**Article:**

Di Gangi, M, Watling, D orcid.org/0000-0002-6193-9121 and Di Salvo, R (2020) Modeling Evacuation Risk Using a Stochastic Process Formulation of Mesoscopic Dynamic Network Loading. IEEE Transactions on Intelligent Transportation Systems. ISSN 1524-9050

https://doi.org/10.1109/TITS.2020.3038478

# Modelling evacuation risk using a Stochastic Process formulation of Mesoscopic Dynamic Network Loading

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**Abstract**

One of the actions usually conducted to limit exposure to a hazardous event is the evacuation of the area that is subject to the effects of the event itself. This involves modifications both to demand (a large number of users all want to move together) and to supply (the transport network may experience changes in capacity, unusable roads, etc.). In order to forecast the traffic evolution in a network during an evacuation, a natural choice is to adopt an approach based on Dynamic Traffic Assignment (DTA) models. However, such models typically give a deterministic prediction of future conditions, whereas evacuations are subject to considerable uncertainty. The aim of the present paper is to describe an evacuation approach for decision support during emergencies that directly predicts the time-evolution of the probability of evacuating users from an area, formulated within a discrete-time stochastic process modelling framework. The approach is applied to a small artificial case as well as a real-life network, where we estimate users' probabilities to reach a desired safe destination and analyze time dependent risk factors in an evacuation scenario.

**Index Terms**

Dynamic traffic assignment, Evacuation, Stochastic process, Risk assessment.

## I. Introduction

### A. State of the art

ONE of the actions usually conducted to limit exposure due to a hazardous event consists of the evacuation of the area that is subject to the effects of the event itself. Considering a transport system, a hazardous event may involve modifications in terms both of demand (a large amount of users all want to move together) and of supply (the event can modify the transport system conditions i.e. changes in capacity, unusable roads, etc.). Concerning the latter issue it is crucial how information on the effects of the event on the transport system are notified to users and how users accordingly adapt their behavior.

Information must be provided by the appropriate means [1], especially in the cases where changes in the network are present [2]. Information can be managed by means of ITS (Intelligent Transportation Systems) and can be provided in real-time either independently of the position of the user in the network (e.g. via smartphone), or in preset points within the network (e.g. via a Variable Message Sign, VMS). In both cases, information provided to the users may involve changes in path choice [3]–[5]. In the latter case, variations in the followed path occur at specific points in the network, whereas in the former case changes are possible at any point of the network.

Several authors have considered user behavior. De Jong and Helsloot [6] analyzed how some responses are more susceptible to influence by information than others, by considering the effects of real-time communication during an evacuation in three different scenarios, namely a flood threat to urban areas on a coastline, from a river and from lakes. Russo and Chilà [7] studied user behavior in relation to both information type and event type. Hasan and Ukkusuri [8] presented a threshold model of social contagion in order to characterize the effects of social influence on the evacuation decision-making process. Pel et al. [9] showed how traveller information and compliance behavior can be considered in modelling evacuation. Knoopa et al. [10] investigated how travelers change their route when faced with unexpected traffic circumstances. Simonovic and Ahmad [11] developed a simulation model for capturing human behavior during a flood emergency evacuation using a system dynamics approach.

In spite of the importance of path choice, in a more specific manner with respect to the scientific results previously cited, where path choice seems us to result as a consequence of users' behavior, in this paper our attention is focused on the analysis of this crucial aspect in the behavioral context.

In non-evacuation conditions, path choice analysis is generally specified into two phases [12]: choice set generation (paths that meet pre-defined criteria are considered as available to users) and path choice (a probability value is associated to each path). In the case of evacuation, many approaches have been proposed, where theoretical developments can be extended to emergency conditions, as in Vitetta et al. [13] and Polimeni et al. [14]. Hsu and Peeta [15], [16] propose a fuzzy approach

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simulating the evacuee behavior on information-based evacuation strategies. In Pel et al. [9], a path size Logit is used to model path choice under information provision during the evacuation. Tu et al. [17] evaluate the influence of driver behavior on evacuation time. Cova and Johnson [18] propose a model to find the shortest paths for vehicles during an evacuation in order to minimize conflicts at junctions. Wu et al. [19] analyze the path choice behavior of users approaching a hurricane, taking into account real-time user information.

Considering assignment, many approaches have been proposed to simulate traffic evolution in a transport network during an evacuation. To allow the analysis of phenomena connected to temporal variations in terms of both demand and supply (such as rising and scattering of queues due to temporary peaks of demand and/or capacity reductions of infrastructures), Dynamic Traffic Assignment (DTA) models are generally adopted.

This topic will be discussed in more detail in the section concerning the methodology adopted.

However, by its nature, evacuation planning is never made in a repeatable, predictable context, and at best we can predict user behavior and traffic interactions according to a probabilistic law. Therefore, deterministic models are of relatively low value in planning for such eventualities; instead, it is important to evaluate *risk* which is naturally associated with stochastic models.

The evaluation of a measure of the risk connected to an evacuation can be conducted either to take into account network conditions due to the peculiarity of the event [44] or to support decisions in emergency planning [45].

The definition of a risk assessment model for emergency evacuation procedures is a challenging task. Risk awareness in local government based on appropriate and effective measures can affect the management of an evacuation in a disaster situation, so that both risk assessment and evacuation analysis are fundamental to safety management, [46]. Systemic safety countermeasures addressing specific safety issues related to the presence of risk factors on road infrastructures may be derived from a network-wide systematic and proactive risk mapping procedure, [47]. Moreover, the evaluation of factors based on a regional disaster system theory provides an effective framework for preventing the occurrence of urban road traffic accidents, [48]. The definition of tools able to assist a community in making local decisions during an evacuation reveals essential in order to improve the emergency planning. The relationship between users' behavior during an evacuation and individual perceptions of risk has a certain scientific rationality, and agent-based models taking into account alternative evacuation plans based on the understanding of residents' movements according to changes of environment, [49], as well as analyzing the effect of disaster risk perception on various emergency behaviors related to one's own perceived vulnerability, [50], have been proposed.

In [51] the calculation of risk indices assessed by means of a rationally structured risk function in case of fire scenarios is discussed. Such a methodology, which is sensitive to design detail and choices, and is able to aid, directly, decision-making, requires on its part the discretization of the involved risk function in order to be adaptable to the available computer power.

All the mentioned aspects lead us towards a deeper analysis of evacuation processes based on the extension of a classical approach [44], [52] to the stochastic context. The main motivations and benefits related to this choice, and how the development of the topic has been carried out within the paper, are outlined in the following sub-section.

### *B. Aim of the paper*

Since evacuation deals with a very specific circumstance on a specific day, no data referred to the past can be used in order to foresee the state of the system. Equilibrium or even normal day-to-day is not relevant, since in this case it is all about dealing with a very specific circumstance on a specific day.

Decisions can be made only referring to data obtainable in a "very early" past and this makes a Markov process suitable to model operations occurring during an evacuation. After the occurrence of the event (or before its effects are perceived by the population) the state of the network, both in terms of traffic conditions and of efficiency and/or availability of connections, depend on the immediate last time period. It is worth noting that, dealing with an anomalous event, the level of uncertainty in the predictive information can be extremely high, and information sources may be subject to disruption, so that a stochastic approach seems especially relevant.

So, it can be said that the state of the network at time $t$ depends on eventualities occurring up to an earlier time $t - \Delta t$, $\Delta t$ being some fixed time period, whose duration is set *a priori* depending on the observation of the nature of the event and its plausible evolution. Then, users adapt their behavior on the conditions they experience during their trip. Starting from these observations, we state that user behavior can be modeled by means of a stochastic process approach, and realizations of traffic condition on the network can be described by using a dynamic network loading (DNL) model based on a stochastic process approach.

In the diagram depicted in 1, the interactions among the proposed model and all the other considered components of the simulation architecture are illustrated. The proposed model can incorporate any of the methods providing information on the evolution of the system, even if they are considered as external components enhancing the proposed model with some information at certain times.

Even if exogenous real-time information (through radio, mobile phones, variable message signs, word-of-mouth, etc.) could be anticipatory, in that it could forecast the most likely future states, this quality does not violate the Markov assumption, since, as it will be more clear in the description of the proposed process model, anticipatory information rises from a model applied to describe how measurements of historical traffic conditions might change in the future.

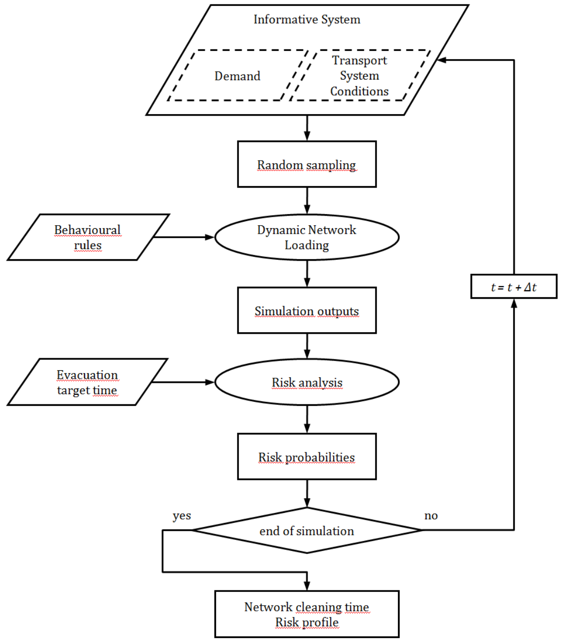


Fig. 1. *Diagram illustrating the simulation architecture.*

So, one of the aim of the paper is about providing a stochastic process formulations of within-day dynamic loading models of transport networks for evacuation modelling. In doing this, at first, a dynamic process formulation of a mesoscopic load model will be proposed; a stochastic process formulation is subsequently specified.

The introduction of a stochastic formulation allows to produce real-time information on the risks with low efforts by giving valuable implications in decision making processes.

In particular, the information strategy implemented in this paper is based on the development of a risk-based evacuation analysis for the entire systems by considering the possibility of information resources at the nodes of the network.

Data obtained by such analysis can be used to define appropriate planning policies and/or control strategies according to the peculiarity of the evacuation scenarios.

Contexts of applicability of the proposed approach include all those diversified situations characterized by the availability of a certain amount of time to be exploited for the evacuation of the area before the effects of the dangerous event definitively take place (e.g. tsunami, flood, slopes, chemical spills). Thus, the only assumption to apply the method concerns the existence of a time interval between the knowledge of the event and its concrete occurrence.

With regard to the stochastic process formulation, it is possible to consider some scenarios where users are evacuating from an area, and study how the probability of evacuation in different time steps looks like. In particular, once defined the probability of evacuation as the likelihood to reach safe areas before the effects of the hazardous event affect the transportation system, a time dependent risk function based on evacuation scenarios is defined. The novel stochastic formulation of the DNL approach allows to pursue the objective of accounting for risk assessment and evaluate effective evacuation strategies.

Concerning the structure of the present paper, in II, a mesoscopic DNL model [52] is considered in order to show how models of such a kind can be formulated by means of a discrete-time dynamical system. In III, user behavior is introduced in order to manage real-time information about en-route diversion and effectively approach emergency conditions in a transportation system. In IV, it is shown a way to define a risk function and, finally, in V, some examples on simple networks are shown in order to illustrate how the proposed formulations are able to model the time evolution of the traffic conditions. At the last, section VI contains some final remarks and comments on the achieved results.

## II. Adopted methodology

In this point, after a concise examination of the network loading models, the methodology adopted to achieve the aim of the paper is described.

### A. *Examination of Dynamic Network Loading models*

Within DTA, Dynamic Network Loading (DNL) models are able to simulate the flow propagation in dynamic assignment. The approaches to solve the problem can be classified according to the function on which they are based (exit function or link travel time function, [20]–[22]) or considering the aggregation level of the assignment model (macroscopic, i.e. [23];

mesoscopic, i.e. [24]; microscopic, i.e. [25]). Relating to the link travel time function, various expressions have been proposed in the literature ranging from linear functions [22] to polynomial functions [26]. Considering the exit function the works of Daganzo [27], Smith [28] and the study conducted by Mahmassani et al. [29] can be cited. Assumptions adopted on evolution of queues can also be used as a method to categorize the DNL models [30]; following this way models can be distinguished between link based and node based. The former [31], [32] are focused on link cost functions, the latter [27], [33], [34] consider flow splitting at nodes.

Concerning the mesoscopic approach, early models considered a packet approach where users are grouped together to form a packet that can be moved along the network so as to realize a discretization of the demand of each O/D pair. It is possible to distinguish between a point packet approach [35]–[37], in which a group of users is concentrated into a single point, and a continuous packet approach [38]–[40] where the users are supposed uniformly distributed in time or space along the packet between the packet edges. Such models have evolved over time both for the various increasingly sophisticated hypotheses that have been considered, and for the growth of computational resources that can be used in the implementation. More recently, Sánchez-Rico et al. [41] propose to solve DNL with a discrete event algorithm based on flow discretization. Celikoglu and Dell'Orco [32] propose a solution algorithm implementing an exit function with respect to capacity constraints and considering acceleration and deceleration on the packets. Linares et al. [42] present a multiclass DNL with a continuous link based approach with discrete demand.

A review of dynamic models to simulate user behavior in evacuation can be found in Pel et al. [43].

The papers reviewed so far all contribute to addressing the behavioral, network and traffic flow aspects of the dynamic evolution of evacuation. Together they provide important components for a decision-support tool for evacuation planning, that would allow alternative contingencies to be evaluated as the event unfolds.

Among DNL models, the mesoscopic approach seems to be the more suitable to our aim, due to the fact that it combines the advantages of dynamic disaggregated traffic modelling (since the packets can be at least composed by a single vehicle modeled individually) with the use of macroscopic speed-density functions, as it is more deeply described in Linares et al. [42], [53].

### *B. Discrete-time dynamical system formulation of DNL*

Let $p_\ell$, $\ell \in \{1, 2, \dots, L\}$, be an acyclic path in the network across all OD movements. Notice that, since in the dynamic network loading process information about the OD is irrelevant (the travel experience is the same, regardless of the OD), such a way of indexing the paths, without additionally specifying the OD, is sufficient for the DNL model.

In the considered class of mesoscopic approaches, traffic is subdivided into discrete packets, any one of which potentially representing several vehicles. In the models taken into account, we suppose packet-size to be common to all paths and OD movements, and constant across the modeled period. In order to switch packets and vehicles, we then only need to define a scalar $\gamma$ converting the packet-size into equivalent passenger car units. This gives the possibility to represent the combined effect of several vehicle types through the use of passenger-car-equivalents. Thus, in our models, we suppose that all packets entering at the same time experience both the same effects on congestion and the same travel times[1].

Suppose that we have a total amount $n$ of packets wishing to travel on one of the acyclic paths, with packet $i$ following path $p(i) = p_\ell$, $\ell \in \{1, 2, \dots, L\}$, $i \in \{1, 2, \dots, n\}$. The departure times (as real clock-times in seconds past time 0, that represents the start of the study period of interest) of the individual packets are denoted by $d(i) \in [0, +\infty)$ $(i = 1, 2, \dots, n)$. Both the path $p(i)$ and the departure time $d(i)$ of each packet $i$ are given inputs of the DNL.

For the implementation of the DNL, a time discretization is made, with $\Delta t$ denoting the duration of one time increment. There is some flexibility on the choice of $\Delta t$, but it must be sufficiently small that it is not possible to enter and exit a link in the same time increment. In order to ensure this, $\Delta t$ should be chosen so as to be smaller than the free-flow travel time of the link with the smallest free-flow travel time.

We denote by $x(i, k)$ the location of the packet $i$ after $k$ time increments (for $k = 0, 1, 2, \dots$), defined as the distance that $i$ has travelled along its pre-defined path $p(i)$ after time $k\Delta t$, having departed at its pre-defined time $d(i)$.

In order to potentially allow spill-back of traffic beyond the start of the journey (*i.e.*, when traffic is so congested that demand must be held in a queue to enter the network), we suppose that each packet $i \in \{1, 2, \dots, n\}$ begins traveling at time $T_0 = 0$ on a virtual link in which it has a negative initial location:

$$x(i, 0) = -x(i, \tilde{k}) \quad \text{s.t.} \quad \tilde{k}\Delta t = d(i). \tag{1}$$

On virtual links, packets move at the speed of one unit of distance per time increment. Thus, provided a packet's real entry-link to the network has sufficient space to allow it, the packet will enter at exactly its desired departure time; otherwise, it will be held in what is effectively a vertical queue until there is space to enter.

[1] This would be different if, for example, we were explicitly representing different compositions of vehicle types in different packets, in which case, not only the impact on congestion, but also the experience of travel time at the same link entry-time could be packet-specific.

Let $|\lambda|$ be the length of some link $\lambda$, for $\lambda = 1, 2, \dots, \Lambda$. We now define the functions $f(x; d, p)$ and $g(x; d, p)$, which are effectively like "look-up tables", stating in a easy way the correspondence between the packet location (in terms of distance a packet has travelled along its route since entering its real entry-link) and the link on which it is located:

$$\begin{cases} f(x; d, p) & := \lambda \\ g(x; d, p) & := \Delta_\lambda \end{cases}, \tag{2}$$

where if $\lambda = f(x(i, k); d(i), p(i))$ then $\lambda$ indexes the link of the packet $i$ at distance $x(i, k)$ along the path $p(i)$, having departed at time $d(i)$, whereas $\Delta_\lambda = g(x(i, k); d(i), p(i)) \in [0, |\lambda|]$ is the distance along $\lambda$ that $i$ has covered.

It will also prove useful to introduce the generic indicator function

$$\delta(\lambda, \mu) = \begin{cases} 1 & \text{if } \lambda = \mu \\ 0 & \text{otherwise} \end{cases}, \tag{3}$$

$\lambda, \mu = 1, 2, \dots, \Lambda$, and the static function $\eta(\lambda, p_\ell)$, defined for any link $\lambda$ that is part of path $p_\ell$ and is not the last link on the path:

$$\eta(\lambda, p_\ell) = \mu, \tag{4}$$

$\mu$ being the link following $\lambda$ on path $p_\ell$ ($\ell = 1, 2, \dots, L$).

Let us suppose that all vehicles on a link travel at the same mean speed at any given time instant and that the common mean speed experienced in the link at any time instant depends only on the instantaneous density of the link. That is to say, for each link $\lambda = 1, 2, \dots, \Lambda$, the common mean speed at any time $\tau$ is a given static function $h_\lambda(y)$ of the density $y = y_\lambda^k := y(\lambda, k)$ of $\lambda$ at time $\tau$. For example, such a function may take the form, [54],

$$h_\lambda(y) = v_{0\lambda} \exp\left(-\zeta_\lambda \left(\frac{y}{y_\lambda^{\max}}\right)^{\xi_\lambda}\right), \tag{5}$$

where, for virtual links, $\zeta_\lambda = 0$ and $v_{0\lambda} := v(\lambda, 0)$ is the constant speed.

To simplify the reading, the management of capacity constraints and queues will be discussed a little further on. Thus, in the $k$-th time increment, a packet on link $\lambda$ traveling at the current common mean link speed, say $v(\lambda, k)$, will travel a distance $v(\lambda, k) \cdot \Delta t$ in a time interval of length $\Delta t$, unless in so doing it would have exited the link. In this latter case, it travels part of the time increment on link $\lambda$ at that link's prevailing speed, and the remainder of the time on the subsequent link of the packet's path at the new prevailing speed.

The fraction of the time increment $[k\Delta t, (k+1)\Delta t)$ spent by the packet $i$ on its current link reads

$$z(i, k) := \min\left\{\frac{\lambda(\alpha(i, k)) - \beta(i, k)}{v(\alpha(i, k), k)\Delta t}, 1\right\}, \tag{6}$$

with:

$\alpha(i, k) = f(x(i, k); d(i), p(i)), i = 1, 2, \dots, n$

$\beta(i, k) = g(x(i, k); d(i), p(i)), i = 1, 2, \dots, n.$

In order to consider a queue within a link and to account for spill-backs, the occupation of vehicles making up the queue is explicitly considered. This can be done by representing the link as a couple of segments "physically" divided by means of a floating section $S$, whose position, to be evaluated at the end of any time increment, depends both on the number of equivalent passenger car units queuing at the ending edge of a link and on their physical occupancy. A scheme of this situation is shown in fig. 2.

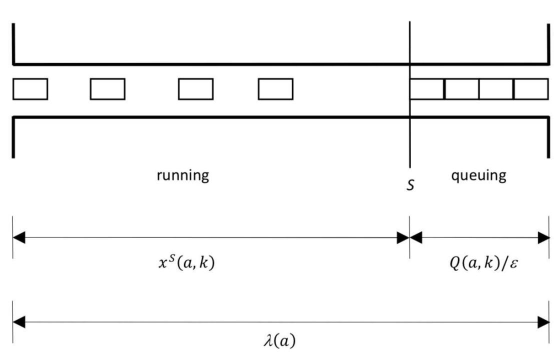


Fig. 2. *Schematic representation of a link with queues and spill-back.*

Formula (6) should then be specified for each segment of the link. In particular, the quantity $\dfrac{\lambda(\alpha(i,k)) - \beta(i,k)}{v(\alpha(i,k),k)\Delta t}$ represents the fraction of the link that can be potentially covered during the time interval; the numerator represents the distance to be covered up to the end of the link and the denominator represents the potential distance that could be covered during the given time interval. Considering fig. 2, instead, for the running segment, the potential distance for link $a$ is given by $x^S(\alpha(i,k),k)) - \beta(i,k)$, whereas the the potentially covered distance is given by $v(\alpha(i,k),k)\Delta t$. For the queing segment the movement of the queue is regulated by the capacity of the ending section, so that the numerator contains the number of vehicles that are in front of the packet and the denominator those that can leave the link during the interval, that is $(\lambda(\alpha(i,k)) - \beta(i,k)) \cdot \epsilon$ and $C(\alpha(i,k),k)\Delta t$ respectively, where $\epsilon$ is the occupation coefficient of equivalent passenger car units forming the queue and $C$ the capacity. This aspect will be described in more detail below.

In order to introduce capacity constraints on the exit from a link, let $C(\lambda,k)$ be the capacity of the link $\lambda$ at time $k\Delta t$, expressed in terms of maximum number of equivalent passenger car units exiting per time step (capacity could also be independent with respect to $k$); moreover, the quantities $Q(\lambda,k)$ and $E(\lambda,k)$ are used to represent the number of equivalent passenger car units queuing on the link $\lambda$ at time $k\Delta t$, and the number of equivalent passenger car units exiting link $\lambda$ during the $k$-th time increment, respectively. $E(\lambda,k)$ is initialized to zero at the beginning of each time increment.

Let $q(i,k)$ be the number of equivalent passenger car units forming the queue in front of packet $i$ in its current link at time $k\Delta t$: when the packet enters the link its value is initialized to $Q(\alpha(i,k),k-1)$, where $\alpha(i,k) = f(x(i,k);d(i),p(i))$.

If the packet covers completely the current link, the residual fraction of the time increment $[k\Delta t,(k+1)\Delta t)$ that packet $i$ should spend within the queue (if the queue is not null) before exiting the current link is given by $(1 - z(i,k))$, with $z(i,k)$ defined as in eq. (6).

The fraction of time increment needed to pass the queue is given by the ratio $\dfrac{q(i,k)}{C(\alpha(i,k),k)\Delta t}$; if this value is lower than $(1 - z(i,k))$, then the packet can exit the link, otherwise it remains within the queue. In order to manage such a situation, let us define a queue indicator function, say

$$w(i,k) = \min\left\{1 - z(i,k), \frac{q(i,k)}{C(\alpha(i,k),k)\Delta t}\right\}. \tag{7}$$

The value of the queue in front of the packet to be covered in the subsequent time interval now reads

$$q(i,k+1) = \max\left\{0, q(i,k) - w(i,k)C(\alpha(i,k),k)\Delta t\right\}. \tag{8}$$

We introduce the following over-saturation indicator function:

$$\Omega(\lambda,k) = \begin{cases} 1 \text{ if } E(\lambda,k-1) > C(\lambda,k-1) \\ 0 \text{ otherwise} \end{cases}, \tag{9}$$

$\lambda = 1,2,\ldots,\Lambda$. When a generic packet $i$ covers completely the link $\alpha(i,k)$ during the $k$-th time increment, it gives a contribution to the quantities $E$ and $Q$ as follows:

$$\begin{cases} E(\alpha(i,k),k) = E(\alpha(i,k),k) + \\ \qquad\qquad\qquad + \gamma(1 - \Omega(\alpha(i,k),k)) \\ Q(\alpha(i,k),k) = \max\{0, Q(\alpha(i,k),k) + \\ \qquad\qquad\qquad + \gamma(2\Omega(\alpha(i,k),k) - 1)\}. \end{cases} \tag{10}$$

In order to take into consideration the physical occupation of vehicles making up the queue, the occupation coefficient of equivalent passenger car units forming the queue should be defined. We consider, for the sake of simplicity, only one user class and define $\epsilon$ as the occupation coefficient of equivalent passenger car units forming the queue, *i.e.* the number of vehicles divided by the length of the lane. The length of the queue in the $k$-th time increment can therefore be obtained as $Q(\lambda,k-1)/\epsilon$, and the position of the section $S$ of the link, measured from its initial node, is then given by

$$x^S(\lambda,k) = |\lambda| - \frac{Q(\lambda,k-1)}{\epsilon}. \tag{11}$$

The movement of packets within the link is conducted into two steps, the first one concerning the running segment and the second one concerning the queue segment. Thus, in the $k$-th time increment, a packet currently on the running segment of the link $\lambda$ travels the running segment at the common link speed $v(\lambda,k)$, and it will travel the distance $v(\lambda,k)\Delta t$ in a time interval of length $\Delta t$, unless in so doing it would enter the queuing segment. In this latter case, it travels part of the time increment on the queuing segment of $\lambda$ at the "queue equivalent speed" $C(\lambda,k)/\epsilon$, and the remainder of the time increment (if any) on the subsequent link of the packet's path at the corresponding prevailing speed and "queue equivalent speed".

Since followed path and departure time are uniquely associated to each packet $i$, in order to simplify the notation, hereafter arguments $d$ and $p$ are omitted within the location functions $f$ and $g$ defined in (2).

The following segment location function is introduced in order to identify the segment of the link occupied by the packet $i$:

$$\sigma(x(i,k)) = \begin{cases} 1 \text{ if } g(x(i,k)) < x^S(f(x(i,k)),k) \\ 0 \text{ otherwise} \end{cases}, \tag{12}$$

$i = 1, 2, \ldots, n$. From the definition given in eq. (12), it follows that the condition $\sigma(x(i,k)) = 1$ implies that the packet $i$ is on the running segment of the link $f(x(i,k))$, otherwise it is located on the queuing segment of the same link.

The distance covered by the packet $i$ during the interval $[k\Delta t, (k+1)\Delta t)$ is stored for convenience in a temporary variable $dx(i,k)$, initialized to zero at the beginning of each time increment. The value assumed at the end of the time increment by the variable $dx$ is then added to $x(i,k)$ in order to evaluate the location of $i$ at the beginning of the subsequent temporal increment.

The procedure `NEXT_LINK(i,k)` evaluates $i$'s possibility to move to the subsequent link of its path $\eta(f(x(i,k)), p(i))$. In order to be moved to the subsequent link of its path, first of all it is checked the condition $w(i,k) > 0$, that is a check on the fact that the generic packet $i$ on link $f(x(i,k))$ has a residual fraction of the time interval $[k\Delta t, (k+1)\Delta t)$ to be spent. The way the node management is performed in this paper is briefly described in the Appendix.

Packet $i$ on link $f(x(i,k))$ is allowed to enter the subsequent link of its path $\eta(f(x(i,k)), p(i))$ if this link can receive packets and is not over-saturated, *i.e.*,

$$\begin{cases} x^S(\eta(f(x(i,k)), p(i)), k) > 0, \\ \omega(f(x(i,k)), k) > 0. \end{cases} \tag{13}$$

If one of the conditions in (13) is not true, packet $i$ spends the remainder of the considered temporal interval within the queue of the link $f(x(i,k))$, and the number of equivalent passenger car units forming the packet is added to the car units queuing on the link, $Q(f(x(i,k)), k)$.

If packet $i$ on link $f(x(i,k))$ is allowed to enter the subsequent link of its path $\eta(f(x(i,k)), p(i))$, the number of equivalent passenger car units forming the packet is added to the car units exiting link, $E(f(x(i,k)), k)$, and subtracted from the car units (if any) queuing on the link, $Q(f(x(i,k)), k)$.

The fact that the subsequent link of the path $\eta(f(x(i,k)), p(i))$ can be covered is formally indicated by adding an infinitesimal amount $\delta$ to the distance $dx(i,k)$ covered by packet $i$ during the $k$-th time increment.

Based on the definitions, assumptions and notation so far defined, the resulting DNL model considering queues and spill-backs may be succinctly schematized in the sequence of operations depicted in fig. 3, where called procedures, to enhance readability, are described separately in fig. 4.

## III. Modelling user behavior: path choice and re-routing

Generally path choice and re-routing are based on the hypothesis that users are familiar with the network configuration that is network knowledge is related to ordinary situations. In emergency conditions changes to the network (i.e. interrupted arches, contra flows, etc.) can upset user perception of the network and how they react to unexpected events. Without going in depth into the behavioral aspects, not being the purpose of this article,

in this section we propose a way to manage cases of real time information to effectively approach (not ordinary) emergency conditions in a transportation system. Such behavioral aspects consist of time dependent path choices and en-route adaptations of the chosen path to cope with eventualities occurring during the trip. In particular en-route diversion is important, as it is the only way in which drivers can really learn about exceptional conditions through their own experience or from exogenous information.

In order to take into account of unpredictable behavior of traveller, stochasticity in path choice and re-routing is introduced and the formulation of the loading model is extended by removing some of the constraints introduced above. The extensions considered here can be applied to each one of the proposed models.

### A. Computation of travel time

In II-B it has been defined that, in the $k$-th time increment, all vehicles on a link $\lambda$ travel at the same speed $v(\lambda, k)$, and that this common speed experienced on the link depends on the density of the link. Let us introduce a random variable $V^u(k) \sim Normal(\frac{1}{v(\lambda,k)}, \rho \cdot \frac{1}{v(\lambda,0)})$, with $\rho$ a real parameter, representing the time needed by user $u$ to travel a unit length of the link at interval $k$. Let $tt^u(\lambda, k)$ be the travel time (cost) of the link $\lambda$ ($\lambda = 1, 2, \ldots, \Lambda$) experienced by the user $u$ during the time increment $k\Delta t$, say

$$tt^u(\lambda, k) = v^u(k) \cdot |\lambda|, \tag{14}$$

$v^u(k)$ being a realization of the random variable $V^u(k)$, and $|\lambda|$ the length of the link $\lambda$. As product of a scalar times a Normal random variable, also the travel time has a Normal distribution.

In the most general case, there is more than one path connecting an OD pair. With the aim of defining the path $p(i)$ followed by the packet $i$ (leaving at time $d(i)$) during the time increment $k\Delta t$, we consider the OD pair $rs(i)$, and we fix $P_{rs(i)}$ to be the set of possible paths connecting the OD pair.

**procedure** DNL($(d,p)$)
    $d(i), p(i) \quad \forall i$
    $x(i,0) \leftarrow -d(i) \quad q(i,0) \leftarrow 0 \quad \forall i$
    $\pi(a,0) \leftarrow 0 \quad Q(a,-1) \leftarrow 0 \quad \forall a$
    $k \leftarrow 0$
    **while** $k < SimulationIntervals$ **do**
        $w(i,k) \leftarrow 1 \quad dx(i,k) \leftarrow 0 \quad \forall i$
        $E(a,k) \leftarrow 0 \quad Q(a,k) \leftarrow Q(a,k-1) \quad \forall a$
        $x^S(a,k) \leftarrow |\lambda| - \frac{Q(a,k-1)}{\epsilon} \quad \forall a$
        $\alpha(i,k) \leftarrow f(x(i,k); d(i), p(i)) \quad \forall i$
        $\beta(i,k) \leftarrow g(x(i,k); d(i), p(i)) \quad \forall i$
        $\sigma(i,k) \leftarrow$ SEGMENT((i,k)) $\quad \forall i$
        $y(a,k) \leftarrow$DENSITY$(a, k, x^S(a,k), k, \sigma) \quad \forall a$
        $v(a,k) \leftarrow h_a(y(a,k)) \quad \forall a$
        $\pi(a,k) \leftarrow$ SATURATION((a,k)) $\quad \forall a$
        **while** $w(i,k) > 0 \quad \forall i$ **do**
            MOVE_ON_RUNNING(i,k) $\quad \forall i$
            MOVE_ON_QUEUING(i,k) $\quad \forall i$
            NEXT_LINK(i,k) $\quad \forall i$
        **end while**
        $x(i,k+1) \leftarrow x(i,k) + dx(i,k) \quad \forall i$
        $k \leftarrow k+1$
    **end while**
**end procedure**

**function** SEGMENT(i,k)
    **return** $\begin{cases} 1 \text{ if } g(x(i,k)) < x^S(f(x(i,k)), k) \\ 0 \text{ otherwise} \end{cases}$
**end function**

**function** DENSITY$(a, k, L, \sigma)$
    **return** $\frac{\gamma}{L} \Sigma_{i=1,n} \delta(a, \alpha(i,k)) \cdot \sigma(i,k)$
**end function**

**function** SATURATION(a,k)
    **return** $\begin{cases} 1 \text{ if } E(a,k-1) > C(a,k-1) \\ 0 \text{ otherwise} \end{cases}$
**end function**

Fig. 3. *Loading procedure considering queues and spill-back.*

Let $A_{\lambda q}$ ($q \in P_{rs(i)}$) be the generic element of the link-path incidence matrix, defined as

$$A_{\lambda q} = \begin{cases} 1 & \text{if } \lambda \text{ belongs to } q \\ 0 & \text{otherwise.} \end{cases} \tag{15}$$

Thus, a realization of the travel time (depending on $k$) for the whole path can be computed as

$$T(q,k) = \sum_{\lambda=1}^{\Lambda} A_{\lambda q} \cdot tt^u(\lambda, k). \tag{16}$$

### *B. Path choice*

As seen above, the travel time (cost) of the link $\lambda$ ($\lambda = 1, 2, \ldots, \Lambda$) experienced during the time increment $(k-1)\Delta t$ is marginally distributed as a Normal variable; thus, it is possible to introduce the vector

$$\boldsymbol{\pi}_{rs(i)}(k) = [\pi(q, k, T(q, k-1), \theta)]_{q \in P_{rs(i)}}, \tag{17}$$

containing all the path choice probabilities computed by means of a MonteCarlo technique.

**procedure** MOVE_ON_RUNNING(i,k)
  $\Delta x \leftarrow x(i,k) + dx(i,k)$
  $dw \leftarrow \frac{x^S(f(\Delta x,k) - g(\Delta x)}{v(f(\Delta x),k) \cdot \Delta t}$
  $z_r(i,k) \leftarrow \sigma(x(i,k)) \cdot \min\{dw, w(i,k)\}$
  $w(i,k) \leftarrow w(i,k) - z_r(i,k)$
  $dx(i,k) \leftarrow dx(i,k) + [z_r(i,k) \cdot v(f(\Delta x),k)] \cdot \Delta t$
**end procedure**

**procedure** MOVE_ON_QUEUING(i,k)
  $\Delta x \leftarrow x(i,k) + dx(i,k)$
  $dw \leftarrow \frac{(\lambda(f(\Delta x,k) - g(\Delta x) \cdot \epsilon}{C(f(\Delta x),k) \cdot \Delta t}$
  $z_q(i,k) \leftarrow (1 - \sigma(\Delta x)) \cdot \min\{dw, w(i,k)\}$
  $w(i,k) \leftarrow w(i,k) - z_q(i,k)$
  $dx(i,k) \leftarrow dx(i,k) + [z_q(i,k) \cdot \frac{C(\alpha(i,k),k)}{\epsilon}] \cdot \Delta t$
**end procedure**

**procedure** NEXT_LINK(i,k)
  **if** $w(i,k) > 0$ **then**
    $\Delta x \leftarrow x(i,k) + dx(i,k)$
    $\zeta \leftarrow f(\Delta x, k)$
    **if** $x^s(\eta(\zeta) > 0 \quad and \quad \pi(\zeta) = 0$ **then**
      $E(\Delta x,k) \leftarrow E(\Delta x,k) + \gamma$
      $Q(\Delta x,k) \leftarrow \max 0, Q(\Delta x,k) - \gamma$
      $dx(i,k) \leftarrow dx(i,k) + \delta$
    **else**
      $w(i,k) = 0$
      $Q(\zeta) \leftarrow Q(\zeta) + \gamma$
    **end if**
  **end if**
**end procedure**

Fig. 4. *Procedures called within DNL.*

The set of paths $P_{rs(i)}$ above defined can be described as a Directed Acyclic Graph (DAG) $\Gamma_{rs(i)}(k)(\mathcal{N}^{rs(i)} \subset \mathcal{N}, \mathcal{A}^{rs(i)} \subset \mathcal{A})$ derived from the set of links belonging to the feasible paths connecting the Origin-Destination pair $rs(i)$ computed for the time increment $k\Delta t$.

The generic packet $i$, whose departure time is within time interval $[k\Delta t, (k+1)\Delta t)$, is associated with one of the paths connecting its OD pair, say $p_k(i)$, once the path probabilities are known, by means of a stochastic sampling approach (see Appendix).

The operations connected to the definition of path followed by each departing packet are synthesized by means of a path generation function:

$$p_k(i) = \varphi(\Gamma_{rs(i)}(k), \boldsymbol{\pi}_{rs(i)}(k-1)), \quad i = 1, 2, \ldots, n, \tag{18}$$

where $\Gamma_{rs(i)}(k)$ is the above described DAG and $\boldsymbol{\pi}_{rs(i)}(k-1)$ is the vector of the path choice probabilities corresponding to the paths connecting $rs(i)$ computed as described above. Path are initialized by considering a fictitious path $p_0(i)$.

### *C. Re-routing*

Re-routing operations here introduced consider that, at every node, users can modify their path depending on available information on network conditions. It is straightforward to consider this occurrence only in some defined nodes and/or some time intervals. In case of re-routing there is no more a univocal relationship between OD pair and followed path. Then the two variables $O(i)$ and $D(i)$, representing the origin and the destination associated to the packet $i$, respectively, are introduced in order to index the OD pair. In addition to the functions $f$ and $g$ defined in (2), here is introduced another law related to the link on which packet $i$ is located, say

$$e(f(x; d, p)) = N_f, \tag{19}$$

giving the ending node $N_f$ of the link identified by $f(x; d, p)$. Let $k_d(i)$ be the time increment at which departure of packet $i$ occurs, *i.e.*

$$k_d(i) = k \qquad \text{if } d(i) \in [k\Delta t, (k+1)\Delta t). \tag{20}$$

It is also functional to the re-routing procedure to introduce a temporary origin variable $O_t(i,k)$, representing the next downstream node at which traffic may reroute, whose values, for a generic packet $i$ traveling between the origin $O(i)$ and the destination $D(i)$, can be defined as follows:

$$O_t(i,k) = \begin{cases} O(i) & \text{if } k \leq k_d(i), \\ e(f(x;d,p)) & \text{if } k > k_d(i). \end{cases} \tag{21}$$

Considering a generic packet $i$ at time increment $k\Delta t$, re-routing is implemented by conducting a path choice (using the symbolism introduced in section III-A) within the set of paths $P_{O_t(i,k)D(i)}$ described by the DAG $\Gamma_{O_t(i,k)D(i)}(k)$. Re-routing is performed by choosing the path between $O_t(i,k)$ and $D(i)$ given by

$$p_k^F(i) = \varphi(\Gamma_{O_t(i,k)D(i)}(k), \boldsymbol{\pi}_{O_t(i,k)D(i)}(k-1)), \tag{22}$$

$i = 1, 2, \ldots, n$, where the superscript $F$ is for "Forward". The path travelled by packet $i$ during the $k$-th time increment can be extracted from $p_{k-1}(i)$ (by means of a suitable function $\vartheta$) and indicated as

$$p_k^B(i) = \vartheta(p_{k-1}(i)), \quad i = 1, 2, \ldots, n.$$

where superscript $B$ is for "Backward". Obviously, if $k \leq k_d(i)$, $p_k^B(i)$ results in an empty set. Thus, the path $p_k(i)$, resulting from the re-routing operation, to be associated to packet $i$ at time increment $k\Delta t$ consists of:

$$p_k(i) = p_k^B(i) \cup p_k^F(i), \qquad i = 1, 2, \ldots, n. \tag{23}$$

### D. Some considerations

The assumptions here described are realistic in those context where a mesoscopic approach is applicable in a mono-modal context. As an example, they are not suitable to model pedestrian evacuation and need more specifications to take into account for a multi-modal context. The scope of our model is to support decisions in case of planning and / or management of an evacuation following a foreseeable event and it is not applicable to all those emergencies where there is no time occurring between time between the knowledge of the event and the occurrence of the effects (i.e. earthquake). The evaluation of evacuation demand is not addressed since the study is focused on the interaction between demand and supply.

## IV. Estimation of risk during an evacuation

Classically risk is defined as the product of three factors: probability, vulnerability and exposure (of populations, goods, etc.) [55]. In order to reduce such a risk, attention may be directed towards the two main components of the probability that the event occurs and the vulnerability of the infrastructure. In the following, in order to show how the proposed approach works, an application to a very simple network is presented. We remark that the most immediate way to mitigate risk, in the case a time interval from the occurrence of the event to the propagation of its effects to population exists, consists in operating on the exposure component when evacuating the whole (or a part of the) population. Thus, considering a generic scenario, it is possible to define a time interval between the time $t_i$ at which the event actually occurs and the time $t_f$ at which the effects of such an event have an effect on the population. Depending on the length of the time interval between these two instants, $t_f - t_i$, we can act in order to rescue those people which are present in the interested area. For some kind of disasters (*i*.e., tsunami, presence of a bomb,...), during such time interval it is possible to intervene to reduce risk. On the contrary, for other kind of events where the duration of this time interval is very short (*e*.g., earthquakes), evacuation can take place only after the effects of the disaster have occurred (post-event evacuation). In the following, only events with delayed effects towards the population have been taken into consideration. In particular, by considering scenarios where users are evacuating from an area, *i*.e., they have a single destination, it will be studied how the probability of evacuating within the time interval $[t_i, t_f]$ varies at different time steps. Therefore, consider $t_f$ as the target time within which the evacuation should take place; let the evacuation probability $\eta^u(\tau)$ be the probability, evaluated at time $\tau$, that some user $u$ (located somewhere within the network) reaches his destination (safe place) within the instant $t_f$. Such a probability depends on the travel time and on the path choice. Considering the assumptions and definitions introduced in section III, it is possible to compute this probability by means of the cumulative distribution function of the normal distribution

$$F(x) = \Phi\left(\frac{x-\mu}{\sigma}\right) \tag{24}$$

where $\Phi(x)$ is the cumulative distribution function of the standard normal distribution, given by:

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}}\, dt \tag{25}$$

In the considered case, $x = t_f$ and $\mu$ and $\sigma$ are respectively the mean value and the standard deviation of the travel time needed to the generic user to reach his desired destination starting from his position at a generic time $t$.

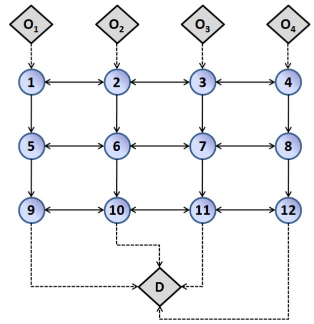


Fig. 5. *A two-dimensional regular lattice with 4 origins and one destination.*

## V. Applications

In this section a sort of measure of "endogenous" risk is derived. In a stochastic process formulation, we now look at an evacuation scenario from an area with a single way out, and analyze probabilities and risk factors, representing the situation by means of a suitable Markov chain. Referring to fig. 5, a completely general and simple situation is schematized as a regular grid and packets are traveling from the origins $O(i) \quad i = 1, .., 4$ to the same safe destination $D$. In the applications here described, users' choice concerns only the path to reach their safe place, since all the users are assumed to be aware about the evacuation procedure to follow. All the choices made before leaving the origins are assumed as exogenous.

The values for the parameters involved in the model are chosen as follows. We consider a regular demand, say a packet made up of two users leaving from each origin every 5 seconds from time 0 to time 60, and, referring to picture fig. 5, we set the arc length $\lambda$ equal to 100 $m$ for the arcs represented as vertical lines (and those connecting destination) and equal to $70m$ for those arcs represented as horizontal lines. Free flow speed $v_{0,\lambda}$ is assumed equal to 60 $km/h$ for all the arcs, except for the connectors from the origins, in which cases it is decreased to the value of 30 $km/h$. Capacity is considered equal to 2500 $veh/hr$ for every arc, excluding the connectors from origins, where the capacity is assumed equal to 100000 $veh/hr$. The formulation of the speed-density function for each link $\lambda$ follows the definition given in eq. (5), with $\zeta_\lambda = 5$, $\xi_\lambda = 2$, $y_\lambda^{\max} = 120$. Adopting the definitions set out in section III, mean value and variance of the distribution of the travel time needed by packet $i$ at interval $k$ to reach the safe destination starting from its current position read:

$$\begin{aligned} \mu_i &= \Sigma_{\lambda \in p_k^F(i)} \quad |\lambda| \cdot \frac{1}{v(\lambda, k)} \\ \sigma_i^2 &= \Sigma_{\lambda \in p_k^F(i)} \quad |\lambda|^2 \cdot \frac{\rho}{v(\lambda, k)} \end{aligned}, \tag{26}$$

where $v(\lambda, k)$ is the speed, at the $k$-th time increment, of the vehicles on link $\lambda$, $\rho$ is a real parameter, whilst $|\lambda|$ represents the length of the link $\lambda$. As described in section II, when user $i$ moves along the link $\lambda$, we take into account the quantity $|\lambda| - \beta(i, k)$, where $\beta(i, k)$ is the distance covered on the link at time $k \cdot \Delta t$. At every simulation interval $k$, it is possible to associate to each packet $i$ a distribution function $F_{ik}(x)$ by considering eq. (24) defined above. Thus, a measure of risk, representing the probability that packet $i$ is unable to reach the safe destination until the time $t_f$, can be computed as

$$r_{ik} = 1 - F_{ik}(t_f). \tag{27}$$

An indicator of risk at time interval $k$, $\bar{r_k}$, is now derived from a weighted mean of the values $r_{ik}$ in which the weights are the numbers $u_i$ of equivalent passenger car units aggregated in packet $i$:

$$\bar{r_k} = \frac{\Sigma_{i=1 \dots N_k} \quad u_i \cdot r_{ik}}{\Sigma_{i=1 \dots N_k} \quad u_i}. \tag{28}$$

To test the proposed approach, several evacuation scenarios (starting at time $t_i = 0$ and for different values of $t_f$) have been considered. We assume, in particular, that $t_f = [160 \dots 300 \ \ step \ \ 10]$ and $\rho = 0.1$. Each trial consists of conducting 1000 simulations, each one representing a realization of the stochastic process; reported results are obtained from the expected values of involved variables. Simulations are stopped until target time is reached. The numerical simulations on the test network has been conducted by considering two situations: in the first one users are continuously informed on the evolution of network conditions and they may change their route according to the communications about the conditions of the downstream network (re-routing), in the second one users, relying on their knowledge of the network, follow the path they decided at the beginning of the evacuation (no re-routing). Obtained results compare the effects of re-routing on the time evolution of the outflow models and on the risk. The variables of interest include the number of packets able to reach the safe destination within the instant $t_f$ and the risk $r_{ik}$, computed for each packet $i$ at every simulation interval $k$. The results, shown in fig. 6, are reported in terms of average rate of arrivals obtained in correspondence of different values of the target time. When choosing $t_f = 160$

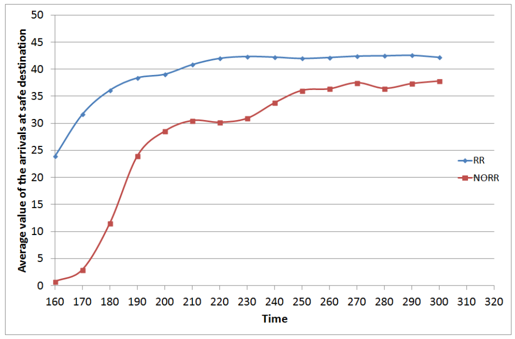


Fig. 6. *Average value of packets reaching the destination within the target time.*

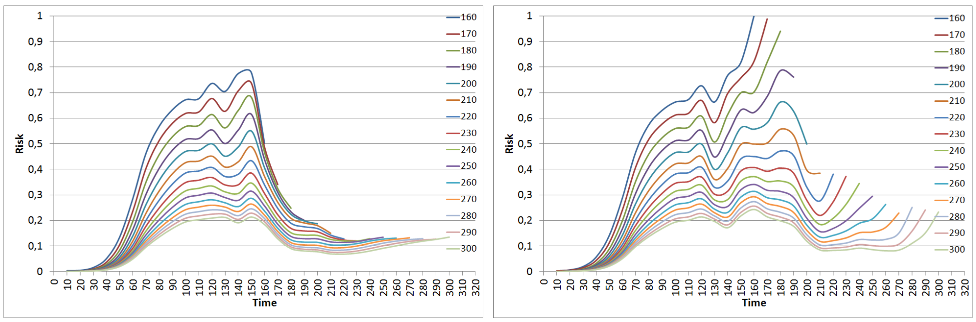


Fig. 7. *Time evolution of the indicator of risk $\bar{r_k}$ with re-routing (top) and without re-routing (bottom) for the different values of $t_f$.*

only 1.5% of packets that do not change their route are able to reach a safe destination in time in comparison with the 49,8% of those that adopt re-routing; considering $t_f = 300$ fractions are respectively 78.8 % for no re-routing users and 87.9% for re-routing users.

The time evolution of the indicator of risk $\bar{r_k}$ for several values of $t_f$ associated with the whole set of packets is depicted in fig. 7, where the horizontal axis refers to the current clock-time (at which any driver has completed part of his planned route), and the vertical axis to the average risk for the remainder of the routes. Each curve refers to a specific target time; it can be seen how higher values of target time are associated with a flattening of the corresponding risk curve. Moreover, when re-routing is included in the model (see fig. 7-*top*), information provided to users allows spreading of packets on the network, with corresponding reduction of congestion and travel times; this implies that the probability of reaching the safe destination within the target time increases, and risk values tends to vanish. On the other hand, in the case in which re-routing is not taken into account (fig. 7-*bottom*), as expected the decrease of risk after a certain amount of time steps is not guaranteed due to an increase of the network emptying time causing that some packet may not be able to leave the network within fixed target time.

In fig. 8 the approach here described has been compared, in terms of number of users reaching safe destination within the considered target times, with two classic mesoscopic loadings: in the first case travel time is evaluated randomly (in a manner which is analogous to the one previously described), whereas in the second case eq. (5) has been used. In the deterministic case, *i.e.* in the absence of random perturbations, when the target time is greater than the maximum travel time of course all users are able to reach their destination, while the random case can be considered as a realization of the process, in fact the values oscillate around the average values. Obviously, in both these cases it is not possible to estimate a risk probability.

Another numerical simulations has been conducted on a real network of the town of Capo d'Orlando (see fig. 9), located in the northern coast of Sicily, under the hypothesis of evacuating the city center following a high probability of flooding; the safe areas identified for the considered type of event are the highest parts of the town, highlighted in red. The number of evacuees is 5448, users evacuate by car, every packet is formed by a single car carrying 3 people and departures take place within 15 minutes. The evaluation of the risk indicator (depicted in fig. 10, where times are expressed in seconds) measures the probability, computed at each time step, that the safe destination is not reached within the target time. Since this application does not account for re-routing, considerations on the obtained results are analogous to those done for fig. 7-bottom. From the comparison of the two information scenarios considered in the test network it can be seen how information strategies result useful to increase the number of users who are able to leave the network within the target time. Furthermore, the effectiveness of a control strategy can be estimated by assessing the associated risk; in the example considered it is highlighted by the trend of the risk as a function of time in the two considered scenarios. In our view, the comparison between other existing methods and the approach proposed in this paper highlights how a focus on the time evolution of the level of risk, rather than the mere evaluation of evacuation times, can provide productive support to control strategies, allowing to build more effective planning

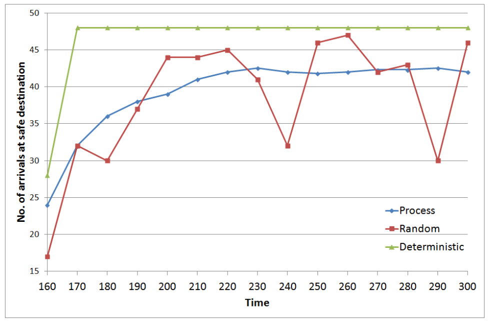


Fig. 8. *Comparison among approaches in terms of arrivals to destination within the target time.*

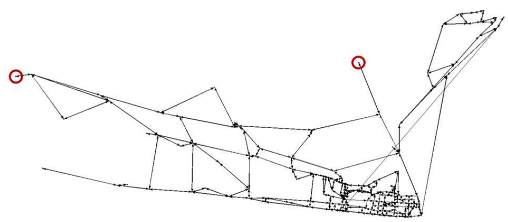

Fig. 9. *Graph of the real size network of Capo d'Orlando*

policies.

## VI. Conclusions

In this paper a method that contribute to the analysis of evacuation modelling and/or planning introducing risk assessment has been described. It is based on a stochastic process formulation of within-day dynamic loading models of transport networks. The introduction of a stochastic formulation allows to produce real-time information on the risks with low efforts by giving valuable implications in decision making processes. Path choice an re-routing have been introduced to simulate user behavior in order to manage case of real time information to effectively approach (not ordinary) emergency conditions in a transportation system. Respect to other existing methods, considering the time evolution of the level of risk can allow to build more effective planning policies and/or control strategies. Some examples of evacuation, conducted as an application on a test network and a real sized one, show the capabilities of the proposed approach providing satisfactory results both in terms of description of outflow conditions and in terms of risk assessment. Further developments concerning the application of the approach to other real size networks, the destination choice in case of different safe sites, and the identification of other kind of risk indicators are in progress.

## Acknowledgment

The authors would like to thank the anonymous reviewers for their careful reading of our manuscript and their many insightful comments and suggestions.

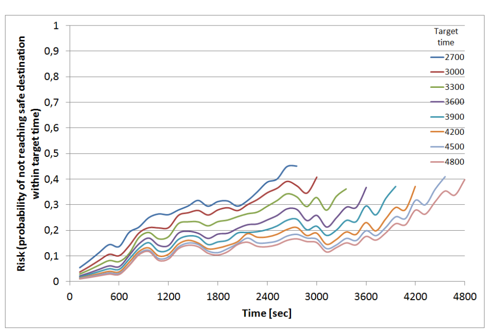


Fig. 10. *Time evolution of the indicator of risk $\bar{r_k}$ without re-routing in the real size network for different values of $t_f$.*

## APPENDIX

### NODE MODEL

Here is explicitly described the node model embedded within NEXT_LINK procedure depicted in Fig. 4.

When a packet $i$, during the $k$-th time increment, reaches the node at the end of a link $\lambda$, the next link $\lambda^+$ considered to continue the trip is defined following (23). To understand if the trip of packet $i$ can continue in the next link of its path until its residual moving time within time interval $k$ expires, some checks are performed:

*Spillback check:* to move packet $i$ on the next link $\lambda^+$, it is necessary that the length of the running segment of link $\lambda^+$ be not null, that is $x^S(\lambda^+, k)) > 0$.

If it happens, packet $i$ may enter link $\lambda^+$; otherwise it means that the entire length of link $\lambda^+$ is occupied by a queue and packet $i$ remains on the queue of link $\lambda$ until the queue length on link $\lambda^+$ be smaller than the length of the link, that is, for the time until the condition $x^S(\lambda^+, k)) > 0$ be verified.

*Capacity check:* if the next link $\lambda^+$ can accept packet $i$, it must be verified that the residual capacity of link $\lambda$ is able to allow the exit.

Let $\tau \in (0, \Delta t]$ the time of interval $k$ when packet $i$ is about to leave link $\lambda$, $C(\lambda, k)$ the capacity of the final section of link $\lambda$ at time interval $k$, expressed in terms of maximum number of equivalent passenger car units exiting per time step (capacity could also be independent with respect to $k$), and $E_\tau(\lambda, k)$ the number of equivalent passenger car units exiting link $\lambda$ during the $k$-th time increment until time $\tau$.

Packet $i$ may leave link $\lambda$ if $[E_\tau(\lambda, k) + \gamma]/\tau < C(\lambda, k)/\Delta t$, where $\gamma$ is the number of equivalent passenger car units of the packet. If this happens, packet $i$ can leave link $\lambda$ otherwise it means that link $\lambda$ is saturated and packet $i$ remains on the queue of link $\lambda$.

### STOCHASTIC SAMPLE

This procedure is used to choose among alternatives to whom a choice probability is associated. Let consider a set of alternatives $A_i$, $i = 1, ..., m$ (*i.e.*, the set of paths connecting an OD pair, links exiting a node, etc.), and suppose that each of these alternatives has an assigned probability value $\pi_i$. Thus, alternatives are mapped as contiguous segments in a grid (see fig. 11) such that each segment is equal in size to the probability of the alternative. A number $r$ is randomly generated in the range $(0, 1]$; $r$ identifies a point in the cumulate probability axis and the corresponding alternative is selected. In fig. 11, the case of the choice among four alternatives having probability 0.4, 0.1, 0.2 and 0.3, respectively, is graphically represented.

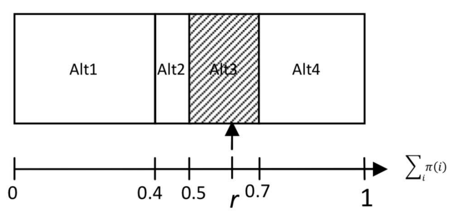


Fig. 11. *Scheme of the stochastic sampling approach adopted for the link choice.*

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