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Optimal tax policy and expected longevity: a mean and variance approach

Marie-Louise LEROUX [1] and Grégory PONTHIERE[2]

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Abstract

This paper studies the normative problem of redistribution between agents who can influence their survival probability through private health spending, but who differ in their attitude towards the risks involved in the lotteries of life to be chosen. For that purpose, we develop a two-period model where agents's preferences on lotteries of life can be represented by a mean and variance utility function allowing, unlike the expected utility form, some – agent-specific – sensitivity to what Allais (1953) calls the 'dispersion of psychological values'. It is shown that if agents ignore the impact of their health expenditures on the return of their savings, the decentralization of the first-best optimum requires not only intergroup lump-sum transfers, but, also, group-specific taxes on health spending. Under asymmetric information, we find that a subsidy on savings is optimal, whereas group-specific taxes on health spending are of ambiguous signs.

Keywords: longevity, risk, lotteries of life, expected utility theory, health spending.

JEL Classification: D81, H21, I12, I18, J18

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1 Introduction

Whereas human longevity depends on factors of various natures - genetic, environmental or sociocultural -, a large demographic literature also emphasizes the crucial influence of the individuals' lifestyles on their longevity.[1] Clearly, *how long* one lives is not independent from *how* one lives. Individual longevity depends on the extent to which one is willing to 'make an effort' to improve or preserve his health, and differences in the amount of efforts carried out by individuals tend to be reflected by longevity differentials.[2]

What should a utilitarian government do in front of such a heterogeneity of lifestyles and longevities? The answer clearly depends on the source of the heterogeneity and on whether longevity is exogenous or not. However, it also depends crucially on the form of the individual's preferences. For instance, Bommier *et al.* (2007 a,b) show that if longevity depends on exogenous health endowments, it is optimal to redistribute from short-lived toward long-lived individuals only when individuals have additively separable preferences, while relaxing this latter assumption, agents should be compensated for their poor longevity. As they explain in their work, assuming additively separable preferences leads to an implicit assumption of net *risk neutrality toward the length of life* which leads to strong (and disputable) conclusions in terms of redistribution.[3]

Starting from the works of Bommier *et al.* (2007 a,b), we might, on the opposite, examine the redistributive consequences of another potential source of heterogeneity in preferences: the attitude of agents towards *risk* and, more precisely, towards *risk on longevity.* This source of heterogeneity is generally ignored since most economic models assume both expected

[1] See Vallin *et al* (2002).

[2] Health-improving efforts can take various forms: the effort can be either *temporal* (e.g. physical activity, see Surault 1996 and Kaplan et al. 1987), *physical* (e.g. abstinence of food, see Solomon and Manson, 1997), or *monetary* (e.g. health services, see Poikolainen and Eskola, 1986).

[3] See Bommier (2005) on the notion of risk neutrality toward the length of life.

utility and additive lifetime welfare which, as we already mentionned, presupposes that *all* agents exhibit risk-neutrality with respect to the length of life.[4] But it is not difficult to see that the attitude towards risk plays a crucial role here, so that assuming a generalized net risk-neutrality with respect to longevity is a quite strong postulate.

Clearly, when an individual chooses how much to invest in his health, he does not choose a certain length of life, but, rather, expresses a preference for a particular *lottery of life*, whose different scenarii involve different lengths of life.[5] The chosen level of health-improving effort will not be a guarantee of a longer life, but only of a longer *expected* length of life with also some possible consequences on the *variance* of the length of life.[6] Thus, in the context of risk about the length of life, individual choices of health-improving efforts may reflect their attitudes towards risk about the length of life, so that the making of a *uniform* assumption on the attitude towards risk may oversimplify the problem of the optimal public intervention.

Let us illustrate this with the following example (see Figure 1). A person of age 50, who has a disease, can choose between two possible lotteries of life: either lottery A, 'no medical treatment', or lottery B, 'medical treatment' (assumed to be costless).[7] Under no medical treatment, the patient is certain to live the next 10 years for sure, but not longer. On the contrary, under the medical treatment, the patient can die during the intervention with a probability 1/2, but can, if the intervention is a success, live until the age of 70 years with a probability 1/2. What will the patient choose?

It is not straightforward to see what the patient will decide. Actually, each lottery exhibits the same expected length of life, equal to 60 years, but different degrees of risk about the length of life: whereas lottery A is

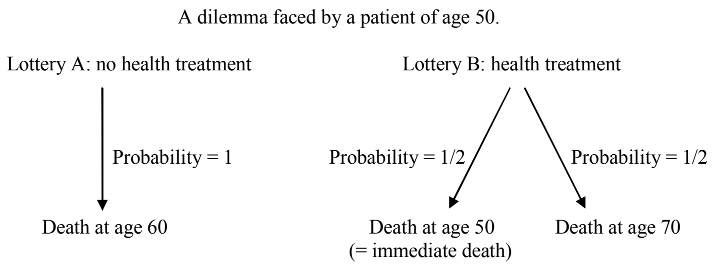
[4] One of such models is Leroux (2007).

[5] Strictly speaking, we should talk of a lottery on the length of life rather than of a lottery of life.

[6] To be precise, the chosen level of health-improving effort is likely to influence the dispersion of the ages at death.

[7] Consumptions per life-period are assumed to be the same under the two lotteries.

risk-free, lottery B is risky regarding the length of life.[8]



Choice between two lotteries of life

Under net risk-neutrality with respect to the length of life, a patient would be totally indifferent between lotteries A and B, and would toss a coin to decide whether he will undergo the medical treatment or not. However, such an indifference is highly unlikely, because the degree of risk about the length of life is a non-neutral information for decision-makers. Thus, it is likely that individuals differ largely regarding their attitude towards risk, and do not all exhibit risk-neutrality with respect to the length of life. Obviously, some patients, who are risk-averse with respect to the length of life, will choose no medical treatment (lottery A), while some others, who are risk-lover, will choose the medical treatment (lottery B).[9] As this example illustrates, the observed inequality in health-influencing efforts is likely to reflect the heterogeneity of preferences, and, in particular, the heterogeneity of individual attitudes towards risk. But this raises the difficult question of the optimal public policy in that context: what should a utilitarian government do in front of such a heterogeneity in the attitude towards risk?

[8] Note that, in general, the choice of an effort level influences not only the expected length of life and the variance of the age at death but also per period utility.

[9] Alternatively, if the treatment had the virtue not to raise, but to reduce the variance of the age at death, risk-averse agents would *ceteris paribus* opt for the treatment.

The goal of this paper is to examine the optimal public policy in an economy where agents can influence their survival by exerting some monetary effort, but differ regarding their attitude towards risk with respect to the length of life. For simplicity, we assume that individuals live for a maximum of two periods, the first one being certain while the second one is conditional on survival. Naturally, by choosing their health expenditures, agents choose a specific lottery of life, which reflects their attitude toward risk.

So as to account for individuals' attitude toward risk on longevity, we model individual preferences using a 'mean and variance' utility function, and we assume that individuals have different sensitivities to the variance of lifetime welfare.[10] As this is well-known since Bommier's (2005) work, there exist two broad ways to depart from net risk-neutrality with respect to the length of life. One way is to relax additive lifetime welfare, as in Bommier's (2005) works; the alternative solution is to relax the expected utility hypothesis. The former approach has the advantage to keep on relying on the - convenient - expected utility theory, but suffers from a lack of intuition behind non-additive lifetime welfare. This is why, in this paper, we prefer to keep additive lifetime welfare but to relax the expected utility hypothesis. Thus, lifetime welfare is still assumed to be additive in temporal welfare (without pure time preferences), but the expected utility hypothesis is here replaced by a less restrictive postulate.

More precisely, it is assumed that agents's preferences on lotteries of life can be represented by a 'mean and variance' utility function of the kind defended by Allais (1953) in his seminal paper. Actually, Allais emphasized that, given that the dispersion of psychological values is 'the specific element of the psychology of risk' (Allais, 1953, p. 512), it follows that '[...] even in a first approximation, one should take into account the second order moment

[10] Actually, the 'mean and variance' utility form is a special case of what Machina (2007) calls the 'Moments of Utility' approach in non-EU theory, followed by Hagen (1979) and Munera and de Neufville (1983).

of the distribution of psychological values' (1953, p. 513).[11] Moreover, it was also quite clear in Allais's mind that '[...] one cannot regard as irrational a psychological attitude in front of risk that takes the dispersion of psychological values into account.'(see Allais, 1953, p. 520).[12] We shall thus postulate a mean and variance utility function, which is a simple generalization of the EU form accounting for Allais's intuition. Naturally, other forms could be chosen instead (see Stigum and Wenstop, 1983; Schmidt, 2004), but the mean and variance utility function has the advantage of simplicity.[13]

The rest of the paper is organized as follows. Section 2 presents the model and derives the laissez-faire equilibrium. Section 3 studies the first best social optimum and its decentralisation. The second-best problem is considered in Section 4. Conclusions are drawn in Section 5.

2 The model

2.1 Environment

Let us consider a population of individuals who live a first period of life (whose length is normalized to one) with certainty, but survive to the second period only with a probability π. This probability depends positively on some monetary investment m:

$$\pi = \pi(m)$$

equivalently, m can be regarded as a private health expenditure made by the agent in the first period of his life, so as to increase his survival probability. We assume here that agents have the same survival function $\pi(.)$ with $\pi'(.) \geq 0$ and $\pi''(.) \leq 0$.

[11] Original version: '[...] *même dans une première approximation*, on doit tenir compte du moment d'ordre deux de la distribution des valeurs psychologiques'.

[12] Original version: '[...] on ne saurait considérer comme irrationnelle une attitude psychologique devant le risque qui tient compte de la *dispersion* des valeurs psychologiques.'

[13] Moreover, that functional form shall, unlike the expected utility function, allow some risk-aversion with respect to the length of life, even under additive lifetime welfare.

However, agents are assumed to differ in their preferences. In order to introduce these differences in preferences, we assume that individual preferences can be represented by a function having the 'mean and variance' utility form (see Allais, 1953), and that agents exhibit different degrees of sensitivity to the volatility of welfare:[14]

$$U_\gamma = \bar{u}_\gamma - \gamma var(u_\gamma)$$

where $\bar{u}_\gamma$ is the expected lifetime welfare of an agent with type γ, while $var(u_\gamma)$ is the variance of his lifetime welfare.[15] The parameter γ reflects the sensitivity to the variance of lifetime welfare exhibited by a lottery of life. Under complete insensitivity, γ equals 0 and we are back to standard expected utility theory. On the contrary, if γ is positive, the agent prefers, *ceteris paribus*, lotteries with a lower variance of lifetime welfare across scenarios, while a negative γ reflects the tastes of 'variance-lover' agents.[16]

Under a zero utility from death and additive lifetime welfare (with no pure time preferences), the expected lifetime welfare $\bar{u}_\gamma$ is:

$$\begin{aligned} \bar{u}_\gamma &= \pi(m_\gamma)\left[u(c_\gamma) + u(d_\gamma)\right] + (1 - \pi(m_\gamma))\left[u(c_\gamma)\right] \\ &= u(c_\gamma) + \pi(m_\gamma)u(d_\gamma) \end{aligned}$$

where c_γ and d_γ denote, respectively, first and second period consumptions of an individual with type γ. The function u is increasing and strictly concave. Moreover, we assume that for all consumption levels that are considered we have:

$$\frac{c_\gamma u'(c_\gamma)}{u(c_\gamma)} < 1$$

[14] Note that this function, although more general than the usual expected utility function, could still be generalized by taking into account higher moments of the distribution of lifetime welfare across scenarios of lotteries of life. For more general functions, see Hagen (1979) and Machina (1983).

[15] Note that preferences represented by a function of that form do not necessarily satisfy the independence axiom, as the initial ordering between two lotteries may be inverted by a convex combination of those lotteries.

[16] By *abus de langage*, it could also be said that agents with a higher γ are more 'risk-averse' than agents with lower γ levels.

which is standard in the litterature that studies the welfare benefits related to longevity extension.[17] In this two-scenarios world, the variance of lifetime welfare takes a quite simple form:

$$\begin{aligned} var(u_\gamma) &= [(u(c_\gamma)+u(d_\gamma))-(u(c_\gamma)+\pi(m_\gamma)u(d_\gamma))]^2 \\ &\quad +[u(c_\gamma)-(u(c_\gamma)+\pi(m_\gamma)u(d_\gamma))]^2 \\ &= [u(d_\gamma)]^2\left[(1-\pi(m_\gamma))^2+(\pi(m_\gamma))^2\right] \end{aligned} \tag{1}$$

where, for ease of notation, we denote $var(u_\gamma) \equiv var\,(d_\gamma, m_\gamma)$ in the following. At this stage, let us note the ambiguous effect of the effort level on the variance of lifetime welfare. Actually,

$$\frac{\partial var\,(d_\gamma, m_\gamma)}{\partial m_\gamma} = 2\pi'(m_\gamma)\,[u(d_\gamma)]^2\,[2\pi(m_\gamma)-1] \begin{array}{c} > \\ = \\ < \end{array} 0 \Longleftrightarrow \pi(m_\gamma) \begin{array}{c} > \\ = \\ < \end{array} \frac{1}{2}$$

Hence, a higher effort level tends to raise the variance of lifetime welfare if $\pi(m_\gamma)$ exceeds 1/2, whereas it tends to lower it if $\pi(m_\gamma)$ is lower than 1/2.

While the effect of health effort on the variance of lifetime welfare is ambiguous, one expects, intuitively, that agents tend generally not to prefer lotteries of life with a lower life expectancy to lotteries with a higher life expectancy, so that $\frac{\partial U_\gamma}{\partial \pi}$ is, in general, non-negative. Imposing the condition $\frac{\partial U_\gamma}{\partial \pi} \geq 0$ amounts to assume:

$$\frac{\partial U_\gamma}{\partial \pi} = u(d_\gamma) - 2\gamma\,[u(d_\gamma)]^2\,[2\pi(m_\gamma)-1] \geq 0$$

which, under $\gamma > 0$, is true for all levels of d_γ if and only if $\pi(m_\gamma) \leqslant 1/2$ for all levels of effort m_γ. Thus, we shall, throughout this paper, assume that $\pi(m_\gamma) \leqslant 1/2$ for all levels of m_γ. A corollary of this postulate is that $var\,(d_\gamma, m_\gamma)$ is *decreasing* in m_γ.

2.2 The laissez-faire

Agents of type γ choose first period and second period consumptions, as well as health expenditure so as to maximize their objective function subject to

[17] See Murphy and Topel (2006) and Becker et al. (2005).

their budget constraint:

$$\max_{c_\gamma, d_\gamma, m_\gamma} U_\gamma(c_\gamma, d_\gamma, m_\gamma)$$
$$\text{s.to} \left\{ \begin{array}{c} c_\gamma = w - s_\gamma - m_\gamma \\ d_\gamma = R s_\gamma \end{array} \right.$$

where lifetime utility takes the following form

$$U_\gamma(c_\gamma, d_\gamma, m_\gamma) = u(c_\gamma) + \pi(m_\gamma) u(d_\gamma) - \gamma var\,(d_\gamma, m_\gamma) \tag{2}$$

and $var\,(d_\gamma, m_\gamma)$ is equal to (1). We assume that savings are entirely invested in private annuities and that R is the return of an annuity. The wealth endowment w is exogenous and identical for all agents. Note also that there is no pure time preference, and that the interest rate is zero.

Assuming actuarially fair prices (i.e. $R = 1/\pi(m_\gamma)$), the laissez-faire allocation for an agent of type γ satisfies the following conditions:

$$u'(\,c_\gamma) = u'(d_\gamma) - \gamma var_d\,(d_\gamma, m_\gamma)\,/\pi(m_\gamma \tag{3}$$
$$u'(\,c_\gamma) = \pi'(m_\gamma) u(d_\gamma) - \gamma var_m\,(d_\gamma, m_\gamma) \tag{4}$$

where $var_x\,(x, y)$ and $var_y\,(x, y)$ are partial derivatives of the variance of lifetime welfare with respect to x and y. Condition (3) characterizes the optimal saving decision. In the absence of any sensitivity to the variance of lifetime welfare ($\gamma = 0$), each agent would choose to smooth consumption over time (i.e. $c_\gamma = d_\gamma\ \forall\gamma$), because of the conjunction of no pure time preference, an actuarially fair annuity price and a zero interest rate. However, under a positive γ, $c_\gamma > d_\gamma\ \forall\gamma$ since $var_d\,(d_\gamma, m_\gamma)$ is always positive ; thus the sensitivity of agents to the variance of lifetime welfare makes them consume more in the first period. Actually, consuming more during the first period is a simple way to insure oneself against undergoing a big loss of welfare if one dies at the end of the first period. Thus, concentrating consumption in the first period is a straightforward way to protect oneself against a too large variation of lifetime welfare across scenarios of the lottery of life. Note also that the higher γ is, the steeper the intertemporal consumption profile will be *ceteris paribus*, because the more variance-sensitive the agent is, the more he will use that trick to avoid big welfare losses. This result is presented in the proposition below:

Proposition 1 *If the market of annuities is actuarially fair, $c_\gamma > d_\gamma$ for any individual with type $\gamma > 0$.*

Condition (4) characterizes the level of health expenditure chosen by the individual in the equilibrium. Under traditional expected utility theory, this condition would collapse to $u'(c_\gamma) = \pi'(m_\gamma)u(d_\gamma)$, stating that the optimal health expenditure is such that the marginal welfare gain due to health expenditure (in terms of the second period of life) should equalize the marginal welfare cost of such an effort. However, under a positive γ, the marginal lifetime utility from health expenditure depends also on its impact on the variance of lifetime welfare (second term in brackets), which is always positive since we assume that $\pi(m_\gamma)$ is lower than 1/2. Thus, under positive sensitivity to the variance in welfare, the level of health investment is always greater than under expected utility theory. Note also that in the Laissez-Faire, the individual does not take into account the impact of health expenditures on the return of his savings, $R = 1/\pi(m_\gamma)$ so that the individual chooses a level of health expenditures which is too high compared to its optimal level.[18]

We can now study the equilibrium levels of consumptions and of health expenditure between individuals with different sensitivities to the variance in welfare. To simplify, let assume two individuals, 1 and 2, with sensitivity to the lifetime variance such that $\gamma_1 > \gamma_2$. Our results are summarized in the following proposition:

[18] This result is highlighted in Becker and Philipson (1998). Actually, each agent tends to consider that his own health effort will not affect the return of the annuity whereas at the aggregate level it does. This is also emphasized in Sheshinski (2007, chapter 7).

Proposition 2 *Provided the market for annuities is actuarially fair, the Laissez-Faire allocation is such that, for any two individuals with sensitivity to the variance in welfare such that* $\gamma_1 > \gamma_2$*,*

(i) if $c_1 = c_2$*, then* $d_1 < d_2$ *and* $m_1 \geq m_2$ *or* $d_1 \leq d_2$ *and* $m_1 > m_2$*,*

(ii) if $d_1 = d_2$*, then* $c_1 > c_2$ *and* $m_1 \leq m_2$ *or* $c_1 \geq c_2$ *and* $m_1 < m_2$*.*

Note first that, given the postulated general functional forms for $u(c)$ and $\pi(m)$, it is not possible, in the present model, to fully describe the optimal levels of consumptions and health expenditures for the two types of agents. Depending on the particular functional forms chosen for $u(c)$ and $\pi(m)$, agents's consumptions and efforts $(c_\gamma, d_\gamma, m_\gamma)$ may *a priori* vary in different ways. This is why we equalize first or second period consumptions between individuals so as to determine how the chosen variables $(c_\gamma, d_\gamma, m_\gamma)$ differ across agents.

If first-period consumption is equal for the two types of agents, then it is necessarily the case that an agent who has a larger sensitivity to the dispersion of psychological values chooses a lower second-period consumption and more health spending than an agent with a lower γ. The intuition behind that result is the following. For a more sensitive agent, having a lower second-period consumption and spending more on health is a rational way to reduce the variance of lifetime welfare since the potential loss (i.e. second-period utility) would be smaller and this would happen with a lower probability. Agents with a lower γ do not have the same concerns, and thus choose, for an equal first period consumption, a higher second-period consumption and a lower health effort.

If, alternatively, it is second-period consumption that is equal for both types of agents, then, without surprise, agents who are more sensitive to the variance will consume more in the first period in comparison to less variance-sensitive agents, so that, given the budget constraint faced, they will also invest less in health in comparison with agents with a lower γ. That result is not surprising, as consuming more in the first period is a standard way to insure oneself against a too large volatility of lifetime welfare.

Thus, one cannot say, under general functional forms, whether agents with a higher γ will spend more or fewer ressources in health. A higher sensitivity to the dispersion of psychological value may imply that an agent spends more on health (at the cost of second-period consumption) or on the contrary, spends less on health (to favour first-period consumption). That indeterminacy can be explained as follows. The two ways to protect oneself against a high volatility of lifetime welfare are either to spend a lot on health or to spend a lot in first-period consumption. Which solution dominates depends on the curvatures of $u(c_\gamma)$ and $\pi(m_\gamma)$. If $\pi'(m_\gamma)$ is large and $u'(c_\gamma)$ is low, then agents with a higher γ will opt for the first way to avoid lifetime welfare variance; on the contrary, if $\pi'(m_\gamma)$ is low and $u'(c_\gamma)$ is large, agents with a higher γ will opt for the second as a more efficient way to avoid lifetime welfare volatility.

3 The first best problem

3.1 The social optimum

In this section, we assume that the social planner is utilitarian and that he perfectly observes individuals' types.[19] The social planner can lend or borrow at a zero interest rate in order to balance the budget at any given period. The resource constraint of the economy is thus:

$$\int_{\gamma\min}^{\gamma_{\max}} \left(c_\gamma + \pi\left(m_\gamma\right) d_\gamma + m_\gamma\right) f\left(\gamma\right) d\gamma \leq w \tag{5}$$

where $f\left(\gamma\right)$ is the distribution function of the γs in the population. Thus, the social planner chooses consumption paths as well as health investments

[19]Note that the standard Benthamite utilitarian criterion exhibits various limitations in general, and in the particular context of endogenous longevity (see Broome, 2004). Thus, it is used here on the mere grounds of analytical conveniency.

levels for each type of individuals in order to maximize

$$\int_{\gamma \min}^{\gamma_{\max}} \left(u(c_\gamma) + \pi(m_\gamma)u(d_\gamma) - \gamma var\left(d_\gamma, m_\gamma\right)\right) f\left(\gamma\right) d\gamma$$

subject to (5).

The first order conditions yield:

$$\begin{aligned}
u'(c_\gamma) &= \lambda && (6)\\
u'(d_\gamma) - \gamma \frac{var_d\left(d_\gamma, m_\gamma\right)}{\pi(m_\gamma)} &= \lambda && (7)\\
\pi'(m_\gamma)u(d_\gamma) - \gamma var_m\left(d_\gamma, m_\gamma\right) &= \lambda\left[1 + \pi'\left(m_\gamma\right) d_\gamma\right] && (8)
\end{aligned}$$

Combining (6) and (7), we obtain the optimal trade-off between present and future consumptions; this is identical to our Laissez-Faire condition (3) when the price of the annuity is actuarially fair. Thus, first-period consumption is still preferred to future consumption in the first best. On the contrary, (8) together with (6) differs from (4) by a term $-\lambda\pi'\left(m_\gamma\right) d_\gamma$. In the first best, the social planner realizes that the level of health expenditure also modifies the budget set. Indeed, a higher level of effort m_γ not only increases direct utility through higher survival but also decreases consumption possibilities as $\pi\left(m_\gamma\right)$ increases in (5). Thus, in the first best optimum, the social planner induces the individual to exert lower effort so as to limit the negative impact of m_γ over the individual's budget set; this was not the case in the Laissez-Faire as the individual was taking the annuity return, R as given. These first results are summarized in the following proposition:

Proposition 3 *The first best allocation is such that, for any individual with sensitivity* $\gamma > 0$

(i) $m_\gamma^{FB} < m_\gamma^{LF}$,

(ii) $c_\gamma > d_\gamma$.

where m_γ^{FB} and m_γ^{LF} are the level of health expenditures in the first best and Laissez-Faire respectively. We now turn to the allocation of consumptions and of health expenditure according to individuals types. Obviously, first-period consumption is equalized across individuals. However, considering (7) and (8), there is no reason for second period consumptions and health expenditure to be identical across individuals. A priori, it is impossible to rank health expenditures and consumptions depending on individuals' types, unless some additional assumptions are made. Therefore, we assume two individuals with types γ_1 and γ_2 such that one is sensitive to the variance in lifetime welfare and the other is not and obtain the following results:[20]

Proposition 4 *Consider two types of individuals with sensitivity to the variance in welfare such that $\gamma_1 > 0$ and $\gamma_2 = 0$. The first best yields:*

(i) $c_1 = c_2 = \bar{c}$*,*

(ii) $d_1 < d_2$*,*

(iii) $m_1 \gtrless m_2$*.*

In the first best, first period consumption is equalized across individuals while second period consumption and health expenditures are differentiated between individuals.

The individual with a zero sensitivity to the variance obtains a higher level of second-period consumption than a variance-sensitive agent. This result is not surprising, as second-period consumption tends necessarily to raise lifetime welfare variance.[21] Given that lifetime welfare variance enters type 1's utility negatively, it does not come as a surprise that the social optimum implies $d_1 < d_2$.

However, it is not clear whether health expenditures should be higher or lower for the individual with higher sensitivity to the variance. This indeterminacy can be explained as follows. On the one hand, higher health investment reduces the lifetime welfare variance of variance-sensitive agents, which matters for *those* agents (unlike for agents of type 2), and, as such,

[20]This proposition is proven in Appendix B.

[21]Clearly, in the extreme case where second-period consumption equals 0, and $u(0) = 0$, there is a zero lifetime welfare variance despite the risk about the length of life.

is justified on the grounds of social welfare maximization. On the other hand, dedicating more resources to the health of variance-sensitive agents has, given $d_1 < d_2$, a *smaller* impact on the expected lifetime welfare of agents of type 1 than on the expected lifetime welfare of agents of type 2. Hence, whether m_1 exceeds m_2 or not depends on which effect dominates. As shown in the Appendix, if γ^1 is extremely large, one necessarily has $m_1 > m_2$, because the social welfare gain from dedicating more resources to the health of agents of type 1 is here large (given the extreme sensitivity of those agents to the variance of lifetime welfare) and thus largely compensates the social welfare loss due to the lower second-period utility exhibited by the life of agents of type 1.

3.2 Decentralisation

We now study how to decentralise the above optimum through a tax-and-transfer scheme. In the following, we assume that instruments available for the social planner are a tax on savings, a tax on health expenditures and lump sum transfers. We still assume that the annuity market is actuarially fair so that $R = 1/\pi(m_\gamma)$ at equilibrium. The individual's problem is then to maximize:

$$u\left(w - s_\gamma\left(1 + t_\gamma\right) - m_\gamma\left(1 + \theta_\gamma\right) + T_\gamma\right) + \pi\left(m_\gamma\right)u\left(Rs_\gamma\right) - \gamma var\left(Rs_\gamma, m_\gamma\right)$$

where t_γ is the tax on savings, θ_γ the tax on health expenditures and T_γ is a monetary transfer for any individual with sensitivity γ. Deriving first order conditions with respect to s_γ and m_γ and rearranging them, we obtain

$$\begin{aligned} \left|MRS_{c,d}\left(c_\gamma, d_\gamma, m_\gamma\right)\right| &\equiv \frac{\pi\left(m_\gamma\right)u'\left(d_\gamma\right) - \gamma var_d\left(d_\gamma, m_\gamma\right)}{u'\left(c_\gamma\right)} \\ &= \pi\left(m_\gamma\right)\left(1 + t_\gamma\right) \qquad (9) \\ \left|MRS_{c,m}\left(c_\gamma, d_\gamma, m_\gamma\right)\right| &\equiv \frac{\pi'(m_\gamma)u(d_\gamma) - \gamma var_m\left(d_\gamma, m_\gamma\right)}{u'\left(c_\gamma\right)} = \left(1 + \theta_\gamma\right) \qquad (10) \end{aligned}$$

where $\left|MRS_{c,d}\left(c_\gamma, d_\gamma, m_\gamma\right)\right|$ and $\left|MRS_{c,m}\left(c_\gamma, d_\gamma, m_\gamma\right)\right|$ account for the marginal rates of substitution between c and d and between c and m expressed in absolute value. Thus comparing these conditions with both (6), (7) and (8), we find that the optimal tax on savings is always zero for any type of individual, e.g. $t_\gamma = 0$ but the optimal level of the tax on health expenditures θ_γ should be equal to $\pi'(m_\gamma) d_\gamma > 0$. This can be related to Becker and Philipson (1998); by implementing a positive tax on health expenditures, one limits health expenditures and make it tend toward its first best level.

Back to the results of Proposition 4, in the specific case where $\gamma_1 > 0$ and $\gamma_2 = 0$, one should have that $\theta_1 = \pi'(m_1) d_1 < \theta_2 = \pi'(m_2) d_2$ so that $m_1 > m_2$. This is the case whenever γ_1 is high. Otherwise, $\theta_1 \lesseqgtr \theta_2$ if $m_1 < m_2$. We also find that if $m_1 < m_2$, the level of expected consumption, defined by

$$\bar{c} + \pi(m_\gamma)d_\gamma + m_\gamma$$

is always greater for individual with type-2 than for the individual with type-1. In this case, the first best optimum transfers resources from the individual with higher sensitivity to the variance to the individual with the lowest one and $T_1 < T_2$. On the opposite, if $m_1 \gtrless m_2$, the direction of transfers is ambiguous and $T_1 \gtrless T_2$.

4 The second best problem

In this section, we now relax the assumption that the social planner observes individuals' sensitivity to the variance in lifetime welfare. Using results of Proposition 4, if the social planner was offering first best bundles, individuals might have interest in claiming to be of the other type so as to get higher consumption and/or benefit from higher health expenditures (depending on first best levels of $c_\gamma, d_\gamma, m_\gamma$).[22] This is why, in the following, we write a general second best problem in which we prevent any individual from

[22] For instance, if the first best allocation is such that $d_1 < d_2$ and $m_1 < m_2$, individual 1 may have interest in claiming to be of type-2 (only if increasing d_2 does not increase too much his variance and/or if γ_1 is sufficiently low) . On the contrary, if $d_1 < d_2$ and $m_1 > m_2$ in the first best, one or the other (or both) type(s) might lie on his (their) type(s) so as to get higher consumption or higher health expenditures.

mimicking the other:

$$\max \int_{\gamma \min}^{\gamma_{\max}} U_\gamma\left(c_\gamma, m_\gamma, d_\gamma\right) f(\gamma)\, d\gamma$$

$$\text{s. to} \left\{ \begin{array}{c} \int_{\gamma_{\min}}^{\gamma_{\max}} \left(w - c_\gamma - \pi\left(m_\gamma\right) d_\gamma - m_\gamma\right) f(\gamma)\, d\gamma \geq 0 \\ u(c_\gamma) + \pi(m_\gamma)u(d_\gamma) - \gamma var\left(m_\gamma, d_\gamma\right) \geq \\ u(c_{\gamma'}) + \pi(m_{\gamma'})u(d_{\gamma'}) - \gamma var\left(m_{\gamma'}, d_{\gamma'}\right) \forall \gamma, \gamma' \end{array} \right.$$

where the last constraint is the incentive constraint and states that any individual with type γ should always be better-off with his bundle $(c_\gamma, d_\gamma, m_\gamma)$ than with the allocation designed for any other type γ'. As we show in Appendix, the continuum of above incentive constraints can be transformed into a local incentive compatibility constraint which has the following expression

$$\dot{U} = -var\left(m_\gamma, d_\gamma\right) < 0$$

where a dot means that U is derived with respect to γ and the second order local conditions are $\dot{m}_\gamma \leq 0$ and $\dot{d}_\gamma \geq 0$.[23] In the Appendix, we show that the second best optimum yields the following trade-offs between 2-period consumptions and between health investment and first period consumption, in absolute value:

$$\begin{aligned} \left|MRS_{c,d}\left(c_\gamma, d_\gamma, m_\gamma\right)\right| &= \pi\left(m_\gamma\right)\left[1 - \frac{\mu(\gamma)\, var_d\left(m_\gamma, d_\gamma\right)}{\pi\left(m_\gamma\right)\lambda f(\gamma)}\right] && (11) \\ \left|MRS_{c,m}\left(c_\gamma, d_\gamma, m_\gamma\right)\right| &= 1 + \pi'\left(m_\gamma\right) d_\gamma - \frac{\mu(\gamma)\, var_m\left(m_\gamma, d_\gamma\right)}{\lambda f(\gamma)} && (12) \end{aligned}$$

with λ, the Lagrangian multiplier associated to the ressource constraint and $var_d\left(m_\gamma, d_\gamma\right) > 0$, $var_m\left(m_\gamma, d_\gamma\right) < 0$. Comparing (11) and (12) with (9) and (10) of the decentralized problem, , we find that the second best optimum could be decentralized by implementing taxes on savings and on

[23] From now on, we assume that second order local conditions are satisfied (i.e. that $\dot{m}_\gamma \leq 0$ and $\dot{d}_\gamma \geq 0$). If this was not the case on some interval, one would have bunching over this interval.

health expenditures equal to

$$\begin{aligned} t_\gamma &= -\frac{\mu(\gamma)\, var_d(m_\gamma, d_\gamma)}{\pi(m_\gamma)\,\lambda f(\gamma)} \\ \theta_\gamma &= \pi'(m_\gamma)\, d_\gamma - \frac{\mu(\gamma)\, var_m(m_\gamma, d_\gamma)}{\lambda f(\gamma)} \end{aligned}$$

where the co-state variable associated to the local incentive constraint, $\mu(\gamma)$ has the following expression

$$\mu(\gamma) = \lambda \int_\gamma^{\gamma_{\max}} \left(\frac{1}{\lambda} - \frac{1}{u'(c_\gamma)}\right) f(\gamma)\, d\gamma \tag{13}$$

with $\mu(\gamma_{\max}) = \mu(\gamma_{\min}) = 0$ from the transversality conditions.

Let now study the level of these taxes. First, the usual result of no distortion at the top and at the bottom holds; for the individuals with the highest sensitivity and the ones with the lowest sensitivity, the trade-offs between two-period consumption and between consumption and health expenditures are equivalent to the first best ones. Thus, the tax on savings is zero in this case ($t_{\gamma_{\min}} = t_{\gamma_{\max}} = 0$) and health expenditures are taxed in the same way as in the first best, i.e. $\theta_{\gamma_{\min}} = \pi'\left(m_{\gamma_{\min}}\right) d_{\gamma_{\min}}$ and $\theta_{\gamma_{\max}} = \pi'\left(m_{\gamma_{\max}}\right) d_{\gamma_{\max}}$.

On the contrary, for any other type $\gamma \in]\gamma_{\min}, \gamma_{\max}[$, the signs of t_γ and θ_γ are not clear and depend on the sign of $\mu(\gamma)$. In our model, $\mu(\gamma)$ represents the social net marginal welfare gain of increasing the tax on savings for individuals with types above γ. Indeed, increasing the tax on savings first generates a gain in increased revenue, $1/u'(c_\gamma)$ per person but it also generates a loss in welfare measured in units of revenue equal to $1/\lambda$. Since $\mu(\gamma_{\max}) = \mu(\gamma_{\min}) = 0$, it is straightforward to show that $\mu(\gamma)$ first decreases and then increases in γ so that $\mu(\gamma)$ has always a negative sign.[24] Thus, in the second best, it is always optimal to subsidize savings, and the level of this subsidy will be higher for individuals with types in the middle ranges. Note also that the level of this tax depends on the mass of individuals with types above γ, represented by $\pi(m_\gamma) f(\gamma)$. Imposing a subsidy

[24] Our analysis is similar to Atkinson and Stiglitz (1980).

on savings is a way to relax the incentive constraint and avoid mimicking behavior.

However, the sign of θ_γ is uncertain and might be positive or negative depending on the size of two countervailing effects. On the one hand, $\pi'(m_\gamma)\, d_\gamma > 0$ which corresponds to the Becker-Philipson effect; as we already mentionned in the first best, imposing a tax on health expenditures is a way to limit individuals' investment in health and thus its negative impact on their budget set. On the other hand, $-\mu(\gamma)\, var_m(m_\gamma, d_\gamma)/\lambda f(\gamma) < 0$ and is related to the incentive effect; in order to avoid mimicking behaviours, the social planner would like to subsidize health expenditures. For instance, if individuals could perfectly see the impact of their health investment on their budget set, the Becker Philipson effect would be absent and in the second best, subsidization of health expenditures would be optimal; the level of this subsidy would be higher for individuals with middle range sensitivities. Thus, for any individual with types $\gamma \in]\gamma_{\min}, \gamma_{\max}[$, the overall effect on the sign of θ_γ is ambiguous and depends on the magnitude of both the Becker-Philipson and the incentive effects.

Our findings are summarized in the proposition below:

Proposition 5 *Assume a population of individuals with sensitivity to the variance $\gamma \in [\gamma_{\min}, \gamma_{\max}]$ and with density function $f(\gamma)$. Under asymmetric information,*

- *The "no distortion at the top and at the bottom" result holds.*
- *A subsidy on savings is optimal and is equal to $t_\gamma = -\mu(\gamma)\frac{var_d(m_\gamma, d_\gamma)}{\pi(m_\gamma)\lambda f(\gamma)}$ for any $\gamma \in]\gamma_{\min}, \gamma_{\max}[$.*
- *A tax on health expenditures is optimal and equal to $\theta_\gamma = \pi'(m_\gamma)\, d_\gamma - \mu(\gamma)\, var_m(m_\gamma, d_\gamma)/\lambda f(\gamma)$ for any $\gamma \in]\gamma_{\min}, \gamma_{\max}[$; but whether it is positive or negative is ambiguous.*

5 Conclusion

This paper aims at studying the optimal taxation policy in an economy where agents can influence their longevity through health efforts but who differ in their attitude towards lotteries of life. For that purpose, we set up a two-period model in which the agents influence their survival probability by means of first-period health spending. Moreover, the heterogeneity of agents is captured by assuming that preferences on lotteries of life can be represented by various sensitivities to the variance of lifetime utilities.

It is shown that, in the laissez-faire, a higher sensitivity to the dispersion of psychological value implies that an agent spends more on health (at the cost of second-period consumption), or, on the contrary, spend less on health (to favour first-period consumption). The choice between those two ways to protect oneself against a high volatility of lifetime welfare depends on the specific functional forms for $u(c_\gamma)$ and $\pi(m_\gamma)$.

At the social optimum, first-period consumptions are equalized across agents, whereas agents with a higher sensitivity to the variance should have a lower second-period consumption. It is not obvious to see whether more variance-sensitive agents should benefit from higher or lower health expenditures, as, from a social point of view, the welfare gain from reducing the variance of their lifetime welfare is to be compared with the lower expected welfare associated with the survival of those agents (given their lower second-period consumption). The social optimum can be decentralized by means of group-specific taxes on health spending (to internalize the Becker-Philipson effect) and by adequate lump-sum transfers, whose directions depend on whether more sensitive agents should have higher health spending or not.

Under asymmetric information, it is shown that a subsidy on savings constitutes a simple way to avoid mimicking across agents. However, the sign of (group-specific) taxes on health spending remains ambiguous, and depends on two effects: first, given the non-internalization by agents of the impact of their effort on the return of their savings, it is optimal to tax health expenditures; second, subsidizing health expenditures is also a way to guarantee incentive-compatibility. Hence, the resulting sign of the optimal tax on health is unknown.

To conclude, it should be stressed that the present study, by focusing exclusively on one source of heterogeneity across agents - their sensitivity to variance of utilities - only covers *one* aspect of the design of the optimal taxation policy under endogenous (differentiated) longevity. Undoubtedly, other sources of heterogeneity exist, regarding, for instance, the genetic background, the degree of rationality/myopia, the impatience or the disutility of efforts. Hence, one could hardly hope to provide a complete answer to that problem without considering what the optimal policy becomes when those various sources of heterogeneity coexist. To answer that question, a crucial point will concern how those different characteristics are correlated across individuals types. Thus, in the light of the difficulties faced in the present study where agents differed in only one aspect, one could hardly overestimate the problems raised by a more complete study of optimal taxation in an economy where agents differ in several characteristics influencing longevity.

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Appendix

A Laissez Faire: proof of Proposition 2

The laissez-faire allocation for an individual of type γ satisfies the following FOCs:

$$\begin{aligned} u'(c_\gamma) &= u'(d_\gamma)\left[1-2\gamma u(d_\gamma)\left(2\pi(m_\gamma)-2+\frac{1}{\pi(m_\gamma)}\right)\right] \\ u'(c_\gamma) &= \pi'(m_\gamma)u(d_\gamma)\left[1-2\gamma u(d_\gamma)\left(2\pi(m_\gamma)-1\right)\right] \end{aligned}$$

Suppose now that $\gamma_1 > \gamma_2$. If $c_1 = c_2 = \bar{c}$, we have:

$$\begin{aligned} u'(\bar{c}) &= u'(d_i)\left[1-2\gamma_i u(d_i)\left(2\pi(m_i)-2+\frac{1}{\pi(m_i)}\right)\right] \\ u'(\bar{c}) &= \pi'(m_i)u(d_i)\left[1-2\gamma_i u(d_i)\left(2\pi(m_i)-1\right)\right] \end{aligned}$$

The second condition excludes the cases where (1) $c_1 = c_2$, $m_1 < m_2$ and $d_1 > d_2$, (2) $c_1 = c_2$, $m_1 \leq m_2$ and $d_1 > d_2$, (3) $c_1 = c_2$, $m_1 < m_2$ and $d_1 \geq d_2$. Moreover, the budget constraint excludes the cases where (1) $c_1 = c_2$, $m_1 \geq m_2$ and $d_1 > d_2$ and (2) $c_1 = c_2$, $m_1 > m_2$ and $d_1 \geq d_2$. Hence, it must be the case that $m_1 \geq m_2$ and $d_1 < d_2$ or $m_1 > m_2$ and $d_1 \leq d_2$.

The part (ii) can be proven by similar reasoning.

B First Best

We rewrite (7) and (7) for each type under the assumption that $\gamma_1 > 0$ and $\gamma_2 > 0$:

$$\begin{aligned} u'(d_1)\left[1-2\gamma_1 u(d_1)\left(2\pi(m_1)-2+\frac{1}{\pi(m_1)}\right)\right] &= \lambda \\ u'(d_2) &= \lambda \\ \pi'(m_1)u(d_1)\left[1-2\gamma_1 u(d_1)\left(2\pi(m_1)-1\right)-\lambda\frac{d_1}{u\left(d_1\right)}\right] &= \lambda \\ \pi'(m_2)u(d_2)\left[1-\lambda\frac{d_2}{u\left(d_2\right)}\right] &= \lambda \end{aligned}$$

Combining first two equations, one finds that $d_1 < d_2$. Using two last equations we find that $m_1 \lesseqgtr m_2$. In the specific case where γ^1 is very high, such that

$$1 - 2\gamma^1 u(d_1)\left(2\pi(m_1) - 1\right) - \lambda \frac{d_1}{u\left(d_1\right)} > 1 - \lambda \frac{d_2}{u\left(d_2\right)}$$

for any level of d_i, one has that $m_1 > m_2$.

C Second best problem

C.1 Local incentive constraint

The continuum of global incentive constraints can be transformed into a local incentive constraint by applying the following method. First,

$$\max_{\tilde{\gamma}} u(c_{\tilde{\gamma}}) + \pi(m_{\tilde{\gamma}})u(d_{\tilde{\gamma}}) - \gamma var\left(m_{\tilde{\gamma}}, d_{\tilde{\gamma}}\right)$$

yields

$$\begin{aligned} & u'(c_{\tilde{\gamma}})\dot{c}_{\tilde{\gamma}} + \pi'(m_{\tilde{\gamma}})u(d_{\tilde{\gamma}})\dot{m}_{\tilde{\gamma}} + \pi(m_{\tilde{\gamma}})u'(d_{\tilde{\gamma}})\dot{d}_{\tilde{\gamma}} \\ - \quad & \gamma\left[var_d\left(m_{\tilde{\gamma}}, d_{\tilde{\gamma}}\right)\dot{d}_{\tilde{\gamma}} + var_m\left(m_{\tilde{\gamma}}, d_{\tilde{\gamma}}\right)\dot{m}_{\tilde{\gamma}}\right] = 0 \end{aligned}$$

whenever $\tilde{\gamma} = \gamma$. And solving the following problem,

$$\max_{\gamma} U_\gamma = u(c_\gamma) + \pi(m_\gamma)u(d_\gamma) - \gamma var\left(m_\gamma, d_\gamma\right)$$

where we replace for the above equality, one obtains

$$\dot{U}_\gamma = -var\left(m_\gamma, d_\gamma\right)$$

with $\dot{m}_\gamma \leq 0$ and $\dot{d}_\gamma \geq 0$. This yields the local incentive constraint.[25]

[25] Similar approach can be found in Diamond (1998).

C.2 Second best solution

Using the local incentive constraint, $\dot{U} = -var\,(m_\gamma, d_\gamma)$, the problem can be rewritten as

$$\max \quad \int_{\gamma\min}^{\gamma_{\max}} \left[u(c_\gamma) + \pi(m)u(d_\gamma) - \gamma var\,(m, d_\gamma)\right] f\,(\gamma)\,d\gamma$$

$$s.to \begin{cases} \int_{\gamma\min}^{\gamma_{\max}} \left(w - c_\gamma - \pi\,(m_\gamma)\,d_\gamma - m_\gamma\right) f\,(\gamma)\,d\gamma \geq 0 \\ U_\gamma = u(c) + \pi(m)u(d) - \gamma var\,(m, d) \\ \dot{U}_\gamma = -var\,(m_\gamma, d_\gamma) \\ \dot{m}_\gamma \leq 0 \text{ and } \dot{d}_\gamma \geq 0 \end{cases}$$

We define three control variables c_γ, m_γ and d_γ, and a state variable, U_γ. The Hamiltonian is then

$$\begin{aligned} H \;=\; & U_\gamma f\,(\gamma) + \lambda\,(w - c_\gamma - \pi\,(m_\gamma)\,d_\gamma - m_\gamma)\,f\,(\gamma) \\ & -\alpha\,(\gamma)\,[U_\gamma - u(c_\gamma) - \pi(m_\gamma)u(d_\gamma) + \gamma var\,(m_\gamma, d_\gamma)] \\ & -\mu\,(\gamma)\,[-var\,(m_\gamma, d_\gamma)] \end{aligned}$$

where $\mu\,(\gamma)$ is the co-state variable associated with $\dot{U}_\gamma = -var\,(m_\gamma, d_\gamma)$, $\alpha\,(\gamma)$ is the shadow value of the constraint $U_\gamma = u(c)+\pi(m)u(d)-\gamma var\,(m, d)$ and λ is the lagrange multiplier associated to the resource constraint. From the Pontryagin principle,

$$\dot{\mu}\,(\gamma) = -\frac{\partial H}{\partial U_\gamma} = \alpha\,(\gamma) - f\,(\gamma)$$

Optimizing with repect to c_γ, m_γ and d_γ also yields:

$$\begin{aligned} & \frac{\partial H}{\partial c_\gamma} = -\lambda f\,(\gamma) + \alpha\,(\gamma)\,u'(c_\gamma) = 0 \\ & \frac{\partial H}{\partial d_\gamma} = -\lambda\pi\,(m_\gamma)\,f\,(\gamma) + \mu\,(\gamma)\,var_{_d}\,(m_\gamma, d_\gamma) \\ - \quad & \alpha\,(\gamma)\left[-\pi(m_\gamma)u'(d_\gamma) + \gamma var_d\,(m_\gamma, d_\gamma)\right] = 0 \\ & \frac{\partial H}{\partial m_\gamma} = -\lambda\left(\pi'\,(m_\gamma)\,d_\gamma + 1\right) f\,(\gamma) + \mu\,(\gamma)\,var_{_m}\,(m_\gamma, d_\gamma) \\ - \quad & \alpha\,(\gamma)\left[-\pi'(m_\gamma)u(d_\gamma) + \gamma var_m\,(m_\gamma, d_\gamma)\right] = 0 \end{aligned}$$

The transversality condition are $\mu(\gamma_{\max}) = \mu(\gamma_{\min}) = 0$. Rearranging terms yield

$$\begin{aligned}
\frac{\pi(m_\gamma)u'(d_\gamma) - \gamma var_d(m_\gamma, d_\gamma)}{u'(c_\gamma)} &= \frac{\lambda\pi(m_\gamma) f(\gamma) - \mu(\gamma) var_d(m_\gamma, d_\gamma)}{\lambda f(\gamma)} \\
\frac{\pi'(m_\gamma)u(d_\gamma) - \gamma var_m(m_\gamma, d_\gamma)}{u'(c_\gamma)} &= \frac{\lambda(\pi'(m_\gamma) d_\gamma + 1) f(\gamma) - \mu(\gamma) var_m(m_\gamma, d_\gamma)}{\lambda f(\gamma)}
\end{aligned}$$

where the left hand sides are simply $|MRS_{c,d}(c_\gamma, d_\gamma, m_\gamma)|$ and $|MRS_{c,m}(c_\gamma, d_\gamma, m_\gamma)|$. Note that $\mu(\gamma)$ has the following expression

$$\begin{aligned}
\mu(\gamma) &= -\int_\gamma^{\gamma_{\max}} \dot{\mu}(\gamma)\, d\gamma \\
&= \int_\gamma^{\gamma_{\max}} f(\gamma) - \alpha(\gamma)\, d\gamma \\
&= \lambda \int_\gamma^{\gamma_{\max}} \left(\frac{1}{\lambda} - \frac{1}{u'(c_\gamma)}\right) f(\gamma)\, d\gamma
\end{aligned}$$

where we made use of $\partial H / \partial c_\gamma = 0$.

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