**



**2. Calculate :**

**1 x 6 = \_\_\_\_    3 x 6 = \_\_\_\_    5 x 6 = \_\_\_\_    7 x 6 = \_\_\_\_    9 x 6 = \_\_\_\_**

**12 : 6 = \_\_\_\_    24 : 6 = \_\_\_\_    36 : 6 = \_\_\_\_    48 : 6 = \_\_\_\_    60 : 6 = \_\_\_\_**

**3. Set the signs <, >, =**

**2 x 6 \_\_\_\_ 18 : 6      4 x 6 \_\_\_\_ 24 : 6      1 x 6 \_\_\_\_ 36 : 6      60 : 6 \_\_\_\_ 5 x 6**

**4. Find the unknown factor.**

$X \times 6 = 54$

$X : 6 = 1$

$X \times 6 = 60$

$X : 6 = 8$

$54 : 6 =$

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

$X =$

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

**5. Calculate:**

a) A book costs 9 euros. How much do cost 6 books ?

\_\_\_\_\_

b) 30 notebooks should be equally divided into 6 boxes. How many notebooks would be in each box?

\_\_\_\_\_

c) A kilogram of apples costs 6 euro. How much do cost 6 kilograms of apples ?

\_\_\_\_\_

d) 24 bananas should be equally divided to 6 students . How many bananas would have each student ?

\_\_\_\_\_

e) The production of numbers 8 and 6 reduce for the quotient of numbers 42 and 6.

\_\_\_\_\_

f) The quotient of numbers 6 and 6 sum with the production of numbers 6 and 6.

\_\_\_\_\_

**Appendix D – Teachers questionnaire**

## QUESTIONNAIRE

**The purpose of this questionnaire is to understand whether the teachers use manipulative tools in teaching multiplication and division. Please read all questions carefully, your answers have really worth for me. All responses will be kept confidential. Thank you for your cooperation!**

**1. Age:**

- 23-28     29-36     37-45     46-55     56-65

**2. City/Village:** \_\_\_\_\_

**3. Do you use manipulative tools in teaching multiplication and division?**

- Yes     No

**If No, why?**

- I do not have manipulative tools in our school.  
 I do not have time to use them.  
 There are too many students in my class.  
 I should by them by myself.  
 I do not want to use them

*The following 5 questions could be answer only by the teachers who said they use manipulative tools!*

**4. Which of the manipulative tools do you use the most while teaching multiplication and division?**

- Pictures  
 Blocks  
 Beads  
 Counters  
 Any other materials

**If, any other materials:**

- Class materials
- Materials that I made by myself

**5. How helpful is using manipulative tools in teaching multiplication and division?**

- Very unhelpful
- Unhelpful
- Middle helpful
- Helpful
- Complitley helpful

**6. Do your students enjoy using manipulative tools to learn multiplication and division?**

- Yes, most students do
- Some students do
- No, most students do not

**7. Do you use them**

- Individually
- In pairs
- Small groups
- Whole Class

**8. It's difficult to manage the class while using manipulative tools**

- Strongly agree
- Agree
- Neither
- Disagree
- Strongly disagree

*The following statements could be answer by all the teachers!*

**9. Manipulative tools increase students' achievement.**

- Strongly agree
- Agree
- Neither
- Disagree
- Strongly disagree

**10. Manipulative tools are confusing for students.**

- Strongly agree
- Agree
- Neither
- Disagree
- Strongly disagree