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A Dynamical Scheme for a Large CP-Violating Phase

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Abstract

A dynamical scheme where the third generation of quarks plays a distinctive role is implemented. New interactions with a θ term induce the breaking of the electroweak symmetry and the top-bottom mass splitting. A large CP-violating phase naturally follows from the latter.

1 Introduction

Among the outstanding problems in particle physics are the mechanism for the electroweak symmetry breaking and the closely related question of the origin of the quark masses. The fact that the top quark is very heavy compared to the other quarks, $m_t = 180 \pm 12$ GeV [1], suggests that the third generation may be playing a special role in the dynamics at the electroweak scale. In particular, new strong interactions might exist at this scale which not only distinguish the third generation from the others but also intimately participate in the breaking of the electroweak symmetry. To implement this scenario one may consider effective four-fermion interactions [2]. They can lead to the formation of quark-antiquark bound states which in turn trigger dynamically the breaking of the electroweak symmetry [3, 4].

More than thirty years after its discovery in the $K^0 - \bar{K}^0$ system, it is fair to say that the mechanism for CP violation is not yet understood. But there is little doubt that CP violation is deeply rooted in the peculiar mass spectrum of the quarks. Nowadays, the familiar puzzles related to an unexpectedly heavy top quark and to small mixing angles in the Cabibbo-Kobayashi-Maskawa (CKM) matrix [5] have been turned around into questions concerning tiny quark masses and a large CP-violating phase. The main purpose of this paper is to suggest a dynamical framework where these "new" points of view are connected.

Two approaches are usually considered when introducing CP violation into gauge models: CP-violating phases can be explicit in the parameters of the theory or alternatively, they can be spontaneously generated through complex vacuum expectation values (VEVs) of the scalar Higgs fields. In the standard model with complex parameters in the Lagrangian, only two phases remain physical after field redefinitions, namely, the QCD-induced phase θ_{QCD} and the phase δ in the CKM matrix. The QCD phase will not concern us here [6].

In this paper, our starting assumption will be that the third generation of quarks does indeed experience new forces (symmetric in t and b) and that these new forces also generate a θ term. We then propose a model where this θ term breaks the symmetry between t and b and induces naturally a large δ phase due to the smallness of the m_b/m_t mass ratio.

2 The model

We consider a standard model Higgs sector in combination with an effective new strong interaction acting on the third generation of quarks and characterized by a θ term [7]. We require this new strong interaction to conserve the isospin symmetry between t and b quarks.

Since the electroweak symmetry breaking will eventually be induced by radiative corrections [8] due to top-quark (and possibly, bottom-quark) loops, we may as well neglect the quartic self-interactions of the Higgs field. In this case, the relevant classical Lagrangian for the fundamental scalar field H is given by

$$L_H = D_\mu H^\dagger D^\mu H - m_H^2 H^\dagger H + \left(h_t \bar{\psi}_L t_R H + h_b \bar{\psi}_L b_R \tilde{H} \; + \; \text{h.c.} \right), \tag{1}$$

where $H = \begin{pmatrix} H^0 \\ H^- \end{pmatrix}$, $\tilde{H} = \begin{pmatrix} H^+ \\ -H^{0*} \end{pmatrix}$ and $\psi_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix}$; h_t and h_b are the Yukawa couplings and D_μ is the usual covariant derivative of the standard model.

Next we assume that the interactions acting on the members of the third generation of quarks are strong enough to form $q\bar{q}$ bound states at the electroweak scale. We shall describe the latter in terms of two complex doublet scalar fields

$$\Sigma_t = \begin{pmatrix} \Sigma_t^0 \\ \Sigma_t^- \end{pmatrix} \sim t_R \bar{\psi}_L \; , \qquad \tilde{\Sigma}_b = \begin{pmatrix} \Sigma_b^+ \\ -\Sigma_b^{0*} \end{pmatrix} \sim b_R \bar{\psi}_L \tag{2}$$

and the corresponding effective Lagrangian then reads:

$$L_\Sigma = D_\mu \Sigma_t^\dagger D^\mu \Sigma_t + D_\mu \Sigma_b^\dagger D^\mu \Sigma_b - m^2 (\Sigma_t^\dagger \Sigma_t + \Sigma_b^\dagger \Sigma_b) + g(\bar{\psi}_L t_R \Sigma_t + \bar{\psi}_L b_R \tilde{\Sigma}_b + \text{h.c.}) \; . \tag{3}$$

From Eqs.(1) and (3) it follows that the top and bottom field-dependent masses are given at the tree level by the linear combinations

$$M_t = h_t H^0 + g \Sigma_t^0, \qquad M_b = h_b \tilde{H}^0 + g \tilde{\Sigma}_b^0, \tag{4}$$

respectively (at this stage, mixing with the two light generations are of course neglected).

In the framework of this effective theory, effects of a (new) θ term can, in principle, be described via an arbitrary function of $\det U$, where

$$U \sim \begin{pmatrix} \bar{t}_L t_R & \bar{t}_L b_R \\ \bar{b}_L t_R & \bar{b}_L b_R \end{pmatrix}, \tag{5}$$

or in terms of the composite fields defined in Eq.(2)

$$U = \begin{pmatrix} \Sigma_t^0 & \Sigma_b^- \\ \Sigma_t^+ & -\Sigma_b^{0*} \end{pmatrix}. \tag{6}$$

In analogy with QCD [9], we shall take the Lagrangian form

$$L_\theta = -\frac{\alpha}{4} \left[i \text{Tr} \; \left(\ln U - \ln U^\dagger \right) + 2\theta \right]^2, \tag{7}$$

which typically arises as a leading term in a $1/N$ - expansion.

The total effective Lagrangian of our model is thus given by

$$L = L_H + L_\Sigma + L_\theta \; , \tag{8}$$

with L_H, L_Σ and L_θ defined in Eqs.(1),(3) and (7), respectively. We notice that if $h_t = h_b$ the total Lagrangian (8) conserves an "isospin" symmetry. As we shall see below, the θ angle will provide a dynamical origin for both CP violation and isospin breaking [10], once the neutral components of the three doublets H, Σ_b and Σ_t acquire nonzero VEVs.

3 Electroweak and isospin symmetry breakings

Let us now discuss how the electroweak symmetry breaking and CP violation arise in the present model. Without loss of generality, we take the phase of the neutral Higgs field H^0 to be zero. This can always be achieved by performing a suitable electroweak gauge transformation. We write the VEVs of the neutral components of the fields in the form

$$\left\langle H^0 \right\rangle = \frac{v}{\sqrt{2}}\,, \quad \left\langle \Sigma_t^0 \right\rangle = \frac{\sigma_t}{\sqrt{2}} e^{i\varphi_t}, \quad \left\langle \Sigma_b^0 \right\rangle = \frac{\sigma_b}{\sqrt{2}} e^{i\varphi_b}. \tag{9}$$

Including the radiative corrections (induced by top and bottom quark loops), the effective potential in terms of these VEVs reads

$$V = m_H^2 \frac{v^2}{2} + \frac{m^2}{2}(\sigma_t^2 + \sigma_b^2) - \beta\left(\mu_t^2 + \mu_b^2\right) + \lambda\left(\mu_t^4 + \mu_b^4\right) + \alpha\left(\theta - \varphi_t + \varphi_b\right)^2\,, \tag{10}$$

where

$$\begin{aligned} \mu_t^2 &= |\left\langle M_t \right\rangle|^2 = \frac{1}{2}\left(h_t^2 v^2 + g^2\sigma_t^2 + 2h_t v g \sigma_t \cos\varphi_t\right), \\ \mu_b^2 &= |\left\langle M_b \right\rangle|^2 = \frac{1}{2}\left(h_b^2 v^2 + g^2\sigma_b^2 + 2h_b v g \sigma_b \cos\varphi_b\right), \end{aligned} \tag{11}$$

while β and λ are some effective quadratic and quartic couplings. In what follows we shall assume all couplings and parameters in the potential to be real and positive.

Note that the potential which leads to Eq.(10) can be viewed either as an effective renormalizable interaction or as an expansion up to quartic terms in a cut-off theory.

The extrema conditions $\frac{\partial V}{\partial v} = \frac{\partial V}{\partial \sigma_t} = \frac{\partial V}{\partial \sigma_b} = \frac{\partial V}{\partial \varphi_t} = \frac{\partial V}{\partial \varphi_b} = 0$ imply the following system of equations

$$\begin{aligned} A_H v &= g h_t I_t \sigma_t \cos\varphi_t + g h_b I_b \sigma_b \cos\varphi_b\,, \\ A_t \sigma_t &= g h_t I_t v \cos\varphi_t\,, \\ A_b \sigma_b &= g h_b I_b v \cos\varphi_b\,, \\ g h_t I_t v \sigma_t \sin\varphi_t &= -g h_b I_b v \sigma_b \sin\varphi_b = 2\alpha\left(\theta - \varphi_t + \varphi_b\right), \end{aligned} \tag{12}$$

where $(i = t, b)$

$$\begin{aligned} A_H &= m_H^2 - h_t^2 I_t - h_b^2 I_b \ , \\ A_i &= m^2 - g^2 I_i \ , \\ I_i &= \beta - 2\lambda\mu_i^2 . \end{aligned} \tag{13}$$

Notice the similarity of the last equation in Eq.(12) with the one appearing in QCD [9]. To Eqs.(11)-(13) we should add the normalization condition

$$v_0 = \sqrt{v^2 + \sigma_t^2 + \sigma_b^2} \ = 246 \text{ GeV} \ , \tag{14}$$

coming from the W boson mass.

We shall take the mass parameters m_H and m to be such that

$$m_H^2 - (h_t^2 + h_b^2)\beta > 0 \ , \ m^2 - g^2\beta > 0. \tag{15}$$

In this case A_H, A_t and A_b defined in (13) are always positive.

From Eqs.(12)-(13) follows the "gap" equation

$$\frac{A_t A_H}{h_t^2 g^2 I_t^2} = \cos^2\varphi_t + \eta \cos^2\varphi_b \equiv r \ , \tag{16}$$

where

$$\eta = \frac{A_t h_b^2 I_b^2}{A_b h_t^2 I_t^2} \ . \tag{17}$$

Moreover,

$$\sin^2\varphi_t = \frac{\sigma_b^2(1 - r + \eta)}{\eta\sigma_t^2 I_t^2/I_b^2 + \sigma_b^2} \ , \ \sin^2\varphi_b = \frac{\sigma_t^2(1 - r + \eta)}{\eta\sigma_t^2 + \sigma_b^2 I_b^2/I_t^2} \tag{18}$$

and

$$\sin 2\varphi_t = -\eta \sin 2\varphi_b \ . \tag{19}$$

If the parameter α is large ($\alpha \gg \beta m_t^2$ with m_t the physical mass of the top quark) the last equation in (12) implies the constraint

$$\theta \simeq \varphi_t - \varphi_b \ . \tag{20}$$

Furthermore, if $\theta = 0$, it is easily seen that $\varphi_t = \varphi_b = 0$ is the only solution of our equations and therefore CP is conserved. Hence, CP violation in the context of our model requires θ to be non zero.

Notice also that with $h_b = 0$ and $h_t \neq 0$ the solution to Eqs.(12) is $\sigma_b = 0$, $\theta = \varphi_t = 0$. In this case $m_b = 0$, the composite field Σ_b is superfluous and we have a CP-conserving model with an elementary Higgs boson and a top-quark condensate [11].

We now proceed to solve Eqs.(16)-(19) for the isopin symmetric case $h_t = h_b \equiv h \neq 0$.

To illustrate a particularly simple analytical solution, let us assume that $\beta \gg 2\lambda m_t^2$. Then (cf. Eq.(13)) $I_t \simeq I_b$, $A_t \simeq A_b, \eta \simeq 1$ and Eq.(19) implies $\sin 2\varphi_t = -\sin 2\varphi_b$. The latter equation has two possible solutions, namely $\varphi_t = -\varphi_b$ or $\varphi_t - \varphi_b = \pi/2$. (Of course any shift of the angles by a multiple of 2π is also a solution).

a) If $\varphi_t = -\varphi_b$, $\sigma_t = \sigma_b$ and from Eq.(11) we obtain $m_t = m_b$, which is experimentally excluded.

b) If $\varphi_t - \varphi_b = \pi/2$, Eq.(16) leads to $r = 1$ and Eqs.(18) imply

$$\sin^2\varphi_t \simeq \frac{\sigma_b^2}{\sigma_t^2+\sigma_b^2}\ ,\ \sin^2\varphi_b \simeq \frac{\sigma_t^2}{\sigma_t^2+\sigma_b^2}\ . \tag{21}$$

Clearly the large splitting between the physical values of the bottom and top masses ($m_b \ll m_t$) requires $\sigma_b \ll \sigma_t$ and thus $\varphi_t \simeq 0, \varphi_b \simeq -\pi/2$. This in turn demands that the CP-violating phase θ be close to $\pi/2$.

To put it differently, the *presence of a phase θ close to $\pi/2$ induces both isospin breaking and CP violation* with

$$\sigma_b \ll \sigma_t \neq 0\ ,\ v \neq 0\ ,\ \varphi_t \simeq \sigma_b/\sigma_t\ ,\ \varphi_b \simeq -\pi/2 + \sigma_b/\sigma_t\ . \tag{22}$$

The actual values of the VEVs can be determined from the physical values of the masses m_t, m_b and m_W. As a function of the VEV v of the elementary Higgs we find:

$$\begin{aligned}
\sigma_t^2 &= \frac{(2m_t^2 - h^2v^2)\,(v_0^2 - v^2)}{2\,(m_t^2 + m_b^2 - h^2v^2)}\ ,\\
\sigma_b^2 &= \frac{(2m_b^2 - h^2v^2)\,(v_0^2 - v^2)}{2\,(m_t^2 + m_b^2 - h^2v^2)}\ ,\\
\tan\varphi_t &= \frac{\sigma_b}{\sigma_t} = \sqrt{\frac{2m_b^2 - h^2v^2}{2m_t^2 - h^2v^2}}\ ,
\end{aligned} \tag{23}$$

and v is determined by the equation

$$\left(m_t^2 + m_b^2 - h^2v^2 - \frac{1}{2}g^2\left(v_0^2 - v^2\right)\right)^2 = (hgv)^2\left(v_0^2 - v^2\right). \tag{24}$$

Note that Eqs.(23) imply an upper bound on v, namely, $v \leq \sqrt{2}m_b/h$. This is quite satisfactory since the light quark masses are generated by v only.

For small values of $v \ll \sigma_b, \sigma_t$ we have

$$\begin{aligned}
\sigma_b &\simeq \frac{m_b v_0}{\sqrt{m_t^2+m_b^2}} \ , \ \sigma_t \simeq \frac{m_t v_0}{\sqrt{m_t^2+m_b^2}} \ , \\
\tan\varphi_t &\simeq \frac{m_b}{m_t} \ , \ g^2 \simeq \frac{2\left(m_t^2+m_b^2\right)}{v_0^2} \ . \qquad (25)
\end{aligned}$$

For example, with $m_t = 180$ GeV, $m_b = 5$ GeV we obtain $\sigma_b = 6.8$ GeV, $\sigma_t = 245.9$ GeV, $g = 1.03$ and $\varphi_t = 0.03$, $\varphi_b = -1.54$.

Next we comment on the mass spectrum of the neutral scalars and pseudoscalars present in our model. To find this spectrum we have to consider the 6×6 mass matrix given by

$$M_{ij}^2 = \left. \frac{\partial^2 V}{\partial \Phi_i \partial \Phi_j} \right|_{\langle \Phi \rangle} \ , \qquad (26)$$

where Φ_i denotes any of the real or imaginary parts of the complex doublets H^0, Σ_t^0 and Σ_b^0 .

It is straightforward to find the linear combination of the Φ_i's which corresponds to the Goldstone boson eventually eaten up by the Z^0. In the α-large limit, one of the eigenvalues of the mass matrix will be proportional to $\sqrt{\alpha\left(\sigma_b^{-2}+\sigma_t^{-2}\right)}$ and therefore the corresponding linear combination of the fields will decouple from the theory.

The remaining 4×4 mass matrix can be easily diagonalized to obtain the other 4 mass eigenstates. We find that the standard Higgs scalar h has a mass given by

$$m_h \simeq 2g\sqrt{\lambda} m_t \ . \qquad (27)$$

For $g \sim 1$ and $\lambda \sim 0.1$, it is of the order of 100 GeV as expected [8]. Two of the remaining masses are proportional to $\sqrt{\beta}$ and thus quite large. The last mass, m_A, which corresponds mainly to a $\bar{b}\gamma_5 b$ bound state is very sensitive to the difference $h_t - h_b$. For $h_t = h_b$ we find $m_A \simeq 2g\sqrt{\lambda} m_b$ at the tree level, but as soon as h_t and h_b differ (as expected from higher order corrections) m_A also gets a contribution proportional to $\sqrt{\beta}$.

The spectrum of the charged (pseudo) scalar sector can be similarly analized: apart from the usual $W^{\pm}$ Goldstone bosons, we find two pairs of complex-conjugate charged bosons, all with a mass proportional to $\sqrt{\beta}$.

4 CP violation

We have seen that the origin of CP violation in the present model is in the new interaction characterized by a $\theta \neq 0$ term. This CP-violating effect filters down to the standard model only if $m_b/m_t \neq 0$. Let us now investigate whether this new source of CP violation can be responsible for what is observed in the $K^0-\bar{K}^0$ system.

Let us consider the 3×3 quark mass matrices

$$\begin{aligned} M_u &= (h)_u v + \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix} g\sigma_t e^{i\varphi_t}, \\ M_d &= (h)_d v + \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix} g\sigma_b e^{-i\varphi_b}, \end{aligned} \tag{28}$$

with $(h)_{u,d}$ arbitrary real matrices.

If we neglect $O(h^2v^2)$ terms, $(MM^\dagger)_{u,d}$ are diagonalized by the following unitary matrices:

$$U_u \simeq R_u \begin{pmatrix} 1 & & \\ & 1 & \\ & & e^{-i\varphi_t} \end{pmatrix}, \; U_d \simeq R_d \begin{pmatrix} 1 & & \\ & 1 & \\ & & e^{i\varphi_b} \end{pmatrix}, \tag{29}$$

with $R_{u,d}$ orthogonal. In this approximation, the Cabibbo-Kobayashi-Maskawa mixing matrix reads

$$V \equiv U_u U_d^\dagger \simeq R_u \begin{pmatrix} 1 & & \\ & 1 & \\ & & e^{-i(\varphi_t+\varphi_b)} \end{pmatrix} R_d^T \, . \tag{30}$$

Using the Kobayashi-Maskawa parametrization [5]

$$V_{KM} \equiv R_{23}(\vartheta_2) R_{12}(\vartheta_1) \begin{pmatrix} 1 & & \\ & 1 & \\ & & e^{i\delta_{KM}} \end{pmatrix} R_{23}(\vartheta_3) \, , \tag{31}$$

where $R_{ij}(\vartheta)$ denotes a rotation in the (i,j) plane by an angle ϑ, it is obvious that ϑ_1 is arbitrary, $\vartheta_2 = \vartheta_2(v/m_t), \vartheta_3 = \vartheta_3(v/m_b)$ and, last but not least,

$$\delta_{KM} \simeq -(\varphi_t + \varphi_b) \simeq \pi/2 \, . \tag{32}$$

The nowadays standard parametrization [1] of the CKM mixing matrix is given by

$$V_{standard} \equiv R_{23}(\vartheta_{23}) \begin{pmatrix} 1 & & \\ & 1 & \\ & & e^{i\delta_{13}} \end{pmatrix} R_{13}(\vartheta_{13}) \begin{pmatrix} 1 & & \\ & 1 & \\ & & e^{-i\delta_{13}} \end{pmatrix} R_{12}(\vartheta_{12}) \; . \tag{33}$$

The CP-violating phase δ_{13} is related to δ_{KM} by

$$\sin\delta_{13} \simeq \frac{\vartheta_2}{\vartheta_{23}} \sin\delta_{KM} \; , \tag{34}$$

in the small mixing angles approximation [12]. From $|V_{ij}|_{KM} = |V_{ij}|_{standard}$ we have

$$\begin{aligned} \vartheta_{12} &\sim \vartheta_1 \; , \\ \vartheta_{13} &\sim \vartheta_1 \vartheta_3 \; , \\ \vartheta_{23} &\sim \left(\vartheta_2^2 + \vartheta_3^2 + 2\vartheta_2\vartheta_3 \cos\delta_{KM}\right)^{1/2} . \end{aligned} \tag{35}$$

In particular, we predict

$$\vartheta_{23} \sim \left(\vartheta_2^2 + \vartheta_3^2\right)^{1/2} \tag{36}$$

if $\delta_{KM} \simeq \pi/2$. On the other hand, the experimental constraints [1] on the Cabibbo angle $\vartheta_1 \sim 0.22$ and the ratio

$$\begin{aligned} \left|\frac{V_{ub}}{V_{cb}}\right| &= 0.08 \pm 0.02 \\ &= \frac{\vartheta_{13}}{\vartheta_{23}} \simeq 0.22 \frac{\vartheta_3}{\vartheta_{23}} \end{aligned} \tag{37}$$

require a texture for the h-matrices such that $\vartheta_2 > \vartheta_3$. From Eqs.(36) and (37) we obtain

$$\vartheta_{23} \sim \vartheta_2 \; . \tag{38}$$

Therefore, we conclude that our solution leads indeed to a sizeable CP-violating phase

$$\delta_{13} \simeq \pi/2 \; , \tag{39}$$

which is welcome by phenomenology in $K^0 - \bar{K}^0$ physics [13].

To conclude let us summarize the main point of this note: we assume that there exists a new interaction to which only the third generation of quarks (b and t) participates. This new interaction is completely symmetric in t and b but is supposed to generate a θ term which we take as the unique source of CP violation. With these assumptions we have shown in a simple effective model that θ triggers the breaking of the isospin symmetry between t and b as expected from general theorems [10]. Self consistency in the context of our model fixes θ to be around $\pi/2$. Due to the smallness of the mass ratio m_b/m_t, this in turn implies a large CP-violating phase in the CKM matrix as required phenomenologically. Other effects of this new θ were not considered.

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