UCL-IPT-02-05

Finite Mixing Angles in the Standard Model

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Abstract

We establish the existence of *one-loop finite relations* between the Cabibbo angle and the quark mass ratios, in the Standard Model with one Higgs doublet and two quark generations. Assuming a simple quark-lepton universality, we use the recent SNO results to predict the two-flavour mass spectrum of the neutrinos.

1 Introduction

Searching for relations between the apparently free parameters of the Standard Model (SM) of elementary particles ranks among the most popular unification attempts. Most of those parameters find their origin in the Yukawa sector of the theory. In particular, the six quark masses and the three Cabibbo-like mixing angles exhibit a strongly hierarchical pattern among families. Furthermore, recent experiments tend to confirm the existence of neutrino masses and therefore of new mixing angles in the lepton sector [1]. For decades one has tried to constrain the SM in order to obtain relations between those apparently free parameters. Until now, most attempts consisted in enlarging the symmetry group of the SM by adding a horizontal component to it [2]. The horizontal symmetry imposes constraints on the structure of the Yukawa couplings. After spontaneous breakdown of the symmetry, the fermion mass matrices that are generated still bear the stamp of those constraints and through bidiagonalization, they give rise to relations between mass ratios and mixing angles. Such an implementation guarantees that the relations survive to renormalization; they are called *natural* [3]. However one soon realized [4] that it could not be achieved without

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extending the particle content, namely by considering models with more than one Higgs doublet, thereby increasing the number of couplings...

In this paper we present an alternative way to envisage naturalness in the Yukawa sector. We assume the existence of a tree-level relation between some apparently free parameters, such that it is not spoiled by divergent one-loop radiative corrections. The condition is necessary to get natural relations (it is not sufficient as long as the relation does not hold at all orders in perturbation theory). Moreover, instead of one-loop naturalness, we talk about one-loop finiteness, since the obtained result is in principle not correlated with the presence of an extra symmetry group in the SM.

A first section is devoted to the presentation of a situation (evoked in [5]) where the use of a simple one-loop argument happens to be quite powerful. We show how such a kind of argument, applied to the weak mixing angle θ_W, within the SM, may give an insight of some higher scale *vertical* symmetry, and suggest the existence, at that scale, of a Grand Unified Theory (GUT).

In the second section we focus on the Cabibbo mixing angle θ_C [6] in a SM with one Higgs doublet and two fermion generations. Again we make use of a one-loop argument to show that it is impossible to determine a non-trivial Cabibbo angle as a fixed, calculable, parameter. The argument confirms a well-known result about *horizontal* symmetries with a single Higgs doublet [4].

We then consider a larger set of *a priori* free parameters in the hadronic Yukawa sector of the SM with one Higgs doublet and two fermion generations, and we establish a method to find out the one-loop finite relations between them. We show that there is indeed an infinite set of one-loop finite relations (expressing the Cabibbo angle θ_C as a function of the quark mass ratios m_u/m_c and m_d/m_s) in the single-Higgs-doublet SM. We conclude that those relations cannot originate from any additionnal horizontal symmetry.

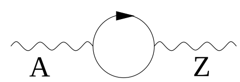
Assuming the existence of a purely Dirac mass term for the neutrinos, we obtain a second infinite set of one-loop finite relations in which the lepton mixing angle θ_L is found to be a non-trivial function of the lepton mass ratios m_{ν_e}/m_{ν_μ} and m_e/m_μ.

Finally, using the Large Mixing Angle (LMA) solution favoured by the new results of the SNO collaboration [7], we predict the two-flavour mass spectrum of the neutrinos from a simple quark-lepton universality.

2 The Weak Angle

The weak angle θ_W defines the physical A and Z^0 neutral gauge bosons of the tree-level $SU(2) \otimes U(1)$ Standard Model. This angle is in principle renormalized by the vacuum polarization diagrams that mix A and Z^0.

Now imagine some new theory, beyond the SM, in which the weak angle is no longer a free parameter. In the absence of counterterm, the radiative corrections to it would then be finite. In particular, at the one-loop level, the fermionic contribution



(1)

would be finite. Therefore, if the matter content of the new theory is identical to the one of the SM, the divergent contributions of the diagram (1) vanish. At the one-loop level, the divergent part of the corrections to θ_W proportional to the number n of fermion generations reads:

$$\delta\,\theta_W \;\sim\; n \left(1 - \frac{8}{3}\sin^2\theta_W\right) \ln\frac{\Lambda^2}{\mu^2} \tag{2}$$

where Λ is a cut-off[1] and μ an arbitrary energy scale. Hence, putting those divergent contributions to zero amounts to impose the non-trivial tree-level relation

$$\sin^2\theta_W = \frac{3}{8} \quad .$$

Such a constraint on the weak angle does not match the measured value

$$\sin^2\theta_W^{exp} \simeq 0.23$$

and, as such, should be thrown away. But our basic assumption was about the existence of some higher scale theory – or some higher scale symmetry which would determine $\sin^2\theta_W$ at that very scale. From this point of view, one notices that $\frac{3}{8}$ is precisely the value of $\sin^2\theta_W$ predicted at the $SU(5)$ GUT scale. So that one can expect $\sin^2\theta_W$ to run from its $SU(5)$ value to its SM value (partially due to the finite part of the diagram (1) if the fermion masses are taken to be non-degenerate within each generation).

[1] We did not write down the quadratically divergent term since it can be made irrelevant in other regularization schemes. However, including this term in equation (2) does not affect the form of the overall coefficient.

Furthermore, equation (2) is also true, independently, for two subsets of fermions running in the loop of the diagram (1), namely the left-handed electron, the left-handed neutrino[2] and the three coloured right-handed *down* quarks on the one hand, and the remaining ten SM fermions of the first generation on the other hand. This simply tells us the minimal way one could distribute the SM fermions in irreducible representations of the higher scale symmetry group, with well-defined dimensions:

$$\nu_L, e_L, d_R^r, d_R^b, d_R^g \rightarrow \mathbf{5} \qquad \text{and} \qquad \text{others} \rightarrow \mathbf{10} \quad .$$

A one-loop argument within the SM provides therefore some information about hypothetical vertical gauge symmetries (GUT's). It is then tempting to adopt the same procedure for the Cabibbo mixing angle.

3 The Cabibbo Angle

In the two-fermion-generation SM, the Cabibbo angle θ_C mixes the isospin eigenstates quarks, to end up with the mass eigenstates ones. At the one-loop level, it is radiatively corrected by a combination of self-energy diagrams of the following type:

. (3)

Now, one may ask again the question: is there a value of θ_C such that the radiative corrections to it are finite? Let us put the divergent contributions of the right combination of the diagrams (3) to zero, to obtain a constraint on the Cabibbo angle. At the one-loop level, in a single-Higgs-doublet model, the divergent part of the corrections to θ_C reads[3]:

$$\delta\theta_C \sim \sin\theta_C \cos\theta_C \, \ln\frac{\Lambda^2}{\mu^2} \tag{4}$$

where Λ is a cut-off and μ an arbitrary energy scale. Hence, putting those divergent contributions to zero amounts to impose the constraint

$$\sin\theta_C = 0 \qquad \text{or} \qquad \cos\theta_C = 0$$

[2] The neutrino has no charge and does not couple to the photon, but we mention it since it is associated to the electron through the $SU(2)$ doublet structure.

[3] The exact expression, given by the first equation of (7), is computed in the Appendix.

that is a trivial mixing. This is not a satisfactory result since it does not correspond to the SM value

$$\sin\theta_C^{exp} \simeq 0.22 \quad .$$

Moreover a trivial mixing angle cannot be associated to any higher scale symmetry, since it could not run to the non-zero SM value (with a trivial mixing, even the finite part of the diagrams (3) vanishes, so that a non-trivial mixing cannot originate from a tree-level trivial mixing[4]).

This simple one-loop argument agrees with a well-known theorem [4], which states that horizontal symmetries cannot lead to the non-trivial determination of the Cabibbo angle in the single-Higgs-doublet SM.

4 The Cabibbo Angle and the Quark Mass Ratios

One may think that the strict vanishing of the divergent corrections to θ_C is too strong a requirement. Inspired by the numerical success of $\tan\theta_C = \sqrt{m_d/m_s}$ [8], one can indeed relax this requirement and try to find a similar relation such that the divergent corrections to θ_C exactly compensate the corrections to the mass ratios. We know that this kind of relation cannot be obtained via the use of horizontal symmetries in the single-Higgs-doublet SM [4]. However, the existence of one-loop finite relations is not necessarily related to the presence of any horizontal symmetry.

We consider the two-fermion-generation SM in its minimal realization, namely built up with one single Higgs doublet. The hadronic Yukawa sector, which we are interested in, contains three useful free parameters: the Cabibbo angle θ_C and the two mass ratios $r_u = m_u/m_c$ and $r_d = m_d/m_s$. We assume the *a priori* existence of a relation between the Cabibbo angle and the mass ratios, whose general form reads:

$$F(\theta_C) = G(r_u, r_d) \tag{5}$$

so that

$$F(\theta_C + \delta\,\theta_C) = G(r_u + \delta\,r_u, r_d + \delta\,r_d) \tag{6}$$

where $\delta\,\theta_C$, $\delta\,r_u$ and $\delta\,r_d$ are the divergent parts of the radiative corrections to θ, r_u and r_d respectively. Thus

$$F_{,\theta_C}\,\delta\,\theta_C = G_{,r_u}\,\delta\,r_u + G_{,r_d}\,\delta\,r_d$$

[4]A tree-level trivial mixing implies the existence of a $U(1)$ symmetry for each generation, which guarantees the absence of mixing at all orders in perturbation theory.

where $f_{|x}$ denotes the (partial) derivative of f with respect to x. Computing the divergent part of the one-loop radiative corrections to the Yukawa couplings (see Appendix), one obtains:

$$\delta\,\theta_C = \epsilon\left[\tfrac{1+r_u^2}{1-r_u^2}(m_d^2-m_s^2)+\tfrac{1+r_d^2}{1-r_d^2}(m_u^2-m_c^2)\right]\sin\theta_C\cos\theta_C$$

$$\delta\,r_u = \epsilon\,r_u\,[(m_d^2-m_s^2)\cos 2\theta_C-(m_u^2-m_c^2)] \tag{7}$$

$$\delta\,r_d = \epsilon\,r_d\,[(m_u^2-m_c^2)\cos 2\theta_C-(m_d^2-m_s^2)]$$

with $\epsilon = \frac{3}{4}\frac{2}{v^2}\left(\frac{1}{4\pi^2}\ln\frac{\Lambda^2}{\mu^2}\right)$, Λ being a cut-off, μ an arbitrary energy scale and v the vacuum expectation value of the scalar field. Introducing those expressions into equation (6) splits it into two independent equations (since we are only interested in relations involving the mass ratios r_u and r_d):

$$F_{|\theta_C}\sin\theta_C\cos\theta_C\,\frac{1+r_u^2}{1-r_u^2} = G_{|r_u}\,r_u\cos 2\theta_C - G_{|r_d}\,r_d \tag{8}$$

$$F_{|\theta_C}\sin\theta_C\cos\theta_C\,\frac{1+r_d^2}{1-r_d^2} = G_{|r_d}\,r_d\cos 2\theta_C - G_{|r_u}\,r_u \tag{9}$$

which turn out to be compatible if and only if

$$\cos 2\theta_C = \frac{\frac{1-r_u^2}{1+r_u^2}\,r_d\,G_{|r_d}-\frac{1-r_d^2}{1+r_d^2}\,r_u\,G_{|r_u}}{\frac{1-r_u^2}{1+r_u^2}\,r_u\,G_{|r_u}-\frac{1-r_d^2}{1+r_d^2}\,r_d\,G_{|r_d}}\quad. \tag{10}$$

Now this *must* be the relation (5) whose existence has been assumed, i.e.[5]

$$F(\theta_C) = \cos 2\theta_C \tag{11}$$

and

$$G(r_u,r_d) = \frac{\frac{1-r_u^2}{1+r_u^2}\,r_d\,G_{|r_d}-\frac{1-r_d^2}{1+r_d^2}\,r_u\,G_{|r_u}}{\frac{1-r_u^2}{1+r_u^2}\,r_u\,G_{|r_u}-\frac{1-r_d^2}{1+r_d^2}\,r_d\,G_{|r_d}}\quad. \tag{12}$$

[5] One checks that the arbitrary character of those identifications will not show itself in the expected solution. To prove it, we imagine (11) would rather read $f(F(\theta_C)) = \cos 2\theta_C$. One should then replace G by $f(G)$ in the left-hand side of (12). But the right-hand side of it is invariant under $G \mapsto f(G)$. Namely, one can solve (12) with respect to the variable $f(G)$ which we eventually identify to $f(F(\theta_C)) = \cos 2\theta_C$.

Exploiting the remaining information in (8) and (9), and using (10) and (11), yields

$$\begin{cases} \frac{1+r_u^2}{1-r_u^2}G - r_u G_{|r_u} + \frac{1+r_d^2}{1-r_d^2} = 0 \\ \frac{1+r_d^2}{1-r_d^2}G - r_d G_{|r_d} + \frac{1+r_u^2}{1-r_u^2} = 0 \quad . \end{cases} \tag{13}$$

This system – from which one obviously recovers equation (12) – is integrable, and the general solution reads

$$G(r_u, r_d) = \frac{-(1+r_u^2)(1+r_d^2) + 2\lambda_C\, r_u r_d}{(1-r_u^2)(1-r_d^2)}$$

with λ_C a flavour-blind integration constant! One concludes that, if a relation of the kind suggested in (5) exists, it necessarily belongs to the following class:

$$\cos 2\theta_C = \frac{-(m_u^2+m_c^2)(m_d^2+m_s^2) + 2\lambda_C\, m_u m_c m_d m_s}{(m_u^2-m_c^2)(m_d^2-m_s^2)} \quad . \tag{14}$$

We have reached our goal to express the Cabibbo angle as a function of the quark mass ratios. However, there still is the unknown integration constant λ_C which must be greater or equal to 2. One checks that under the interchange $m_u \leftrightarrow m_c$ (or $m_d \leftrightarrow m_s$), the Cabibbo angle of equation (14) moves to its complementary, as expected. In the realistic limit $m_u << m_c$ and $m_d << m_s$, equation (14) implies

$$\cos^2\theta_C \simeq \lambda_C \frac{m_u m_d}{m_c m_s} \tag{15}$$

and requires therefore a large-valued λ_C to get $\cos^2\theta_C$ close to its experimental value.

5 The Lepton Mixing Angle and Neutrino Masses

The analysis we have conducted here can be applied to the leptons, provided that the neutrinos are massive, their mass being of the Dirac type exclusively. We thus consider the minimal extension of the SM that allows for massive neutrinos. We add right-handed neutrinos to the matter content and maintain the lepton number conservation, thereby forbidding Majorana mass terms. For the SM with two fermion generations and one Higgs doublet, we perform the same calculation[6] as in the quark sector. We conclude

[6] The calculation depends on the sole structure of the Yukawa couplings and is therefore identical.

that any natural relation between the lepton mixing angle θ_L and the lepton mass ratios must be of the following type:

$$\cos 2\theta_L = \frac{-(m^2_{\nu_e} + m^2_{\nu_\mu})(m^2_e + m^2_\mu) + 2\lambda_L\, m_{\nu_e} m_{\nu_\mu} m_e m_\mu}{(m^2_{\nu_e} - m^2_{\nu_\mu})(m^2_e - m^2_\mu)} \tag{16}$$

λ_L being again a flavour-blind integration constant greater or equal to 2. If $m_{\nu_e} << m_{\nu_\mu}$, then one has the approximate relation

$$\cos^2\theta_L \simeq \lambda_L \frac{m_{\nu_e} m_e}{m_{\nu_\mu} m_\mu} \tag{17}$$

analogous to (15). If $m_{\nu_e} >> m_{\nu_\mu}$, we simply interchange the neutrino masses in (17).

6 Quark-lepton Universality

A one-loop calculation does not determine neither λ_C nor λ_L. These dimensionless parameters could *a priori* take any real value greater than 2. But their flavour independence prompts us to assume a simple quark-lepton universality such that

$$\lambda_C = \lambda_L \quad .$$

Combining equations (14) and (16) in terms of the mass ratios r_u, r_d, $r_\nu \equiv m_{\nu_e}/m_{\nu_\mu}$ and $r_\ell \equiv m_e/m_\mu$, we end up now with a one-loop finite relation

$$\frac{(1+r^2_\nu)(1+r^2_\ell) + (1-r^2_\nu)(1-r^2_\ell)\cos 2\theta_L}{(1+r^2_u)(1+r^2_d) + (1-r^2_u)(1-r^2_d)\cos 2\theta_C} = \frac{r_\nu r_\ell}{r_u r_d} \quad . \tag{18}$$

involving only "measurable" quantities. In the limit $r_u << 1$, $r_d << 1$, $r_\nu << 1$ and $r_\ell << 1$, equation (18) yields

$$\frac{\cos^2\theta_L}{\cos^2\theta_C} \simeq \frac{r_\nu r_\ell}{r_u r_d} \quad . \tag{19}$$

Introducing reasonable mass ratios [9], the Cabibbo mixing angle and the LMA solution [7], one finds:

$$\frac{m_{\nu_e}}{m_{\nu_\mu}} \simeq \frac{1}{35} \quad .$$

The mass splitting $\Delta m^2 = m^2_{\nu_\mu} - m^2_{\nu_e}$ associated to the LMA solution [7] implies $m_{\nu_\mu} \simeq 7 \cdot 10^{-3}$eV and $m_{\nu_e} \simeq 2 \cdot 10^{-4}$eV.

7 Conclusion

We have derived two infinite sets of one-loop finite relations from the Yukawa sector. Each of them expresses a mixing angle as a function of fermion mass ratios. Determined by equations (14) and (16), thoses sets are parametrized in terms of two dimensionless constants λ_C and λ_L. At the one-loop level, one has no further theoretical argument to constrain the value of those constants; and one cannot evade the difficulty by asking one of the quark (or lepton) masses to vanish, since it would then lead to a cosine smaller than minus one[7]. However, by identifying λ_C and λ_L, which are flavour-blind, we are then able to predict the two-flavour mass spectrum of the neutrinos.

Stating that finite relations potentially exist in a single-Higgs-doublet model apparently contradicts previous results obtained in the context of family symmetries [4]. But since we do not appeal to such kind of symmetries, we do not expect our result to respect the conclusions derived in their context. Namely, the one-loop finite relations (14) or (16) cannot be associated with the presence of any extra horizontal symmetry. One should examine the validity of the results at higher loop level; then look for some possible "determination principle" of it outside or beyond the SM – just as the $SU(5)$ GUT determines the "one-loop conjectured" value of the weak angle θ_W at the GUT scale.

Whe should stress that the existence of non-trivial solutions to the system (13) crucially relies on the expression of the divergent one-loop radiative corrections to the Yukawa parameters, and would definitively be invalidated if one modifies a single coefficient in those corrections.

The extension of the present calculation to a three-generation SM seems to be doomed to failure because of the complexity of the one-loop radiative corrections to the Cabibbo-Kobayashi-Maskawa parameters [6, 10]. Those corrections are indeed too cumbersome to be manipulated and introduced into a solvable partial differential equations system. We will however expound, in a forthcoming paper, an alternative approach to derive the one-loop finite relations in the n-generation SM.

[7]Consequently we may already conclude that it is not possible to naturally set the mass of one single quark to zero, together with the requirement of the existence of a natural relation between the Cabibbo angle and the quark mass ratios. In this context, by excluding $m_u = 0$ our result rules out the natural vanishing of the QCD T-violating parameter θ_S.

Acknowledgments

This work is supported by the *Fonds pour la Formation à la Recherche dans l'Industrie et dans l'Agriculture* (FRIA). We would like to thank J.-M. Gérard and J. Weyers for a critical reading of the manuscript.

Appendix

A Introductory remark

We start from a SM with one Higgs doublet and n fermion generations. The approach we put forward is based on the calculation of the divergent one-loop radiative corrections to the quark mass matrices, which exclusively involves self-energy and tadpole diagrams. More precisely, since we are interested in natural relations between up-type quark mass ratios and down-type quark mass ratios on the one hand, and mixing angles on the other hand, we will solely compute the divergent one-loop radiative corrections to those specific parameters. This considerably simplifies our task. One indeed notices that neither QED nor QCD, which are flavour-blind, will bring in divergent contributions that would affect the mixing angle or the mass ratios. The same argument holds for the diagrams involving the transverse polarizations of the Z^0 and of the $W^\pm$ vector bosons, as well as the tadpoles. To convince oneself, it is worth checking that the contribution of the latter diagrams to the renormalization of the quark mass ratios reads

$$\frac{m_u}{m_c} \longmapsto \frac{(1 + C_\gamma + C_G + C_{Z^0} + C_{W^\pm} + C_T) m_u}{(1 + C_\gamma + C_G + C_{Z^0} + C_{W^\pm} + C_T) m_c} = \frac{m_u}{m_c}$$

where C_γ, C_G, C_{Z^0} and $C_{W^\pm}$ respectively originate from the interventions of the photon, the gluons, the transverse Z^0 and the transverse $W^\pm$ in the up-type quark self-energies, while C_T originates from the tadpole diagrams. Those C's are identical for the mass renormalization of any up-type quark – as far as the divergent part is concerned. The same reasoning can be applied to the down-type quarks. For the mixing angles, the proof is even more direct since none of those diagrams leads to divergent non-diagonal correction to the tree-level diagonal mass matrices. In other words, the only diagrams one has to consider are the quark self-energies due to the exchange of the scalars (Higgs and would-be-Goldstone bosons) – for the complete list of the relevant divergent diagrams, see figure 1. This is not astonishing since the scalars are the only fields that know about the difference between the fermion families.

B Self-energies in the Yukawa sector

After spontaneous breakdown of the symmetry, the Yukawa sector Lagrangian for quarks reads:

$$\begin{array}{rcccccccc} \mathcal{L}_Y & = & \bar{u}_L\Gamma_d d_R\phi^+ & + & \bar{d}_L\Gamma_d d_R\phi^0 & + & \bar{d}_R\Gamma_d^\dagger u_L\phi^- & + & \bar{d}_R\Gamma_d^\dagger d_L\phi^{0\star} \\ & + & \bar{u}_L\Gamma_u u_R\phi^{0\star} & - & \bar{d}_L\Gamma_u u_R\phi^- & + & \bar{u}_R\Gamma_d^\dagger u_L\phi^0 & - & \bar{u}_R\Gamma_u^\dagger d_L\phi^+ \\ & + & \bar{d}_L M_d d_R & + & \bar{u}_L M_u u_R & + & \bar{d}_R M_d^\dagger d_L & + & \bar{u}_R M_d^\dagger u_L \end{array} .$$

The Lagrangian fields and parameters are renormalized:

$$\begin{array}{rcccl} u_{L,R} & \longmapsto & u'_{L,R} & = & (Z^u_{L,R})^{-\frac{1}{2}} u_{L,R} \\ d_{L,R} & \longmapsto & d'_{L,R} & = & (Z^d_{L,R})^{-\frac{1}{2}} d_{L,R} \\ M_u & \longmapsto & M'_u & = & (Z^u_L)^{\frac{1}{2}}(Z_{M_u})^{-1}M_u(Z^u_R)^{\frac{1}{2}} \\ M_u & \longmapsto & M'_d & = & (Z^d_L)^{\frac{1}{2}}(Z_{M_d})^{-1}M_d(Z^d_R)^{\frac{1}{2}} \end{array} .$$

The one-loop calculation of the fermion self-energies leads to (the first term corresponding to the neutral current intervention ; the second, to the charged current intervention – see figure 1):

$$\begin{array}{cc} Z^u_L = 1 - \frac{\epsilon}{2}[\Gamma_u\Gamma_u^\dagger + \Gamma_d\Gamma_d^\dagger] & Z^d_L = 1 - \frac{\epsilon}{2}[\Gamma_d\Gamma_d^\dagger + \Gamma_u\Gamma_u^\dagger] \\ Z^u_R = 1 - \frac{\epsilon}{2}[\Gamma_u\Gamma_u^\dagger + \Gamma_u\Gamma_u^\dagger] & Z^d_R = 1 - \frac{\epsilon}{2}[\Gamma_d\Gamma_d^\dagger + \Gamma_d\Gamma_d^\dagger] \\ Z_{M_u} = 1 - \epsilon[0 + \Gamma_d\Gamma_d^\dagger] & Z_{M_d} = 1 - \epsilon[0 + \Gamma_u\Gamma_u^\dagger] \end{array} \tag{20}$$

with $\epsilon = \left(\frac{1}{4\pi^2}\ln\frac{\Lambda^2}{\mu^2}\right)$ where Λ is a cut-off and μ an arbitrary energy scale (one checks that $Z^u_L = Z^d_L$ as expected). Those results are true only up to a term proportional to the identity in the flavour space, which would take into account the electromagnetic, weak transversal and strong contributions, as well as the tadpole ones. But since this term would factor out in the final result, which is supposed to involve exclusively mixing parameters and *up*- or *down*-type mass ratios, we chose not to write it down. The finite parts of the diagrams are omitted.

From (20), one derives the corrections to the mass matrices in the weak base

$$M'_u = M_u + \epsilon\,[M_d M_d^\dagger M_u - M_u M_u^\dagger M_u]$$
$$M'_d = M_d + \epsilon\,[M_u M_u^\dagger M_d - M_d M_d^\dagger M_d]$$

and in the physical base

$$U_L^\dagger M_u' U_R = D_u + \epsilon\,[K D_d^2 K^\dagger D_u - D_u^3]$$
$$V_L^\dagger M_d' V_R = D_d + \epsilon\,[K^\dagger D_u^2 K D_d - D_d^3]$$

where we have absorbed a $\frac{3}{4}\frac{2}{v^2}$ factor in ϵ, v being the scalar VEV, and where D_u and D_d are the tree-level diagonal mass matrices while K is the tree-level Cabibbo-Kobayashi-Maskawa matrix. Let us repeat that those expressions do not include the finite parts of the radiative correction, and that they account for the sole (neutral and charged) scalar exchanges in the fermion self-energies.

One can rewrite those last expressions as follows

$$M_u'' = D_u + \epsilon_u$$
$$M_d'' = D_d + \epsilon_d$$

and proceed to the diagonalization of M_u'' and M_d'', i.e.

$$U_L'^\dagger M_u'' U_R' = D_u'$$
$$V_L'^\dagger M_d'' V_R' = D_d'$$

where, in the two-fermion generations case,

$$D_u' = \begin{pmatrix} m_u + \epsilon_{u\,11} & \\ & m_c + \epsilon_{u\,22} \end{pmatrix} \qquad U_L' = \begin{pmatrix} 1 & \theta_u \\ -\theta_u & 1 \end{pmatrix}$$

$$D_d' = \begin{pmatrix} m_d + \epsilon_{d\,11} & \\ & m_s + \epsilon_{d\,22} \end{pmatrix} \qquad V_L' = \begin{pmatrix} 1 & \theta_d \\ -\theta_d & 1 \end{pmatrix}$$

with

$$\theta_u = \frac{m_u \epsilon_{u\,21} + m_c \epsilon_{u\,12}}{m_c^2 - m_u^2} \qquad \text{and} \qquad \theta_d = \frac{m_d \epsilon_{u\,21} + m_s \epsilon_{u\,12}}{m_s^2 - m_d^2} \quad .$$

The one-loop mixing matrix is defined by

$$K' = U_L'^\dagger K V_L'$$

so that the one-loop mixing angle reads

$$\theta_C' = \theta_C - \theta_u + \theta_d \quad .$$

Inserting the value of ϵ_u and ϵ_d in θ_C', D_u' and D_d', leads to equations (7).

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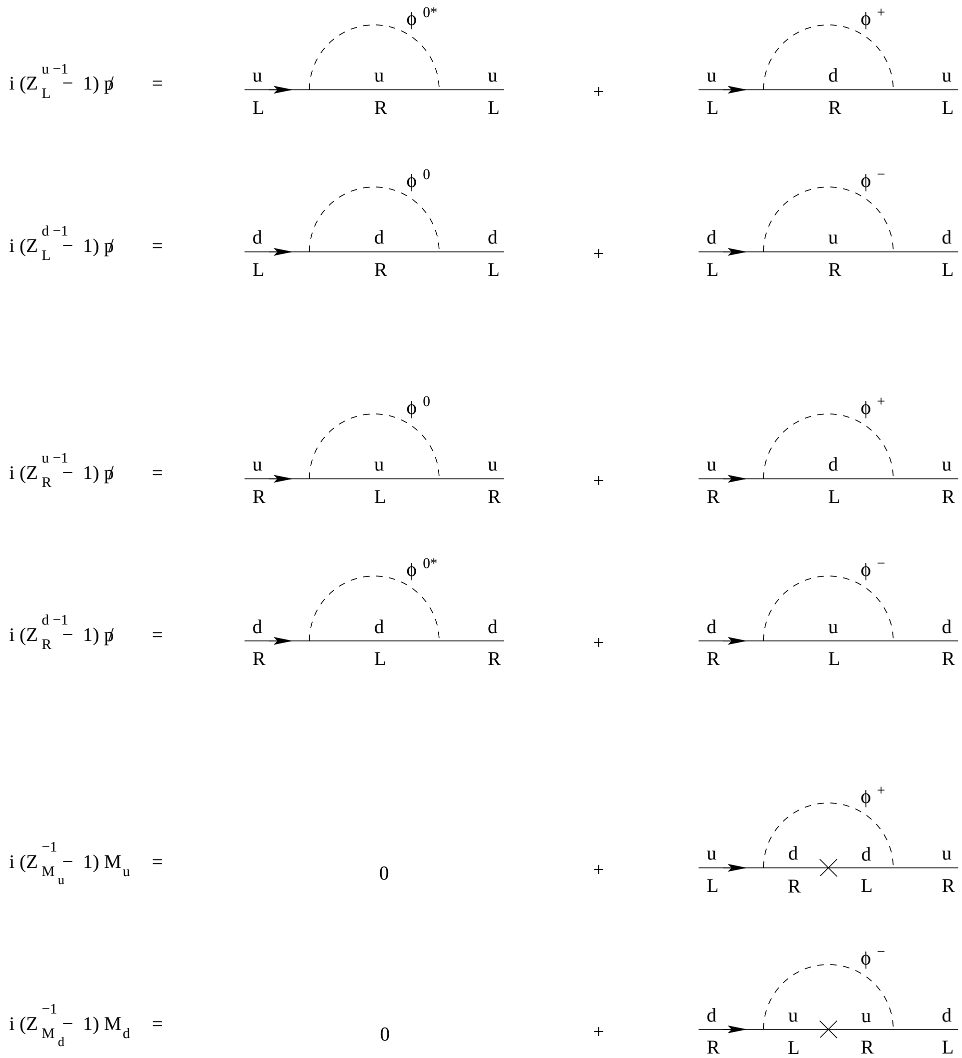

Fig. 1: relevant divergent diagrams involved in the calculation of the renormalization constants.