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Characterizations of Pareto-efficient, fair, and
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# Characterizations of Pareto-efficient, fair, and strategy-proof allocation rules in queueing problems

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## Abstract

A set of agents with possibly different waiting costs have to receive the same service one after the other. Efficiency requires to maximize total welfare. Equity requires to at least treat equal agents equally. One must form a queue, set up monetary transfers to compensate agents having to wait, and not a priori arbitrarily exclude agents from positions. As one may not know agents' waiting costs, they may have no incentive to reveal them. We identify the only rule satisfying Pareto-efficiency, a weak equity axiom as equal treatment of equals in welfare or symmetry, and strategy-proofness. It satisfies stronger axioms, as no-envy and anonymity. Further, its desirability extends to related problems. To obtain these results, we prove that even non-single-valued rules satisfy Pareto-efficiency of queues and strategy-proofness if and only if they select Pareto-efficient queues and set transfers in the spirit of Groves (1973). This holds in other problems, provided the domain of quasi-linear preferences is rich enough.

**Keywords**: queueing problems, efficiency, fairness, strategy-proofness.

**JEL Classification**: D63, C72

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# 1 Introduction

Consider an ice storm in Rochester, NY. After the storm, many business firms urge for individual repair of their electrical systems at the same time. Rochester Gas and Electric eventually provides the repair service in each firm. Yet, each firm having to wait, incurs a cost that linearly increases with time, namely its hourly loss of revenue.

In such a situation, agents must queue. One may *a priori* recommend several queues and may set up monetary transfers. Efficiency requires to maximize total welfare. Equity requires to treat equal agents equally. As one may not know agents' waiting costs, they may have no incentive to reveal them.

Our objective is to identify solutions to such *queueing problems* satisfying the following axioms. First is *Pareto-efficiency*. On the domain of linear preferences in transfers, we can decompose it into two distinct axioms, *Pareto-efficiency of queues*, i.e. queues should minimize total waiting cost, and *balancedness*, i.e. transfers should sum up to zero.

Second, we should treat agents with equal waiting costs equally. As we cannot serve agents simultaneously, we cannot give two agents with equal waiting costs equal positions. Yet, using transfers, we can compensate the agent having to wait. Selecting more than one queue, we can avoid *a priori* arbitrarily excluding an agent from the position of the other. We impose *equal treatment of equals in welfare*, i.e. agents with equal waiting costs should have equal welfares, and *symmetry*, i.e. agents with equal waiting costs should have the possibility to receive the same bundles.

Both axioms are important. In particular, they are necessary conditions for axioms embodying further basic equity criteria. *No-envy*, i.e. no agent should prefer another agent's bundle to her own, implies the former. *Anonymity*, i.e. agents' names should not matter, implies the latter. *A priori* recommending not more than one way to allocate positions and transfers, we may satisfy *equal treatment of equals in welfare*, not *symmetry*. Thus, because we cannot serve agents simultaneously, *anonymity* is possible if and only if we allow selecting more than one allocation.

Third is *strategy-proofness*, i.e. each agent should find revealing her unit waiting cost at least as desirable as misrepresenting it. The impossibility to serve agents simultaneously and equity impose allowing to select more than one allocation. Agents compare non-empty sets of allocations. We assume that an agent finds such a set as desirable as another if and only if she finds (*i*) the worst bundle she may receive from the former as desirable as the worst bundle she may from the latter, and (*ii*) the worst bundle she may receive from the former as desirable as the worst bundle she may from the latter.

We identify the only allocation *rule* satisfying these axioms on the domain of linear preferences in positions and transfers. For each problem, it selects all *Pareto-efficient* queues and sets transfers considering each pair of agents in turn, making each agent in the pair pay the cost she imposes on the pair, and distributing the sum of these two payments equally among the others. We refer to it as the *Largest Equally Distributed Pairwise Pivotal (LEDPP) rule*. Further, we prove that it satisfies *no-envy*, *anonymity*, and stronger incentive compatibility criteria. Indeed, it is such that each agent always finds any bundle she receives when revealing her unit waiting cost at least as desirable as any bundle she receives when misrepresenting it.

To reach our objective, we first characterize the class of rules satisfying *Pareto-efficiency of queues* and *strategy-proofness*. In such queueing problems, as in any public decision-making problem with additively separable preferences with respect to transfers, there are *single-valued* rules satisfying these axioms (Groves, 1973). Also, as in any public decision-making problem with convex preferences, only these *single-valued* rules satisfy them (Holmström, 1979).[2] Yet, for the cases where equity considerations make multiple choices compelling, or simply for a concern to *a priori* impose the least, one need not want to impose *single-valuedness*.

We prove that even a non-*single-valued* rule satisfies *Pareto-efficiency of queues* and *strategy-proofness* if and only if for each problem, it selects *Pareto-efficient* queues and it sets each agent's transfer in the spirit of Groves (1973). Indeed, for each bundle that maximizes or minimizes her utility, her transfer is equal to the others' total waiting cost plus an amount only depending on these agents' unit waiting costs. This characterization holds in public decision-making problems other than the queueing problems we study, provided the domain of quasi-linear preferences is rich enough.

Second, we identify the class of rules satisfying *Pareto-efficiency, equal treatment of equals in welfare*, and *strategy-proofness*. We prove that one of these rules, the LEDPP rule, is the only rule satisfying *Pareto-efficiency, symmetry*, and *strategy-proofness*. Thus, while *Pareto-efficiency* and *strategy-proofness* leave us with a large class of rules, adding a weak equity axiom as *equal treatment of equals in welfare* or *symmetry*, imposes a unique transfers list. One also obtains it following Suijs (1996), Mitra and Sen (1998), and Mitra (2001).

The intuition for this characterization is simple. We obtain any rule that satisfies *Pareto-efficiency of queues* and *strategy-proofness*, selecting the *Pareto-efficient* queues appropriately and setting each agent's transfer equal to the

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[2] Holmström (1979) proves this result on the domain of public decision-making problems with smoothly connected preferences, i.e. for any two profiles in the domain, there is a differentiable deformation of one profile into the other that is also in the domain. Green and Laffont (1977) prove it on the universal domain.

cost she imposes on the others plus an appropriately chosen amount that only depends on the others' waiting costs. By *balancedness*, a desirable rule redistributes these costs' sum.[3] By equity, it selects all *Pareto-efficient* queues and it redistributes this sum fairly. By *strategy-proofness*, it redistributes this sum, so that each agent's share only depends on the others' waiting costs.

This is exactly what the LEDPP rule does. It selects all *Pareto-efficient* queues (so it is efficient and fair). It sets each agent's transfer considering each pair of agents in turn, making each agent in the pair pay the cost she imposes on the pair, and distributing the sum of these two payments (so it is efficient) equally (so it is fair) among the others (so it is *strategy-proof*).

The framework's key feature, on which we build this rule, is as follows. We can express the sum of the costs each agent imposes on the others, i.e. the sum that *Pareto-efficiency of queues*, *strategy-proofness*, and *balancedness* impose to redistribute, as the sum of the costs each agent imposes on each element of a partition of the others. This is what makes these axioms and equity compatible in the particular queueing problems we study, not in others.[4]

We conjecture that in general public decision-making problems with additively separable preferences with respect to transfers, in which we can express the sum of welfare changes each agent imposes on the others likewise, this compatibility be possible. In particular, the appropriate generalization of the LEDPP rule should still satisfy these axioms, if not still be the only one. It is true in queueing problems, in which agents enjoy some benefit once they receive the service; in queueing problems, in which agents may differ in processing times; and in assignment problems of at most one indivisible good per agent, in which agents rank goods identically. (See Extensions.)

The allocation problems we study are thus among the few, in which *Pareto-*

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[3] As each of these costs is always greater or equal to zero, with equality holding only for the last agent in the queue, their sum is always strictly positive.

[4] In public decision-making problems with additively separable preferences with respect to transfers, *Pareto-efficiency of queues* and *strategy-proofness* imply transfers that need not sum to zero and hence, an incompatibility with *balancedness* (Green and Laffont, 1977). In queueing problems with linear preferences in transfers, combinatorial and independence conditions over the structure of waiting costs are necessary and sufficient for these axioms' compatibility (Mitra and Sen, 1998; Mitra, 2001, 2002). For instance, if preferences are also linear in positions, there are rules satisfying these axioms (Suijs, 1996; Mitra and Sen, 1998). The combinatorial condition requires each agent to always have her $(n-1)th$ order difference of waiting costs at the first position in the queue equal to zero, where $n$ is the number of agents. The independence condition requires that if an agent precedes another in a *Pareto-efficient* queue and a third agent leaves, the former should still precedes the other in a *Pareto-efficient* queue for the reduced problem.

*efficiency*, equity, and *strategy-proofness* are compatible. In general social choice problems, no equity axiom is compatible with *strategy-proofness* (Gibbard, 1973; Satterthwaite, 1975). If rules satisfy equity axioms and *strategy-proofness* in restricted classes of problems, they violate *Pareto-efficiency*.

The other exceptions are as follows. In allocation problems of a public good chosen in an interval, over which preferences are continuous and single-peaked, only the Generalized Condorcet rules satisfy *Pareto-efficiency*, *anonymity*, and *strategy-proofness* (Moulin, 1980).[5] In allocation problems of infinitely divisible goods, in which preferences are continuous and single-peaked, only the Uniform rule satisfies *Pareto-efficiency*, *equal treatment of equals in welfare*, and *strategy-proofness*, as well as *Pareto-efficiency*, *symmetry*, and *strategy-proofness* (Sprumont, 1991; Ching, 1994).[6] In allocation problems of infinitely divisible goods produced by a linear technology, only the Equal Income Walrasian rule satisfies *Pareto-efficiency*, *equal treatment of equals in welfare*, and *strategy-proofness* (Maniquet and Sprumont, 1999).

The allocation problems we study distinguish themselves from the former ones. When allocating a public good or an infinitely divisible good, we can give equal agents equal bundles. Then, *equal treatment of equals in welfare* is equivalent to *symmetry*. In queueing problems, as we cannot serve agents simultaneously, it is not the case. Then, efficiency, equity, and incentive compatibility simultaneously may still produce ties. A solution satisfying such criteria hence differs from the former ones. In particular, it should not *a priori* arbitrarily break these ties, instead allow multiple choices.

In Section 2, we formally introduce the model. In Section 3, we define the axioms we impose on rules. In Section 4, we identify the only rule satisfying these axioms. We prove it satisfies further axioms. In Section 5, we discuss extensions of the problems we study. Finally, we give concluding comments.

## 2  Model

There is a finite set of agents $N$. Let $n = |N|$. Each agent $i \in N$ may consume a *position* $\sigma_i \in \mathbb{N}$ in a queue and a positive or negative *transfer* $t_i \in \mathbb{R}$.

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[5] The Generalized Condorcet rule associated with a list of $(n-1)$ parameters in the interval, always selects the median of these parameters and the agents' peaks.
[6] The Uniform rule always selects its allocation as follows. If the sum of the agents' peaks is greater or equal to the amount of goods available, an agent receives her peak if it is smaller than a common bound, otherwise she receives this bound, chosen so the allocation is feasible. If the sum of the agents' peaks is less or equal to the amount of goods available, she receives her peak if it is greater than a common bound, otherwise she receives this bound, chosen so the allocation is feasible.

Preferences are linear in position and transfer. Let $c_i \in \mathbb{R}_{++}$ be the *unit waiting cost of* $i \in N$. If $i$ is served $\sigma_i$-th, her *total waiting cost* is $(\sigma_i - 1)c_i$. We can represent her preferences by the following function: for each $(\sigma_i, t_i) \in \mathbb{N} \times \mathbb{R}$, we have $u(\sigma_i, t_i; c_i) = -(\sigma_i - 1)c_i + t_i$. For each $(\sigma_i, t_i) \in \mathbb{N} \times \mathbb{R}$, we use the following notational shortcut. For $c_i \in \mathbb{R}_{++}$, let $u_i(\sigma_i, t_i) \equiv u(\sigma_i, t_i; c_i)$; for $c'_i \in \mathbb{R}_{++}$, let $u'_i(\sigma_i, t_i) \equiv u(\sigma_i, t_i; c'_i)$; and so on. A (queueing) *problem* is a list $c \equiv (c_i)_{i \in N} \in \mathbb{R}^N_{++}$.

An *allocation* is a pair $(\sigma, t) \equiv (\sigma_i, t_i)_{i \in N} \in (\mathbb{N} \times \mathbb{R})^N$ such that $(i)$ for each pair of agents, their positions in queue $\sigma$ differ, i.e. for each $\{i, j\} \subset N$ with $i \neq j$, we have $\sigma_i \neq \sigma_j$, and $(ii)$ the sum of transfers $t$ is non-positive, i.e. $\sum_{i \in N} t_i \leq 0$. Let $Z$ be the set of all allocations. An (allocation) *rule* $\varphi$ is a correspondence that associates with each problem $c \in \mathbb{R}^N_{++}$ a non-empty set of allocations $\varphi(c) \subseteq Z$.

We end this section with some more notation. For $S \subseteq N$, let $\overline{\Sigma}(S) \equiv \{(\sigma_i)_{i \in S} \in \mathbb{N}^S \mid$ for each $i, j \in S$ with $i \neq j$, we have $\sigma_i \neq \sigma_j\}$. For $\sigma \in \overline{\Sigma}(N)$ and $i \in N$, let $P_i(\sigma) \equiv \{j \in N \mid \sigma_j < \sigma_i\}$ be the set of all agents *preceding* $i$ in $\sigma$ and $F_i(\sigma) \equiv \{j \in N \mid \sigma_j > \sigma_i\}$ be the set of all agents *following* $i$ in $\sigma$. For $j \in N \backslash \{i\}$, let $B_{ij}(\sigma) \equiv \{l \in N \mid \min\{\sigma_i, \sigma_j\} < \sigma_l < \max\{\sigma_i, \sigma_j\}\}$ be the set of all agents served *between* $i$ and $j$ in $\sigma$. Let $\mathcal{Z}$ be the set of all non-empty sets of allocations. For $c_i \in \mathbb{R}_{++}$ and $X \in \mathcal{Z}$, let $\min u_i(X) \equiv \min_{(\sigma,t) \in X} u_i(\sigma_i, t_i)$, let $\arg\min u_i(X) \equiv \{(\sigma, t) \in X \mid u_i(\sigma_i, t_i) = \min u_i(X)\}$, and likewise for $\max u_i(X)$ and $\arg\max u_i(X)$. For $c \in \mathbb{R}^N_{++}$, let $c_{-i} \equiv (c_l)_{l \in N \backslash \{i\}}$ be the list of the unit waiting costs of $N \backslash \{i\}$.

## 3 Properties of rules

In this section, we define the axioms we impose on rules. Let $\varphi$ be a rule.

Efficiency is standard. There should be no allocation each agent finds at least as desirable as a selected allocation and at least one agent prefers. Formally,

**Pareto-efficiency:** *For each* $c \in \mathbb{R}^N_{++}$ *and each* $(\sigma, t) \in \varphi(c)$, *there is no* $(\sigma', t') \in Z$ *such that for each* $i \in N$, *we have* $u_i(\sigma'_i, t'_i) \geq u_i(\sigma_i, t_i)$ *and for at least one* $j \in N$, *we have* $u_j(\sigma'_j, t'_j) > u_j(\sigma_j, t_j)$.

Remark that $(\sigma, t) \in Z$ is *Pareto-efficient for* $c \in \mathbb{R}^N_{++}$ if and only if $(i)$ $\sigma$ is *Pareto-efficient for* $c$, i.e. for each $\sigma' \in \overline{\Sigma}(N)$, we have $\sum_{i \in N}(\sigma'_i - 1)c_i \geq \sum_{i \in N}(\sigma_i - 1)c_i$, and $(ii)$ $t$ is *balanced*, i.e. $\sum_{i \in N} t_i = 0$. Thus, queues minimize the agents' total waiting cost independently of transfers. Such queues are unique up to any permutation of agents with equal unit waiting costs following

6

one another. Let $\overline{\Sigma}^*(c)$ be the set of all *Pareto-efficient* queues for $c$. For each $c \in \mathbb{R}_{++}^N$ and each $(\sigma, t) \in Z$, we have $\sigma \in \overline{\Sigma}^*(c)$ if and only if for each $\{i, j\} \subset N$ with $i \neq j$, if $\sigma_i < \sigma_j$, then $c_i \geq c_j$. Following this remark, we decompose *Pareto-efficiency* into two axioms.

**Pareto-efficiency of queues:** *For each $c \in \mathbb{R}_{++}^N$ and each $(\sigma, t) \in \varphi(c)$, we have $\sigma \in \overline{\Sigma}^*(c)$.*

**Balancedness:** *For each $c \in \mathbb{R}_{++}^N$ and each $(\sigma, t) \in \varphi(c)$, we have $\sum_{i \in N} t_i = 0$.*

Equity requires to treat agents with equal unit waiting costs equally. As we cannot serve agents simultaneously, we cannot give two agents with equal unit waiting costs equal bundles. We require agents with equal unit waiting costs to have equal welfares and to be treated symmetrically, i.e. if there is an allocation different from a selected allocation in that we exchange the bundles of two of such agents and keep the bundles of the others unchanged, it should be selected. Formally,

**Equal treatment of equals in welfare:** *For each $c \in \mathbb{R}_{++}^N$, each $(\sigma, t) \in \varphi(c)$, and each $\{i, j\} \subset N$ with $i \neq j$ and $c_i = c_j$, we have $u_i(\sigma_i, t_i) = u_j(\sigma_j, t_j)$.*

**Symmetry:** *For each $c \in \mathbb{R}_{++}^N$, each $(\sigma, t) \in \varphi(c)$, and each $\{i, j\} \subset N$ with $i \neq j$ and $c_i = c_j$, if $(\sigma', t') \in Z$ such that $(\sigma'_i, t'_i) = (\sigma_j, t_j)$, $(\sigma'_j, t'_j) = (\sigma_i, t_i)$, and for each $l \in N \backslash \{i, j\}$, we have $(\sigma'_l, t'_l) = (\sigma_l, t_l)$, then $(\sigma', t') \in \varphi(c)$.*

Both axioms are important. In particular, axioms embodying further basic equity criteria imply them. *Equal treatment of equals in welfare* is necessary for no agent to prefer another agent's bundle to her own. *Symmetry* is necessary for agents' names not to matter. This second axiom requires that if we permute agents' unit waiting costs, we should permute the selected bundles accordingly. Formally, let $\Pi$ be the set of all permutations on $N$. For $\pi \in \Pi$ and $c \in \mathbb{R}^N$, let $\pi(c) \equiv (c_{\pi(i)})_{i \in N}$ and $\pi(\sigma, t) \equiv (\sigma_{\pi(i)}, t_{\pi(i)})_{i \in N}$.

**No-envy:** *For each $c \in \mathbb{R}_{++}^N$, each $(\sigma, t) \in \varphi(c)$, and each $i \in N$, there is no $j \in N \backslash \{i\}$ such that $u_i(\sigma_j, t_j) > u_i(\sigma_i, t_i)$.*

**Anonymity:** *For each $c \in \mathbb{R}_{++}^N$, each $(\sigma, t) \in \varphi(c)$, and each $\pi \in \Pi$, we have $\pi(\sigma, t) \in \varphi(\pi(c))$.*

*Single-valued* and non-*single-valued* rules may satisfy *equal treatment of equals in welfare*. Yet, no *single-valued* rule satisfies *symmetry*. Thus, because we

cannot serve agents simultaneously, we may impose *anonymity* if and only if we allow rules to select more than one allocation.

Strategic considerations motivate the last axiom. We may not know agents' unit waiting costs. As agents may behave strategically when announcing them, we may not attain efficiency nor equity. We require each agent to find revealing her unit waiting cost as desirable as misrepresenting it.

As rules may select more than one allocation, a strategic agent compares non-empty sets of allocations. It is crucial to determine this comparison's outcomes. The agent's preferences are defined on the set of positions and transfers. It is not immediate how these preferences and her preferences on the set of non-empty sets of allocations relate. Yet, selecting a multi-valued set of allocations is a first step. By definition, the allocations it contains are mutually exclusive. *In fine*, only one materializes. Thus, the comparison's outcomes depend on the agent's predictions regarding how, when a rule produces such a tie, it is broken, and on her aversion toward the uncertainty such a context involves.

Given the equity considerations guiding our analysis, we assume the tiebreaker to be at least a random device. We assume the agents know it. Besides, as allocations are mutually exclusive, we assume each agent's preferences on the set of non-empty sets of allocations *extend* her preferences on the set of allocations, in the sense that her singletons' ranking matches her original ranking.[7] Then, each plausible extension of an agent's preferences on the set of allocations to the set of non-empty sets of allocations is entirely determined by her ranking of these sets' best and worst elements (Kannai and Peleg, 1984).[8] Further, if the agent is neutral with respect to allocations' names, there is only one such extension that is as follows (Bossert, 1989).

An agent finds a non-empty set of allocations as desirable as another if and only if she finds (*i*) the worst bundle she may receive from the former as desirable as the worst bundle she may from the latter, and (*ii*) the worst

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[7] In any social choice problem with mutually exclusive alternatives, it is natural to assume that each agent's preferences on the set of non-empty sets of alternatives is an *extension* of her preferences on the set of alternatives. Then, from the many possible tiebreaker predictions and uncertainty aversions, there are many possible such extensions. See e.g. Fishburn (1972), Pattanaik (1973), Gärdenfors (1976, 1979), Barberá (1977), Kelly (1977), Feldman (1979), Duggan and Schwartz (2000), Barberá et al. (2001), and Ching and Zhou (2002).

[8] Plausible in the problem we study, as their result holds under the following assumptions. Preferences on the set of alternatives are an ordering on this set. They include a linear ordering on a subset of at least six alternatives. Preferences on the set of non-emtpy sets of alternatives is a quasi-ordering on this set. They may embody any attitude toward uncertainty, but extreme ones. They do not reveal complementarity type relationships between alternatives.

bundle she may receive from the former as desirable as the worst bundle she may from the latter. Formally, for $c_i \in \mathbb{R}_{++}$, let $R_i(c_i)$ be such that for each $X, X' \in \mathcal{Z}$, we have $X\ R_i(c_i)\ X'$ if and only if $\min u_i(X) \geq \min u_i(X')$ and $\max u_i(X) \geq \max u_i(X')$. Then,

**Strategy-proofness:** *For each $c \in \mathbb{R}_{++}^N$, each $i \in N$, and each $c_i' \in \mathbb{R}_{++}$, we have $\varphi(c)\ R_i(c_i)\ \varphi(c_i', c_{-i})$.*

We end this section with two remarks on this axiom. First, a *single-valued* rule satisfies it if and only if for each $c \in \mathbb{R}_{++}^N$, each $i \in N$, and each $c_i' \in \mathbb{R}_{++}$, if $(\sigma, t) = \varphi(c)$ and $(\sigma', t') = \varphi(c_i', c_{-i})$, then $u_i(\sigma_i, t_i) \geq u_i(\sigma_i', t_i')$.

Second, as e.g. in Pattanaik (1973), it requires each agent to find the worst bundle she may receive when revealing her unit waiting cost at least as desirable as the worst bundle she may receive when misrepresenting it. As e.g. in Feldman (1979), it requires each agent to find the best bundle she may receive when revealing her unit waiting cost at least as desirable as the best bundle she may receive when misrepresenting it. This second requirement is a necessary condition for further basic incentive compatibility criteria, in particular for rules to be implementable in undominated strategies by bounded mechanisms (Jackson, 1992) and for each agent not to find misrepresenting her unit waiting cost more desirable than revealing it via addition or deletion of allocations (Ching and Zhou, 2002). [9]

## 4 Characterizations

In this section, we identify the only rule satisfying the axioms we impose. Further, we prove that it satisfies *no-envy*, *anonymity*, and stronger incentive compatibility criteria. Indeed, it is such that each agent always finds any bundle she receives when revealing her unit waiting cost at least as desirable as any bundle she receives when misrepresenting it.

Therefore, we characterize the class of rules satisfying *Pareto-efficiency of queues* and *strategy-proofness* (Theorem 1). Then, we identify the class of rules satisfying *Pareto-efficiency*, *equal treatment of equals in welfare*, and *strategy-proofness* and we prove that these rules satisfy *no-envy* (Theorem 3, Statements 1 and 2). Finally, we prove that one of these rules is the only

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[9] Formally, an agent does not find misrepresenting her unit waiting cost more desirable as revealing it via addition or deletion of allocations if there is no $c \in \mathbb{R}_{++}^N$, $i \in N$, and $c_i' \in \mathbb{R}_{++}$ such that for $(\sigma, t) \in \varphi(c_i', c_{-i}) \backslash \varphi(c)$, we have $u_i(\sigma_i, t_i) > \min u_i(\varphi(c))$ or for $(\sigma, t) \in \varphi(c) \backslash \varphi(c_i', c_{-i})$, we have $\max u_i(\varphi(c_i', c_{-i})) > u_i(\sigma_i, t_i)$.

rule satisfying *Pareto-efficiency*, *symmetry*, and *strategy-proofness*, and that it satisfies *anonymity* (Theorem 3, Statements 3 and 4).

We begin by proving that a rule satisfies *Pareto-efficiency of queues* and *strategy-proofness* if and only if for each problem, it selects *Pareto-efficient* queues and it sets each agent's transfer in the spirit of Groves (1973). Indeed, for each bundle that maximizes or minimizes her utility, her transfer is equal to the others' total waiting cost plus an amount only depending on these agents' unit waiting costs.

The intuition is as follows. Assume that is not the case and that if an agent with some waiting cost announces some other waiting cost, the change in her utility would be greater (smaller) than the efficiency loss. Then, there would always be a waiting cost for which she should receive the same position and transfer as with the latter (former) waiting cost and for which announcing the former (latter) waiting cost would increase her utility. We apply this rationale first to the selected bundles that are the arguments of the utility maximum, then to those that are the arguments of the utility minimum. Yet, the proof is not symmetric. In the latter case, we must use a neighborhood argument.

Formally, let $D$ be the set of all correspondences that associate with each $c \in \mathbb{R}^N_{++}$ a non-empty set of *Pareto-efficient* queues for $c$, i.e. for each $d \in D$ and each $c \in \mathbb{R}^N_{++}$, we have $d(c) \neq \emptyset$ and $d(c) \subseteq \overline{\Sigma}^*(c)$. Let $H$ be the set of all lists of functions that associate with each $i \in N$ and each $c_{-i} \in \mathbb{R}^{N \setminus \{i\}}_{++}$ a real number, i.e. for each $(h_i)_{i \in N} \in H$, each $i \in N$, and each $c_{-i} \in \mathbb{R}^{N \setminus \{i\}}_{++}$, we have $h_i(c_{-i}) \in \mathbb{R}$. A rule $\varphi$ is a *Groves rule* if and only if there are $d \in D$ and $\{\underline{h}, \overline{h}\} \subset H$ such that for each $c \in \mathbb{R}^N_{++}$, (*i*) $d(c) \equiv \{\sigma \in \overline{\Sigma}(N) \mid$ there is $t \in \mathbb{R}^N$ such that $(\sigma, t) \in \varphi(c)\}$ and (*ii*) for each $i \in N$, if $(\sigma, t) \in \arg\min u_i(\varphi(c))$, then $t_i = -\sum_{l \in N \setminus \{i\}} (\sigma_l - 1) c_l + \underline{h}_i(c_{-i})$ and if $(\sigma, t) \in \arg\max u_i(\varphi(c))$, then $t_i = -\sum_{l \in N \setminus \{i\}} (\sigma_l - 1) c_l + \overline{h}_i(c_{-i})$.

**Theorem 1** *Let $\varphi$ be a rule. Then, $\varphi$ satisfies* Pareto-efficiency of queues *and* strategy-proofness *if and only if it is a Groves rule.*

### Proof.

**Part 1:** *If $\varphi$ is a Groves rule, then it satisfies* Pareto-efficiency of queues *and* strategy-proofness.

Pareto-efficiency of queues: *Assume that $\varphi$ is a Groves rule. By definition, there is $d \in D$ such that for each $c \in \mathbb{R}^N_{++}$, if $(\sigma, t) \in \varphi(c)$, then $\sigma \in d(c) \subseteq \overline{\Sigma}^*(c)$.*

Strategy-proofness: *Assume that $\varphi$ is a Groves rule. By definition, there is $d \in D$ such that (**i**) for each $c \in \mathbb{R}^N_{++}$, if $(\sigma, t) \in \varphi(c)$, then $\sigma \in d(c) \subseteq \overline{\Sigma}^*(c)$. Also, there is $\{\underline{h}, \overline{h}\} \subset H$ such that (**ii**) for each $c \in \mathbb{R}^N_{++}$ and each $i \in N$,*

10

- if $(\sigma, t) \in \arg\min u_i(\varphi(c))$, then $t_i = -\sum_{l \in N \setminus \{i\}} (\sigma_l - 1)c_l + \underline{h}_i(c_{-i})$;
- if $(\sigma, t) \in \arg\max u_i(\varphi(c))$, then $t_i = -\sum_{l \in N \setminus \{i\}} (\sigma_l - 1)c_l + \overline{h}_i(c_{-i})$.

Let $c \in \mathbb{R}^N_{++}$, $i \in N$, and $c'_i \in \mathbb{R}_{++}$. Then, the following holds.

(iii) *For each $(\sigma, t) \in \arg\min u_i(\varphi(c))$, we have $u_i(\sigma_i, t_i) \geq \min u_i(\varphi(c'_i, c_{-i}))$. Indeed, let $(\sigma, t) \in \arg\min u_i(\varphi(c))$ and $(\sigma', t') \in \arg\min u'_i(\varphi(c'_i, c_{-i}))$. By (i), $-\sum_{l \in N}(\sigma_l - 1)c_l \geq -\sum_{l \in N}(\sigma'_l - 1)c_l$. Thus, $-(\sigma_i - 1)c_i - \sum_{l \in N \setminus \{i\}}(\sigma_l - 1)c_l + \underline{h}_i(c_{-i}) \geq -(\sigma'_i - 1)c_i - \sum_{l \in N \setminus \{i\}}(\sigma'_l - 1)c_l + \underline{h}_i(c_{-i})$. By (ii), $u_i(\sigma_i, t_i) \geq u_i(\sigma'_i, t'_i)$. By assumption, $u_i(\sigma'_i, t'_i) \geq \min u_i(\varphi(c'_i, c_{-i}))$. Thus, $u_i(\sigma_i, t_i) \geq \min u_i(\varphi(c'_i, c_{-i}))$.*

(iv) *For each $(\sigma, t) \in \arg\max u_i(\varphi(c))$, we have $u_i(\sigma_i, t_i) \geq \max u_i(\varphi(c'_i, c_{-i}))$. Indeed, let $(\sigma, t) \in \arg\max u_i(\varphi(c))$ and $(\sigma', t') \in \arg\max u'_i(\varphi(c'_i, c_{-i}))$. Let $(\tilde{\sigma}, \tilde{t}) \in \arg\max u_i(\varphi(c'_i, c_{-i}))$ and $g_i(c'_i, c_{-i}) \in \mathbb{R}$ be such that $\tilde{t}_i = -\sum_{l \in N \setminus \{i\}}(\tilde{\sigma}_l - 1)c_l + g_i(c'_i, c_{-i})$. By assumpion, $u'_i(\sigma'_i, t'_i) \geq u'_i(\tilde{\sigma}_i, \tilde{t}_i)$. By (ii), $-(\sigma'_i - 1)c'_i - \sum_{l \in N \setminus \{i\}}(\sigma'_l - 1)c_l + \overline{h}_i(c_{-i}) \geq -(\tilde{\sigma}_i - 1)c'_i - \sum_{l \in N \setminus \{i\}}(\tilde{\sigma}_l - 1)c_l + g_i(c'_i, c_{-i})$. By (i), $-(\sigma'_i - 1)c'_i - \sum_{l \in N \setminus \{i\}}(\sigma'_l - 1)c_l = -(\tilde{\sigma}_i - 1)c'_i - \sum_{l \in N \setminus \{i\}}(\tilde{\sigma}_l - 1)c_l$. Thus, $\overline{h}_i(c_{-i}) \geq g_i(c'_i, c_{-i})$. By (i), $-\sum_{l \in N}(\sigma_l - 1)c_l \geq -\sum_{l \in N}(\tilde{\sigma}_l - 1)c_l$. Thus, $-(\sigma_i - 1)c_i - \sum_{l \in N \setminus \{i\}}(\sigma_l - 1)c_l + \overline{h}_i(c_{-i}) \geq -(\tilde{\sigma}_i - 1)c_i - \sum_{l \in N \setminus \{i\}}(\tilde{\sigma}_l - 1)c_l + g_i(c'_i, c_{-i})$. By (ii) and as $\tilde{t}_i = -\sum_{l \in N}(\tilde{\sigma}_l - 1)c_l + g_i(c'_i, c_{-i})$, we have $u_i(\sigma_i, t_i) \geq u_i(\tilde{\sigma}_i, \tilde{t}_i)$. Thus, $u_i(\sigma_i, t_i) \geq \max u_i(\varphi(c'_i, c_{-i}))$.*

By (iii) and (iv), $\varphi(c) \; R_i(c_i) \; \varphi(c'_i, c_{-i})$.

**Part 2:** *If $\varphi$ satisfies* Pareto-efficiency of queues *and* strategy-proofness, *then it is a Groves rule.*

*Assume that $\varphi$ satisfies* Pareto-efficiency of queues *and* strategy-proofness. *By* Pareto-efficiency of queues, *for each $c \in \mathbb{R}^N_{++}$, if $(\sigma, t) \in \varphi(c)$, then $\sigma \in \overline{\Sigma}^*(c)$. Thus, there is $d \in D$ such that for each $c \in \mathbb{R}^N_{++}$, we have $d(c) \equiv \{\sigma \in \overline{\Sigma}(N) \mid \text{there is } t \in \mathbb{R} \text{ such that } (\sigma, t) \in \varphi(c)\}$. In Claims 1 and 2, we prove that* Pareto-efficiency of queues *and* strategy-proofness *imply that there is $\{\underline{h}, \overline{h}\} \subset H$ such that for each $i \in N$,*
- if $(\sigma, t) \in \arg\min u_i(\varphi(c))$, then $t_i = -\sum_{l \in N \setminus \{i\}}(\sigma_l - 1)c_l + \underline{h}_i(c_{-i})$;
- if $(\sigma, t) \in \arg\max u_i(\varphi(c))$, then $t_i = -\sum_{l \in N \setminus \{i\}}(\sigma_l - 1)c_l + \overline{h}_i(c_{-i})$.

**Claim 1:** *There is $\overline{h} \in H$ such that for each $c \in \mathbb{R}^N_{++}$ and each $i \in N$, if $(\sigma, t) \in \arg\max u_i(\varphi(c))$, then $t_i = -\sum_{l \in N \setminus \{i\}}(\sigma_l - 1)c_l + \overline{h}_i(c_{-i})$.*

*Let $(\overline{g}_i : \mathbb{R}^N_{++} \to \mathbb{R})_{i \in N}$ be a list of real-valued functions such that (i) for each $c \in \mathbb{R}^N_{++}$ and each $i \in N$, if $(\sigma, t) \in \arg\max u_i(\varphi(c))$, then $t_i = -\sum_{l \in N \setminus \{i\}}(\sigma_l - 1)c_l + \overline{g}_i(c)$. By contradiction, assume that (ii) for $c \in \mathbb{R}^N_{++}$, $i \in N$, and $c'_i \in \mathbb{R}$, we have $\Delta \equiv \overline{g}_i(c) - \overline{g}_i(c'_i, c_{-i}) > 0$. (The symmetric case is immediate.) Let $(\sigma, t) \in \arg\max u_i(\varphi(c))$ and $(\sigma', t') \in \arg\max u'_i(\varphi(c'_i, c_{-i}))$.*

*By* strategy-proofness, *$u_i(\sigma_i, t_i) \geq \max u_i(\varphi(c'_i, c_{-i}))$ and $\max u'_i(\varphi(c)) \leq u'_i(\sigma'_i, t'_i)$. By assumption, $\max u_i(\varphi(c'_i, c_{-i})) \geq u_i(\sigma'_i, t'_i)$ and $u'_i(\sigma_i, t_i) \leq$*

11

$\max u'_i(\varphi(c))$. Thus, $u_i(\sigma_i, t_i) \geq u_i(\sigma'_i, t'_i)$ and $u'_i(\sigma_i, t_i) \leq u'_i(\sigma'_i, t'_i)$. By **(i)**, $-(\sigma_i - 1)c_i - \sum_{l \in N\setminus\{i\}}(\sigma_l - 1)c_l + \overline{g}_i(c) \geq -(\sigma'_i - 1)c_i - \sum_{l \in N\setminus\{i\}}(\sigma'_l - 1)c_l + \overline{g}_i(c'_i, c_{-i})$ and $-(\sigma_i - 1)c'_i - \sum_{l \in N\setminus\{i\}}(\sigma_l - 1)c_l + \overline{g}_i(c) \leq -(\sigma'_i - 1)c'_i - \sum_{l \in N\setminus\{i\}}(\sigma'_l - 1)c_l + \overline{g}_i(c'_i, c_{-i})$. Thus,

**(iii)** $(\sigma_i - \sigma'_i)c'_i + \sum_{l \in N\setminus\{i\}}(\sigma_l - \sigma'_l)c_l \geq \Delta \geq (\sigma_i - \sigma'_i)c_i + \sum_{l \in N\setminus\{i\}}(\sigma_l - \sigma'_l)c_l$.

By Pareto-efficiency of queues, *for each* $S \subseteq N$, *if for each* $\{k, k'\} \subset S$ *with* $k \neq k'$, *we have* $c_k = c_{k'}$ *and there is no* $k'' \in N\setminus S$ *such that* $k'' \in B_{kk'}(\sigma) \cup B_{kk'}(\sigma')$, *then* $-\sum_{l \in S}(\sigma_l - 1)c_l = -\sum_{l \in S}(\sigma'_l - 1)c_l$. *Also, there is* $j \in N$ *such that* $\sigma_j = \sigma'_j$. *Let* $sign : \mathbb{R} \to \{-1, 0, 1\}$ *be such that for each* $a \in \mathbb{R}$, $sign(a) = -1$ *if and only if* $a < 0$, $sign(a) = 0$ *if and only if* $a = 0$, *and* $sign(a) = 1$ *if and only if* $a > 0$. *Then,*

**(iv)** $\sum_{l \in N\setminus\{i\}}(\sigma_l - \sigma'_l)c_l = -sign(\sigma_i - \sigma'_i)(\sum_{l \in B_{ij}(\sigma)} c_l + c_j)$.

*Assume* $c'_i > c_i$. *(The symmetric case is immediate.)* By Pareto-efficiency of queues, $\sigma_i - \sigma'_i \geq 0$ *and* $c_j \geq c_i$. *Distinguish three cases.*

**Case 1:** $(\sigma_i - \sigma'_i) = 0$. *By* **(iii)** *and* **(iv)**, $\Delta = 0$, *contradicting* **(ii)**.

**Case 2:** $(\sigma_i - \sigma'_i) = 1$. *By* **(ii)**, **(iii)**, **(iv)**, *and* Pareto-efficiency of queues, **(v)** $c'_i - c_j \geq \Delta > 0 \geq c_i - c_j$. *Let* $c''_i \in \mathbb{R}_{++}$ *be such that* **(vi)** $\Delta > c''_i - c_j > 0$. *Let* $(\sigma'', t'') \in \arg\max u''_i(\varphi(c''_i, c_{-i}))$. *By* **(v)** *and* **(vi)**, $c'_i > c''_i > c_j$. *By* Pareto-efficiency of queues, $\sigma''_i = \sigma'_i$. *Thus,* **(vii)** $\sigma_i - \sigma''_i = 1$ *and by Case 1's logic,*[10] $\overline{g}_i(c''_i, c_{-i}) = \overline{g}_i(c'_i, c_{-i})$. *Together,* **(vi)** *and* **(vii)** *imply* $\overline{g}_i(c) - \overline{g}_i(c''_i, c_{-i}) > (\sigma_i - \sigma''_i)c''_i - sign(\sigma_i - \sigma''_i)(\sum_{l \in B_{ij}(\sigma)} c_l + c_j)$. *Thus,* $-(\sigma_i - 1)c''_i - \sum_{l \in N\setminus\{i\}}(\sigma_l - 1)c_l + \overline{g}_i(c) > -(\sigma''_i - 1)c''_i - \sum_{l \in N\setminus\{i\}}(\sigma''_l - 1)c_l + \overline{g}_i(c''_i, c_{-i})$. *By* **(i)**, $u''_i(\sigma_i, t_i) > u''_i(\sigma'', t'')$. *By assumption,* $\max u''_i(\varphi(c)) \geq u''_i(\sigma_i, t_i)$. *Thus,* $\max u''_i(\varphi(c)) > u''_i(\sigma'', t'')$, *contradicting* strategy-proofness.

**Case 3:** $(\sigma_i - \sigma'_i) > 1$. *Let* $\delta \in \mathbb{N}$ *be such that* $\sigma_i - \sigma'_i = \delta$. *For each* $s \in \{1, 2, ..., \delta - 1\}$, *let* $c^s_i \in \mathbb{R}_{++}$ *and* $(\sigma^s, t^s) \in \arg\max u^s_i(\varphi(c^s_i, c_{-i}))$ *be such that* $\sigma^s_i = \sigma'_i + s$. *By* Case 2's logic,[11] $\overline{g}_i(c^1_i, c_{-i}) = \overline{g}_i(c'_i, c_{-i})$, $\overline{g}_i(c^2_i, c_{-i}) = $

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[10] By *strategy-proofness*, $u'_i(\sigma'_i, t'_i) \geq \max u'_i(\varphi(c''_i, c_{-i}))$ and $\max u''_i(\varphi(c'_i, c_{-i})) \leq u''_i(\sigma''_i, t''_i)$. By assumption, $\max u'_i(\varphi(c''_i, c_{-i})) \geq u'_i(\sigma''_i, t''_i)$ and $u''_i(\sigma'_i, t'_i) \leq \max u''_i(\varphi(c'_i, c_{-i}))$. Thus, $u'_i(\sigma'_i, t'_i) \geq u'_i(\sigma''_i, t''_i)$ and $u''_i(\sigma'_i, t'_i) \leq u''_i(\sigma''_i, t''_i)$.

[11] By *strategy-proofness*, $u^1_i(\sigma^1_i, t^1_i) \geq \max u^1_i(\varphi(c'_i, c_{-i}))$ and $\max u'_i(\varphi(c^1_i, c_{-i})) \leq u'_i(\sigma'_i, t'_i)$; for each $s \in \{2, ..., \delta - 1\}$, we have $u^s_i(\sigma^s_i, t^s_i) \geq \max u^s_i(\varphi(c^{s-1}_i, c_{-i}))$ and $\max u^{s-1}_i(\varphi(c^s_i, c_{-i})) \leq u^{s-1}_i(\sigma^{s-1}_i, t^{s-1}_i)$; ...; and $u_i(\sigma_i, t_i) \geq \max u_i(\varphi(c^{\delta-1}_i, c_{-i}))$ and $\max u^{\delta-1}_i(\varphi(c)) \leq u^{\delta-1}_i(\sigma^{\delta-1}_i, t^{\delta-1}_i)$. By assumption, $\max u^1_i(\varphi(c'_i, c_{-i})) \geq u^1_i(\sigma'_i, t'_i)$ and $u'_i(\sigma^1_i, t^1_i) \leq \max u'_i(\varphi(c^1_i, c_{-i}))$; for each $s \in \{2, ..., \delta - 1\}$, we have $\max u^s_i(\varphi(c^{s-1}_i, c_{-i})) \geq u^s_i(\sigma^{s-1}_i, t^{s-1}_i)$ and $u^{s-1}_i(\sigma^s_i, t^s_i) \leq \max^{s-1}_i(\varphi(c^s_i, c_{-i}))$; ...; and $\max u_i(\varphi(c^{\delta-1}_i, c_{-i})) \geq u_i(\sigma^{\delta-1}_i, t^{\delta-1}_i)$ and $u^{\delta-1}_i(\sigma_i, t_i) \leq \max u^{\delta-1}_i(\varphi(c))$. Thus, $u^1_i(\sigma^1_i, t^1_i) \geq u^1_i(\sigma'_i, t'_i)$ and $u'_i(\sigma^1_i, t^1_i) \leq u'_i(\sigma'_i, t'_i)$; for each $s \in \{2, ..., \delta - 1\}$, we have $u^s_i(\sigma^s_i, t^s_i) \geq u^s_i(\sigma^{s-1}_i, t^{s-1}_i)$ and $u^{s-1}_i(\sigma^s_i, t^s_i) \leq u^{s-1}_i(\sigma^{s-1}_i, t^{s-1}_i)$; ...; and $u_i(\sigma_i, t_i) \geq u_i(\sigma^{\delta-1}_i, t^{\delta-1}_i)$ and $u^{\delta-1}_i(\sigma_i, t_i) \leq u^{\delta-1}_i(\sigma^{\delta-1}_i, t^{\delta-1}_i)$.

$\overline{g}_i(c_i^1, c_{-i})$, ..., $\overline{g}_i(c) = \overline{g}_i(c_i^{\delta-1}, c_{-i})$. Thus, $\overline{g}_i(c) = \overline{g}_i(c_i', c_{-i})$, *contradicting* **(ii)**.

**Claim 2:** *There is $\underline{h} \in H$ such that for each $c \in \mathbb{R}_{++}^N$ and each $i \in N$, if $(\sigma, t) \in \arg\min u_i(\varphi(c))$, then $t_i = -\sum_{l \in N \setminus \{i\}} (\sigma_l - 1) c_l + \underline{h}_i(c_{-i})$.*

*Let $(\underline{g}_i : \mathbb{R}_{++}^N \to \mathbb{R})_{i \in N}$ be a list of real-valued functions such that for each $c \in \mathbb{R}_{++}^N$ and each $i \in N$, if $(\sigma, t) \in \arg\min u_i(\varphi(c))$, then $t_i = -\sum_{l \in N \setminus \{i\}} (\sigma_l - 1) c_l + \underline{g}_i(c)$. By contradiction, assume that for $c \in \mathbb{R}_{++}^N$, $i \in N$, and $c_i' \in \mathbb{R}$, we have $\underline{g}_i(c) - \underline{g}_i(c_i', c_{-i}) \neq 0$. Let $(\sigma, t) \in \arg\min u_i(\varphi(c))$ and $(\sigma', t') \in \arg\min u_i'(\varphi(c_i', c_{-i}))$. Then, the following holds.*

(**i**) *If $u_i(\sigma_i, t_i) \geq u_i(\sigma_i', t_i')$ and $u_i'(\sigma_i, t_i) > u_i'(\sigma_i', t_i')$, or $u_i(\sigma_i, t_i) < u_i(\sigma_i', t_i')$ and $u_i'(\sigma_i, t_i) \leq u_i'(\sigma_i', t_i')$, then for $c_i'', c_i''' \in \mathbb{R}_{++}$ and sufficiently small $\epsilon, \epsilon' \in \mathbb{R}$ such that $c_i'' = c_i + \epsilon$ and $c_i''' = c_i' + \epsilon'$ and for each $(\sigma'', t'') \in \arg\min u_i''(\varphi(c_i'', c_{-i}))$ and $(\sigma''', t''') \in \arg\min u_i'''(\varphi(c_i''', c_{-i}))$, we have $u_i''(\sigma_i'', t_i'') \geq u_i''(\sigma_i''', t_i''')$ and $u_i'''(\sigma_i'', t_i'') \leq u_i'''(\sigma_i''', t_i''')$. Indeed, assume $u_i(\sigma_i, t_i) \geq u_i(\sigma_i', t_i')$ and $u_i'(\sigma_i, t_i) > u_i'(\sigma_i', t_i')$, or $u_i(\sigma_i, t_i) < u_i(\sigma_i', t_i')$ and $u_i'(\sigma_i, t_i) \leq u_i'(\sigma_i', t_i')$. Let $c_i'', c_i''' \in \mathbb{R}_{++}$ and sufficiently small $\epsilon, \epsilon' \in \mathbb{R}$ be such that $c_i'' = c_i + \epsilon$ and $c_i''' = c_i' + \epsilon'$. Let $(\sigma'', t'') \in \arg\min u_i''(\varphi(c_i'', c_{-i}))$ and $(\sigma''', t''') \in \arg\min u_i'''(\varphi(c_i''', c_{-i}))$. By strategy-proofness, $u_i''(\sigma_i'', t_i'') \geq \min u_i''(\varphi(c_i''', c_{-i}))$ and $\min u_i'''(\varphi(c_i'', c_{-i})) \leq u_i'''(\sigma_i''', t_i''')$. By Pareto-efficiency of queues, for each $(\tilde{\sigma}, \tilde{t}), (\tilde{\sigma}', \tilde{t}') \in \varphi(c_i'', c_{-i})$, we have $\tilde{\sigma}_i = \tilde{\sigma}_i'$. Also, for each $(\tilde{\sigma}, \tilde{t}), (\tilde{\sigma}', \tilde{t}') \in \varphi(c_i''', c_{-i})$, we have $\tilde{\sigma}_i = \tilde{\sigma}_i'$. Thus, $\arg\min u_i''(\varphi(c_i''', c_{-i})) = \arg\min u_i'''(\varphi(c_i''', c_{-i}))$ and $\arg\min u_i'''(\varphi(c_i'', c_{-i})) = \arg\min u_i''(\varphi(c_i'', c_{-i}))$. Thus, $u_i''(\sigma_i'', t_i'') \geq u_i''(\sigma_i''', t_i''')$ and $u_i'''(\sigma_i'', t_i'') \leq u_i'''(\sigma_i''', t_i''')$.*

(**ii**) *For each $c_i'', c_i''' \in \mathbb{R}_{++}$, if for sufficiently small $\epsilon, \epsilon' \in \mathbb{R}$, we have $c_i'' = c_i + \epsilon$ and $c_i''' = c_i' + \epsilon'$, then $\underline{g}_i(c) = \underline{g}_i(c_i'', c_{-i})$ and $\underline{g}_i(c_i', c_{-i}) = \underline{g}_i(c_i''', c_{-i})$. Indeed, let $c_i'' \in \mathbb{R}_{++}$ be such that for sufficiently small $\epsilon \in \mathbb{R}$, we have $c_i'' = c_i + \epsilon$. By contradiction, assume $\underline{g}_i(c) - \underline{g}_i(c_i'', c_{-i}) \neq 0$. (The other case is immediate.) Let $(\sigma, t) \in \arg\min u_i(\varphi(c))$ and $(\sigma'', t'') \in \arg\min u_i''(\varphi(c_i'', c_{-i}))$. By strategy-proofness, $u_i(\sigma_i, t_i) \geq \min u_i(\varphi(c_i'', c_{-i}))$ and $\min u_i''(\varphi(c)) \leq u_i''(\sigma_i'', t_i'')$. As $c_i'' = c_i + \epsilon$, we have $\arg\min u_i(\varphi(c_i'', c_{-i})) = \arg\min u_i''(\varphi(c_i'', c_{-i}))$ and $\arg\min u_i''(\varphi(c)) = \arg\min u_i(\varphi(c))$. Thus, $u_i(\sigma_i, t_i) \geq u_i(\sigma_i'', t_i'')$ and $u_i'(\sigma_i, t_i) \leq u_i''(\sigma_i'', t_i'')$. By Claim 1's logic, we obtain a contradiction. [12]*

*By (**i**) and (**ii**), w.l.o.g. assume $u_i(\sigma_i, t_i) \geq u_i(\sigma_i', t_i')$ and $u_i'(\sigma_i, t_i) \leq u_i'(\sigma_i', t_i')$. By Claim 1's logic, we obtain a contradiction. [12]* ∎

**Nota 1** *By Theorem 1, a* single-valued *rule satisfies* Pareto-efficiency of

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[12] Claim 2's extended proof, using at this point Claim 1's logic, is available to the reader upon request to us.

queues *and* strategy-proofness *if and only if it is a* single-valued *Groves rule. As preferences are convex, this corrolary follows from Holmström's result (1979). Thus, in Theorem 1, we prove that even in the more general case, in which one does not impose* single-valuedness, *only Groves rules satisfy these axioms. This characterization holds on many other domains.*

*Indeed, for* single-valued *rules, the main argument is that for any pair of the privately known parameters, there is a preference reversal on the alternatives selected, one for each parameter. Thus, if the domain is rich enough, there always is another parameter such that the same alternatives are selected, one for this parameter, one for one of the former parameters, and there is no preference reversal (Holmström, 1979). For non-*single-valued *rules, the existence of such preference reversals is not guaranteed. We prove that in the cases where for some pair of parameters, it is not guaranteed, if the domain is rich enough, there always is another pair of parameters such that the same alternatives are selected, and there is such a preference reversal.*

The class of Groves rules is large. We may distinguish subclasses. In particular, a Groves rule associated with a unique $h \in H$ is such that for each $c \in \mathbb{R}^N_{++}$, if $(\sigma, t)$ is selected, then for each $i \in N$, we have $t_i = -\sum_{l \in N \setminus \{i\}} (\sigma_l - 1)c_l + h_i(c_{-i})$.

For instance, a *Pivotal rule* selects for each problem, *Pareto-efficient* queues and sets transfers such that each agent pays what she imposes on the others if she is pivotal on the choice of the queue, and nothing otherwise (Clarke, 1971). Each agent is always pivotal on the choice of a *Pareto-efficient* queue. Indeed, for $\sigma \in \overline{\Sigma}(N)$ and $i \in N$, let $\sigma^{-i} \in \overline{\Sigma}(N \setminus \{i\})$ be such that for each $l \in P_i(\sigma)$, we have $\sigma_l^{-i} = \sigma_l$ and for each $l \in F_i(\sigma)$, we have $\sigma_l^{-i} = \sigma_l - 1$. For each $c \in \mathbb{R}^N_{++}$, each $\sigma \in \overline{\Sigma}^*(c)$, each $S \subseteq N$, the *cost that $i$ imposes on $S$* is $(\sum_{l \in S}(\sigma_l - 1)c_l) - (\sum_{l \in S \setminus \{i\}}(\sigma_l^{-i} - 1)c_l) = \sum_{l \in S \cap F_i(\sigma)} c_l$. Thus, the cost an agent imposes on the others is always equal to the sum of the costs her followers incur by waiting one more unit of time. Formally, Pivotal rules are Groves rules associated with $h \in H$ such that for each $c \in \mathbb{R}^N_{++}$, if $(\sigma, t)$ is selected, then for each $i \in N$, we have $h_i(c_{-i}) = \sum_{l \in N \setminus \{i\}} (\sigma_l^{-i} - 1)c_l$.

By Theorem 1, a rule satisfies *Pareto-efficiency* and *strategy-proofness* if and only if it is a Groves rules and it satisfies *balancedness*. Yet, in two-agent problems, no Groves rule satisfies *balancedness* (Suijs, 1996). [13] Thus, from now on, we focus on problems with more than two agents.

We introduce a new rule. For each problem, it selects all *Pareto-efficient* queues and sets transfers considering each pair of agents in turn, making each

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[13] Suijs (1996) proves this impossibility for *single-valued* rules. One can prove it for any rules using Suijs' result's logic, Theorem 1, and the fact that if the agents unit costs differ, by *balancedness*, a bundle maximizes one agent's utility if and only if the corresponding bundle minimizes the other's.

agent in the pair pay what a Pivotal rule recommends for the reduced problem consisting of these two agents, and distributing the sum of these two payments equally among the others. For each problem and each selected *Pareto-efficient* queue, each agent pays the cost she imposes on each pair of agents she could be in, and she receives $(1/(n-2))th$ of the cost each other agent imposes on each pair of agents this agent is and she is not. Thus, each agent pays the unit waiting costs of her followers, and she receives $(1/(n-2))th$ of the unit waiting costs of each other agent's followers, but her. Formally,

**The Largest Equally Distributed Pairwise Pivotal (LEDPP) rule,** $\varphi^{\mathbf{L}}$**:** For each $c \in \mathbb{R}^N_{++}$, we have $(\sigma, t) \in \varphi^L(c)$ if and only if $\sigma \in \overline{\Sigma}^*(c)$ and for each $i \in N$, we have $t_i = -\sum_{l \in F_i(\sigma)} c_l + \frac{1}{(n-2)} \sum_{j \in N \setminus \{i\}} \sum_{l \in F_j(\sigma^{-i})} c_l$.

As there may be several *Pareto-efficient* queues for a problem, this rule has several subcorrespondences. We refer to these and to the LEDPP rule as *Equally Distributed Pairwise Pivotal (EDPP) rules*.

We now prove that for each problem and each selected *Pareto-efficient* queue, we obtain the transfers of the LEDPP rule, or of any other EDPP rule, in three other ways. First, making each agent pay what a Pivotal rule recommends for the problem and giving each agent $(1/(n-2))th$ of the others' minimum total waiting cost if she was not there. Second, giving each agent $(1/(n-2))th$ of her predecessors' total waiting cost and making each agent pay $(1/(n-2))th$ of her followers' waiting gain from not being last in the queue (Mitra and Sen, 1998; Mitra, 2001). Third, giving each agent $1/2$ of her predecessors' unit waiting cost and making each agent pay $1/2$ of her followers' unit waiting cost plus $(1/2(n-2))th$ of the difference in the unit waiting costs of each agent, but her and each of this agent's predecessors's, but her (Suijs, 1996).

**Theorem 2** *Let $\varphi$ be a rule. The following statements are equivalent.*

*(1) $\varphi$ is an Equally Distributed Pairwise Pivotal rule.*

*(2) $\varphi$ is a Groves rule associated with $h \in H$ such that for each $c \in \mathbb{R}^N_{++}$, if $(\sigma, t) \in \varphi(c)$, then for each $i \in N$, we have $h_i(c_{-i}) = \sum_{l \in N \setminus \{i\}} (\sigma_l^{-i} - 1)c_l + \frac{1}{(n-2)} \sum_{l \in N \setminus \{i\}} (\sigma_l^{-i} - 1)c_l$.*

*(3) $\varphi$ is such that for each $c \in \mathbb{R}^N_{++}$, if $(\sigma, t) \in \varphi(c)$, then $\sigma \in \overline{\Sigma}^*(c)$ and for each $i \in N$, we have $t_i = \sum_{l \in P_i(\sigma)} \frac{(\sigma_l - 1)}{(n-2)} c_l - \sum_{l \in F_i(\sigma)} \frac{(n - \sigma_l)}{(n-2)} c_l$.*

*(4) $\varphi$ is such that for each $c \in \mathbb{R}^N_{++}$, if $(\sigma, t) \in \varphi(c)$, then $\sigma \in \overline{\Sigma}^*(c)$ and for each $i \in N$, we have $t_i = \sum_{l \in P_i(\sigma)} \frac{c_l}{2} - \sum_{l \in F_i(\sigma)} \frac{c_l}{2} - \sum_{l \in N \setminus \{i\}} \sum_{k \in P_l(\sigma) \setminus \{i\}} \frac{c_k - c_l}{2(n-2)}$.*

**Proof.** *Assume that $\varphi$ is as in Statement 1. Let $c \in \mathbb{R}^N_{++}$, $(\sigma, t) \in \varphi(c)$, and $i \in N$. Then,*

$t_i = -\sum_{l \in F_i(\sigma)} c_l + \frac{1}{(n-2)} \sum_{j \in N \setminus \{i\}} \sum_{l \in F_j(\sigma^{-i})} c_l$  *(1.)*

15

$$= -\sum_{l \in F_i(\sigma)} c_l + \frac{1}{(n-2)} \sum_{l \in N \setminus \{i\}} (\sigma_l^{-i} - 1) c_l$$

$$= -\sum_{l \in N \setminus \{i\}} (\sigma_l - 1) c_l + \sum_{l \in N \setminus \{i\}} (\sigma_l^{-i} - 1) c_l + \frac{1}{(n-2)} \sum_{l \in N \setminus \{i\}} (\sigma_l^{-i} - 1) c_l \quad (2.)$$

$$= -\sum_{l \in F_i \sigma} c_l + \frac{1}{n-2} \sum_{l \in P_i(\sigma)} (\sigma_l - 1) c_l + \frac{1}{(n-2)} \sum_{l \in F_i(\sigma)} (\sigma_l - 2) c_l$$

$$= \sum_{l \in P_i(\sigma)} \frac{(\sigma_l - 1)}{(n-2)} c_l + \sum_{l \in F_i(\sigma)} \frac{(\sigma_l - 2) - (n-2)}{(n-2)} c_l$$

$$= \sum_{l \in P_i(\sigma)} \frac{(\sigma_l - 1)}{(n-2)} c_l - \sum_{l \in F_i(\sigma)} \frac{(n - \sigma_l)}{(n-2)} c_l \quad (3.)$$

$$= \sum_{l \in P_i(\sigma)} \frac{c_l}{2} - \sum_{l \in F_i(\sigma)} \frac{c_l}{2} - \sum_{l \in N \setminus i} \frac{(n - 2\sigma_l) c_l}{2(n-2)}$$

$$= \sum_{l \in P_i(\sigma)} \frac{c_l}{2} - \sum_{l \in F_i(\sigma)} \frac{c_l}{2} - \left[ \sum_{l \in N \setminus \{i\}} \frac{(n - \sigma_l - 1) c_l}{2(n-2)} - \sum_{l \in N \setminus \{i\}} \frac{(\sigma_l - 1) c_l}{2(n-2)} \right]$$

$$= \sum_{l \in P_i(\sigma)} \frac{c_l}{2} - \sum_{l \in F_i(\sigma)} \frac{c_l}{2} - \sum_{l \in N \setminus \{i\}} \sum_{k \in P_l(\sigma) \setminus \{i\}} \frac{c_k - c_l}{2(n-2)}. \quad (4.) \; \blacksquare$$

We now come to our central result. We prove that only EDPP rules satisfy *Pareto-efficiency*, *equal treatment of equals in welfare*, and *strategy-proofness*. They satisfy *no-envy*. Only the LEDPP rule satisfies *Pareto-efficiency*, *symmetry*, and *strategy-proofness*. It satisfies *anonymity*. Thus, as *no-envy* implies *equal treatment of equals in welfare* and *anonymity* implies *symmetry*, we prove that only the LEDPP rule satisfies the axioms we impose.

This central result holds for variants of the incentive compatibility property we impose. First, in Statement 2's proof, we prove that any EDPP rule is such that each agent always finds any bundle she may receive when revealing her unit waiting cost at least as desirable as any bundle she may receive when misrepresenting it. Second, in Statement 1's proof, we prove that if a rule satisfies *Pareto-efficiency*, *equal treatment of equals in welfare*, and is such that each agent always finds the worst bundle she may receive when revealing her unit waiting cost at least desirable as the worst bundle she may receive when misrepresenting it, it is an EDPP rule. The same holds, considering best bundles only. Thus, our result holds for stronger, as well as weaker incentive criteria, embodying extreme uncertainty aversions.

**Theorem 3** *Let $\varphi$ be a rule.*

*(1) If $\varphi$ satisfies* Pareto-efficiency, equal treatment of equals in welfare, *and* strategy-proofness, *then it is a subcorrespondence of the Largest Equally Distributed Pairwise Pivotal rule.*

*(2) If $\varphi$ is a subcorrespondence of the Largest Equally Distributed Pairwise Pivotal rule, then it satisfies* Pareto-efficiency, no-envy, *and* strategy-proofness.

*(3) If $\varphi$ satisfies* Pareto-efficiency, symmetry, *and* strategy-proofness, *then it is the Largest Equally Distributed Pairwise Pivotal rule.*

*(4) If $\varphi$ is the Largest Equally Distributed Pairwise Pivotal rule, then it satisfies* Pareto-efficiency, anonymity, *and* strategy-proofness.

**Proof.**

**Statement 1:** *Assume that $\varphi$ satisfies the axioms of Theorem 3.1. Let $c \in \mathbb{R}^N_{++}$. By Pareto-efficiency, for each $(\sigma, t) \in \varphi(c)$, we have $\sigma \in \overline{\Sigma}^*(c)$. By Theorem 1,* Pareto-efficiency *and* strategy-proofness *imply that there is $\{\underline{h}, \overline{h}\} \subset H$ such that for each $i \in N$,*
*- if $(\sigma, t) \in \arg\min u_i(\varphi(c))$, then $t_i = -\sum_{l \in N\setminus\{i\}}(\sigma_l - 1)c_l + \underline{h}_i(c_{-i})$;*
*- if $(\sigma, t) \in \arg\max u_i(\varphi(c))$, then $t_i = -\sum_{l \in N\setminus\{i\}}(\sigma_l - 1)c_l + \overline{h}_i(c_{-i})$.*

*Let $\{(\underline{\gamma}_i)_{i\in N}, (\overline{\gamma}_i)_{i\in N}\} \subset H$ be such that for each $i \in N$,*
*- if $(\sigma, t) \in \arg\min u_i(\varphi(c))$, then $\underline{h}_i(c_{-i}) = \sum_{l \in N\setminus\{i\}}(\sigma_l^{-i} - 1)c_l + \underline{\gamma}_i(c_{-i})$;*
*- if $(\sigma, t) \in \arg\max u_i(\varphi(c))$, then $\overline{h}_i(c_{-i}) = \sum_{l \in N\setminus\{i\}}(\sigma_l^{-i} - 1)c_l + \overline{\gamma}_i(c_{-i})$.*

*W.l.o.g. assume $N = \{1, ..., n\}$ and $c_1 \geq ... \geq c_n$. For $S \subseteq N$, let $c_S \equiv (c_l)_{l \in S}$ be the list of the unit waiting costs of the members of $S$ and $c_n^S \equiv (c_n)_{l \in S}$ be the list of the unit waiting costs of the members of $S$ when for each $l \in S$, we have $c_l = c_n$. Let $\sigma^S \in \overline{\Sigma}^*(c_S) \equiv \{(\sigma_i)_{i \in S} \in \mathbb{N}^S \mid (i)$ for each $\{i, j\} \subset S$, we have $\sigma_i \neq \sigma_j$ and (ii) for each $(\sigma'_i)_{i \in S} \in \mathbb{N}^S$ such that for each $\{i, j\} \in S$, we have $\sigma'_i \neq \sigma'_j$, we have $\sum_{i\in S}(\sigma'_i - 1)c_i \geq \sum_{i\in S}(\sigma_i - 1)c_i\}$.*
*For each $\tilde{c} \in \{(c_n, ..., c_n), (c_1, c_n, ..., c_n), ..., (c_1, ..., c_{n-1}, c_n)\}$, each $(\tilde{\sigma}, \tilde{t}) \in \varphi(\tilde{c})$, and each $(\gamma_i)_{i \in N} \in H$, if for each $i \in N$, we have $\tilde{t}_i = -\sum_{l \in N\setminus\{i\}}(\tilde{\sigma}_l - 1)\tilde{c}_l + \sum_{l \in N\setminus\{i\}}(\tilde{\sigma}_l^{-i} - 1)\tilde{c}_l + \gamma_i(\tilde{c}_{-i})$, the following holds.*

**Step 0:** $(c_n, ..., c_n) \in \mathbb{R}^N_{++}$.
*By* Pareto-efficiency, $\sum_{i\in N} \gamma_i(c_n^{N\setminus\{i\}}) = \frac{n(n-1)}{2}c_n$. *By* equal treatment of equals in welfare, $\gamma_1(c_n^{N\setminus\{1\}}) = ... = \gamma_n(c_n^{N\setminus\{n\}})$. *Thus, for each $i \in N$, we have $\gamma_i(c_n^{N\setminus\{i\}}) = \frac{(n-1)}{2}c_n$.*

**Step 1:** $(c_1, c_n, ..., c_n) \in \mathbb{R}^N_{++}$.
*By* Pareto-efficiency, $\gamma_1(c_n^{N\setminus\{1\}}) + \sum_{i\in N\setminus\{1\}} \gamma_i(c_1, c_n^{N\setminus\{1,i\}}) = \frac{(n-1)n}{2}c_n$. *By Step 0, $\gamma_1(c_n^{N\setminus\{1\}}) = \frac{(n-1)}{2}c_n$. By* equal treatment of equals in welfare, $\gamma_2(c_1, c_n^{N\setminus\{1,2\}}) = ... = \gamma_n(c_1, c_n^{N\setminus\{1,n\}})$. *Thus, for each $i \in N\setminus\{1\}$, we have $\gamma_i(c_1, c_n^{N\setminus\{1,i\}}) = \frac{(n-1)}{2}c_n$.*
*By the same logic, for each $l \neq 1 \in N$ and each $i \in N\setminus\{l\}$, we have $\gamma_i(c_l, c_n^{N\setminus\{l,i\}}) = \frac{(n-1)}{2}c_n$.*

**Step 2:** $(c_1, c_2, c_n, ..., c_n) \in \mathbb{R}^N_{++}$.
*By* Pareto-efficiency, $\sum_{i\in\{1,2\}} \gamma_i(c_{\{1,2\}\setminus\{i\}}, c_n^{N\setminus\{1,2\}}) + \sum_{i\in N\setminus\{1,2\}} \gamma_i(c_{\{1,2\}}, c_n^{N\setminus\{1,2,i\}}) = c_2 + \frac{(n-2)(n+1)}{2}c_n$. *By Step 1, for each $i \in \{1, 2\}$, we have $\gamma_i(c_{\{1,2\}\setminus\{i\}}, c_n^{N\setminus\{1,2\}}) = \frac{(n-1)}{2}c_n$. By* equal treatment of equals in welfare, $\gamma_3(c_{\{1,2\}}, c_n^{N\setminus\{1,2,3\}}) = ... = \gamma_n(c_{\{1,2\}}, c_n^{N\setminus\{1,2,n\}})$. *Thus, for each $i \in N\setminus\{1, 2\}$, we have $\gamma_i(c_{\{1,2\}}, c_n^{N\setminus\{1,2,i\}}) = \frac{1}{(n-2)}c_2 + \frac{(n-3)n}{2(n-2)}c_n$.*
*By the same logic, for each $\{l, l'\} \neq \{1, 2\} \subset N$ and each $i \in N\setminus\{l, l'\}$, we have $\gamma_i(c_{\{l,l'\}}, c_n^{N\setminus\{l,l',i\}}) = \sum_{k\in\{l,l'\}} \frac{(\sigma_k^{\{l,l'\}}-1)}{(n-2)}c_k + \frac{(n-3)n}{2(n-2)}c_n$.*

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17

**Step s:** $(c_1, ..., c_s, c_n, ..., c_n) \in \mathbb{R}^N_{++}$.

By Pareto-efficiency, $\sum_{i \in \{1,...,s\}} \gamma_i(c_{\{1,...,s\}\setminus\{i\}}, c_n^{N\setminus\{1,...,s\}}) + \sum_{i \in N\setminus\{1,...,s\}} \gamma_i(c_{\{1,...,s\}}, c_n^{N\setminus\{1,...,s,i\}}) = \sum_{i \in \{1,...,s\}} (\sigma_i^{\{1,...,s\}} - 1)c_i + \frac{(n-s)(n+s-1)}{2}c_n$.

By Step $s-1$, for each $i \in \{1, ..., s\}$, we have $\gamma_i(c_{\{1,...,s\}\setminus\{i\}}, c_n^{N\setminus\{1,...,s\}}) = \sum_{l \in \{1,...,s\}\setminus\{i\}} \frac{(\sigma_l^{\{1,...,s\}\setminus\{i\}} - 1)}{(n-2)}c_l + \frac{(n-(s-1)-1)(n+(s-1)-2)}{2(n-2)}c_n$. By equal treatment of equals in welfare, $\gamma_{s+1}(c_{\{1,...,s\}}, c_n^{N\setminus\{1,...,s,s+1\}}) = ... = \gamma_n(c_{\{1,...,s\}}, c_n^{N\setminus\{1,...,s,n\}})$.

Thus, for each $i \in N\setminus\{1, ..., s\}$, we have $\gamma_i(c_{\{1,...,s\}}, c_n^{N\setminus\{1,...,s,i\}}) = \sum_{l \in \{1,...,s\}} \frac{(\sigma_l^{\{1,...,s\}} - 1)}{(n-2)}c_l + \frac{(n-s-1)(n+s-2)}{2(n-2)}c_n$.

By the same logic, for each $S \neq \{1, ..., s\} \subset N$ with $|S| = s$ and each $i \in N\setminus S$, we have $\gamma_i(c_S, c_n^{N\setminus(S\cup\{i\})}) = \sum_{l \in S} \frac{(\sigma_l^S - 1)}{(n-2)}c_l + \frac{(n-s-1)(n+s-2)}{2(n-2)}c_n$.

...

**Step n − 1:** $(c_1, ..., c_{n-1}, c_n) \in \mathbb{R}^N_{++}$.

By Pareto-efficiency, $\sum_{i \in \{1,...,n-1\}} \gamma_i(c_{\{1,...,n-1\}\setminus\{i\}}, c_n) + \gamma_n(c_{\{1,...,n-1\}}) = \sum_{i \in \{1,...,n-1\}} (\sigma_i^{\{1,...,n-1\}} - 1)c_i + (n-1)c_n$. By Step $n-2$, **(i)** for each $i \in \{1, ..., n-1\}$, we have $\gamma_i(c_{\{1,...,n-1\}\setminus\{i\}}, c_n) = \sum_{l \in \{1,...,n-1\}\setminus\{i\}} \frac{(\sigma_l^{\{1,...,n-1\}\setminus\{i\}} - 1)}{(n-2)}c_l + c_n$. Thus, **(ii)** $\gamma_n(c_{\{1,...,n-1\}}) = \sum_{l \in \{1,...,n-1\}} \frac{(\sigma_l^{\{1,...,n-1\}} - 1)}{(n-2)}c_l$. By **(i)** and **(ii)**, for each $i \in N$, we have $\gamma_i(c_{-i}) = \sum_{l \in N\setminus\{i\}} \frac{(\sigma_l^{N\setminus\{i\}} - 1)}{(n-2)}c_l = \sum_{l \in N\setminus\{i\}} \frac{(\tilde{\sigma}_l^{-i} - 1)}{(n-2)}c_l$.

By this step process' logic, for each $i \in N$, if $(\sigma, t) \in \arg\min u_i(\varphi(c)) \cup \arg\min u_i(\varphi(c))$, then $t_i = -\sum_{l \in N\setminus\{i\}} (\sigma_l - 1)c_l + \sum_{l \in N\setminus\{i\}} (\sigma_l^{-i} - 1)c_l + \frac{1}{(n-2)} \sum_{l \in N\setminus\{i\}} (\sigma_l^{-i} - 1)c_l$.

Let $i \in N$, $(\underline{\sigma}, \underline{t}) \in \arg\min u_i(\varphi(c))$, $(\overline{\sigma}, \overline{t}) \in \arg\max u_i(\varphi(c))$, and $(\sigma, t) \in \varphi(c)$. By Pareto-efficiency of queues, $\sum_{l \in N} (\underline{\sigma}_l - 1)c_l = \sum_{l \in N} (\overline{\sigma}_l - 1)c_l$ and $\sum_{l \in N\setminus\{i\}} (\underline{\sigma}_l^{-i} - 1)c_l = \sum_{l \in N\setminus\{i\}} (\overline{\sigma}_l^{-i} - 1)c_l$. Thus, $u_i(\underline{\sigma}_i, \underline{t}_i) = u_i(\overline{\sigma}_i, \overline{t}_i)$. Thus, $t_i = -\sum_{l \in N\setminus\{i\}} (\sigma_l - 1)c_l + \sum_{l \in N\setminus\{i\}} (\sigma_l^{-i} - 1)c_l + \frac{1}{(n-2)} \sum_{l \in N\setminus\{i\}} (\sigma_l^{-i} - 1)c_l$.

By Theorem 2, $t_i = -\sum_{l \in F_i(\sigma)} c_l + \frac{1}{(n-2)} \sum_{j \in N\setminus\{i\}} \sum_{l \in F_j(\sigma^{-i})} c_l$. [14]

**Statement 2:**

Pareto-efficiency: *Assume that for each $c \in \mathbb{R}^N_{++}$, we have $\varphi(c) \subseteq \varphi^L(c)$. Let*

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[14] Assume $(\sigma, t) \notin \arg\min u_i(\varphi(c))$. (The "max" case is immediate.) Let $(\Delta_k)_{k \in N} \in \mathbb{R}^N$ be such that for each $k \in N$, we have $t_k = -\sum_{l \in N\setminus\{k\}} (\sigma_l - 1)c_l + \sum_{l \in N\setminus\{k\}} (\sigma_l^{-k} - 1)c_l + \frac{1}{(n-2)} \sum_{l \in N\setminus\{k\}} (\sigma_l^{-k} - 1)c_l + \Delta_k$. By *Pareto-efficiency of queues*, $\sum_{l \in N} (\underline{\sigma}_l - 1)c_l = \sum_{l \in N} (\sigma_l - 1)c_l$ and $\sum_{l \in N\setminus\{i\}} (\sigma_l^{-i} - 1)c_l = \sum_{l \in N\setminus\{i\}} (\sigma_l^{-j} - 1)c_l$. Thus, $\Delta_i > 0$. By *balancedness*, there is $j \in N$ such that $\Delta_j < 0$. Let $(\underline{\sigma}, \underline{t}) \in \arg\min u_j(\varphi(c))$. By *Pareto-efficiency of queues*, $\sum_{l \in N} (\underline{\sigma}_l - 1)c_l = \sum_{l \in N} (\sigma_l - 1)c_l$ and $\sum_{l \in N\setminus\{j\}} (\underline{\sigma}_l^{-j} - 1)c_l = \sum_{l \in N\setminus\{j\}} (\sigma_l^{-j} - 1)c_l$. Thus, $u_j(\underline{\sigma}_j, \underline{t}_j) > u_j(\sigma_j, t_j)$, contradicting $(\underline{\sigma}, \underline{t}) \in \arg\min u_j(\varphi(c))$. Thus, $t_i = -\sum_{l \in N\setminus\{i\}} (\sigma_l - 1)c_l + \sum_{l \in N\setminus\{i\}} (\sigma_l^{-i} - 1)c_l + \frac{1}{(n-2)} \sum_{l \in N\setminus\{i\}} (\sigma_l^{-i} - 1)c_l$.

$c \in \mathbb{R}^N_{++}$ and $(\sigma, t) \in \varphi(c)$. By definition of $\varphi^L$, we have $\sigma \in \overline{\Sigma}^*(c)$. By Theorem 2, for each $i \in N$, we have $t_i = -\sum_{l \in F_i(\sigma)} c_l + \frac{1}{(n-2)} \sum_{j \in N \setminus \{i\}} \sum_{l \in F_j(\sigma^{-i})} c_l$

Thus, $\sum_{i \in N} t_i = \sum_{i \in N} [-\sum_{l \in F_i(\sigma)} c_l + \frac{1}{(n-2)} \sum_{j \in N \setminus \{i\}} \sum_{l \in F_j(\sigma^{-i})} c_l]$

$$= \sum_{i \in N} [-\sum_{l \in F_i(\sigma)} c_l + \frac{1}{(n-2)} \sum_{l \in N \setminus \{i\}} (\sigma_l^{-i} - 1) c_l]$$

$$= -\sum_{i \in N} \sum_{l \in F_i(\sigma)} c_l + \frac{1}{(n-2)} \sum_{i \in N} \sum_{l \in N \setminus \{i\}} (\sigma_l^{-i} - 1) c_l$$

$$= -\sum_{i \in N} (\sigma_i - 1) c_i + \frac{1}{(n-2)} \sum_{i \in N} (n-2)(\sigma_i - 1) c_i$$

$$= 0.$$

No-envy: [15] *Assume that for each $c \in \mathbb{R}^N_{++}$, we have $\varphi(c) \subseteq \varphi^L(c)$. Let $c \in \mathbb{R}^N_{++}$, $(\sigma, t) \in \varphi(c)$, and $\{i, j\} \subset N$ with $i \neq j$. By definition of $\varphi^L$, we have $\sigma \in \overline{\Sigma}^*(c)$. By Theorem 2, $t_i = \sum_{l \in P_i(\sigma)} \frac{(\sigma_l - 1)}{(n-2)} c_l - \sum_{l \in F_i(\sigma)} \frac{(n - \sigma_l)}{(n-2)} c_l$ and $t_j = \sum_{l \in P_j(\sigma)} \frac{(\sigma_l - 1)}{(n-2)} c_l - \sum_{l \in F_j(\sigma)} \frac{(n - \sigma_l)}{(n-2)} c_l$. Then, distinguish two cases.*

**Case 1:** *$\sigma_i < \sigma_j$. Let $\delta \in \mathbb{N}$ be such that $\sigma_j = \sigma_i + \delta$. Then, as by assumption, $1 \leq \sigma_i < \sigma_j \leq n$, we have $\delta \leq n - \sigma_i$. Also, as $\sigma \in \overline{\Sigma}^*(c)$, for each $l \in B_{ij}(\sigma)$, we have $c_i \geq c_l \geq c_j$. Thus,*

$$u_i(\sigma_i, t_i) - u_i(\sigma_j, t_j) = (-(\sigma_i - 1)c_i - \sum_{l \in B_{ij}(\sigma)} \frac{(n - \sigma_l)}{(n-2)} c_l - \frac{(n - \sigma_j)}{(n-2)} c_j)$$

$$- (-(\sigma_j - 1)c_i + \frac{(\sigma_i - 1)}{(n-2)} c_i + \sum_{l \in B_{ij}(\sigma)} \frac{(\sigma_l - 1)}{(n-2)} c_l)$$

$$= \frac{(n-2)\delta - (\sigma_i - 1)}{(n-2)} c_i - \frac{(n-1)}{(n-2)} \sum_{l \in B_{ij}(\sigma)} c_l - \frac{(n - \sigma_i - \delta)}{(n-2)} c_j$$

$$\geq (\frac{(n-2)\delta - (\sigma_i - 1) - (n-1)(\delta - 1) - (n - \sigma_i - \delta)}{(n-2)}) c_i$$

$$= 0.$$

**Case 2:** *$\sigma_i > \sigma_j$. Let $\delta \in \mathbb{N}$ be such that $\sigma_i = \sigma_j + \delta$. Then, as by assumption, $n \geq \sigma_i > \sigma_j \geq 1$, we have $\delta \leq n - \sigma_j$. Also, as $\sigma \in \overline{\Sigma}^*(c)$, for each $l \in B_{ji}(\sigma)$, we have $c_i \leq c_l \leq c_j$. Thus,*

$$u_i(\sigma_i, t_i) - u_i(\sigma_j, t_j) = (-(\sigma_i - 1)c_i + \frac{(\sigma_j - 1)}{(n-2)} c_j + \sum_{l \in B_{ji}(\sigma)} \frac{(\sigma_l - 1)}{(n-2)} c_l)$$

$$- (-(\sigma_j - 1)c_i - \sum_{l \in B_{ji}(\sigma)} \frac{(n - \sigma_l)}{(n-2)} c_l - \frac{(n - \sigma_i)}{(n-2)} c_i)$$

$$= \frac{-(n-2)\delta + (n - \sigma_j - \delta)}{(n-2)} c_i + \frac{(n-1)}{(n-2)} \sum_{l \in B_{ji}(\sigma)} c_l + \frac{(\sigma_j - 1)}{(n-2)} c_j$$

$$\geq (\frac{-(n-2)\delta + (n - \sigma_j - \delta) + (n-1)(\delta - 1) + (\sigma_j - 1)}{(n-2)}) c_i$$

$$= 0.$$

Strategy-proofness: *Assume that for each $c \in \mathbb{R}^N_{++}$, we have $\varphi(c) \subseteq \varphi^L(c)$. By definition, (**i**) for each $c \in \mathbb{R}^N_{++}$ and each $(\sigma, t) \in \varphi(c)$, we have $\sigma \in \overline{\Sigma}^*(c)$. By Theorem 2, there is $h \in H$ such that (**ii**) for each $c \in \mathbb{R}^N_{++}$, each $(\sigma, t) \in \varphi(c)$, and each $i \in N$, we have $t_i = -\sum_{l \in N \setminus \{i\}} (\sigma_l - 1) c_l + h_i(c_{-i})$. Let $c \in \mathbb{R}^N_{++}$, $i \in N$, $c_i' \in \mathbb{R}_{++}$, $(\sigma, t) \in \varphi(c)$, and $(\sigma', t') \in \varphi(c_i', c_{-i})$. By (**i**), $-\sum_{l \in N} (\sigma_l - 1) c_l \geq -\sum_{l \in N} (\sigma_l' - 1) c_l$. Thus, $-(\sigma_i - 1) c_i - \sum_{l \in N \setminus \{i\}} (\sigma_l - 1) c_l + h_i(c_{-i}) \geq -(\sigma_i' - 1) c_i - \sum_{l \in N \setminus \{i\}} (\sigma_l' - 1) c_l + h_i(c_{-i})$. By (**ii**), $u_i(\sigma_i, t_i) \geq u_i(\sigma_i', t_i')$.*

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[15] A rule $\varphi$ to satisfy *Pareto-efficiency* and *no-envy* if and only if for each $c \in \mathbb{R}^N_{++}$ and each $(\sigma, t) \in \varphi(c)$, we have $\sigma \in \Sigma^*(c)$, $\sum_{i \in N} t_i = 0$, and for each $\{i, j\} \subset N$, if $\sigma_j = \sigma_i + 1$, then $c_i \geq t_j - t_i \geq c_j$ (Chun, 2006b). An alternative proof consists in proving that *EDPP* rules satisfy this condition. Rules in Suijs (1996) satisfy it (Katta and Sethuraman, 2006). Thus, by Theorem 2, so do *EDPP* rules.

This holds for each $(\sigma, t) \in \varphi(c)$ and each $(\sigma', t') \in \varphi(c'_i, c_{-i})$. Thus, $\varphi(c) \, R_i(c_i) \, \varphi(c'_i, c_{-i})$.

**Statement 3:** *Assume that $\varphi$ satisfies the axioms of Theorem 3.3. Let $c \in \mathbb{R}^N_{++}$ and $(\sigma, t) \in \varphi(c)$. By Pareto-efficiency, $\sigma \in \overline{\Sigma}^*(c)$. By Theorem 1,* Pareto-efficiency *and* strategy-proofness *imply that there is $\{\underline{h}, \overline{h}\} \subset H$ such that for each $i \in N$, if $(\underline{\sigma}, \underline{t}) \in \arg\min u_i(\varphi(c))$, then $\underline{t}_i = -\sum_{l \in N \setminus \{i\}} (\underline{\sigma}_l - 1)c_l + \underline{h}_i(c_{-i})$ and if $(\overline{\sigma}, \overline{t}) \in \arg\max u_i(\varphi(c))$, then $\overline{t}_i = -\sum_{l \in N \setminus \{i\}} (\overline{\sigma}_l - 1)c_l + \overline{h}_i(c_{-i})$. By* symmetry, *for each $\{i, j\} \subset N$, if $c_{-i} = c_{-j}$, then $\underline{h}_i(c_{-i}) = \underline{h}_j(c_{-j})$ and $\overline{h}_i(c_{-i}) = \overline{h}_j(c_{-j})$. Thus, for each $\{i, j\} \subset N$, if $c_i = c_j$, then $\underline{h}_i(c_{-i}) = \underline{h}_j(c_{-j})$ and $\overline{h}_i(c_{-i}) = \overline{h}_j(c_{-j})$. By Statement 1's step process and* Pareto-efficiency *of queues, for each $i \in N$, we have $t_i = -\sum_{l \in F_i(\sigma)} c_l + \frac{1}{(n-2)} \sum_{j \in N \setminus \{i\}} \sum_{l \in F_j(\sigma^{-i})} c_l$. Thus, $\varphi(c) \subseteq \varphi^L(c)$. By* symmetry, *$\varphi(c) = \varphi^L(c)$.*

**Statement 4:** *Assume that for each $c \in \mathbb{R}^N_{++}$, we have $\varphi(c) = \varphi^L(c)$. By Theorem 3.2, $\varphi$ satisfies* Pareto-efficiency *and* strategy-proofness. *Also, $\varphi$ does not depend on agents' names. In particular, $t_i$ has the same structure for each $i \in N$. Thus, $\varphi$ satisfies* anonymity. ∎

**Nota 2** *By Theorem 3, a* single-valued *rule satisfies* Pareto-efficiency, *equal treatment of equals in welfare, and* strategy-proofness *if and only if it is a single-valued EDPP rule. Thus, in Theorem 3, we prove that in all cases, even in those where, for any reasons (political, social, practical, ...) one imposes to systematically select only one alternative and hence forces to make arbitrary choices, at the expense of basic equity considerations, there is a unique transfers list. One obtains it considering each pair of agents in turn, making each agent in the pair pay the cost she imposes on the pair, and distributing the sum of these two payments equally among the others.*

Finally, *Pareto-efficiency, no-envy,* and *strategy-proofness* are independent of one another. Rules such that for each $c \in \mathbb{R}^N_{++}$, if $(\sigma, t)$ is selected, then $\sigma \in \overline{\Sigma}^*(c)$ and for $\alpha \in \mathbb{R}^N_{++}$ such that if $i \in N$ with $\sigma_i \neq 1$ and $\{j, k\} \subset N$ are such that $\sigma_j = \sigma_i - 1$ and $\sigma_k = \sigma_i + 1$, then $\alpha_i \in [c_j, c_k]$ and $t_i = \sum_{l \in P_i(\sigma) \cup \{i\}} \alpha_l$ and if $i \in N$ with $\sigma_i = 1$, then $t_i = \alpha_i$ and $\sum_{l \in N} t_l = 0$, satisfy all axioms, but *strategy-proofness* (Chun, 2006b; Katta and Sethuraman, 2006). Groves rules associated with $h \in H$ such that for each $c \in \mathbb{R}^N_{++}$ and for $\lambda \in \mathbb{R}$ with $\lambda \neq 0$, if $(\sigma, t)$ is selected, $h_1 = \sum_{l \in N \setminus \{1\}} (\sigma_l^{-1} - 1)c_l + \frac{1}{n-2} \sum_{l \in N \setminus \{1\}} (\sigma_l^{-1} - 1)c_l + \lambda$ and for each $i \in N \setminus \{1\}$, we have $h_i = \sum_{l \in N \setminus \{i\}} (\sigma_l^{-i} - 1)c_i + \frac{1}{n-2} \sum_{l \in N \setminus \{i\}} (\sigma_l^{-i} - 1)c_l - \frac{\lambda}{(n-1)}$ satisfy all axioms, but *no-envy*. Groves rules associated with $h \in H$ such that for each $c \in \mathbb{R}^N_{++}$ and for $\lambda \in \mathbb{R}_{++}$, if $(\sigma, t)$ is selected, then for each $i \in N$, we have $h_i = \sum_{l \in N \setminus \{i\}} (\sigma_l^{-i} - 1)c_i + \frac{1}{n-2} \sum_{l \in N \setminus \{i\}} (\sigma_l^{-i} - 1)c_l - \lambda$ satisfy all axioms, but *Pareto-efficiency*.

Also, *Pareto-efficiency*, *anonymity*, and *strategy-proofness* are independent of one another. The rule that selects all *Pareto-efficient* queues and sets each agent's transfer equal to the Shapley value of the associated coalitional game, where the worth of a coalition is the minimum possible sum of its members waiting costs (Maniquet, 2003), satisfies all axioms, but *strategy-proofness*. Proper subcorrespondences of a rule that is the union of all the *single-valued* rules that are Groves rules associated with $h \in H$ and that satisfy *balanced-ness*, satisfies all axioms, but *anonymity*. Any rule such that for each $c \in \mathbb{R}^N_{++}$ and for $\lambda \in \mathbb{R}_+$, we have that $(\sigma, t)$ is selected if and only if $\sigma \in \{1, 2, ..., n\}^N$ and for each $i \in N$, we have $t_i = -\lambda$, satisfy all axioms, but *Pareto-efficiency*.

## 5   Extensions

From the previous sections, we know that the problems we study are among the few allocation problems, in which *Pareto-efficiency*, weak equity axioms as *equal treatment of equals in welfare* or *symmetry*, and *strategy-proofness* are compatible. The natural step now is to determine if this compatibility extends to other problems and when it does, if the appropriate generalization of the rule we introduce, is still a desirable rule.

First, assume that agents enjoy some benefit, once they receive the service. We may exclude agents, not force them to participate. In problems, in which these axioms are compatible, the appropriate generalization of the LEDPP rule remains the only rule satisfying them. Further, it guarantees each agent voluntarily participate to the rule.[16] The intuition is simple. As *Pareto-efficiency* may impose excluding an agent when considering $N$, not when considering subsets of $N$, we cannot always express the sum of the costs each agent imposes on the others, as the sum of the costs each agent imposes on each element of a partition of the others. *A contrario*, we can, if *Pareto-efficiency* imposes excluding an agent when considering $N$ if and only if it does when considering any pair of agents including her.

Second, assume that agents may differ in processing times. As one easily checks the accuracy of these processing times' announcements once one serves the agents, it is natural to assume that only information on waiting costs is private. The appropriate generalization of the LEDPP rule still satisfies these axioms.

Indeed, when agents have equal unit waiting costs and processing times, to treat agents equally, a desirable rule selects queues and sets transfers in the spirit of the LEDPP rule. When agents differ in unit waiting costs, not in

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[16] We thank an anonymous referee for recommending this extension. These claims' proofs, using Theorem 3's logic, are available to the reader upon request to us.

processing times, to give agents incentives to reveal their waiting cost, it still selects queues and sets transfers in the spirit of this rule. When agents differ in unit waiting costs and processing times, it still selects all *Pareto-efficient* queues and it sets *balanced* transfers such that for each agent and each bundle that maximizes or minimizes her utility, her transfer is equal to the others' total waiting cost plus an amount only depending on these agents' unit waiting costs. Setting them in the spirit of the LEDPP rule is an obvious example.

Finally, assume that instead of organizing queues, we assign objects, tasks, time shifts, etc. For instance, think of allocation problems of student houses. Students value a house according to how close it is to the campus. This is an assignment problem of at most one indivisible good per agent, in which agents rank goods identically. As queueing problems are such problems, our results apply. Efficiency, equity, and incentive compatibility need not be simultaneously possible. Yet, in problems, in which they are, the appropriate generalization of the LEDPP rule satisfies them, if it is not the only one.

## 6  Concluding comments

Our objective was to identify allocation rules for queueing problems satisfying efficiency, equity, and incentive compatibility criteria simultaneously. We proved that on the domain of linear preferences in positions and transfers, the Largest Equally Distributed Pairwise Pivotal (LEDPP) rule is the only such rule. It is characterized by *Pareto-efficiency*, *equal treatment of equals in welfare*, *symmetry*, and *strategy-proofness*. Further, it satisfies stronger equity and incentive compatibility criteria.

*Since the day that humans gathered into societies, there have been queues* (Hall, 1991). In particular situations, one may hardly equate the costs from having to wait, to the monetary costs waiting almost inescapably implies. Think of a patient listed for an organ transplant. Yet, in many cases, agents value their waiting costs in such monetary terms. Obvious examples are costs that continue running while waiting (wages, inventory costs, legal constraints, telephone charges, "burned" gaz, etc.) or opportunity costs (wages, shortfalls, etc.). As Hall (1991) puts it, *every minute that an employee spends waiting for another department and every minute that a job spends waiting to be processed is money wasted.* Such especially short term problems involving cashflows are frequent in daily lifes of individuals, businesses, and institutions. One then approximates waiting costs with a linear function of time. [17]

Throughout our analysis, we had the concern to *a priori* impose the least to

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[17] As agents discount future, those exponentially increase with time in the long run.

have stronger results. First, we considered any rules, i.e. *single-valued* or not. Doing so, we made our study more general in the following sense. It provides answers even for those cases where, for any reasons (political, social, practical, ...) one imposes to systematically select only one alternative and hence, forces to make arbitrary choices, at the expense of basic equity considerations.

Second, the equity and incentive compatibility axioms we imposed are weak. These may hence encompass extreme positions. For instance, if agents have exactly the same costs, our equity axioms require to treat these agents equally, whereas if agents have even only slightly different costs, they do not require anything. The underlying idea was to first establish what happens when we impose weak axioms. Only then, we may strenghten them and instead, impose axioms that we find as appealing, if not more, as *no-envy*, *anonymity*, or the incentive compatibility axiom according to which each agent always finds any bundle she receives when revealing her unit waiting cost at least as desirable as any bundle she receives when misrepresenting it.

Doing so, we determined each axiom's role and hence, we made the axiomatic analysis more transparent. Eventually, we proved that in the queueing problems we studied, one does not need to impose stronger equity or incentive compatibility axioms as they are "for free". Thus, *a priori* requiring the least, we made our characterizations stronger in the following sense. When such results hold for weak axioms, they hold for stronger axioms, not inversely.

Our analysis formalized solutions as allocation rules. These associate with each problem a non-empty set of allocations. As explained when defining *strategy-proofness*, selecting a multi-valued set of allocations is a first step. By definition, the allocations it contains are mutually exclusive. *In fine*, only one materializes. The interpretation we gave is that eventually, a random device selects one of these allocations as the final outcome. An alternative formalization of solutions, in which chance may play a role, is the one of *decision schemes*. These associate with each problem a lottery on the set of allocations. One may reformulate all our results in terms of decision schemes.[18] We used the allocation rule formalization as it offers more possible interpretations.

We draw three lessons from our results. First, the problems we studied are among the few allocation problems, in which *Pareto-efficiency*, weak equity

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[18] In particular, only *LEDPP schemes*, i.e. shemes such that for each problem, an allocation is in the support selected for this problem if and only if it is in the set selected by the LEDPP rule for this problem, satisfy the efficiency, equity, and incentive compatibility axioms we imposed. There is no need for further assumptions regarding agents' uncertainty aversions, especially that agents are *expected utility maximizers*. A stronger version of *symmetry* requires all symmetric allocations with respect to equal agents to be equally probable. Then, only the *uniform LEDPP scheme* satisfies these axioms.

axioms as *equal treatment of equals in welfare* or *symmetry*, and *strategy-proofness* are compatible. In other problems, these axioms need not be simultaneously possible. Yet, in problems, in which they are, the appropriate generalization of the LEDPP rule satisfies them, if it is not the only one.

Second, while *Pareto-efficiency* and *strategy-proofness* leave us with a large class of rules, adding a weak equity axiom as *equal treatment of equals in welfare* or *symmetry*, imposes a unique transfers list. To underline this contrast, we identified the class of non-*single-valued* rules that *Pareto-efficiency of queues* and *strategy-proofness* recommend. We proved that as in the *single-valued* case, this class is large. It contains the rules that select *Pareto-efficient* queues and set transfers in the spirit of Groves (1973). This result holds in other problems, provided the domain of quasi-linear preferences is rich enough.

Third, in the queueing problems we studied, simply requiring to treat equal agents equally in addition to *Pareto-efficiency* and *strategy-proofness*, guarantees further basic equity criteria. It prevents agents from envying one another. It prevents agents' names to matter. It guarantees each agent with a minimal welfare level. Indeed, in allocation problems of at most one indivisible good per agent, *no-envy* implies the *identical-preferences lower bound*, i.e. each agent should find her bundle at least as desirable as any bundle *Pareto-efficiency* and *equal treatment of equals in welfare* recommend when the others have her preferences (Beviá, 1996). It also satisfies other equity criteria specific to queueing problems (Katta and Sethuraman, 2006).

In allocation problems of private goods, *equal treatment of equals in welfare* and *coalition strategy-proofness*, i.e. no coalition should gain by simultaneously misrepresenting their preferences, imply *no-envy* (Moulin, 1993). In general public decision-making problems with strictly monotonically closed preferences, *equal treatment of equals in welfare*, *strategy-proofness*, and *non-bossiness*, i.e. if a change in an agent's waiting cost does not alter her bundle, it should not alter other agents' bundles, imply *no-envy* (Fleurbaey and Maniquet, 1997). These results do not apply to queueing problems. *Pareto-efficiency* is incompatible with *coalition strategy-proofness* (Kayı and Ramaekers, 2006; Mitra and Mutuswami, 2008), and with *strategy-proofness* and *non-bossiness* (Kayı and Ramaekers, 2006).

For further equity axioms, as those relative to changes in the set of agents or waiting costs, one must know agents' waiting costs. Indeed, on the domain of linear preferences in positions and transfers, only rules that select *Pareto-efficient* queues and set each agent's transfer equal to the Shapley value of some associated coalitional game, satisfy these axioms, and the efficiency and equity axioms we imposed (Maniquet, 2003; Chun, 2006a; Katta and Sethuraman, 2006). Yet, such rules do not satisfy *strategy-proofness*. Further, these axioms, *Pareto-efficiency*, and *no-envy* are incompatible (Chun, 2006b).

24

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