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Are $B \to \pi K$ CP-asymmetries quantized ?

Jean-Marc Gérard[1] and Christopher Smith[2]

Institut de Physique Théorique, Université catholique de Louvain
Chemin du Cyclotron, 2, B-1348, Louvain-la-Neuve, Belgium

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Abstract

Search for patterns in the numerous B-decay modes now available is necessary in order to test the Cabibbo-Kobayashi-Maskawa theory of CP-violation. In particular, the well-structured pattern of $B \to \pi K$ branching ratios may lead to a quantized spectrum for direct CP-asymmetries, providing in this way a rather unique opportunity to discriminate between hadronic final state interaction models.

[1]gerard@fyma.ucl.ac.be
[2]smith@fyma.ucl.ac.be

1 Introduction

In 2001, soon after the starting of two dedicated B-factories, CP-violation was observed for the first time in B-meson decays, more precisely in the interference of mixing and decay of $B^0 - \bar{B}^0$ states, by the BaBar Collaboration at SLAC [1] and the Belle Collaboration at KEK [2]. This rather spectacular result definitely promoted the Cabibbo-Kobayashi-Maskawa (CKM) parametrization [3] to the first theory of microscopic irreversibility.

In 1999, direct CP-violation was eventually established in K-meson decays by two other experiments (NA48 at CERN [4] and KTeV at Fermilab [5]), after tremendous efforts. Today, the ε'/ε parameter is accurately known experimentally, but is still of no use to constrain the unitary CKM mixing matrix.

Strong dynamics at low scale is mainly responsible for this paradoxical situation in weak decay physics. In particular, the so-called $\Delta I = 1/2$ rule leads to large hadronic uncertainties in $K \to \pi\pi$ decay amplitudes. At the B-mass scale, we expect genuine patterns to emerge for branching ratios of two-body weak decay processes related by hadronic flavor symmetries such as isospin.

So, the future of B-meson phenomenology may look bright in view of the opening of so many decay channels. However, if we want to shed some new light on CP-violation, we have somehow to rely on a quark-gluon picture. But the Operator Product Expansion methodology inevitably involves many hadronic matrix elements. Moreover, present technologies within QCD do not provide rigorous predictions on the CP-conserving strong phases induced by final state interactions (FSI) at the finite B-mass scale. Consequently, we usually have to rely on specific models for hadron dynamics to probe the CKM mixing matrix through our beloved direct CP-asymmetries.

Many bounds on the CKM angle γ from $B \to \pi K$ decay rates and CP-asymmetries have been derived since five years [6]. However, they all require some assumption on the origin and size of the CP-conserving strong phases involved. In this letter, we would like to illustrate, without theoretical prejudice beyond isospin invariance, how the observed $B \to \pi K$ decay modes might become soon a reliable test for the rather simple though controversial SU(2)-elasticity and on-shell $c\bar{c}$ rescattering hypotheses.

2 Quantized CP-asymmetries in $K \to \pi\pi$

The measured exclusive semi-leptonic decay width ratio [7]

$$\frac{\Gamma\left(K^+ \to e^+\pi^0\nu_e\right)}{\Gamma\left(K^0 \to e^+\pi^-\nu_e\right)} \simeq 0.52 \tag{1}$$

is very close to the value expected from isospin symmetry. A factor 1/2 originates indeed from the $1/\sqrt{2}$ Clebsch-Gordan coefficient for π^0. So, the large K^+ lifetime observed [7]

$$\frac{\tau\left(K^+\right)}{\tau\left(K^0\right)} \simeq 70 \tag{2}$$

requires a dominance of the $\Delta I = 1/2$ amplitude A_0 over the $\Delta I = 3/2$ amplitude A_2:

$$\left|\frac{A_2}{A_0}\right|^{\exp} \approx \frac{1}{22} \tag{3}$$

if we adopt the standard parametrization for the hadronic K-decay amplitudes

$$\begin{aligned} A\left(K^{+}\rightarrow\pi^{0}\pi^{+}\right) &= \sqrt{\frac{3}{2}}A_{2}e^{i\delta_{2}} \\ A\left(K^{0}\rightarrow\pi^{-}\pi^{+}\right) &= \sqrt{\frac{2}{3}}A_{0}e^{i\delta_{0}}+\sqrt{\frac{1}{3}}A_{2}e^{i\delta_{2}} \\ A\left(K^{0}\rightarrow\pi^{0}\pi^{0}\right) &= \sqrt{\frac{1}{3}}A_{0}e^{i\delta_{0}}-\sqrt{\frac{2}{3}}A_{2}e^{i\delta_{2}}\ . \end{aligned} \tag{4}$$

The CKM theory for CP-violation requires a link between the hadron world and its quark-gluon representation. In the free-quark approximation, the effective $\Delta S=1$ weak Hamiltonian for a semi-leptonic or hadronic decay simply factorizes into two currents. However, hadronic decays also involve non-factorizable gluon exchanges between these currents.

In particular, factorizable (F) quark diagrams easily explain Eq.(1) but are unable, alone, to produce the $\pi^0\pi^0$ final state (see Fig.1a). Hence

$$\left(\frac{A_2}{A_0}\right)^{F}=\frac{1}{\sqrt{2}} \tag{5}$$

and this neutral state is only reachable through either $\pi^-\pi^+$ hadronic rescattering ($\delta_0\neq\delta_2$) or non-factorizable (NF) quark diagrams (see Fig.1b).

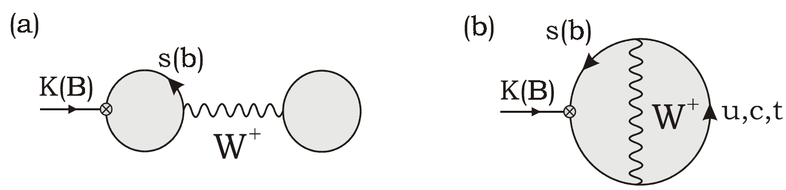

*Fig.*1 To first order in G_F and to all orders in α_S: (a) factorizable and (b) non-factorizable $\Delta S\,(\Delta B)=1$ quark diagrams, before final state hadronization.

Nowadays, we are convinced that strong dynamics is indeed fully responsible for the empirical $\Delta I=1/2$ rule [7]

$$\begin{aligned} \frac{Br\left(K_S\rightarrow\pi^0\pi^0\right)}{Br\left(K_S\rightarrow\pi^-\pi^+\right)} &= 0.458\pm0.004 \\ &\approx \frac{1}{2}\left\{1-3\sqrt{2}\,\mathrm{Re}\left(\frac{A_2}{A_0}\right)\cos\left(\delta_2-\delta_0\right)\right\} \end{aligned} \tag{6}$$

and provides the necessary ingredients to generate non-zero $K\rightarrow\pi\pi$ CP-asymmetries

$$A_{CP}^{K}\left(\pi\pi\right)\equiv\frac{\Gamma\left(\bar{K}\rightarrow\pi\pi\right)-\Gamma\left(K\rightarrow\pi\pi\right)}{\Gamma\left(\bar{K}\rightarrow\pi\pi\right)+\Gamma\left(K\rightarrow\pi\pi\right)}\ \div\ \mathrm{Im}\left(\frac{A_2}{A_0}\right)\sin\left(\delta_2-\delta_0\right) \tag{7}$$

within the Standard Model. The CP-conserving δ_I can be extracted from $\pi\pi$-scattering data at the K-mass scale. On the other hand, the CP-violating prefactor is now under control thanks to the recently measured direct CP-violation parameter ([4], [5])

$$\varepsilon'=\frac{i}{\sqrt{2}}\,\mathrm{Im}\left(\frac{A_2}{A_0}\right)e^{i\left(\delta_2-\delta_0\right)}\ . \tag{8}$$

So the CP-asymmetries defined in Eq.(7) are calculable today. However, the hadronic parametrization introduced in Eqs.(4) was already sufficient to predict (long time ago) what we will henceforth call quantized CP-asymmetries

$$\frac{A_{CP}^{K}\left(\pi^{0}\pi^{0}\right)}{A_{CP}^{K}\left(\pi^{-}\pi^{+}\right)}=-2 \tag{9}$$

in the phenomenological limit ($A_0 >> A_2$) of structured hadronic K^0-decay branching ratios

$$\frac{Br\left(\pi^{0}\pi^{0}\right)}{Br\left(\pi^{-}\pi^{+}\right)}=\frac{1}{2}\,. \tag{10}$$

As we shall see, this seemly academic exercise turns out to be fruitful in B-physics where numerous decay modes are now available.

3 Unquantized CP-asymmetries in $B \to \pi\pi$

The isospin decomposition of the hadronic B-decays into two pions is identical to the one already given in Eqs.(4). However, comparison of the inclusive lifetime ratio [7]

$$\frac{\tau\left(B^{+}\right)}{\tau\left(B^{0}\right)}=1.08\pm0.02 \tag{11}$$

with Eq.(2) strongly suggests that non-factorizable $\Delta B = 1$ quark diagrams (see Fig.1 with b substituted for s) are much less efficient at the B-mass scale. The present B^+/B^0 pattern displayed by exclusive $B \to \pi\pi$ branching ratios ([8], [9], [10]):

$$\begin{aligned} Br\left(B^{+}\to\pi^{0}\pi^{+}\right)&=(5.8\pm1.0)\times10^{-6}\\ Br\left(B^{0}\to\pi^{-}\pi^{+}\right)&=(4.7\pm0.5)\times10^{-6}\\ Br\left(B^{0}\to\pi^{0}\pi^{0}\right)&=(2.0\pm0.7)\times10^{-6} \end{aligned} \tag{12}$$

tends to confirm this expectation, though no real structure emerges yet. In particular, the rather large central value for the $\pi^0\pi^+$ mode definitely excludes a new $\Delta I = 1/2$ rule. On the other hand, the surprisingly large central value for the $\pi^0\pi^0$ mode apparently requires a sizeable CP-conserving phase $(\delta_2 - \delta_0)$ [11] to dilute the fully factorized hierarchy (see Eq.(5)) predicted by Fig.1a. More precise measurements might of course change the whole pattern in $B \to \pi\pi$. However, at this point we should also seriously question the validity of the parametrization (4) used before arguing for large, unquantized CP-asymmetries. Let us therefore briefly recall what the theoretical assumptions behind it are.

Assuming CPT-invariance, the **S**-matrix for the $P^0 \to \pi\pi$ multiplet of decay amplitudes

$$W=\begin{pmatrix} P^{0}\to\pi^{-}\pi^{+}\\ P^{0}\to\pi^{0}\pi^{0}\end{pmatrix} \tag{13}$$

reads

$$\mathbf{S}=\begin{pmatrix} 1 & iW^{t}\\ iCP(W) & S\end{pmatrix} \tag{14}$$

with

$$S=\left(\begin{array}{cc}\pi^-\pi^+\to\pi^-\pi^+ & \pi^-\pi^+\to\pi^0\pi^0\\ \pi^0\pi^0\to\pi^-\pi^+ & \pi^0\pi^0\to\pi^0\pi^0\end{array}\right). \tag{15}$$

Imposing then the unitarity of $\mathbf{S}$, i.e. $\mathbf{S}^\dagger\mathbf{S}=\mathbf{S}\mathbf{S}^\dagger=\mathbf{1}$, for this subset of final states, we obtain (see [12] for more details) the "generalized Watson theorem" [13]

$$\begin{array}{c}W=\sqrt{S}W_b\\ CP\left(W\right)=\sqrt{S}W_b^*\end{array} \tag{16}$$

with W_b the bare amplitudes denoted as

$$W_b=\left(\begin{array}{c}P^0\to\{\pi^-\pi^+\}\\ P^0\to\left\{\pi^0\pi^0\right\}\end{array}\right). \tag{17}$$

Because bare amplitudes simply get complex conjugated under CP (see Eqs.(16)), they obviously do not contain any strong phase. In other words, the FSI effects are contained in $\sqrt{S}$ and factorize. Consequently, the bare amplitudes are real (up to CKM factors) and arise from the quark diagrams projected on specific light hadronic states.

Our restriction to isospin-multiplets is called the SU(2)-elastic hypothesis since the unitarity of the $\mathbf{S}$-matrix implies then the probability conservation:

$$|W|^2=\left|\sqrt{S}W_b\right|^2=|W_b|^2\ . \tag{18}$$

The isospin symmetry relates the bare final states to the isospin states

$$\left(\begin{array}{c}|0,0\rangle\\ |2,0\rangle\end{array}\right)=\underbrace{\left(\begin{array}{cc}\sqrt{2/3} & \sqrt{1/3}\\ \sqrt{1/3} & -\sqrt{2/3}\end{array}\right)}_{O_{SU(2)}}\left(\begin{array}{c}\{\pi^-\pi^+\}\\ \left\{\pi^0\pi^0\right\}\end{array}\right). \tag{19}$$

In the isospin state basis, the rescattering matrix S is diagonal

$$S_{diag}=\left(\begin{array}{cc}e^{2i\delta_0} & 0\\ 0 & e^{2i\delta_2}\end{array}\right) \tag{20}$$

such that

$$\begin{aligned}W&=\sqrt{S}.O^t_{SU(2)}.\left(\begin{array}{c}A_0\\ A_2\end{array}\right)\\ &=O^t_{SU(2)}.\sqrt{S_{diag}}.\left(\begin{array}{c}A_0\\ A_2\end{array}\right)\end{aligned} \tag{21}$$

and we recover the standard parametrization (4).

SU(2)-elasticity follows from imposing a bloc-diagonal form for S, each bloc corresponding to an isospin multiplet. This assumption is obviously reasonable at the K-mass scale, but certainly questionable at the B-mass scale where so many channels are open. Systematic cancellations among many final state rescatterings cannot be excluded and theoretical estimates based on Regge theory tend to support this picture at the D-mass scale (see [14]). For $B\to\pi\pi$ decays, this approach predicts a rather small phase shift such that large non-factorizable contributions (see Fig.1b) are needed to enhance the $\pi^0\pi^0$ mode. Waiting eagerly on better precision measurements for this mode, we now would like to argue that the $B\to\pi K$ decays already provide us with a very interesting laboratory to test (and also extend) the phenomenological SU(2)-elasticity assumption.

4 Quantized CP-asymmetries in $B \to \pi K$

In $B \to \pi K$ decays, the well-established CKM mixing hierarchy may supply for strong dynamics to ensure an effective $\Delta I = 0$ rule. Indeed, charm and top quark contributions in Fig.1b are double-Cabibbo-enhanced

$$(\sin\theta_c)^2 \sim \frac{1}{20} \tag{22}$$

compared to the other non-factorizable and factorizable diagrams. In that sense, the $B \to \pi K$ system is similar to the $K \to \pi\pi$ one (see Eq.(3)) and we easily understand the measured branching ratios ([8], [9], [10])

$$\begin{aligned} Br\left(B^+ \to \pi^0 K^+\right) &= (12.7 \pm 1.2) \times 10^{-6} \\ Br\left(B^+ \to \pi^+ K^0\right) &= (18.1 \pm 1.7) \times 10^{-6} \\ Br\left(B^0 \to \pi^0 K^0\right) &= (10.2 \pm 1.5) \times 10^{-6} \\ Br\left(B^0 \to \pi^- K^+\right) &= (18.5 \pm 1.0) \times 10^{-6} \end{aligned} \tag{23}$$

which clearly exhibit a structured pattern with 1/2 factors originating from the $\pm 1/\sqrt{2}$ Clebsch-Gordan coefficients for π^0.

4.1 SU(2)-elasticity

Encouraged by such a $\Delta I = 0$ rule, let us first proceed as for $K \to \pi\pi$ and assume

$$\pi K \rightleftharpoons \pi K \tag{24}$$

SU(2)-elastic rescatterings. We then have the following hadronic decay amplitudes:

$$\begin{aligned} A^{0+} &= \sqrt{\frac{1}{3}}\left(A'_{1/2} + A_{1/2}\right) e^{i\delta_{1/2}} + \sqrt{\frac{2}{3}} A_{3/2} e^{i\delta_{3/2}} \\ A^{+0} &= \sqrt{\frac{2}{3}}\left(A'_{1/2} + A_{1/2}\right) e^{i\delta_{1/2}} - \sqrt{\frac{1}{3}} A_{3/2} e^{i\delta_{3/2}} \\ A^{00} &= -\sqrt{\frac{1}{3}}\left(A'_{1/2} - A_{1/2}\right) e^{i\delta_{1/2}} + \sqrt{\frac{2}{3}} A_{3/2} e^{i\delta_{3/2}} \\ A^{-+} &= \sqrt{\frac{2}{3}}\left(A'_{1/2} - A_{1/2}\right) e^{i\delta_{1/2}} + \sqrt{\frac{1}{3}} A_{3/2} e^{i\delta_{3/2}} \end{aligned} \tag{25}$$

with $A'_{1/2}$ $(A_{1/2}, A_{3/2})$ the reduced matrix elements of the isosinglet (isotriplet) weak Hamiltonian, respectively [15].

Straightforward projections on appropriate color-singlet states link then the $A'_{1/2}$ bare amplitude to the Cabibbo-enhanced non-factorizable quark diagrams (see Fig.2a).

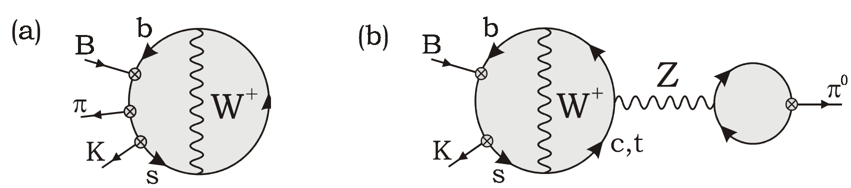

*Fig.*2 Double-Cabibbo-enhanced $B \to \{\pi K\}$ decay diagrams:
(a) non-factorizable and (b) hybrid.

Notice that the second order "hybrid" diagrams shown in Fig.2b only produce a neutral pion in the final state. We conclude from Eqs.(25) with $\delta_I = 0$ that they contribute to the $A_{1/2}$ and $A_{3/2}$ bare amplitudes, but not to $A'_{1/2}$.

In the phenomenological limit ($A'_{1/2} >> A_{1/2}, A_{3/2}$) of perfectly structured branching ratios, the direct CP-asymmetries

$$A^B_{CP}(\pi K) \equiv \frac{\Gamma\left(\bar{B} \to \pi \bar{K}\right) - \Gamma\left(B \to \pi K\right)}{\Gamma\left(\bar{B} \to \pi \bar{K}\right) + \Gamma\left(B \to \pi K\right)} \doteq \mathrm{Im}\left(\frac{A_{3/2}}{A'_{1/2}}\right) \sin\left(\delta_{3/2} - \delta_{1/2}\right) \tag{26}$$

derived from Eqs.(25) are quantized in a way similar to Eq.(9):

$$A^B_{CP}\left(\pi^0 K^+\right) : A^B_{CP}\left(\pi^+ K^0\right) : A^B_{CP}\left(\pi^0 K^0\right) : A^B_{CP}\left(\pi^- K^+\right) = +2 : -1 : -2 : +1 \,. \tag{27}$$

Such a simple pattern is certainly not excluded by present preliminary data from CLEO, Belle and BaBar experiments, whose current combined averages are ([8], [9], [10])

$$\begin{aligned} A^B_{CP}\left(B^+ \to \pi^0 K^+\right) &= -0.10 \pm 0.08 \\ A^B_{CP}\left(B^+ \to \pi^+ K^0\right) &= +0.05 \pm 0.08 \\ A^B_{CP}\left(B^0 \to \pi^0 K^0\right) &= +0.03 \pm 0.37 \\ A^B_{CP}\left(B^0 \to \pi^- K^+\right) &= -0.08 \pm 0.04 \,. \end{aligned} \tag{28}$$

The observation of a large departure from the quantized pattern Eq.(27) would indicate sizeable FSI effects beyond SU(2)-elasticity. Let us therefore anticipate such a possibility by illustrating how far one can go in that direction without invoking intricate fits.

4.2 Beyond SU(2)-elasticity

As we already said, SU(2)-elasticity requires a bloc-diagonal form for $\sqrt{S}$ such that states belonging to different iso-multiplets do not communicate. From a phenomenological point of view, it is interesting to have a formalism in hand allowing for more general rescatterings. So, let us assume that B-mesons can also decay into some other unsuppressed $I = 1/2$ bare mode $\{XY\}$ (see Fig.3) which then rescatters into the physical πK states.

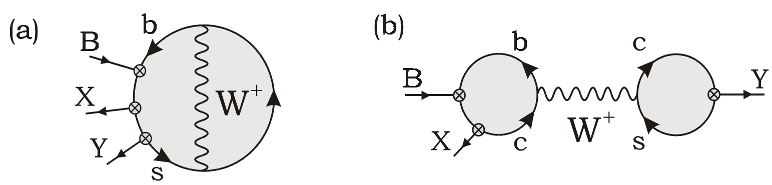

*Fig.*3 Cabibbo-favored $B \to \{XY\}$ decay diagrams:
(a) non-factorizable and (b) factorizable.

In that case, we have to enlarge the isospin content such that

$$\begin{pmatrix} |1/2', \pm 1/2\rangle \\ |1/2, \pm 1/2\rangle \\ |3/2, \pm 1/2\rangle \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & \pm\sqrt{1/3} & \sqrt{2/3} \\ 0 & \sqrt{2/3} & \mp\sqrt{1/3} \end{pmatrix}}_{O^{\overset{+}{0}}_{SU(2)}} \begin{pmatrix} \{XY\}^{\overset{+}{0}} \\ \{\pi^0 K^{\overset{+}{0}}\} \\ \{\pi^{\pm} K^{\overset{0}{+}}\} \end{pmatrix} . \tag{29}$$

Without breaking SU(2), we can introduce new rescattering channels by mixing the two 1/2 representations (see [12])

$$\left(\begin{array}{c} |C_1\,(1/2,\pm 1/2)\rangle \\ |C_2\,(1/2,\pm 1/2)\rangle \\ |C_3\,(3/2,\pm 1/2)\rangle \end{array}\right) = \underbrace{\left(\begin{array}{ccc} \cos\chi & \sin\chi & 0 \\ -\sin\chi & \cos\chi & 0 \\ 0 & 0 & 1 \end{array}\right)}_{O_\chi} \left(\begin{array}{c} |1/2',\pm 1/2\rangle \\ |1/2,\pm 1/2\rangle \\ |3/2,\pm 1/2\rangle \end{array}\right) . \tag{30}$$

We then obtain

$$\begin{aligned} W^{\stackrel{+}{0}} &= \sqrt{S_\chi^{\stackrel{+}{0}}}.O^{\stackrel{+}{0}\,t}_{SU(2)}. \left(\begin{array}{c} B^{\pm}_{1/2} \\ A'_{1/2} \pm A_{1/2} \\ A_{3/2} \end{array}\right) \\ &= O^{\stackrel{+}{0}\,t}_{SU(2)}.O^t_\chi.\sqrt{S_{diag}}.O_\chi. \left(\begin{array}{c} B^{\pm}_{1/2} \\ A'_{1/2} \pm A_{1/2} \\ A_{3/2} \end{array}\right) \end{aligned} \tag{31}$$

since the rescattering matrix is now diagonal in the C_i-state basis:

$$S_{diag} = \left(\begin{array}{ccc} e^{2i\delta_{C_1}} & 0 & 0 \\ 0 & e^{2i\delta_{C_2}} & 0 \\ 0 & 0 & e^{2i\delta_{C_3}} \end{array}\right) . \tag{32}$$

Let us emphasize that the rescattering remains elastic with respect to the full set of states $(\{XY\}, \{\pi K\})$ since $\sqrt{S_\chi}$ is unitary.

If $\chi = 0$, we recover the standard SU(2)-elastic parametrization given in Eqs.(25), with the identifications $\delta_{C_2} = \delta_{1/2}$ and $\delta_{C_3} = \delta_{3/2}$. On the other hand, if χ is an arbitrary angle, we are in general left with two unrelated CP-conserving phase differences and, consequently, with unquantized CP-asymmetries. However, $B \to \pi K$ quantized CP-asymmetries are recovered in two special limits for the mixing angle χ.

4.2.1 Large mixing angle

Let us first consider the case where $\{\pi K\}$ and $\{XY\}$ are generated by hadronization of the same (Cabibbo-enhanced) non-factorizable quark diagrams (see Fig.2a and Fig.3a, respectively). The corresponding bare amplitudes $(B^{\pm}_{1/2})^{NF}$ are of the same order as $(A'_{1/2})^{NF}$. In the SU(3) limit (see [12]), this amounts to consider

$$\left\{ \begin{array}{l} \pi K \rightleftharpoons \pi K \\ \eta_8 K \rightleftharpoons \pi K \end{array} \right. \tag{33}$$

elastic rescatterings with a large mixing

$$\tan\chi = -3 \tag{34}$$

and only two independent eigenphases:

$$\sqrt{S_{diag}} = diag\left(e^{i\delta_8}, e^{i\delta_{27}}, e^{i\delta_{27}}\right) . \tag{35}$$

The resulting CP-asymmetries proportional to $\sin(\delta_{27}-\delta_8)$ result from interferences between NF and F bare amplitudes and obey the following quantization pattern

$$A^B_{CP}\left(\pi^0 K^+\right) : A^B_{CP}\left(\pi^+ K^0\right) : A^B_{CP}\left(\pi^0 K^0\right) : A^B_{CP}\left(\pi^- K^+\right) = +2 : -\frac{1}{2} : -\frac{3}{2} : +1\ . \quad (36)$$

Notice that this SU(3)-elastic pattern is not very different from the SU(2)-elastic one given in Eq.(27). However, we know that the $\eta_8 - \eta_0$ mixing is not negligible. New disconnected quark diagrams have to be taken into account and the observed $B \to \eta' K$ branching ratios are fairly large. So this rather academic exercise is principally a consistency check of the rescattering formalism presented here to go beyond SU(2)-elastic FSI (see Eq.(31)).

4.2.2 Small mixing angle

Let us turn to the more interesting case where $\{X_{\bar{c}} Y_c\}$ is generated by hadronization of the (Cabibbo-favored) factorizable quark diagram (see Fig.3b). Now, the corresponding bare amplitude $\left(B_{1/2}\right)^F$ dominates over $(A'_{1/2})^{NF}$ (with $B_{1/2} \equiv B^+_{1/2} = B^-_{1/2}$). For illustration, if we consider

$$\bar{D} D_s \rightleftharpoons \pi K \quad (37)$$

rescatterings, the $B \to \bar{D} D_s$ branching ratios are indeed of the order of 1% [16]. On the other hand, we expect the mixing angle χ to be rather small since $\left\{\bar{D} D_s\right\} \to \pi K$ can only proceed through the annihilation of the $c\bar{c}$ pair into a light $q\bar{q}$ pair in a semi-inclusive picture. Therefore, the three independent eigenphases should approximately be those of the isospin basis, i.e.

$$\sqrt{S_{diag}} \approx diag\left(e^{i\delta^D_{1/2}}, e^{i\delta_{1/2}}, e^{i\delta_{3/2}}\right)\ . \quad (38)$$

Notice that the SU(4) symmetry is <u>not</u> invoked in any sense, seeing that χ is now assumed very small (compare with Eq.(34)). We have in fact what we may call an <u>enlarged</u> SU(2)-elasticity since charmed mesons decouple and CP-asymmetries obey the SU(2)-elastic quantization pattern (27) in the limit $\chi = 0$.

In the complementary limit $\delta_{3/2} - \delta_{1/2} = 0$, charmed meson rescatterings dominate and the CP-asymmetries proportional to $\sin(2\chi)\sin(\delta^D_{1/2} - \delta_{1/2})$ result from interferences between $\left(B_{1/2}\right)^F$ and $(A'_{1/2}, A_{1/2}, A_{3/2})^F$ bare amplitudes (to leading order in the CKM mixing, the amplitudes $(B_{1/2})^F$ and $(A'_{1/2})^{NF}$ have no relative CP-violating phase). But factorizable quark diagrams cannot produce a neutral K^0 in the final state (see Fig.1a). Consequently, the CP-asymmetries obey a new quantization pattern:

$$A^B_{CP}\left(\pi^0 K^+\right) : A^B_{CP}\left(\pi^+ K^0\right) : A^B_{CP}\left(\pi^0 K^0\right) : A^B_{CP}\left(\pi^- K^+\right) = +1 : 0 : 0 : +1 \quad (39)$$

if, again, $B \to \pi K$ branching ratios are taken perfectly structured.

The enlarged isospin-invariant formalism based on Eq.(31) allows us to treat pure SU(2)-elastic and $\bar{D} D_s$ rescatterings on an equal footing. A thorough analysis of $B \to \pi K$ combining these rescattering processes will be presented elsewhere. For now, let us just emphasize that the effects of $\bar{D} D_s \rightleftharpoons \pi K$ can be factorized and absorbed into a redefinition of the isospin amplitudes appearing in Eqs.(25). Using $\sin\chi << 1$, Eq.(31) is indeed equivalent to Eqs.(25) if the bare amplitude $A'_{1/2}$ is replaced by an effective decay amplitude including now a CP-conserving strong phase:

$$A'_{1/2} \to \left(A'_{1/2}\right)^{eff} = A'_{1/2} - \left(1 - e^{i(\delta^D_{1/2} - \delta_{1/2})}\right) \chi B_{1/2} \; . \tag{40}$$

It is of course tempting to interpret Eq.(40) as the hadronic representation for intermediate on-shell $c\bar{c}$ rescatterings (see Fig.2a). In this way, the duality correspondence between the quark-level picture and the hadron-level picture already advocated [17] for (semi-) inclusive processes is implemented in the specific case of $B \to \pi K$ exclusive decays.

5 Conclusion

If the B meson were infinitely heavy, one would argue [18] that there would be no time for the final hadrons to rescatter and all the direct CP-asymmetries would simply vanish.

For a finite B mass, the $\bar{D}D_s \rightleftharpoons \pi K$ elastic scheme represents an attempt to isolate the dominant (heavy) intermediate contributions since the measured $B \to \bar{D}D_s$ branching ratios are of the order of 1%.

On the other hand, the simple SU(2) and SU(3)-elastic approximations are more difficult to justify because of the large energy release in B decays. The πK and $\eta_8 K$ states are indeed only a small subset of all possible (light) intermediate states. However, it might be worth recalling that these approximations do not assert that each specific inelastic channel is small, but only assume that the average over all inelastic channels still vanishes when the B mass is taken finite. As a matter of fact, the recent isospin analysis of the measured $B \to D\pi$ decays [19] does not contradict this hypothesis.

The present experimental data do not allow us to exclude one of these hadronic models for final state interactions. Consequently, we propose a phenomenological test without theoretical prejudice beyond isospin invariance.

In the phenomenologically reasonable limit of perfectly structured $B \to \pi K$ branching ratios, we have obtained three possible sets of quantized CP-asymmetries displayed in Table 1.

	BR	CP-asymmetries		
		SU(2)-elastic	SU(3)-elastic	On-shell $c\bar{c}$
		$\pi K \rightleftharpoons \pi K$	$\pi K \rightleftharpoons \pi K$ $\eta_8 K \rightleftharpoons \pi K$	$\bar{D}D_s \rightleftharpoons \pi K$
$B^+ \to \pi^0 K^+$	1/2	+2	+2	+1
$B^+ \to \pi^+ K^0$	1	−1	−1/2	0
$B^0 \to \pi^0 K^0$	1/2	−2	−3/2	0
$B^0 \to \pi^- K^+$	1	+1	+1	+1

Table 1 Structured branching ratios and quantized CP-asymmetries normalized to the $B^0 \to \pi^- K^+$ mode.

We do not really expect that one of these "ideal" quantization patterns for CP-asymmetries will eventually emerge from future measurements. In particular, $B \to \pi K$ branching ratios are not perfectly structured since Cabibbo-suppressed factorizable quark diagrams (see Fig.1a) and second-order hybrid quark diagrams (see Fig.2b) do also contribute to them. However, their relative weight can only be determined through SU(3) arguments which are in principle beyond the scope of the present work. Yet, extracting the factorizable contribution from the available $B \to \pi\pi$ data, we checked numerically that corrections to Table 1 cannot exceed 20% in the full flavor-SU(3) limit, if the CKM angle γ is larger or equal to 60°.

To our surprise, relative signs between CP-asymmetries are <u>not</u> sufficient to exclude models for FSI (see Table 1). Nevertheless, we hope that more precise measurements of CP-asymmetries in $B \to \pi K$ decays will allow us to discriminate soon between the various possible scenarii for final state interaction effects discussed in the present paper. If such is the case, the CKM angle γ will be directly accessible and confidently confronted with its best-fit value extracted from the standard unitary triangle.

We also hope that our treatment of intermediate charm at the hadronic level will be of some use for the forthcoming global fits on B-decays into two light pseudoscalars [20].

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