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Visible mathematics pedagogy: A model for transforming classroom practice

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Abstract:

This paper focuses on the development of a model of research and professional development which aims to bring about transformations in classroom practice in situations that have previously proved resistant to change. We explore the reasons why conventional approaches to research have failed to address one such situation, the continuing predominance of teacher-centred pedagogies in mathematics classrooms. We review conventional models of professional development from the perspective of teacher agency and engagement in and with research processes. We highlight how a critical model of participatory action research offers the potential for transforming situations in which more conventional approaches have failed. We draw on the findings from the Visible Mathematics Pedagogy research project to highlight how this model can be refined to enhance its potential to bring about changes in classroom practice. We report on research tools and processes that were developed, distinct from those commonly used in research, including the organisation of research team meetings around participatory principles, the active involvement of teachers in designing and employing data collection tools, and in generating protocols associated with video-stimulated reflection. We demonstrate how these research tools and processes enhanced collaboration and teacher agency, the trustworthiness of the research findings and teachers' critical reflection on existing practice. We argue that our refined model of participatory action research can inform and support teachers and researchers wishing to bring about transformations in classroom practice, especially in situations analogous to many mathematics classrooms, in which conventional approaches have so far had little impact.

Keywords:

Visible mathematics pedagogy; participatory action research; transforming classroom practice; critical reflection; video-stimulated reflection.

Introduction

This paper focuses on the development of a model of research and professional development which we argue will inform teachers and researchers wishing to bring about transformations in classroom practice, in situations that have previously proved resistant to change. It builds on a critical model of participatory action research (Skovsmose & Borba, 2004) that the authors took as a methodological starting point for the Visible Maths Pedagogy (VMP) research project, a collaboration between an academic researcher (first author) and two teacher researchers (co-authors). The project had two aims, the first of which was to investigate the extent to which making 'progressive' mathematics pedagogies more visible to students impacted on their appreciation of how to be successful mathematical learners. The term 'progressive' in this paper refers to open-ended, discursive, collaborative, problem-solving teaching approaches that are favoured by those exhibiting a 'progressive educator' ideology (Ernest, 1991). The second aim, which is the focus for this paper, was to develop and refine the model referred to above. Whilst we provide some background and context to the VMP project, this is limited to those aspects that explain the formulation of the model, demonstrate its efficacy and exemplify the associated research tools and processes that were developed. We describe the process of making progressive pedagogies more visible to learners, and its impact on students, more fully elsewhere (Wright, 2019; 2020; Wright, Fejzo, & Carvalho, 2020).

The problem with conventional mathematics education research

In this section we highlight an example of one situation that has proved resistant to change, i.e. the predominance of teacher-centred pedagogies in many mathematics classrooms in England. We consider why conventional approaches to research and professional development have done little to address this situation and why alternative approaches are needed that can bring about transformations in classroom practice.

Prominent mathematics education researchers have called repeatedly for the adoption of more progressive teaching approaches that promote greater engagement and enjoyment amongst learners and deeper understanding of the application of procedures and underlying mathematical concepts (Boaler, 2009; Foster, 2013; 2018; Hudson, 2018; Skemp, 1972; Swan, 2006). Despite these calls, most mathematics lessons continue to be characterised by the teacher demonstrating a mathematical procedure to students, followed by students completing a series of almost identical closed questions before being tested on their retention (Skovsmose, 2011). Whilst teacher-centred approaches are not universal, they are more likely to be experienced by students from disadvantaged backgrounds (Gutstein, 2006; Straehler-Pohl & Gellert, 2013) and lead to students exhibiting anxiety towards studying mathematics, and to growing alienation and disengagement from the subject (Grootenboer, 2013; Nardi & Steward, 2003; Williams & Choudhury, 2016). The predominance of teacher-centred pedagogies is therefore linked to the persistent and significant association between students' socio-economic backgrounds and their mathematical attainment (Boaler, Altendorf, & Kent, 2011), an indication of the high levels of inequity that have existed for many years within the mathematics classroom (Noyes, 2009). There is strong evidence to suggest that adopting more progressive teaching approaches is a necessary component of a more equitable approach to mathematics teaching (Boaler, 2008; Gutstein, 2006; Smith, Lee, & Newmann, 2001; Wright, 2016; 2017). Despite inequity being an important focus for researchers for several decades, socio-economic disadvantage remains the most decisive factor that determines students' success in school mathematics (Ernest, 2016; Jorgensen, 2016).

To understand why conventional approaches to research have failed to address the situation in many mathematics classrooms, it is important to appreciate the socio-political nature of school mathematics and mathematics education research. Ernest and Sriraman (2016), along with others adopting critical perspectives, challenge the notion that mathematics is neutral and value-free and argue that agency, empowerment and critical understanding of learners should be important considerations in the teaching and learning of mathematics. Conventional research that claims to be objective and bias-free is merely denying the ideologies and power relationships that permeate the field (Valero, 2004) and failing to recognise the importance of social class as a factor in limiting some children's mathematical success (Jorgensen, 2016). Such research is often conducted in 'prototypical' classroom situations which fail to take account of typical classroom contexts, challenges and constraints teachers face on a daily basis (Bishop, 1998; Elliott, 2009; Skovsmose, 2011). Engaging with critical frameworks, such as Bourdieu's theory of 'social reproduction' (Bourdieu & Passeron, 1990), helps to explain the resistance of mathematics classrooms to change by highlighting how existing practice often serves to preserve the interests of powerful groups in society and maintain the exploitation of others (Gutstein, 2006; Skovsmose, 2011).

A review of conventional models of professional development

We now turn to common models of teacher professional development and consider why these have had relatively little impact on mathematics classroom practice. Imants and Van der Wal (2020, p. 2) define 'professional development' as "those processes and activities designed to enhance the professional knowledge, skills and attitudes of teachers on an individual level so that they might, in turn, improve the learning of students". Boylan, Coldwell, Maxwell and Jordan (2018) analysed five contemporary theoretical models of professional development they considered most significant and found that most paid attention to teachers' knowledge, beliefs and attitudes, and classroom and school contexts. However, all five models failed to take account of wider social forces and ideologies, which they claim is problematic given the pervasiveness of neo-liberalism and discourses of performativity in schools. They highlight how models of professional development tend to ignore teachers' perceptions of their own professional learning and how this learning is embodied in classroom practice. Most models fail to take account of collaborative professional learning and how teachers' identities might change over an extended period of time as they experiment with different ways of being a teacher.

Clarke and Hollingsworth's (2002) 'interconnected model of teacher professional growth' was the only one of the five models referred to above to emphasise teacher agency, by incorporating teachers' reflective participation in professional development practices. This is based around cycles of 'enactment', i.e. trying out new ideas and putting into practice developing beliefs, and 'reflection', i.e. careful consideration of actions and their implications leading to further changes in beliefs and practices. Imants and Van der Wal (2020) consider four contemporary models of professional development from the perspective of teacher agency. They concur that Clarke and Hollingsworth's (2002) model is the only one to embrace teacher agency, although it was lacking in other aspects, such as focusing too much on individual teachers. They highlight how teacher agency has recently received much greater attention with a growing awareness of the role that teachers play as change agents in school improvement and reform. They highlight how most models of professional development tend to: cast teachers as passive (rather than active) participants; view content as a fixed entity (rather than variable according to teachers' enactment); present the learning process as linear with outcomes seen as end-results (rather than events in a continuing cycle of change); and neglect wider contexts such as school cultures and the national policy frameworks.

Bryan and Burstow (2018) highlight a growing recognition that school-based research is integral to school improvement, e.g. through the recent establishment in England of 'teaching schools' and 'research schools' (both required to undertake research), the 'Chartered College of Teaching' and 'ResearchEd' (both set up independently with a focus on supporting teacher-led research). This new enthusiasm for research is seen as an integral part of the professional development process and a vehicle for sharing best practice. However, they highlight how schools claiming to embrace research-related activity often focus on using existing knowledge to improve teachers' practice, rather than generating new knowledge through systematic inquiry. Teachers tend to work mostly "within the boundaries of the social norms of teaching" (ibid., p.117) with limited opportunities to reflect on the research process and little understanding of its ethical dimensions.

'Lesson study', which involves focusing on a research question arising from classroom practice in the collaborative planning, teaching, observation and evaluation of a lesson (Takahashi & McDougal, 2016), has grown rapidly in popularity in recent years. It is heavily promoted by the government-funded 'Maths Hubs' in England, no doubt helped by its association with 'mastery' approaches to teaching in East Asian countries that perform highly in international comparisons of mathematics attainment (Lewis, Perry, & Friedkin, 2009). Lesson study has been a traditional feature of teachers' professional learning in various East Asian countries for more than 100 years, although it has only recently been theorised (Takahashi & McDougal, 2016). It incorporates aspects that are generally accepted as essential for effective professional development, including being collaborative, sustained over a long period, research-informed, and drawing on evidence from lesson observations to evaluate practice (Wake, Swan, & Foster, 2016). Japanese lesson study provides a structured model, with protocols to be followed through consecutive phases (Takahashi & McDougal, 2016).

Leat, Lofthouse and Reid (2014) distinguish between engagement in research, as a social practice, and engagement with research, as a body of knowledge. Unfortunately, most teachers' experiences of research are restricted to the latter, often being required to implement unquestioningly recommendations for changes in practice from research about which they know little or nothing. This might be through targets set as part of school performance management processes, leading to teachers viewing new initiatives based on research findings as tacit monitoring or promoting a political agenda (Hammersley, 2004; Thomas, 2004).

Rationale for adopting a critical model of participatory action research

It is unlikely that the models of research and professional development described above will bring about significant changes in teaching practice in situations such as that outlined earlier, in which teacher-centred mathematics pedagogies have proved so resistant to change. We therefore seek an alternative model that will challenge current orthodoxies and social norms within mathematics classrooms and develop teachers' capacity to reflect critically on existing policies and practice, rather than adopting 'what works' protocols (Winch, Oancea, & Orchard, 2013). Leat, Lofthouse and Reid (2014) argue that teachers who engage 'in', as well as 'with', research are more likely to generate new knowledge and insight, adopt more critical perspectives and affect significant changes in their practice. Cordingley (2013) highlights how teachers can be encouraged to take risks, and explore practices that are effective in different classroom contexts, by engaging in collaborative inquiry facilitated by external agents. Action research, which embraces the teacher agency and dynamic cycles of enactment and reflection proposed in Clarke and Hollingsworth's (2002) growth model, is one form of practitioner inquiry that appears to be growing in popularity, with an increasing number of academic publications and research groups in schools and universities (Noffke, 2009). Wyse, Brown, Oliver and Poblete (2018) highlight the growing interest in 'close-to-practice' research, involving teachers and researchers working together on practice-based problems of mutual interest, which is seen as having the potential to foster professional learning and challenge existing practice. However, they highlight how a common perception of action research, along with other less well-established forms of close-to-practice research, is that it is limited in scale and lacking in quality. Whilst there are numerous published reports of action research projects, there is little guidance available to practitioners on how to conduct participatory action research in a systematic and rigorous way.

One approach to action research, for which there is a relatively well-formulated model, is that of lesson study (see previous section). Whilst it is clear that it promotes the personal and professional development of teachers, through generating individual and collective knowledge, the extent to which lesson study addresses the socio-political dimensions of mathematics education is questionable (Lewis, Perry, & Friedkin, 2009). Japanese Lesson Study is concerned primarily with implementing a problem-solving teaching approach, for which there is already largescale consensus (Takahashi & McDougal, 2016). Noffke (2009) argues that lesson study foregrounds the professional dimension of action research, although there is no reason why it couldn't include a political dimension as it allows for questions relating to the nature and purpose of teaching to be raised (Lewis, Perry, & Friedkin, 2009). However, Elliott (2019, p. 185) warns that, in importing lesson study models from their traditional cultural contexts, some of their original critical characteristics may have been lost as “the vision of the teacher as a curriculum developer appears to have been replaced by the idea of the teacher as a curriculum implementer”.

Participatory action research, on the other hand, is overtly political in character. It goes much further than Clarke and Hollingsworth's (2002) model in that it seeks to generate knowledge of greater relevance to practitioners and develop a deeper understanding of theory-in-practice (Brydon-Miller, Greenwood, & Maguire, 2003). It involves a genuine collaboration between teachers and academics which recognises both the latter's research expertise and teachers' in-depth knowledge of the classroom situation (Atweh, 2004). It offers a “systematic approach to personal, organizational, and structural transformation, and an intentionally and transparently political endeavour that places human self-determination, the development of critical consciousness, and positive social change as central goals of social science research” (Brydon-Miller & Maguire, 2009, p. 80). Skovsmose and Borba's (2004) critical model of participatory action research takes account of socio-political perspectives on mathematics education and facilitates collaboration between teachers and academics. Its starting point is that existing practices should not be taken as given and it therefore offers a useful model for challenging the predominance of teacher-centred pedagogies.

Skovsmose and Borba (2004)'s model involves a series of participatory action research cycles, each built around three 'situations': the 'current situation', i.e. existing practices that are in need of change; the 'imagined situation', i.e. an alternative vision of what better practices might look like; and the 'arranged situation', i.e. an attempt to put into practice aspects of the imagined situation. Their model makes three key research processes explicit and describes how they relate to these situations. 'Pedagogical imagination' is the process of developing a critical understanding of the current situation, through drawing on research literature and practical knowledge, so that an imagined situation can be articulated. 'Practical organisation' involves teachers and academics cooperating in trying out some aspects of the alternative vision in an arranged situation, taking account of the realities and constraints of the classroom. 'Explorative reasoning' involves evaluating the arranged situation in order to better understand the current situation and to assess the feasibility of the imagined situation. Through these key research processes, the current and imagined situations thus evolve in subsequent cycles. Whilst the framework offers a theoretical model that promises to facilitate transformations in classroom practice, there is limited guidance on what these key processes might look like in practice. Our aim in this paper is therefore to draw on the findings from the VMP project to suggest refinements to Skovsmose and Borba's (2004)

model that might inform and support other teachers and researchers wishing to address situations that have proved resistant to change.

Developing and refining the research model

In this section we draw on the findings from the VMP project to highlight how research tools and processes that we developed might be used to refine Skovsmose and Borba's (2004) model and enhance its potential to bring about changes in mathematics classroom practice.

The VMP project was a collaboration between an academic researcher, Pete (first author), and two teacher researchers, Alba and Tiago (co-authors), who share an interest in employing progressive mathematics pedagogies and a commitment towards equity and social justice. We refer to ourselves collectively as the 'researchers' or the 'research team'. The research team was established in November 2017, after Tiago approached Pete to express a desire (which he shared with his colleague, Alba) to engage in practice-based research. Having worked as a secondary mathematics teacher for 15 years, Pete had become a teacher educator at UCL Institute of Education, which involved being a tutor on an initial teacher education course and conducting other collaborative research projects with teachers. Tiago was in his second year of teaching at Stoke Newington School, a multi-ethnic inner-city comprehensive secondary (age 11-18) school in London with relatively high levels of disadvantaged students. He had previously been a student on the course on which Pete was a tutor. Alba was in her fifth year of teaching at Stoke Newington School and had previously been a mentor on the same course.

We met initially to discuss our shared interests and a possible focus for the research project. We considered Lubienski's (2004) findings that working-class children were generally less able to decipher the 'rules of the game' under 'reform-oriented' progressive teaching approaches adopted in the US. In drawing on Bernstein's (2000) notion of 'invisible pedagogy', Lubienski argues that concerns over already-marginalised children being further disadvantaged by experiencing less structured progressive pedagogies, in which the rules that need to be followed by students to realise success are generally implicit, might help to explain the predominance of teacher-centred pedagogies in mathematics classrooms. We therefore decided to explore strategies for making 'progressive' mathematics pedagogies more visible to students, particularly those from disadvantaged backgrounds, and how these might impact on their appreciation of how to be successful mathematical learners. We also discussed possible methodological approaches and agreed to adopt Skovsmose and Borba's (2004) critical model of participatory action research as a starting point. We describe below how our research model evolved and developed, focusing on those refinements which we consider to be distinctive and relevant to the focus of this paper.

Research team meetings

Given the constraints on teachers' time, we decided to meet at the school during periods when both Alba and Tiago were not teaching. Most of these meetings were approximately one hour in duration, although the school agreed to release Alba and Tiago for some half-day meetings during the second year of the project (meetings 8, 11 and 15). To ensure research team meetings were focused whilst, at the time, conducted in a collaborative and participatory spirit, Pete's role was largely facilitative. For example, he compiled an agenda that included items that the teacher researchers were invited to submit and circulated notes summarising key decisions and action points from the meetings. The research team meetings served a variety of purposes, as outlined

below, although each meeting incorporated several of these different functions. All meetings were characterised by negotiation and agreement with all researchers making significant contributions to the discussions. The full programme of meetings is included in Figure 1.

Timescale	Action
1st and 29th November 2017	Two initial research team meetings (agreed research design and overall structure; presented and discussed research literature on progressive pedagogies and equity)
January 2018	Beginning of action research cycle 1
16th January	Research team meeting 1 (planned first research lessons, devised strategies, designed surveys)
31st January	First research lessons (both video-recorded; trialled 'advocating' and 'separating' strategies)
31st January	Student survey (administered immediately after research lessons)
7th February	Research team meetings 2 and 3 (evaluated 'advocating' and 'separating' strategies; reviewed surveys and videos)
20th March	Research team meeting 4 (overall evaluation of cycle 1)
May 2018	Beginning of action research cycle 2
9th May	Research team meeting 5 (planned second research lessons, devised strategies, designed interviews)
16th May	Second research lessons (both video-recorded; trialled 'scribing', 'annotating' and 'classifying' strategies)
17th/18th May	Semi-structured interviews (conducted with 3 students in each class)
13th June	Research team meeting 6 (evaluated 'scribing', 'annotating' and 'classifying' strategies; reviewed interviews and videos)
2nd July	Research team meeting 7 (overall evaluation of cycle 2)
November 2018	Beginning of action research cycle 3
22nd November	Research team meeting 8 (presented and discussed research literature on VSR; discussed VSR protocols; presented and discussed initial findings from year 1)
5th December	Research team meeting 9 (planned third research lessons, devised strategies, designed surveys and interviews)
16th January	Research team meeting 10 (evaluated 'model solution' and 'card sort' strategies; reviewed surveys, interviews and videos using VSR)
11th March	Research team meeting 11 (presented and discussed research literature on critical reflection; reviewed VSR protocols; discussed data analysis methods from year 1)
March 2019	Beginning of action research cycle 4
20th March	Research team meeting 12 (planned fourth research lessons, devised/refined strategies, reviewed design of surveys and interviews)
3rd April	Research team meeting 13 (evaluated 'boxing up' and 'card sort' strategies; reviewed surveys, interviews and videos using VSR)
1st May	Research team meeting 14 (overall evaluation of cycles 3 and 4; reflected on methods and research tools developed)
8th July	Research team meeting 15 (final evaluation of project; planned programme for disseminating findings)

Figure 1: Structure of action research cycles for the VMP research project

'Design meetings' were used for strategic discussions, e.g. refining the research questions, clarifying the aims of the project, identifying constraints and resolving how these might be overcome. They also considered logistical arrangements and decisions relating to the finer details of the research design, e.g. the timings of meetings and research lessons, which classes and students might participate, which data collection and analytical tools to use, and resources that might be required.

'Review meetings' focused on reviewing current practice within teacher researchers' own classrooms, their departments and schools, and practice in other settings. In order to facilitate critical reflection on existing practice, we took it in turn to read and present research articles for

discussion. These articles were selected by Pete from those he considered relevant to the aims of the project and to the research methodology and methods adopted by the research team.

‘Planning meetings’ were used to formulate the strategies for making progressive pedagogy more visible to learners, which were tried out in the classroom as part of the action research cycles. We refined the plans for the research lessons during which these strategies would be trialled. We also selected and developed the data collection tools (see below) to be used to evaluate the strategies, e.g. by deciding the questions to be asked in surveys and interviews. Later in the project, these meetings were used to plan how to disseminate the findings through joint presentations at events, co-authoring publications and through a designated website.

‘Evaluation meetings’ focused on evaluating the impact of the strategies on students’ appreciation of how to be successful mathematical learners. The data generated from the research lessons, i.e. video recordings, survey responses and interview transcripts, were discussed and related to the teacher researchers’ own reflections on the lessons through the use of ‘video-stimulated reflection’ (see later section). These evaluations were used to inform subsequent review and planning meetings. All research team meetings were audio-recorded and the data from these recordings were used for the analysis on which the findings in this paper are based (see ‘Data collection and analysis’ section).

Data collection and evaluation tools

We employed a range of different tools that provided complementary forms of evidence used to evaluate the impact of the strategies. Each one of us kept our own research journal, which was seen as an effective means of capturing thoughts and experiences relating to the research project. We recorded our immediate responses to, and reflections on, the research lessons, as well as details of any other thoughts, events or conversations considered to be significant and relevant. The research journals were useful in stimulating discussion during research team meetings, and helped us to recall our thoughts and experiences, evaluate the strategies and disseminate the findings from the project.

Involving the teacher researchers in designing and administering the data collection tools, particularly the surveys and interviews, was seen as a way of enhancing the trustworthiness of the research findings. Because of their in-depth knowledge of students, they were seen as best placed to choose the most appropriate questions, and forms of language, to enable students to articulate their thoughts and experiences. It was felt that the relationships of mutual trust and rapport already established between teachers and students would help students feel more comfortable, and less anxious, about answering questions openly. Consideration was given to the possibility that students would say what they thought their teachers wanted to hear and ways of mitigating against this, e.g. by triangulating data.

Anonymous student surveys were seen as useful for evaluating the impact of the project on all students participating in the research. We experimented with using these during the first cycle, immediately after the research lessons, to assess students’ awareness of their success during the lesson. The questions included:

How well do you think you’ve done in today’s maths lesson? [on a scale of 1 to 5]

How do you know?

In the second cycle, instead of surveys, we explored the use of semi-structured individual interviews conducted by teacher researchers with three students in their class identified as ‘pupil premium’ (a measure of disadvantage) shortly after the second research lessons. After evaluating the use of data collection tools during cycles 1 and 2, we agreed that using both surveys and interviews in cycles 3 and 4 would provide stronger evidence to evaluate the strategies. We decided to administer the surveys at the beginning of cycle 3 and the end of cycle 4 so that we could evaluate the development of students’ thinking over an extended period. We agreed to amend the survey questions to explore more general views of success:

How successful do you think you are in maths in general? [on a scale of 1 to 5].

How do you know?

What do you think you can do to be more successful in maths?

We decided that semi-structured interviews would allow us to explore students’ awareness of the teachers’ reasons for using visible pedagogies in greater depth. We adopted an ‘empathetic’ approach, which aims to build rapport and trust between interviewer and interviewee, to allow more meaningful representations of students’ experiences to emerge (Fontana & Frey, 2008). This allowed for responses from the interviewer such as *“That’s interesting, can you tell me more?”*. The interviews were administered shortly after the third and fourth research lessons and included questions tailored to the pedagogies employed during the lesson similar in structure to the following:

Why do you think I asked you to ...?

Video-stimulated reflection

One of the most distinctive features of our developing research model was the use of video recordings to stimulate critically reflective and evaluative discussions. We experimented with video in the first research lessons taught by Alba and Tiago, which Pete observed. A single video camera was set up and operated by Pete at the back of the classroom, enabling the teachers to retain their focus on the lesson. During each lesson, Pete also recorded key events relevant to the aims of the project on a timeline, making it easier to locate these when reviewing the video recording. In subsequent cycles, we decided it would be more manageable for Alba and Tiago to take Pete’s place in observing each other’s research lessons, operating the video camera and generating the timeline.

The video recordings were used during evaluation meetings, along with survey responses and interview transcripts, to stimulate reflective discussions focusing on evaluating the success of the strategies. Over the course of the project we developed and refined a series of protocols for using the videos, informed by various research papers that we engaged with during review meetings (Coles, 2013; Geiger, Muir, & Lamb, 2016; Hollingsworth & Clarke, 2017; Sherin & Dyer, 2017). We believed that the benefits of videos went far beyond a mere memory prompt, since they provided an alternative view from the perspective of the students and allowed the teacher to see things that they weren’t in a position to notice during the lesson. We therefore decided to refer to ‘video-stimulated reflection’, rather than ‘video-stimulated recall’ (which is commonly referred to in the literature). The protocols we developed included: the teacher and observer each select one short clip from both research lessons; the reason for selecting each clip is explained; the researchers view each clip in turn; they discuss how evidence from the video clip addresses the

aims of the project; they focus first on factual discussion of what happened ('descriptive phase') before moving on to inferences that can be drawn ('evaluative phase'); the timeline is used to locate other parts of the lesson which the researchers feel might be useful to view to inform the discussions.

We felt it important that teachers retained ownership of the video recordings and for ethical reasons these were not considered part of the data. Note that all surveys, interviews and video recordings were conducted in accordance with the school's child protection and data protection policies and under the approval of UCL Institute of Education's Research Ethics Committee. The schedule for the surveys, interviews and research lessons is included in the appendix.

Data collection and analysis

The interviews were audio-recorded and transcribed to ensure the anonymity of students. All research team meetings were audio-recorded and selected extracts were transcribed (mostly those relating to planning and evaluating the strategies). Thematic analyses of the data were carried out using a combination of deductive coding (derived from the theoretical background) and inductive coding (derived from a reading of the data), with different coding schemes used for the surveys, interviews and research group meetings. NVivo software was used to code the data and to facilitate the thematic analyses. The thematic analysis of the research team meetings was mostly deductive with the initial coding scheme drawn from: the research tools employed; the strategies tried out; the key processes identified in Skovsmose and Borba's (2004) model (i.e. pedagogical imagination, practical organisation, explorative reasoning); characteristics of participatory action research (e.g. collaborative, participatory, bringing about positive social change). The coding scheme was modified through familiarisation with the data and during the coding process, as it became evident that some codes were redundant and that others were required (Fereday & Muir-Cochrane, 2006). The analysis involved reading and re-reading extracts of text assigned the same code, looking for patterns in the coding, and exploring 'commonalities', 'differences' and 'relationships' between codes to enable further themes to be identified from the data (Gibson & Brown, 2009). Whilst much of the analysis was carried out by the academic researcher, the trustworthiness of the findings was enhanced by regularly sharing and discussing the transcripts, coding schemes and data analysis with teacher researchers (Lincoln & Guba, 2003).

Findings from thematic analysis of the research team meetings

We report elsewhere (Wright, 2019; 2020; Wright, Fejzo, & Carvalho, 2020) on the success of the strategies developed and employed by the teacher researchers during the project. This success is evidenced by analyses of students' survey and interview responses. To summarise these findings briefly, over the course of the project, students were increasingly able to articulate valid reasons for why their teachers adopted 'progressive' teaching approaches and were increasingly likely to attribute their enjoyment of mathematics and their mathematical success to their engagement with such pedagogies.

Given the focus of this paper, we concentrate in this section on evaluating the efficacy of the research model in facilitating transformations in classroom practice. We present four themes that emerged from the thematic analysis of the research team meetings, which highlight aspects of the model that appeared to have a particularly powerful impact on the teacher researchers'

thinking and practice. Evidence to support the findings is numbered to facilitate references back in the 'Discussion' section.

1) Reflections on classroom practice

1.1) The teacher researchers described the significant impact the project had on their classroom practice and their awareness of the issues the project focused on:

Pete: So, do you think being part of the project has made you more disposed towards discussing 'why' [you adopt particular teaching approaches] with them [students]?

Alba: Yes, I think it's become a much more emphatic part of the lesson, and a much more consistent part of the lesson, than it was before.

Tiago: For me, I would say that I was already predisposed to justifying, for reasons of engagement, because I don't have that intuition of just going: "Ah, I will just give my rules" ...

Pete: So, you might be more predisposed towards doing it, but were you doing it as much, is the other question?

Tiago: I think what changed with the project is becoming more aware, and noticing more ...

(Meeting 11)

1.2) Pete's questioning, related to students' understanding of the reasons behind their pedagogical choices, prompted the teacher researchers to re-examine their rationale for using progressive pedagogies:

Pete: Getting the students to consider what they think are the most important reasons for using a particular teaching approach, ... in doing that, does it actually make you think more about: "Why am I doing this?"

Tiago: ... What this does is making us be more aware of that process [...] "What do we want them to get from this lesson? And in which way? What skills do we want them to develop? Is it working collaboratively? Why?"

Alba: [...] Not just in that lesson, but what habits do we want them to build in the long-term? Whether that's working with other people, whether that's how they write the work down, whether that's the conversations that they have in discussion with me or with others. [...]

Tiago: All of the ... approaches, the pedagogies that we are using, for this lesson, are coming from things that we already were doing. [...] We're questioning: "We've been using this. Do they understand why we do this?"

(Meeting 10)

1.3) Reviewing and reflecting on their reasons for adopting particular teaching approaches enabled teacher researchers to articulate these reasons more clearly:

Pete: In order to assess whether they [students] have the same idea of the purpose of the teaching approach as you do, you need to be really clear in advance what your rationale is for using it ... I think that comes across very clearly in that discussion [in the video clip], that you are able to articulate ... a much stronger rationale for why you're doing something, than would normally happen in a maths lesson.

Alba: Yes. Because it was so considered, so well-considered before, through our planning, through our intentions, through wanting a certain result from it, from them, as well.

(Meeting 13)

1.4) Initial worries teacher researchers had about devoting valuable classroom time to discussing their pedagogical rationale seem to have been allayed through finding ways of incorporating these discussions into their routine practice:

Alba: Now we've had a period of time to put this in place, you don't have to be doing it every lesson. Once you've really invested their time in that lesson, the next lesson I only need to mention "Now we are copying down a model solution", and maybe, here and there, question them on why that's important, just to refresh that. But I don't need to be doing a half-an-hour starter to reinforce it ... But I see the value in it, definitely, more than enough to want to keep doing it.

(Meeting 10)

1.5) Pete's presentation of the initial findings from the data analysis helped teacher researchers to appreciate the extent to which their thinking, practice and understanding of research processes had grown during the project:

Alba: What you've picked out are the key elements, I feel, of what happened in that first year. And I think it's also brought out to me how much more we've been able to build on it this year, as well. Like refine the model of professional development and come up with a protocol. And really zone into reasons and rationales behind the pedagogies and make them explicit amongst us. So that, then, we can go back to the classroom, make them explicit to the students, in return. I think those are some things that we've grown this year.

(Meeting 15)

2) Reflections on surveys and interviews

2.1) The researchers considered that both the surveys and interviews provided valuable evidence in evaluating the success of the strategies tried out. The surveys gave an indication of what the majority of students felt about the lesson, avoiding possibly unrepresentative feedback received from a small vocal minority:

Alba: I thought the surveys were a great tool for seeing an overview for the whole class.

(Meeting 7)

2.2) The interviews were seen as valuable for evaluating the experiences of the target (disadvantaged) students, who appeared to engage positively with the interview questions:

Alba: I really noticed that, with the interviews of the three PPI [disadvantaged] students ... that was just as powerful. Their responses were really articulate.

(Meeting 7)

Tiago: I think I made that very explicit in the lesson and they got the idea. And the interviews just show that ...

(Meeting 14)

2.3) The researchers acknowledged how the combination of surveys and interviews provided stronger evidence and hence both should be adopted for subsequent cycles:

Alba: So, I think both were really interesting for different reasons [...] Doing both, at the same time, would be even better, because then we can get an idea of more.

Pete: If we'd done both at the start, and then both again at the end, we might really notice the differences between the two. Whereas, it's quite hard to compare what you got from the interviews at the end with what you got from the surveys at the beginning.

(Meeting 7)

2.4) In reviewing the interview transcripts, the researchers recognised the benefits of teachers conducting interviews with their own students, with students feeling comfortable about being interviewed by their own teacher. For example, students appeared willing to take their time before responding to questions:

Tiago: When I was doing one of the interviews with Neal, he was taking long pauses, basically, to think. Very long pauses, sometimes like 30 seconds ... I didn't want to influence the response and I was just giving him time. Sometimes I would say things like 'it's fine', I would say 'brilliant' quite a lot, it appears that I did say that.

Pete: [...] Because you know them, you can put them at ease, you can support them. [...] You have a relationship with them. And you will know when that person is just thinking, and when they're feeling really uncomfortable inside because they don't know what to say.

(Meeting 14)

2.5) There was also an appreciation that the involvement of the teacher researchers in designing the interview questions elicited more meaningful responses from students:

Pete: Because you know the students better. You know what happened in the lesson better, because you were there. So, does that make the interviews more valid? [...]

Tiago: I think that ... because we planned the lesson, we planned the questions for the interview, so they're more meaningful for us, because we're thinking "What is it that we want to get?" [...] But I think the fact that we are there ... there's a different relationship with the student than someone outside, it's going to be a different interview. I think they're going to be more keen on answering. [...]

Alba: I think it makes it more valid ... If someone from the outside saw the video ... and came in and asked those questions, the students didn't know this person, I don't know, I don't think they would respond in as an engaged way. [...] And because there's a rapport and trust there, I feel they fully engage with the interview, as much as they can [...] give answers that are quite intricate and quite elaborate.

(Meeting 14)

2.6) The researchers recognised power relations existing between the teacher (as interviewer) and student (as interviewee) and considered how these might impact on students' openness in responding to questions:

Pete: If I was playing Devil's advocate, I'd say: "OK, you've got a good relationship with these students, they're buying into the project, so does that make them more prone to say

things are more successful than they really are, because they want to say things to please you?"

Alba: There's definitely a sense of they want to say the right thing, but I don't know how you would distinguish between 'this is what they know' and 'this is what they're saying'.

Pete: It's whether they genuinely believe what they're saying or whether they actually think something different, but they're saying what they think you want to hear.

Alba: The thing is, because they explain themselves quite well ... in the interviews. And they were able to reason and back up their answer, and go into quite a lot of detail ...

Pete: So, you think the detail they went into would have made it quite difficult for them to keep up a pretence of something different to what they actually thought?

Alba: Yeah, I thought that it was quite genuine.

(Meeting 14)

3) Video-stimulated reflection (VSR)

3.1) The teacher researchers noted how VSR enabled them to see things that they would not otherwise have noticed because of other things they needed to attend to whilst teaching:

Alba: It allowed me to look at everything from how I verbalized the questions ... to which students were paying attention in the class [...] which students seemed to be responding more [...] to how the discussion was going [...] So, I think, yeah, it was great on so many levels. It gave me a lot of feedback that I used, I think. I'd like to use it more often.

(Meeting 7)

3.2) They reported how VSR allowed them to view their own teaching from different perspectives, including that of their students:

Alba: It's amazing, I think, how much you can find what you'd want to develop, or change or test, when you watch yourself. [...]

Tiago: ... I think that, not only, it allows me to see myself teaching, but also it puts me in the perspective of a student. [...] But now I'm seeing it from ... how do they, students perceive that?

(Meeting 13)

3.3) VSR prompted the researchers to draw on a range of evidence, from the video clips, surveys and interviews, in evaluating the success of the strategies:

Tiago: I think there was evidence of it [the strategy] making it explicit, more from what we saw from the video recordings than what we can deduce from the interview transcripts. Some of them make explicit mention of that in the surveys as well.

(Meeting 14)

3.4) VSR facilitated discussions that enabled the researchers to evaluate the strategies in depth, and to review the way these were implemented in the research lessons:

Alba: They engaged really well at this point, considering how much talking there had been in the lesson ... When we had a discussion the first time, which was much earlier ... at 23 minutes [referring to timeline produced by Tiago], I ask the class what their preferred way of answering the question is [...] It was a brief conversation ... but I think that was

helpful getting them to think what we were trying to do. I don't think this would have been as useful, this conversation at the end, if we didn't have this here. [...]

Tiago: I think it's also the reflection point, isn't it? [...] And that happened at the right time. If you'd left that to the end ... they might not associate to that anymore.

(Meeting 10)

3.5) The researchers reflected critically on the use of VSR, by raising questions about the rigour and practicalities of the protocols and relating these to their previous experiences of using it:

Tiago: I've thought about this ... I think it introduces a bias on the discussions [...] if I select. Because it's just one person thinking about what we want to address. [...] In terms of practicalities, I don't think that it's possible, given our time constraint, to actually watch the whole video and decide in the moment, which moments to analyse. [...]

Alba: [...] I think it's best to pick the moments and then let the discussion be open about what is key about that clip, or that section. [...]

Pete: The timeline has been really useful for facilitating these discussions. [...] And as the discussion goes on, everyone's got the timeline in front of them, they can sort of say: "Well, yeah, to understand this a bit more, why don't we look at this bit of the clip?"

(Meeting 8)

3.6) These reflections were informed by the research literature and resulted in continuous refinement of the VSR protocols. The following exchange followed detailed discussion amongst researchers prompted by presentations of three research papers (Coles, 2013; Geiger, Muir, & Lamb, 2016; Hollingsworth & Clarke, 2017):

Tiago: So, we might want to do this differently, it's just a suggestion [...] each of us could pick a point or two from the lesson with a focus on the strategy ... I like the idea of doing a narrative of what happened ...

Alba: Yeah, I like the idea of that ...

Tiago: ... and then moving on to evaluating.

(Meeting 8)

4) Engagement with research literature

4.1) The researchers appreciated opportunities to reflect carefully on their practice in a depth that would not normally be possible given the time constraints and pressures of teaching. Critical reflection was facilitated by engaging with research literature and relating this to existing practice, the classroom context and to the development of the research tools used in project. The following exchange was prompted by Tiago presenting a paper on socio-political aspects of mathematics learning (Hand, 2012):

Tiago: This idea of dispositions ... Teachers are predisposed [...] they bring their backgrounds. And therefore, they can notice in the way that Mason says of sensing, or seeing, or perceiving, what happens in a classroom. And that can inform their actions ... Should we use video-stimulated reflection to aid noticing of issues of equity, inequity, with our groups? [...] Also, I was thinking about our target students and [...] why they act and interpret things in a certain way. And the questions that we ask them, ... do they reflect

their culture? [...] Should we be thinking, when formulating questions, of making them fair? [...]

Alba: Yeah, that was my concern when I was talking about giving things historical and cultural context earlier, sometimes. And discussing that when we plan the lessons, the cycles.

Tiago: So, it's kind of agreeing, in a Freirean way, of agreeing the terms of our discussion ... we should use the terminology that they would use ... that's their way of naming things ...

Alba: Linguistics should be aligned, yeah.

Pete: Reading the interviews, that's kind of what happens [...] One of you asks a question, and the response shows the child has misinterpreted your question. So, you re-phrase it, and you give them more context.

(Meeting 11)

4.2) The following exchange, which relates the literature to the iterative plan-teach-evaluate cycles of the research design, was prompted by Alba presenting a paper on using critical reflection to promote teacher learning (Liu, 2015):

Alba: Exactly, this iterative process of planning, acting, observing, reflecting, which is what she's done, in that study [...]

Tiago: Which is kind of what we decided to do after the last cycle, when we tried the cards, and we thought "OK, next time we should still use the cards because there's a lot we can do there", but think of another way. [...] There was some element of reflection there, together, and then action. Is that it?

Alba: Yes, that's what she means ... Which is what we've been doing before. But I think we really hummed into it in the last cycle in that there was something we felt we could improve upon and change for next time.

(Meeting 11)

Discussion:

The findings highlight the developmental nature of the research project, with the action research cycles enabling the researchers to evaluate, develop and refine the research tools and processes (see subsections 3.6 and 4.2 in the 'Findings' section above). We discuss below how these tools and processes exemplify how a critical model of participatory action research can be employed to bring about transformations in classroom practice (with references back to the 'Findings' section).

The efficacy of the research model was demonstrated by the impact the project had on teacher researchers' thinking and classroom practice (1.5), with strong evidence to suggest that features of strategies tried out had been incorporated into their routine practice (1.1/1.4). Skovsmose and Borba's (2004) key processes framed the conduct of the research, e.g. 'pedagogical imagination' was evident in the way that teacher researchers, through relating research literature to their classroom experiences (4.1), began to articulate what making progressive pedagogies more visible might look like in practice (1.3). 'Practical organisation' was exemplified by researchers cooperating to develop VSR protocols that enabled focused discussions to take place in the little time available for meetings (3.5). 'Explorative reasoning' was evident in the use the researchers made of evidence, including from surveys and interviews (2.1/2.2/2.3), to re-evaluate their rationale for using progressive pedagogies (1.2).

The research model was participatory in the sense that the teacher researchers played a leading role in the development and refinement of the research design, e.g. in devising and evaluating the strategies (3.3/3.4), designing data collection tools (2.5) and generating VSR protocols (3.5/3.6). This reinforces Leat, Lofthouse and Reid (2014)'s contention that, by engaging in research as a social process, teachers are able to bring about new knowledge and insight and transform their practice. The agency of teacher researchers was evident in the cycles of 'enactment', i.e. trying out new ideas through the strategies generated through critical reflection and discussion (see below), and 'reflection', i.e. evaluation supported by the data collection tools leading to further changes in beliefs and practices (Clarke & Hollingsworth, 2002). The research model appeared to satisfy Imants and Van der Wal's (2020) criteria for teacher agency, e.g. the teacher researchers played an active role in a continuing cycle of change in which the strategies they enacted were open to negotiation, rather than fixed or pre-determined.

There is evidence to suggest that the involvement of the teacher researchers in the design and collection of data to evaluate the success of the strategies resulted in more trustworthy findings (Lincoln & Guba, 2003), i.e. greater 'credibility' (confidence that the phenomena being studied are accurately reported) and 'confirmability' (the reports are derived from the views and experiences of the participants rather than the researchers). This was particularly so for the interviews where the relationship of trust already existing between teacher and student appeared to help establish a safe environment in which students were willing to be forthcoming and open in their responses (2.4/2.5/2.6), more so than if interviewed by a stranger. This 'empathetic' approach to interviewing, based on building rapport between interviewer and interviewee, enables a more meaningful representation of students' views to emerge through interaction and dialogue (Fontana & Frey, 2008; Kvale & Brinkmann, 2009). Engaging with the 'voice' of disadvantaged students is seen as essential in interrupting existing power relations and bringing about changes in classroom practice (Thomson & Gunter, 2009).

The collaborative nature of the research model was apparent in the research team meetings which were characterised by open discussions and collective decision-making. Practitioner/researcher knowledge and expertise were assigned equal value, e.g. with Alba and Tiago playing a leading role in the research design (see above) and Pete taking the lead in analysing data (1.5) and selecting the research literature to discuss (3.6/4.1/4.2). Pete's role in facilitating research team meetings was also vital in ensuring the discussions remained focused on the aims of the research, whilst embracing suggestions and ideas from all researchers. This was often achieved through posing (sometimes challenging) questions to prompt teacher researchers to reflect on their practice and articulate their thinking (1.1/1.2/2.6). This highlights the benefits of including an external researcher in the research team, who can encourage teacher researchers to question their assumptions about existing practice and the extent to which their claims and inferences derived from the data are supported by the evidence. Jaworski (2006) argues that such an external stimulus is essential for establishing a 'community of inquiry' in which researchers are able to develop a critical understanding of, and see beyond established classroom norms. With his knowledge of the field of educational research, Pete was also pivotal in identifying publications, organisations and events through which the research team could jointly present the project's

findings¹. It is essential for insights such as these to be shared more widely so that educational theory can be developed that draws on teachers' practical understanding (Elliott, 2009).

The critical nature of the research model was also enhanced by researchers reading, presenting and discussing the relevance to their practice, of a range of research literature on equity in mathematics education (Boaler, 2008; Hand, 2012; Lubienski, 2004; Wright, 2017), critical reflection (Liu, 2015) and VSR methodology (3.6/4.1/4.2). The data collection tools and the VSR discussions also facilitated critical reflection on practice by enabling teacher researchers to re-evaluate their teaching from different perspectives, e.g. those of (disadvantaged) students (2.1/2.2/2.3/3.1/3.2/3.3). In this way the research model demonstrates the potential for a transformative educational approach in which classrooms become critical sites of inquiry, problematic aspects of schooling are recognised, and teachers strive to realise ideas of equity and social justice through bringing about changes in practice (Cochran-Smith & Lytle, 2009). It offers teachers a model of how to engage in 'educational action', i.e. a form of 'praxis' in which teachers' ethical dispositions inform how they act in applying theory to practice (Carr & Kemmis, 2009).

It is worth emphasising that the research tools and processes referred to above were regarded by the research team as integral to the holistic structure of the research model and should be used together for maximum impact (1.5/2.3/4.2). Whilst considering individual aspects in isolation might lead to some benefits, e.g. using surveys might encourage teachers to pay closer attention to students' opinions, it is unlikely to lead to the deeper understanding of research processes necessary for transforming classroom practice (Bryan & Burstow, 2018).

Conclusion:

We offer in this paper a model of research and professional development which builds on Skovsmose and Borba's (2004) critical model of participatory action research. We believe the model will inform and support teachers and researchers in working collaboratively to bring about transformations in classroom practice, especially in situations where conventional approaches have previously had little impact. We have emphasised aspects of the model that we feel had a particularly powerful impact on the development of teachers' thinking and practice and that are distinct from those of more conventional approaches to research and professional development. These include the organisation and running of a series of research team meetings focusing on project design, reviewing practice, engaging with research literature, planning and evaluating classroom strategies. These meetings supported the active involvement of teachers as co-researchers in developing and refining the research design, devising and employing data collection tools and protocols associated with the effective use of video-stimulated reflection. We have demonstrated how these refinements to Skovsmose and Borba's (2004) critical research model enhance collaboration and teacher agency, the trustworthiness of the research findings and teachers' critical reflection on existing practice.

¹ Details of joint presentations and publications arising from the Visible Maths Pedagogy research project can be found on the designated website: <https://visiblemathspedagogy.wordpress.com/>

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