Artificial Neural Networks for Tours of Multiple Asteroids

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Abstract. Designing multiple near-Earth asteroid (NEA) rendezvous missions is a complex global optimization problem, which involves the solution of a large combinatorial part to select the sequences of asteroids to visit. Given that more than 22,000 NEAs are known to date, trillions of permutations between asteroids need to be considered. This work develops a method based on Artificial Neural Networks (ANNs) to quickly estimate the cost and duration of low-thrust transfers between asteroids. The capability of the network to map the relationship between the characteristics of the departure and arrival orbits and the transfer cost and duration is studied. To this end, the optimal network architecture and hyper-parameters are identified for this application. An analysis of the type of orbit parametrization used as network inputs for best performance is performed. The ANN is employed within a sequence-search algorithm based on a tree-search method, which identifies multiple rendezvous sequences and selects those with lowest time of flight and propellant mass needed. To compute the full trajectory and control history, the sequences are subsequently optimized using an optimal control solver based on a pseudospectral method. The performance of the proposed methodology is assessed by investigating NEA sequences of interest. Results show that ANN can estimate the cost or duration of optimal low-thrust transfers with high accuracy, resulting into a mean relative error of less than 4%.

Keywords: Neural network · Space mission · Trajectory optimization · Near-Earth asteroids.

1 Introduction

In the last decades, Near-Earth Asteroids (NEAs) have caught the attention of the scientific community for planetary defense, technology demonstration, and resource exploitation. The irregularity of their shape, size, gravity and magnetic fields, composition makes each of them unique and worth to be studied [7]. Multiple NEA rendezvous missions allow to visit a larger number of asteroids than single-NEA missions, while reducing the cost for each observation. Low-thrust propulsion systems, such as solar electric propulsion (SEP), are a more efficient technology than chemical propulsion in terms of fuel consumption. The higher specific impulse of low-thrust systems makes them an attractive solution for multi-target missions since they require less propellant for a given velocity increment ΔV [18].

The design of multiple NEA missions requires the solution of a complex global optimization problem to compute the optimal trajectory that meets the mission requirements, such as type of propulsion system and propellant mass available. This problem consists mainly of two sub-problems which are tightly coupled: a large combinatorial part, aiming at the identification of the most convenient asteroid sequences; and a continuous part to identify the optimal trajectory and control history to fly from asteroid to asteroid. The final purpose is to determine an optimal trajectory, which requires the least amount of fuel mass and maximizes the number of asteroids visited within a given time of flight (TOF).

To identify the most promising sequences of asteroids, all the permutations among them need to be considered. Given that more than 22,000 NEAs are known to date, according to the NASA's database[1], trillions of permutations should be analyzed. Moreover, since low-thrust propulsion produces little thrust continuously for a long time, the problem is continuous, thus computationally very intensive to obtain solutions for each asteroid-to-asteroid transfer.

Several methodologies have been proposed to solve this complex problem. The solution advanced by the majority of them requires the use of a simplified model to determine the most convenient asteroid sequences and, successively, convert it into feasible trajectories by means of a low-thrust optimization. For instance, Peloni et al. [10] approximate the low-thrust legs using a shape-based method and find the sequence with a search-and-prune algorithm; while a homotopic approach was used for approximating transfers in Ref. [15].

Previous works show that artificial intelligence can be applied to solve complex problems in aerospace sciences. A method based on an evolutionary algorithm and an artificial neural network (ANN) was employed to determine trajectories to a single NEA using solar sailing as propulsion system.[3] It was proved that this method can find a solution more efficiently than traditional optimal control methods. Machine learning was also successfully used in identifying low-thrust trajectories with minimum fuel consumption between main belt asteroids [5] and in estimating the final mass of the spacecraft after a transfer between two NEAs [8]. Other applications include the accuracy enhancement for pinpoint landing [13] and orbit prediction [11].

This paper investigates the use of ANNs within a sequence-search algorithm to solve the global optimization problem of multiple asteroid missions, by identifying the most convenient asteroid sequences and providing estimates of the cost and duration of each transfer. In essence, the goal of the analysis is to demonstrate that ANN can *quickly* estimate the cost and TOF of low-thrust transfers between NEAs. When trained appropriately, using a neural network

[1] Data available through the link https://cneos.jpl.nasa.gov/orbits/elements.html (accessed on 2020/01/10)

can potentially eliminate the need to prune the database of asteroids as it can generalize the network function to asteroids and departure dates not included in the training.

The ANN design for the tour of multiple asteroids presents multiple challenges. First, a method to compute the training database needs to be identified. With the goal of including a sufficient number of samples in the database, direct and indirect optimization methods result to be very expensive in terms of computational effort. Instead, analytic methods can help in the initial phase of the design, thus when training the network, to provide a quick and reliable, but approximated, description of the trajectory. Secondly, since the network inputs influence the accuracy of the network output with respect to the targets, different parametrizations of the orbits and their effect on the network performance are studied. Also, the extended close-up observation of the NEAs requires that velocity and position of the asteroids are matched by the spacecraft at the departure and arrival point of each transfer; thus, the asteroid phasing should be carefully considered in the inputs. It is paramount to define the topology and hyper-parameters of the network for this application. This is not straightforward and needs to be investigated.

The structure of the paper is the following. In Section 2 the optimal control problem is described, followed by the generation of the ANN training database. The input vector, architecture and hyper-parameters of the network are optimized in Section 3 to achieve the highest performance. The trained ANN is then integrated into a sequence search algorithm, which is detailed in Section 4. Among the sequences obtained, one is optimized, highlighting the main results of the methodology. Finally, Section 6 completes this paper with the conclusions.

2 Optimal Low-Thrust Trajectories

In the following, an optimal control problem (OCP) for low-thrust trajectories is formulated. This identifies the optimal trajectory for each body-to-body leg, i.e., Earth-to-asteroid or asteroid-to-asteroid. For a spacecraft orbiting the Sun, the state vector, $\mathbf{x}$, is expressed in modified equinoctial elements (MEE) [2], adjoined by the spacecraft mass:

$$\mathbf{x} = [p, f, g, h, k, L, m]^T \tag{1}$$

The following set of ordinary differential equations of motion can be defined:

$$\dot{\mathbf{x}}(t) = \mathbf{A}(\mathbf{x})\mathbf{a} + \mathbf{b}(\mathbf{x}) \tag{2}$$

with $\mathbf{a}$ being the acceleration generated by the propulsion system, and $\mathbf{A}(\mathbf{x})$ and $\mathbf{b}(\mathbf{x})$ being, respectively, the matrix and the vector of the dynamics. A full definition of $\mathbf{A}(\mathbf{x})$ and $\mathbf{b}(\mathbf{x})$ can be found in Ref. [1]. The SEP propulsive acceleration can be described as follows:

$$\mathbf{a} = \frac{T_{max}}{m}\mathbf{N} \tag{3}$$

where T_{max} is the maximum thrust that can be generated and $\mathbf{N} = [N_r, N_\theta, N_h]^T$ indicates the acceleration direction and magnitude vector in radial, transverse, out-of-plane frame. The mass of the spacecraft m changes with time while thrusting as described by the following mass differential equation:

$$\dot{m} = -\frac{T_{max}||\mathbf{N}||}{I_{sp} g_0} \tag{4}$$

with $||\mathbf{N}||$ being the magnitude of $\mathbf{N}$ and I_{sp} is the specific impulse of the propulsion system. For the remainder of this work, a solar electric propulsion system with $I_{sp} = 3000$ s and $T_{max} = 0.3$ N is adopted for a spacecraft with initial mass of 1500 kg.

In this case the objective of the optimization is to find the optimal control history $\mathbf{u}(t) \equiv \mathbf{N}(t)$ so that the least amount of propellant mass is used to visit the highest number of NEAs. Thus, the performance index to minimize is:

$$J = \int_{t_0}^{t_f} m(t) \ \mathrm{d}t \tag{5}$$

subject to the constraint:

$$\begin{aligned} &||\mathbf{N}(t)|| \leq 1 \qquad \forall t \in [t_0, t_f] \\ &\mathbf{r}(t_0) = \mathbf{r}_0 \\ &\mathbf{v}(t_0) = \mathbf{v}_0 \\ &\mathbf{r}(t_f) = \mathbf{r}_f \\ &\mathbf{v}(t_f) = \mathbf{v}_f \end{aligned} \tag{6}$$

where $\mathbf{N}$ can vary in magnitude to allow for thrust throttling, with N_r, N_θ, $N_h \in [-1, 1]$. Moreover, to satisfy the rendezvous conditions, the position $\mathbf{r}$ and velocity $\mathbf{v}$ of the spacecraft have to match with the position $\mathbf{r}_0$ and velocity $\mathbf{v}_0$ of the departure body at the departure time t_0, and with the the position $\mathbf{r}_f$ and velocity $\mathbf{v}_f$ of the arrival body at the arrival time t_f.

Several methodologies were proposed to solve the low-thrust OCP. Indirect methods solve the OCP by transforming the problem into a two-boundary value problem, using the Pontryagin's principle [12]. Direct methods transcribe the continuous OCP into a non-linear programming (NLP) problem which discretizes the trajectory into smaller arcs of constant thrust magnitude and direction [14]. Both method are computationally intensive, thus not efficient to generate the network training database where thousands of samples are included.

It is chosen to use a shape-based approach, which can produce a trajectory solution while reducing the required computational effort. This method approximates the shape of the rendezvous trajectory with minimum-cost for the given range of launch dates, TOF, and number of revolutions [4]. The required control history to fly the calculated trajectory is retrieved from the acceleration profile. A genetic algorithm is employed to search for the optimal shaping parameters for the transfer with minimum time of flight.

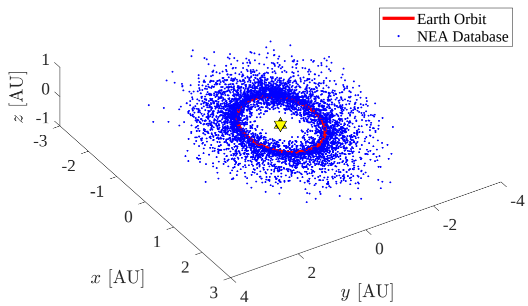

Fig. 1: Visualization of all the asteroids in the training database.

2.1 Database Generation

The training database contains the network inputs and the desired outputs, which are the orbital characteristics of the departure and arrival orbits and the position of the objects along their orbits (*input vector* $\mathbf{x}$) and the cost ΔV and TOF of the transfer between them (*output vector* $\mathbf{y}$), defined as follows:

$$\mathbf{x} = [p_0, f_0, g_0, h_0, k_0, L_0, p_f, f_f, g_f, h_f, k_f, L_f] \tag{7}$$

$$\mathbf{y} = [\Delta V, t_{0,f}] \tag{8}$$

where ΔV indicates the velocity increment and $t_{0,f}$ the time of flight.

The NEA orbital characteristics are obtained from the NASA's Near-Earth Object Program[2]. NEAs that are interesting for scientific reasons, their composition and orbital dynamics are included in the database.

To improve the converge rate of low-thrust OCPs, the highly-inclined ($i \geq 20°$) and highly-eccentric ($e \geq 0.4$) asteroids are excluded. Transfers to those asteroids would have been excluded by the network anyway since they require a longer TOF and considerable amount of propellant. This results in 6286 asteroids, with about 300 Potentially Hazardous Asteroids (PHA) and about 1450 Near-Earth Object Human Space Flight Accessible Targets Study (NHATS). PHAs have an Earth minimum orbit intersection distance lower than 0.05 AU and estimated diameter greater than 150 m, while NHATS are selected by NASA because they might be accessible by future human space flight missions.

Figure 1 shows all the asteroids included in the database with respect to the the orbit of Earth. The reference time of 2019/04/27 (2458600.5 Julian day) is used to determine the position of the asteroids along their orbits.

For the generation of the training database, the permutations among a subset of NEAs including 100 objects are considered for a total of 10,100 transfer samples. The goal is to verify the *generalization* property of the neural network: a successfully trained ANN is able to generalize estimating transfer costs between NEAs not included in the database and with different launch dates.

[2] Data available through the link https://cneos.jpl.nasa.gov/orbits/elements.html (accessed on 2019/06/17)

To verify the generalization property during the training, the database is divided into training set, validation set and test set. The training set is used for the training, while the validation and test sets contain samples that are not included in the training. The validation set is used to verify the overfitting does not occur during the training and the test set is used to test the performance of the network after the training with totally new cases.

For each transfer between two selected bodies, the shape-based method is used to find the ΔV and TOF of the transfer. The training database is generated by storing for each transfer the parametrization of the departure and arrival orbits, the angular position of the relative asteroids at a reference time, and the cost and TOF of the minimum-cost transfer. The launch window is set in the period 01/01/2020 and 30/12/2030. A maximum time of flight of 1500 days per transfer is set as transfers longer than four years are not of interest.

3 Neural Network Design

The topology and the hyper-parameters of the network can affect its performance. Since their best values are not known a priori, it is essential to analyze the ANN performance with respect to its parameters so that the correlation between network outputs and targets is maximized and the network error in the identification of transfer cost and time mapping is minimized. In a regression analysis, the correlation identifies how well the outputs fit the targets, with one and zero indicating perfect or zero fit, respectively. The network error can be defined as the mean square error (MSE) between the output of the network $\mathbf{y}$ and the targets $\mathbf{y}_t$:

$$\mathcal{E}_{MSE} = \frac{1}{N} \sum_{i=1}^{N} ||\mathbf{y}_i - \mathbf{y}_{t,i}||^2 \tag{9}$$

with N being the number of outputs. Since the validation set has samples which are not included in the training database, the validation-set MSE is often used.

3.1 Input Vector Analysis

The input vector of the network needs to define completely the departure and arrival orbits. To this end the orbital parameters can be used. However, several orbit parametrizations exist. Among these, the classical orbital elements (COE), modified equinoctial elements (MEE), equinoctial elements (EE), Cartesian coordinates, Delaunay elements, and eccentricity and angular momentum vector (eH) [19]. In this section, the effect of using different orbit parametrization on the network performance is investigated.

These parametrizations are used as input to a network, whose architecture is taken from Ref. [8], with two hidden layers and 80 neurons per layer to evaluate how the performance changes. The sigmoid is used as activation function and the stochastic gradient-descent algorithm is adopted for the training. The learning

Table 1: Network performance for different parametrizations of the orbit.

ANN input	Correlation	Validation-set MSE
COE	0.855	0.530
EE	0.856	0.487
MEE	0.925	0.236
Cartesian	0.551	0.761
Delaunay	0.694	0.862
eH	0.908	0.221

rate is set to 0.01, which is the highest value that does not cause divergence in the training process, and the database is divided in 70% training set, 15% validation set, and 15% test set. The performance of the network, in terms of correlation and validation-set MSE, is presented in Table 1 for each orbit parametrization.

The highest correlation is obtained when MEE are used as inputs ($C_{MEE} = 0.925$), which presents also a low validation-set error ($e_{MEE} = 0.236$). The latter is slightly lower when the eH parametrization is used, but a poorer correlation is registered in this case. However, priority is given to the highest correlation since this represents the performance of the network in all the three training, validation and test phases. Thus, for the remaining of this paper, MEE are used to describe the departure and arrival orbits as input to the network.

3.2 Architecture Optimization

In this subsection, we aim at finding the best values of the network hyper-parameters. To this end, the response of the network to each of these parameters is analyzed. The architecture of the network is defined by the number of hidden layers and the number of neurons. Other hyper-parameters that can affect the performance are the learning algorithm, activation function for each hidden layer, learning rate or gradient constant and its increase or decrease factor.

To determine the optimal values of the network architecture and hyper-parameters, an optimization procedure needs to be carried out. In theory, the most systematic option would be to optimize all the parameters at the same time by using, for instance, a genetic algorithm. However, the number of parameters to optimize and the need to train the network at every trial make the computational time extremely extensive. For this reason, one parameter at a time is tuned. First, the parameter's values are set to their default values taken from Ref. [8]. Secondly, one parameter is varied individually and the effect on the ANN performance is studied. The parameter is then set equal to the optimal value found, and the next parameter is considered for the same procedure. The default values and search space for each network parameter are detailed in Table 2, where the parameters are presented in the same order of analysis.

A neural network with an appropriate number of layers and neurons per each layer can approximate any continuous linear or non-linear function [6]. A larger number of neurons and layers will increase the flexibility of the network

Table 2: Default values, search space and optimal values for the network hyper-parameters.

ANN Parameter	Default	Search space	Optimal
Number of hidden layers	2	[2,8]	4
Number of neurons	80	[40, 100]	80
Learning algorithm	Gradient Descent	Levenberg-Marquardt Resilient back-propagation Scaled conjugate gradient Gradient descent	Levenberg-Marquardt
Activation function	sigmoid	tansig, sigmoid, ReLu	sigmoid

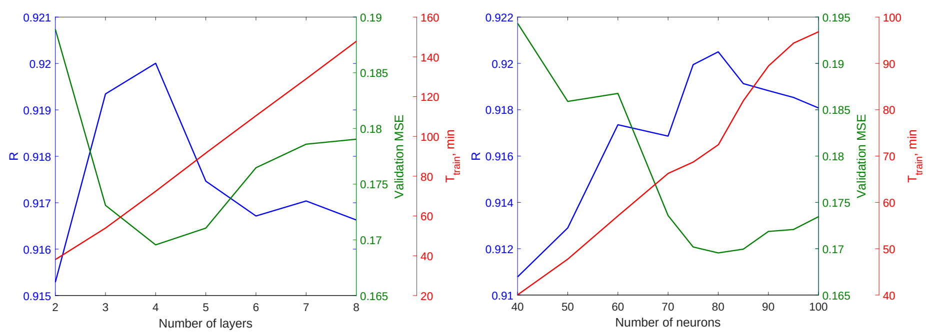

Fig. 2: Effect of varying the network parameters on its performance.

introducing more weights. However, more flexibility can induce to an overfitting of the data, thus to a bad generalization of the network function.

The samples are divided in 75% training set, 15% validation set, and 15% testing set. For the analysis the datasets, weights and biases are initialized with same seed at every evaluation. The effect of changing the number of layers and neurons on the correlation coefficient R, MSE of the validation set and training time T_{train} is shown in Figure 2. Increasing the number of layers and neurons improves the network performance up to certain number of layers and neurons (*peak*), after which the performance starts to degrade. As expected, the time required for the training process increases significantly as the depth of the network grows. The highest correlation coefficient and lowest validation-set MSE occur with number of layers of four and number of neurons of 80.

Different training algorithms and activation functions of the hidden layers are studied. As shown in Table 3, each of them induces some differences in the accuracy and the training speeds. Among the training algorithms, the Levenberg-Marquardt backpropagation offers the best performance at cost of a larger training time, while the sigmoid function performs better as activation function. For the chosen training algorithm, the gradient constant μ influences the state vector $\mathbf{x}$ as follows:

Table 3: Effect of different values of the network parameters on its performance.

Training Algorithm	R	Validation MSE	T_{train} [min]
Levenberg-Marquardt	0.9732	0.1211	59.79
Resilient back-propagation	0.9258	0.1598	2.39
Scaled conjugate gradient	0.9386	0.1467	4.67
Gradient descent	0.9205	0.1584	76.93
Activation Function	**R**	**Validation MSE**	**T_{train} [min]**
tanh	0.9489	0.1609	34.31
sigmoid	0.9732	0.1211	59.79
ReLu	0.9210	0.2295	52.42

$$\mathbf{x}_{k+1} = \mathbf{x}_k - [\mathbf{J}^T\mathbf{J} + \mu\mathbf{I}]^{-1}\mathbf{J}^T\mathbf{e} \tag{10}$$

where $\mathbf{J}$ is the Jacobian matrix of the network error vector $\mathbf{e}$ with respect to the current k-th weights and biases. The initial value of μ is set to 0.001. When μ is large, the algorithm becomes a gradient descent with small step size. However, after each successful step (i.e., the cost function is reduced) μ is reduced of a decrease factor μ_{dec}. The closest μ is to zero, the more the algorithm moves towards the Newton's method, which has a faster convergence and is more accurate. The decrease factor is set equal to 0.1.

From the investigation of all the parameters the optimal structure of the network is defined and detailed in Table 2. As a verification, the algorithm is run again but this time using the optimal values as default values. The test confirms that the obtained values allow the network to achieve the highest performance, with correlation coefficient of 0.9732 and validation-set MSE of 0.1211.

4 Sequence Search

The sequence search identifies the most convenient sequences of asteroids. It starts at the Earth at a fixed departure date. Once the full NEA database is loaded, the asteroid ephemerides are updated at departure time $t_{0,i}$, with i indicating the i^{th} leg of the sequence. The trained ANN is employed to calculate the cost and the TOF of transfers from the Earth to all the NEAs available in the database. Only $N_{max} = 200$ of the best transfers, in terms of largest number of objects visited with lower ΔV, are stored. To ensure close-up observation, a 100-days stay time is added at the arrival asteroid. At this point, the arrival asteroid becomes the departure asteroid of the following transfer, for which the same procedure is iterated, following a *tree-search method*. The sequence is complete when the total mission time reaches 10 years.

The departure date 01/01/2035 is chosen outside of the time frame used to build the network training database, so that the generalization property can be

Table 4: Orbital characteristics of the NEAs visited in the optimized sequence.

Designation Classification	2011 WU2 NHATS	2014 WU200 NHATS	2013 DA1 NHATS	2004 FM32 NHATS	2007 SQ6 NHATS	2019 FU2 PHA
a, AU	1.182	1.028	1.168	1.099	1.043	1.069
e, -	0.0439	0.0715	0.1365	0.1623	0.1456	0.1116
i, deg	3.02	1.27	1.89	3.76	9.10	7.79
Ω, deg	236.78	265.69	142.45	183.99	191.40	189.55
ω, deg	198.59	226.51	348.42	298.55	283.77	287.42
M, deg	341.04	43.17	290.34	66.42	224.21	82.30

Note: M is calculated at 2019/04/27

tested. In [16] it is shown that, in the context of the global optimization problem, using a trained ANN allows a much faster evaluation of the best asteroid sequences. The algorithm is 25 times faster than other methodologies [10] previously used, where the same type of machine was used to compute the same search.

5 Tour of Multiple Asteroids

The sequence search algorithm identifies 200 asteroid sequences in less than 15 hours. Once all the sequences have been characterized, one of the sequences that visits six asteroids, which are NHATS and at least one PHA, in 10 years is chosen to be fully optimized. The algorithm implements the OCP detailed in Sec. 2, optimizing the trajectory leg by leg. It requires an initial guess which is generated by solving a Lambert problem [2] and uses the departure and arrival orbits and TOF identified by the network. To allow for close-up observations and avoid overlapping between legs, a minimum stay time of 20 days is enforced. The OCP is solved by using a discrete NLP together with a variable-order adaptive Radau collocation method [9] and the NLP solver IPOPT [17].

The orbital characteristics of the encountered bodies are detailed in Table 4 for the selected sequence. The characteristics of the multiple NEA rendezvous mission are reported in Table 5. It presents the departure and arrival dates, the cost and TOF resulted from the optimization, in brackets the cost and TOF calculated by the ANN, and the stay time at each object.

To evaluate how well the network performs with respect to the optimization procedure, the values of ΔV and TOF from ANN are compared to the optimal ones. The deviation is quantified by calculating the average percentage error:

$$\mathcal{E}_{\Delta V} = \frac{1}{N}\sum_{i}^{N}\left(\frac{\Delta V_{opt} - \Delta V_{ANN}}{\Delta V_{opt}}\right)\cdot 100 = 2.19\% \tag{11}$$

$$\mathcal{E}_{TOF} = \frac{1}{N}\sum_{i}^{N}\left(\frac{TOF_{opt} - TOF_{ANN}}{TOF_{opt}}\right)\cdot 100 = 4.97\% \tag{12}$$

Table 5: Mission parameters of the optimized NEA sequence.

Leg	Departure Arrival	TOF [days]	ΔV [km/s]	Stay Time [days]
Earth - 2011 WU2	2035/08/24 2037/02/27	553 (503)	5.71 (4.96)	20
2011 WU2 - 2014 WU200	2037/03/19 2039/01/09	661 (631)	5.31 (4.87)	100
2014 WU200 - 2013 DA1	2039/04/19 2040/09/30	530 (580)	4.78 (4.92)	20
2013 DA1 - 2004 FM32	2040/10/20 2042/03/19	515 (518)	4.24 (4.28)	20
2004 FM32 - 2007 SQ6	2042/04/09 2044/01/06	637 (657)	5.27 (5.44)	20
2007 SQ6 - 2019 FU2	2044/01/26 2045/11/06	650 (670)	5.08 (4.23)	–

(.) Results from ANN.

with N being the number of legs in the trajectory.

The low average percentage error for both ΔV and TOF indicates that the network was able to learn the complicated non-linear relationship between the inputs (i.e., departure and arrival orbits and position of the bodies at a reference time) and outputs, which are an estimation of the cost and duration of the low-thrust transfer between these two orbits. Considering also the drastic reduction of computational time that ANN allows, it is possible to conclude that using ANN to search for preliminary multiple-asteroid rendezvous missions improves the speed of the search and guarantees a high accuracy of the results.

6 Conclusions

An artificial neural network is designed to estimate the cost and time of flight of transfers between asteroids with the final purpose of computing low-thrust tours of multiple asteroids. Tuning the architecture and hyper-parameters of the network as well as choosing the best inputs for this application is essential to optimize the network performance.

It is shown that employing an ANN within a sequence search algorithm offers a high accuracy with a relative error of less than 4% on average. In addition, it vastly improves the speed of the algorithm, reducing the computational time of 25 times.

To obtain the flight trajectory and control history, an optimal control problem needs to be solved. However, using this methodology allows to reduce the problem to solve the OCP only for the sequences of interest.

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