UPDATED BIG–BANG NUCLEOSYNTHESIS COMPARED TO WMAP RESULTS

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From the observations of the anisotropies of the Cosmic Microwave Background (CMB) radiation, the WMAP satellite has provided a determination of the baryonic density of the Universe, $\Omega_b h^2$, with an unprecedented precision. This imposes a careful reanalysis of the standard Big–Bang Nucleosynthesis (SBBN) calculations. We have updated our previous calculations using thermonuclear reaction rates provided by a new analysis of experimental nuclear data constrained by R-matrix theory. Combining these BBN results with the $\Omega_b h^2$ value from WMAP, we deduce the light element (^4He, D, ^3He and ^7Li) primordial abundances and compare them with spectroscopic observations. There is a very good agreement with deuterium observed in cosmological clouds, which strengthens the confidence on the estimated baryonic density of the Universe. However, there is an important discrepancy between the deduced ^7Li abundance and the one observed in halo stars of our Galaxy, supposed, until now, to represent the primordial abundance of this isotope. The origin of this discrepancy, observational, nuclear or more fundamental remains to be clarified. The possible role of the up to now neglected ^{7}Be(d,p)2α and ^{7}Be(d,α)^{5}Li reactions is considered.

1. Introduction

Big–Bang nucleosynthesis used to be the only method to determine the baryonic content of the Universe. However, recently other methods have emerged. In particular the analysis of the anisotropies of the cosmic microwave background radiation has provided $\Omega_b h^2$ values with ever increasing precision. (As usual, Ω_b is the ratio of the baryonic density over the critical density and h the Hubble constant in units of 100 km·s^{-1}·Mpc^{-1}.) The baryonic density provided by WMAP[1], $\Omega_b h^2 = 0.0224 \pm 0.0009$, has indeed dramatically increased the precision on this crucial cosmological parameter with respect to earlier experiments: BOOMERANG, CBI, DASI, MAXIMA, VSA and ARCHEOPS. It is thus important to improve the precision on SBBN calculations. Within the standard model of BBN, the only remaining free parameter is the baryon over photon ratio η directly related to $\Omega_b h^2$ [$\Omega_b h^2$=3.6519$\times$10^7 η]. Accordingly, the main source of uncertainties comes from the nuclear reaction rates. In this paper we use the results of a new analysis[2,3] of nuclear data providing improved reaction rates which reduces those uncertainties.

2. Nuclear reaction rates

In a previous paper[4] we already used a Monte–Carlo technique, to calculate the uncertainties on the light element yields (^4He, D, ^3He and ^7Li) related to nuclear reactions. The results were compared to observations that are thought to be representative of the corresponding primordial abundances. We used reaction rates from the NACRE compilation of charged particles reaction rates[5] completed by other sources[6,7,8] as NACRE did not include all of the 12 important reactions of SBBN. One of the main innovative features of NACRE with respect to former compilations[9] is that uncertainties are analyzed in detail and realistic lower and upper bounds for the rates are provided. However, since it is a general compilation for multiple applications, coping with a broad range of nuclear configurations, these bounds had not always been evaluated through a rigorous statistical methodology. Hence, we assumed a simple uniform distribution between these bounds for the Monte–Carlo calculations. Other works [10,11] have given better defined statistical limits for the reaction rates of interest for SBBN. In these works, the astrophysical S–factors (see definition in Ref. [4]) were either fitted with spline functions[10] or with NACRE S–factor fits and data but using a different normalization[11]. In this work, we use a new compilation[2] specifically dedicated to SBBN reaction rates using for the first time in this context nuclear theory to constrain the S–factor energy dependences and provide statistical limits. The goal of the R-matrix method[12] is to parametrize some experimentally known quantities, such as cross sections or phase shifts, with a small number of parameters, which are then used to interpolate the cross section within astrophysical energies. The R-matrix theory has been used for many decades in the nuclear physics community (see e.g. Ref. [13,14] for a recent application to a nuclear astrophysics problem) but this is the first time that it is applied to SBBN reactions. This method can be used for both resonant and non-resonant contributions to the cross section. (See Ref. [2] and references therein for details of the method.) The R-matrix framework assumes that the space is divided into two regions: the internal region (with radius a), where nuclear forces are important, and the external region, where the interaction between the nuclei is governed by the Coulomb force only. The physics of the internal region is parameterized by a number N of poles, which are characterized by energy E_λ and reduced width $\tilde{\gamma}_\lambda$. Improvements of current work on Big Bang nucleosynthesis essentially concerns a more precise evaluation of uncertainties on the reaction rates. Here, we address this problem by using standard statistical methods [15]. This represents a significant improvement with respect to NACRE [5], where uncertainties are evaluated with a simple prescription. The R-matrix approach depends on a number of parameters, some of them being fitted, whereas others are constrained by well determined data, such as energies or widths of resonances. As usual, the adopted parameter set is obtained from the minimal χ^2 value. The uncertainties on the parameters are evaluated as explained in Ref.[15]. The range of acceptable p_i values is such that

$\chi^2(p_i) \leq \chi^2(p_i^{min}) + \Delta\chi^2$, where p_i^{min} is the optimal parameter set. In this equation, $\Delta\chi^2$ is obtained from $P(\nu/2, \Delta\chi^2/2) = 1-p$, where ν is the number of free parameters p_i, $P(a,x)$ is the Incomplete Gamma function, and p is the confidence limit ($p = 0.683$ for the 1σ confidence level)[15]. This range is scanned for all parameters, and the limits on the cross sections are then estimated at each energy. As it is well known, several reactions involved in nuclear astrophysics present different data sets which are not compatible with each other. An example is the ^{3}He$(\alpha,\gamma)^7$Be reaction where data with different normalizations are available. In such a case, a special procedure is used[2].

This new compilation[2] provides 1–σ statistical limits for each of the 10 rates: ^{2}H(p,γ)^{3}He, ^{2}H(d,n)^{3}He, ^{2}H(d,p)^{3}H, ^{3}H(d,n)^{4}He ^{3}H$(\alpha,\gamma)^7$Li, ^{3}He(n,p)^{3}H, ^{3}He(d,p)^{4}He, ^{3}He$(\alpha,\gamma)^7$Be, ^{7}Li(p,α)^{4}He and ^{7}Be(n,p)^{7}Li. The two remaining reactions of importance, n$\leftrightarrow$p and ^{1}H(n,γ)^{2}H come from theory and are unchanged with respect to our previous work[4].

3. SBBN calculations

We performed Monte-Carlo calculations using Gaussian distributions with parameters provided by the new compilation and calculated the ^4He, D, ^3He and ^7Li yield range as a function of η, fully consistent with our previous analysis[4]. The differences with Ref. [11] on the ^7Li yield is probably due to their different normalization procedure of the NACRE S–factors. Figure 1 displays the resulting abundance limits (1-σ) [it was 2-σ in Fig.4 of Ref. [4]] from SBBN calculations compared to primordial ones inferred from observations. Using these results and the WMAP $\Omega_b h^2$ range (quoted WMAP+SBBN in the following), it is now possible to infer the primordial ^4He, D, ^3He and ^7Li abundances.

We obtain (WMAP+SBBN) a deuterium primordial abundance of D/H $= (2.60^{+0.19}_{-0.17}) \times 10^{-5}$ [ratio of D and H abundances by number of atoms] which is in perfect agreement with the average value $(2.78^{+0.44}_{-0.38}) \times 10^{-5}$ of D/H observations in cosmological clouds[16]. These clouds at high redshift on the line of sight of distant quasars are expected to be representative of primordial D abundances. The exact convergence between these two independent methods is claimed to reinforce the confidence in the deduced $\Omega_b h^2$ value.

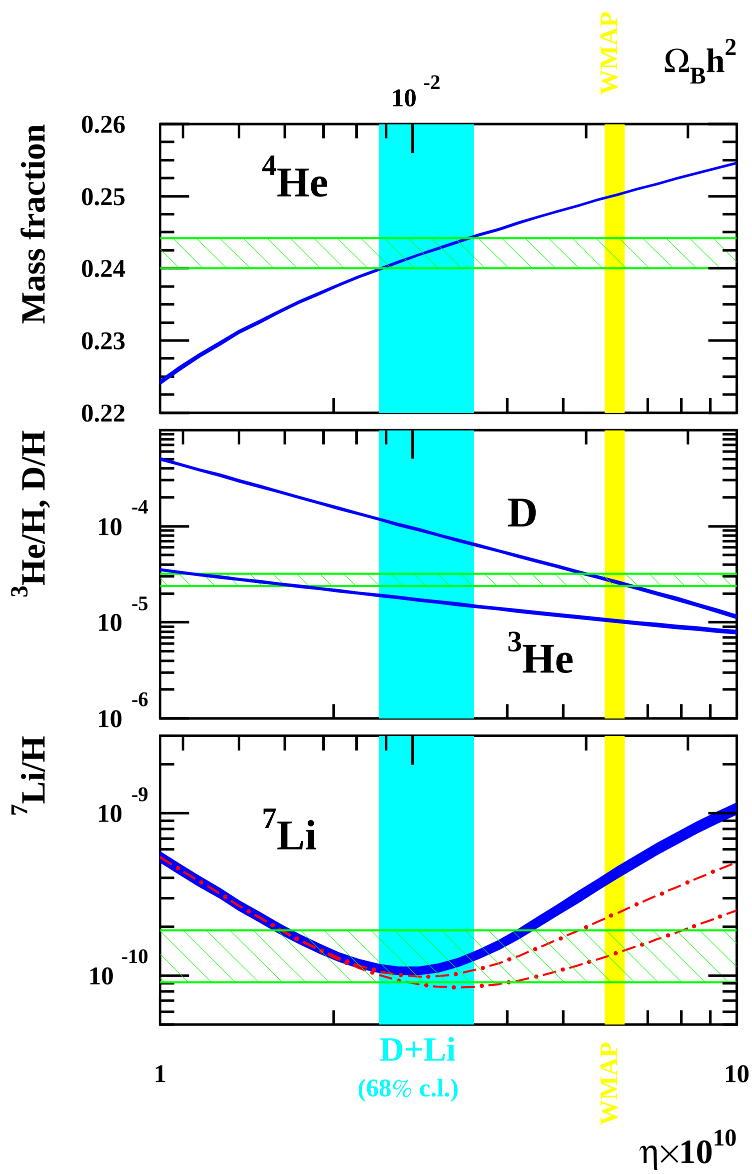

Figure 1. Abundances of ^4He (mass fraction), D, ^3He and ^7Li (by number relative to H) as a function of the baryon over photon ratio η or $\Omega_b h^2$. Limits (1-σ) are obtained from Monte Carlo calculations. Horizontal lines represent primordial ^4He, D and ^7Li abundances deduced from observational data (see text). The vertical stripes represent the (68% c.l.) $\Omega_b h^2$ limits provided by WMAP[1] or deduced from ^7Li and ^4He observations and SBBN calculations. For the dash-dotted lines in the bottom panel: see text.

The other WMAP+SBBN deduced primordial abundances are $Y_P = 0.2457 \pm 0.0004$ for the ^4He mass fraction, ^{3}He/H = $(1.04 \pm 0.04) \times 10^{-5}$ and ^{7}Li/H = $(4.15^{+0.49}_{-0.45}) \times 10^{-10}$. We leave aside ^3He whose primordial abundance cannot be reliably determined because of its uncertain rate of stellar production and destruction[17].

The ^4He primordial abundance, Y_p (mass fraction), is derived from observations of metal–poor extragalactic, ionized hydrogen (H II) regions. Recent evaluations gave a relatively narrow ranges of abundances: $Y_p = 0.2452 \pm 0.0015$ (Izotov et al.[18]), 0.2391 ± 0.0020 (Luridiana et al.[19]). However, recent observations by Izotov and Thuan[20] on a large sample of 82 H II regions in 76 blue compact galaxies have lead to the value of $Y_p = 0.2421 \pm 0.0021$ that we adopt here. With this range, WMAP and SBBN results are hardly compatible. Nevertheless, as systematic uncertainties may prevail due to observational difficulties and complex physics[21] ^4He alone is unsufficient to draw a conclusion.

The ^7Li abundance measured in halo stars of the Galaxy is considered up to now as representative of the primordial abundance as it display a plateau[32] as a function of metallicity (see definition in Ref. [4]). Recent observations[31] have lead to (95% c.l.) Li/H = $(1.23^{+0.68}_{-0.32}) \times 10^{-10}$. These authors have extensively studied and quantified the various sources of uncertainty : extrapolation, stellar depletion and stellar atmosphere parameters. This Li/H value, based on a much larger number of observations than the D/H one was considered[4] as the most reliable constraint on SBBN and hence on $\Omega_b h^2$. However, it is a factor of 3.4 lower than the WMAP+SBBN value. Even when considering the corresponding uncertainties, the two Li/H values differ drastistically. This confirms our[4] and other[11,22] previous conclusions that the $\Omega_b h^2$ range deduced from SBBN of ^7Li are only marginally compatible with those from the CMB observations available by this time (BOOMERANG, CBI, DASI and MAXIMA experiments). It is strange that the major discrepancy affects ^7Li since it could a priori lead to a more reliable primordial value than deuterium, because of much higher observational statistics and an easier extrapolation to primordial values.

Fig. 2 shows a comparison between $\Omega_b h^2$ ranges deduced either from SBBN or WMAP. The curves represent likelihood functions obtained from our SBBN calculations and observed deuterium[16], helium[20] and lithium[31] primordial abundances. These were obtained as in our previous analysis[4] except for the new reaction rates and new D and ^4He primordial abundances. The incompatibility between the D and ^7Li likelihood curves is more obvious than before due to the lower D/H adopted value (Kirkman

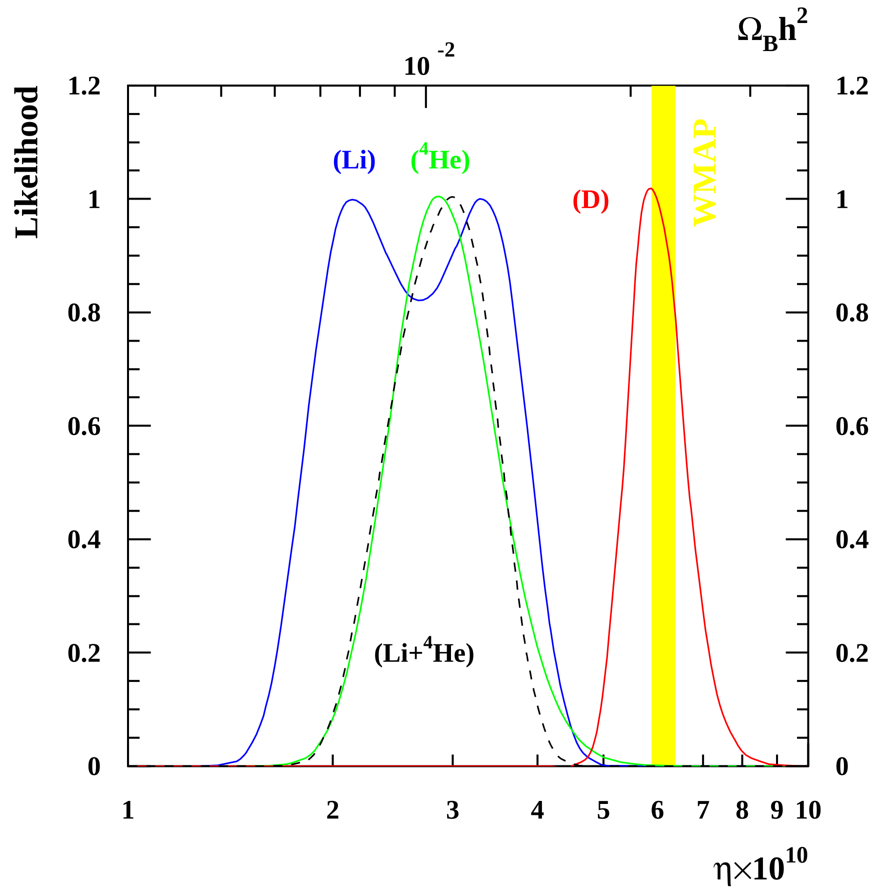

Figure 2. Likelihood functions for D, ^4He and ^7Li (solid lines) obtained from our SBBN calculations and Kirkman et al.[16], Izotov and Thuan[20], and Ryan et al.[31] data for D, ^4He and ^7Li respectively. The dashed curve represent the likelihood function for ^4He and ^7Li while the verical stripe shows the WMAP $\Omega_b h^2$ range[1].

et al., averaged value). On the contrary, the new ^4He adopted value[20] is perfectly compatible with the ^7Li one as shown on Fig. 2 (likelihood curves) and Fig. 1 (abundances). Putting aside, for a moment, the CMB results on the baryonic density, we would deduce the following 68% c.l. intervals: 1.85< η_{10} <3.90 [0.007<$\Omega_b h^2$<0.014] from ^7Li only or 5.4< η_{10} <6.6 [0.020<$\Omega_b h^2$<0.024] from D only. If we now consider ^4He together with ^7Li, we obtain 2.3< η_{10} <3.5 [0.009<$\Omega_b h^2$<0.013]. Hence, including these new ^4He observations favors a low $\Omega_b h^2$ interval as proposed in our previous work[4]. The WMAP result on the contrary definitively favors the upper (D) one. If we now assume that the ^4He constrain is not so tight, because e.g. of systematic errors on this isotope whose weak sensitivity to $\Omega_b h^2$ requires high precision abundance determinations, the origin of the discrepancy on ^7Li remains a challenging issue very well worth further investigations.

4. Possible origins of ^7Li discrepancy between SBBN and CMB

4.1. *Stellar*

Both observers and experts in stellar atmospheres agree to consider that the abundance determination in halo stars, and more particularly that of lithium requires a sophisticated analysis. The derivation of the lithium abundance in halo stars with the high precision needed requires a fine knowledge of the physics of stellar atmosphere (effective temperature scale, population of different ionization states, non LTE (Local Thermodynamic Equilibrium) effects and 1D/3D model atmospheres[35]. However, the 3D, NLTE abundances are very similar to the 1D, LTE results, but, nevertheless, 3D models are now compulsory to extract lithium abundance from poor metal halo stars[36].

Modification of the surface abundance of Li by nuclear burning all along the stellar evolution has been discussed for a long time in the literature. There is no lack of phenomena to disturb the Li abundance: rotational induced mixing, mass loss,...[37,38]. However, the flatness of the plateau over three decades in metallicity and the relatively small dispersion of data represent a real challenge to stellar modeling. In addition, recent observations of ^6Li in halo stars (an even more fragile isotope than ^7Li) constrain more severely the potential destruction of lithium[39].

4.2. *Nuclear*

Large systematic errors on the 12 main nuclear cross sections are excluded[2,3]. However, besides the 12 reactions classically considered in SBBN, first of all the influence of *all* nuclear reactions needs to be evaluated[3]. It is well known that the valley shaped curve representing Li/H as a function of η is due to two modes of ^7Li production. One, at low η produces ^7Li directly via ^{3}H$(\alpha,\gamma)^7$Li while ^7Li destruction comes from ^{7}Li(p,α)^{4}He. The other one, at high η, leads to the formation of ^{7}Be through ^{3}He$(\alpha,\gamma)^7$Be while ^{7}Be destruction by ^{7}Be(n,p)^{7}Li is inefficient because of the lower neutron abundance at high density (^7Be later decays to ^7Li). Since the WMAP results point toward the high η region, a peculiar attention should be paid to ^{7}Be synthesis. In particular, the ^{7}Be+d reactions could be an alternative to ^{7}Be(n,p)^{7}Li for the destruction of ^7Be, by compensating the scarcity of neutrons at high η. Fig. 1 shows (dash–dotted lines) that an increase of the ^{7}Be(d,p)2^{4}He reaction rate by factors of 100 to 300 would remove the discrepancy. The rate for this reaction[9] can be traced to an estimate by Parker[40] who assumed for the astrophysical S–factor a constant value of 10^5 kev.barn. based on the single experimental data available[41]. To derive this S–factor, Parker used this measured differential cross section at 90° and assumed isotropy of the cross section. Since Kavanagh measured only the p_0 and p_1 protons (i.e. feeding the ^{8}Be ground and first excited levels), Parker introduced an additional but arbitrary factor of 3 to take into account the possible population of higher lying levels. Indeed, a level at 11.35 MeV is also reported[42]. This factor should also include the contribution of another open channel in ^{7}Be+d: ^{7}Be(d,α)^{5}Li for which no data exist. In addition, one should note that *no* experimental data for this reaction is available at energies relevant to ^{7}Be Big Bang nucleosynthesis (Fig. 3), taking place when the temperature has dropped below 10^9 K. A seducing possibility[3] to reconciliate, SBBN, ^7Li and CMB observations would then be that new experimental data below $E_d = 700$ keV ($E_{cm} \approx$0.5 MeV) for ^{7}Be(d,p)2^{4}He [and ^{7}Be(d,α)^{5}Li] would lead to a sudden increase in the S–factor as in ^{10}B(p,α)^{7}Be[43,5]. This is not supported by known data, but considering the cosmological or astrophysical consequences, this is definitely an issue to be investigated and an experiment is planned in 2004 at the Cyclotron Research Centre in Louvain-la-Neuve.

4.3. *Cosmology*

Recent theories that could affect BBN include the variation of the fine structure constant[44], the modification of the expansion rate during BBN induced by quintessence[45], modified gravity[46], or leptons asymmetry[47]. However, their effect is in general more significant on ^4He than on ^7Li.

It may not be excluded that some bias exists in the analysis of CMB anisotropies. For instance, it has been argued[48] that a contamination of CMB map by blazars could affect the second peak of the power spectrum on which the CMB $\Omega_b h^2$ values are based.

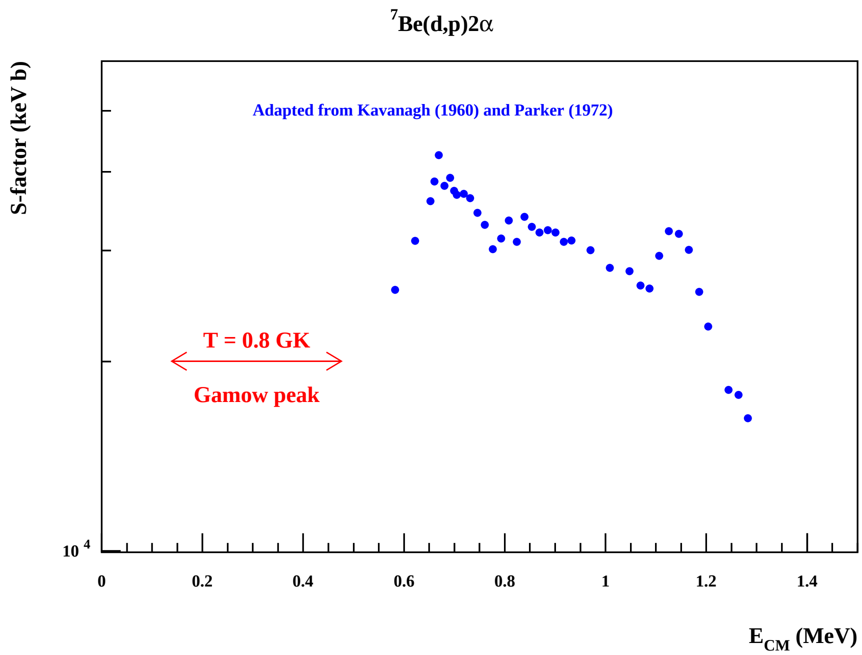

Figure 3. The only experimental data available for the ^{7}Be(d,p)2^{4}H reaction from Kavanagh (1960). The displayed S–factor is calculated as in Parker (1972) from the differential cross section at 90° ($\times 4\pi$) leading to the ground and first ^{8}Be excited states. Note that no data is available at SBBN energies as shown by the Gamow peak for a typical temperature of $T_9 = 0.8$

4.4. *Pregalactic evolution*

We note that between the BBN epoch and the birth of the now observed halo stars, ≈1 Gyr have passed. Primordial abundances could have been altered during this period. For instance, cosmological cosmic rays, assumed to have been born in a burst at some high redshift, could have modified these primordial abundances in the intergalactic medium[49]. This would increase the primordial ^7Li and D abundances trough spallative reactions, increasing in the same time the discrepancy between SBBN calculations and observations instead to reconciliate them.

Another source of alteration of the primordial abundances could be the contribution of the first generation stars (Population III). However, it seems difficult that they could reduce the ^7Li abundance without affecting the D one, consistent with CMB $\Omega_b h^2$.

5. Conclusions

The baryonic density of the Universe as determined by the analysis of the Cosmic Microwave Background anisotropies is in very good agreement with Standard Big–Bang Nucleosynthesis compared to D primordial abundance deduced from cosmological cloud observations. However, it strongly disagrees with lithium observations in halo stars (Spite plateau) and possibly ^4He new observations. The origin of this discrepancy, if not nuclear, is a challenging issue.

References

1. D.N. Spergel, L. Verde, H.V. Peiris, E. Komatsu, M.R. Nolta, C.L. Bennett, M. Halpern, G. Hinshaw, N. Jarosik, A. Kogut, M. Limon, S.S. Meyer, L. Page, G.S. Tucker, J.L. Weiland, E. Wollack and E.L. Wright, *Astrophys. J. S.* **148**, 175 (2003).
2. P. Descouvemont, A. Adahchour, C. Angulo, A. Coc and E. Vangioni–Flam, preprint, http://pntpm.ulb.ac.be/bigbang.
3. A. Coc, E. Vangioni-Flam, P. Descouvemont, A. Adahchour and C. Angulo, *Astrophys. J.*, in press, ArXiv:astro-ph/0309480
4. A. Coc, E. Vangioni–Flam, M. Cassé and M. Rabiet, *Phys. Rev.* **D65**, 043510 (2002).
5. C. Angulo, M. Arnould, M. Rayet et al., *Nucl. Phys.* **A656**, 3 (1999).
6. M.S. Smith, L.H. Kawano, and R.A. Malaney, *Astrophys. J. S.* **85**, 219 (1993).
7. C.R. Brune, K.I. Hahn, R.W. Kavanagh and P.W. Wrean, *Phys. Rev.* **C60**, 015801 (1999).
8. J.-W. Chen, and M.J. Savage, *Phys. Rev.* **C60**, 065205 (1999).
9. G.R. Caughlan and W.A. Fowler, *Atomic Data and Nuclear Data Tables* **40**, 283 (1988).
10. K.M. Nollett and S. Burles (NB), *Phys. Rev.* **D61**, 123505 (2000).
11. R.H. Cyburt, B.D. Fields, and K.A. Olive (CFO), *New Astronomy* **6**, 215 (2001).
12. A.M. Lane and R.G. Thomas, Rev. Mod. Phys. **30**, 257 (1958)
13. F.C. Barker and T. Kajino, Aust. J. Phys. **44** (1991) 369.
14. C. Angulo and P. Descouvemont, *Nucl. Phys.* **A639** 733 (1998).
15. Particle Data Group, K. Hagiwara et al., *Phys. Rev.* **D66**, 010001 (2002)
16. D. Kirkman, D. Tytler, N Suzuki, J.M. O'Meara, and D. Lubin, submitted to *Astrophys. J. S.*, ArXiv:astro-ph/0302006.
17. E. Vangioni–Flam, K.A. Olive, B.D. Fields and M. Cassé, *Astrophys. J.* **585**, 611 (2003).
18. Y.I. Izotov, F.H. Chaffee, C.B. Foltz, R.F. Green, N.G. Guseva, and T.H. Thuan, *Astrophys. J.* **527**, 757 (1999).
19. V. Luridiana et al., *Astrophys. J.* **592**, 846 (2003).
20. Y.I. Izotov and T.X. Thuan, *Astrophys. J.*, in print, ArXiv:astro-ph/0310421.

21. B.D. Fields and K.A. Olive, *Astrophys. J.* **506**, 177 (1998).
22. R.H. Cyburt, B.D. Fields, and K.A. Olive, *Phys. Lett.* **B567**, 227 (2003).
23. S. Burles and D. Tytler, *Astrophys. J.* **499**, 699 (1998).
24. S. Burles and D. Tytler, *Astrophys. J.* **507**, 732 (1998).
25. D. Tytler, X.-M. Fan and S. Burles, S., *Nature* **381** 207 (1996).
26. J.M. O'Meara D. Tytler, D. Kirkman, N. Suzuki, J.X. Prochaska, D. Lubin and A.M. Wolfe, *Astrophys. J.* **552**, 718 (2001).
27. M. Pettini, and D.V. Bowen, 2001, *Astrophys. J.* **560**, 41 (2001).
28. S. D'Odorico, M. Dessauges-Zavadsky, and P. Molaro, *Astron. Astrophys.* **368**, L21 (2001).
29. S.G. Ryan, T. Kajino, T.C. Beers, T.K. Suzuki, D. Romano, F. Matteucci and K. Rosolankova, *Astrophys. J.* **549**, 55 (2001).
30. S.G. Ryan, J. Norris, and T.C. Beers, *Astrophys. J.* **523**, 654 (1999).
31. S.G. Ryan, T.C. Beers, K.A. Olive, B.D. Fields, and J.E. Norris, *Astrophys. J.* **530** L57 (2000).
32. F. Spite, and M. Spite, *Astron. Astrophys.* **115**, 357 (1982).
33. F. Thévenin, C. Charbonnel, J.A. de Freitas Pacheco, T.P. Idiart, G. Jasniewicz, P. de Laverny, B. Plez, *Astron. Astrophys.* **373**, 905 (2001).
34. P. Bonifacio, L. Pasquini, F. Spite, et al., *Astron. Astrophys.* **390**, 91 (2002).
35. M. Asplund, M. Carlsson and A.V. Botnen, *Astron. Astrophys.* **399**, L31 (2003).
36. P.S. Barklem, A.K. Belyaev, and M. Asplund, 2003, astro-ph 0308170.
37. S. Theado and S. Vauclair, *Astron. Astrophys.* **375**, 86 (2001).
38. M.H. Pinsonneault et al., *Astrophys. J.* **574**, 411 (2002).
39. E. Vangioni-Flam, et al., *New Astronomy* **4**, 245 (1999).
40. P.D. Parker, *Astrophys. J.* **175**, 261 (1972).
41. R.W. Kavanagh, *Nucl. Phys.* **18**, 492 (1960).
42. F. Ajzenberg-Selove, *Nucl. Phys.* bf A490, (1988) 1 and TUNL Nuclear Data Evaluation Project, http://www.tunl.duke.edu/nucldata/fas/88AJ01.shtml.
43. C. Angulo, S. Engstler, G. Raimann, C. Rolfs, W.H. Schulte, and E. Somorjai, *Z. Phys.* **A345**, 231 (1993).
44. K.M. Nollett and R.E. Lopez, *Phys. Rev.* **D66**, 063507 (2002).
45. P. Salati, *Phys. Lett.* **B571**, 121 (2003).
46. A. Navarro, A. Serna and J.-M Alimi, Classical and Quantum Gravity, **19**, 4361 (2002).
47. M. Orito, T. Kajino, G.J. Mathews and Y. Wang, *Phys. Rev.*, **D65** 123504 (2002).
48. P. Giommi, S. Colafrancesco, submitted to *Astron. Astrophys.*, ArXiv:astro-ph/0306206.
49. T. Montmerle, *Astrophys. J.* **216**, 620 (1977).