RESONANCE PROPAGATION AND THRESHOLD SINGULARITIES.

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We consider the problem of propagation of an unstable particle in the framework of Quantum Field Theory. Using unitarity, we show that a real renormalization constant free of threshold singularities naturally arises.

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Comparison of the standard model of the electroweak interactions with high precision data involving the production of unstable particles ($W^{\pm}, Z^0$ and hopefully the Higgs in the near future) requires the incorporation of radiative corrections in the theoretical predictions. In particular, the particle's propagator is obtained via Dyson summation of the self-energy $A(s)$:

$$P(s) \; = \; \frac{1}{s - m_0^2 - A(s)},\tag{1}$$

where m_0 stands for the bare mass. In the conventional on-shell renormalization scheme (os) [1], the renormalized mass and width are defined as:

$$M^2 \; = \; m_0^2 \; + \; Re \; A(M^2),\tag{2a}$$

$$M\Gamma^{os} \; = \; - \; \frac{Im \; A(M^2)}{1 \; - \; Re \; A'(M^2)},\tag{2b}$$

and the field renormalization constant is given by:

$$Z_2^{os} \; = \; \frac{1}{1 \; - \; Re \; A'(M^2)}.\tag{3}$$

The on-shell scheme provides a gauge invariant definition of the mass as long as the particle can be considered as stable. This is no longer true when the resonance width can not be neglected [2–4] and the formulation must be corrected in higher orders of perturbation theory by the addition of gauge dependent terms. It has been known for a long time that this problem can be solved by considering the mass m and width Γ of the unstable particle to be defined by the pole of the propagator [2–4]. The position of the propagator's pole s_p is obtained by solving the equation:

$$s_p \; = \; m_0^2 + A(s_p) \; = \; m^2 - im\Gamma\tag{4}$$

with

$$m^2 = m_0^2 + Re\ A(s_p),\qquad(5a)$$

$$m\Gamma = -Im\ A(s_p).\qquad(5b)$$

Another problem of the on-shell scheme is that the renormalization constant suffers from threshold singularities. These singularities appear in $Re\ A'(m^2)$ - but not in $Re\ A(m^2)$, nor in $Im\ A(m^2)$ - in the amplitude describing the S wave two body decays of scalar or vector resonances [5,6], when the mass of the decaying particle approaches from below the mass threshold of the produced particles. Examples where this problem may be relevant have been discussed in [5]. In fact we have to distinguish two different aspects: the first, is the relevance of the threshold singularities in the determination of the mass and width of the unstable particle, which is solved in the pole scheme [6], and second the effect of the threshold singularities on the predictions of the theory for the production and decay rates. For the latter, a solution has been recently proposed, in the context of a gauge theory [5], by Kniehl,Palisoc and Sirlin (hereafter referred as KPS)

It is convenient at this point to recall the arguments used by KPS to find a formulation in which the threshold singularities are avoided. As far as we can see, the central points are the following:

- The pole position, and its interpretation in terms of the physical mass and width of the unstable particle, lead to the relation, valid at the one loop level only:

$$Re\ A'(m^2) = \frac{Im\ A^{(1)}(m^2) - Im\ A^{(1)}(s_p)}{m\Gamma}.\qquad(6)$$

  The superscript refer to the number of quantum loops included in the computation of the self-energy. According to KPS, this serves as a regularized version of $Re\ A'(m^2)$, with the decay width Γ serving as regulator.

- In the **pole scheme** the width of the resonance is defined by Eq.(5b). The KPS approach is based on the following identity:

$$m\Gamma = -\frac{Im\ A(m^2)}{1 - \frac{Im\ A(m^2) - Im\ A(s_p)}{m\Gamma}}\qquad(7)$$

  Comparing with the **on-shell** definition of width Eq.(2b), KPS **propose** that the regularized field renormalization constant should be given at all orders by:

$$\frac{1}{Z_2^{KPS}} = 1 - \frac{Im\ A(m^2) - Im\ A(s_p)}{m\Gamma}.\qquad(8)$$

Introducing Eq.(5b) into Eq.(9), we get

$$\frac{1}{Z_2^{KPS}} = -\frac{Im\ A(m^2)}{m\Gamma}\qquad(8')$$

In this paper, we shall **derive**, at all orders, Eq.(8').

Following Ref. [7], for **real** s we introduce the real and imaginary parts of the self-energy:

$$Re\ A(s) = R(s), \qquad\qquad Im\ A(s) = I(s).\qquad(9)$$

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In terms of the pole position, the propagator, Eq.(1) is expressed as:

$$P(s)_{pole} = \frac{1}{F(s)} \frac{1}{s - s_p};$$

(10)

where

$$F(s) = 1 - \frac{R(s) - R(s_p)}{s - s_p} - i \frac{I(s) - I(s_p)}{s - s_p}.$$

(11)

The field renormalization constant can be obtained from $F(s)$, usually after expanding around some value of $s = s_0$. Notice that if the point $s_0 = s_p$ is chosen, a complex valued $Z_2^{-1}(s_p) = 1 - A'(s_p)$ is obtained. The pole approach is based on the isolation of the pole, as in Eq.(10). We are not forced however to perform an expansion based on s_p of the full quantity of interest (Green function or S matrix element). For example if in the present case such an expansion is carried for $F(s)$ we get a complex valued field renormalization constant, which most authors prefer to avoid. The common procedure is to perform an expansion of everything but the pole based on some real value of s.

Our starting point is the observation that, naively, one would expect in the pole approach a complex field renormalization constant [8] $Z_2^{-1}(s_p) = 1 - A'(s_p)$, in contrast with the real Z_2^{KPS} (Eq.(8)) found by KPS. We remark in this respect that KPS obtain Z_2 by comparing an identity following from the **pole scheme**, with the **on-shell** definition of width where the Z is real. Since the pole and on-shell schemes are equivalent through next to leading order, the procedure is fully justified to that order.

It proofs convenient to consider

$$T(s) = \frac{I(s)}{s - m_0^2 - R(s) - iI(s)}.$$

(12)

$T(s)$ fulfills the unitarity relation: $Im\ T(s) = T(s)T^{\dagger}(s)$, and $P(s) = T(s)/I(s)$. The unitarity of $T(s)$ ensures that it can be expressed in the form:

$$T(s) = \frac{-m\Gamma}{s - m^2 - i\ m\Gamma}e^{2i\delta(s)} + \frac{-y(s)}{1 + iy(s)},$$

(13)

where:

$$e^{2i\delta(s)} = \frac{1 - iy(s)}{1 + iy(s)}$$

(14)

If we want that $P(s) = T(s)/I(s)$, then $y(s)$ is given by:

$$y(s) = \frac{I(s)(s - m^2) + F(s)m\Gamma}{I(s)m\Gamma - F(s)(s - m^2)},$$

(15)

with:

$$F(s) = s - m_0^2 - R(s) = s - m^2 + Re\ R(s_p) - R(s) - Im\ I(s_p).$$

(16)

It is important to remark that $y(s)$ is real for real s. For our purposes it is better to express $T(s)$ not as in Eq.(13) but in the equivalent form:

$$T(s) = \frac{-m\Gamma - (s - m^2)y(s)}{(s - s_p)(1 + iy(s))},$$

(17)

as this allow us to conclude that:

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$$P(s) = \frac{Z_2(s)}{(s - s_p)(1 + iy(s))}, \tag{18}$$

where we have introduced:

$$Z_2(s) = \frac{-m\Gamma - (s - m^2)y(s)}{I(s)} \tag{19}$$

This is our main result. We have renormalized the propagator in the pole scheme, using a real renormalization constant not involving $R'(m^2)$. The following are the main characteristics of our procedure and result:

- The field renormalization we introduced in Eq.(19) is free of threshold singularities as it only depends on $I(s)$ and $R(s)$.

- We have made no assumption about the order of perturbation theory in which the self-energy has to be computed. In principle our result is valid to arbitrary order of perturbation theory.

- For s real, the field renormalization constant Eq.(19) is **real**. Our result has the advantage that any point s_0 can be chosen to expand the Green function. A case of particular interest is $s_0 = m^2$:

$$Z_2(m^2) = -\frac{m\Gamma}{I(m^2)} \tag{20}$$

This is precisely the Z_2 found by KPS (see Eq.(8') above and Eq.(23) in Ref. [5], and recall that $m\Gamma = -Im\, A(s_p)$). Furthermore, when $s_0 = m^2$ is chosen, we obtain for the propagator:

$$P(s) = \frac{Z_2(m^2)}{s - s_p}\frac{1}{1 + iy(m^2)} + ... \tag{21}$$

The ellipsis stand for terms of order $s - m^2$. The term $1 + iy(m^2)$ is not considered by KPS, this can be understood by noticing that $y(m^2)$ can be written as:

$$y(m^2) = \frac{Re\, R(s_p) - R(m^2) - Im\, I(s_p)}{I(m^2)} = \frac{m^2 - m_0^2 - R(m^2)}{I(m^2)} \tag{22}$$

The numerator of Eq.(22) is reminiscent of the on-shell mass definition Eq.(2a). In fact, $y(m^2)$ vanishes when the $m^2 = M^2$ equality holds, which is precisely the equivalence between the on-shell and pole schemes, which is valid only to leading order. Obviously the term $1 + iy(m^2)$ is necessary to fulfill the unitarity requirement.

We can define a real field renormalization constant when $s_0 = s_p$ (see the text beneath Eq.(11)) by expanding $F(s)$:

$$F(s) = 1 - Re\, A'(s_p) - iIm\, A'(s_p) + ... = (1 - Re\, A'(s_p))(1 - i\frac{Im\, A'(s_p)}{1 - Re\, A'(s_p)}) + ... \tag{23}$$

In this case, the ellipsis stand terms of order $(s - s_p)$. If we define:

$$Z_2^{pole} \equiv \frac{1}{1 - Re\, A'(s_p)}, \tag{24}$$

we find

$$P(s)_{pole} = \frac{Z_2^{pole}}{s - s_p}\frac{1}{1 - i\frac{Im\, A'(s_p)}{1 - Re\, A'(s_p)}}. \tag{25}$$

To be compared with the on-shell field renormalization constant Eq. (3) and $Z_2(m^2)$ Eq.(20), while Eq.(25) compares to Eq.(21). Notice that these expressions are equivalent only at the leading order.

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In summary, we considered the problem of threshold singularities using the pole scheme to define the mass and width of the resonance. In this scheme, We introduced a **real** field renormalization constant which is free of threshold singularities and proves the result obtained by Kniehl, Palisoc and Sirlin (see Eqs.(8) and (8')). The advantages of our approach are that *i)* It does not rely upon comparison of the conventional on-shell and pole schemes, *ii)* It is valid to arbitrary order of perturbation theory and *iii)* The field renormalization we introduce is a function of s, which can be expanded around the value of s better suited for each particular calculation.

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[1] A. Sirlin, Phys. Rev. D22 (1980) 971.
[2] A. Sirlin, Phys. Rev. Lett. 67 (1991), 2127.
[3] R.G. Stuart, Phys. Lett. B 262 (1991) 113; Phys. Rev. Lett 70 (1993) 3193.
[4] S. Willembrock and G. Valencia, Phys. Lett. B 259 (1991) 373.
[5] B.A. Kniehl, C.P. Palisoc and A. Sirlin, Nucl. Phys.B 591 (2000) 296.
[6] T. Bhattacharya and S. Willembrock, Phys. Rev. D47 (1993) 4022.
[7] P.A. Grassi, B.A. Kniehl and A. Sirlin. hep-th/0005149.
[8] G. Lopez Castro, J.L. Lucio M and J. Pestieau, Int. J. Mod. Phys. A11 (1996) 563.