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# Quark mixings as a test of a new symmetry of quark Yukawa couplings

H. González[1,3], S.R. Juárez W.[2], P. Kielanowski[1,4] and G. López Castro[5,1]

[1] *Departamento de Física, CINVESTAV del IPN, Apartado Postal 14-740, 070000 México, D.F. México*
[2] *Escuela Superior de Fís. y Mat. del IPN, U.P.-A.L.M. Edif. 9, 07738 México, D.F., México*
[3] *Programa de Mat. y Fís., Universidad Surcolombiana A.A. 385 Neiva, Colombia*
[4] *Department of Physics, Warsaw University, Poland*
[5] *Institut de Physique Théorique, Université catholique de Louvain, B-1348 Louvain-la-Neuve, Belgium*

**Abstract**

Based on the hierarchy exhibited by quarks masses at low energies, we assume that Yukawa couplings of up and down quarks are related by $Y_u \propto Y_d^2$ at grand unification scales. This ansatz gives rise to a symmetrical CKM matrix at the grand unification (GU) scale. Using three specific models as illustrative examples for the evolution down to low energies, we obtain the entries and *asymmetries* of the CKM matrix which are in very good agreement with their measured values. This indicates that the small asymmetry of the CKM matrix at low energies may be the effect of the renormalization group evolution only.

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# 1 Introduction

In the Standard Model (SM) of quark and lepton interactions, all the fermions get their masses from Yukawa couplings after spontaneous breaking of the $SU(2)_L \times U(1)_Y$ gauge symmetry (SSB). Since the intensity of Yukawa couplings is described by arbitrary complex constants, the quark mass matrices induced by SSB are, in general, non-diagonal. The diagonalization of these mass matrices yields non-diagonal (in flavor space) charged weak interactions, which are described by the unitary Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix

$$V_{ckm} = V_u V_d^\dagger, \tag{1}$$

where $V_u$ ($V_d$) is a unitary matrix that diagonalizes the up (down) quark mass matrix.

Thus, the original information contained in the gauge-invariant Yukawa couplings for quarks is drifted to 10 free parameters contained in quark masses and mixings at low energies. Therefore, if additional symmetries are present in the Yukawa couplings before symmetry breaking, one can expect to observe their traces in the structure of quarks masses and mixings at low energies. However, the values of the Yukawa couplings cannot be uniquely recovered from the observable quark masses and the parameters of the CKM matrix. Nevertheless, their measured values can be used as useful constrains on the search for symmetries of Yukawa couplings. In fact, the main motivation of the classical Fritzsch's Ansatz [1] for the Yukawa couplings was the phenomenological compatibility with the observable values of quark masses and mixings. Fritzsch's Ansatz was eventually excluded by the high mass of the top quark and now the structure of the Yukawa couplings for the up and down quarks is expected to be different and mainly motivated by grand unified theories (GUT).

Present information about quark masses and mixings [2] indicates that, at low energies, the unitary CKM matrix is almost symmetric. At the same time, the diagonal form of up and down quark mass matrices satisfies the approximate relation $M_u \approx M_d^2 m_t/m_b^2$. One may wonder if these properties of low energy observables can be correctly reproduced by assuming that the CKM matrix is symmetrical and that the relation

$$Y_u \propto Y_d^2 \tag{2}$$

($Y_{u,d}$ are Yukawa couplings of up and down quarks) are exact properties valid at the energy scales of grand unification. In this letter we adopt this hypothesis. By assuming the validity of Eq. (2) at GUT scales, we find a symmetrical CKM matrix. The evolution of Yukawa couplings down to observable low energies, generates the correct asymmetries and absolute values for the entries of the CKM matrix.

To our knowledge, this approach is different from previous studies (see for example Refs. [3–7]). Current approaches to the problem of the generation of quark masses and mixings, are based on the postulation of specific Yukawa couplings valid at GUT scales which can be chosen in such a way that the corresponding low energy data are reproduced. Usually, one can further assume some textures (zeros) for entries in the mass matrices in order to account for additional symmetries present at GUT scales.

A different view based on the notion of *natural* mass matrices, was introduced recently by Peccei and Wang [8], [9]. Their main idea is to derive the GU textures of quark Yukawa couplings by evolving to high energies the observed values of quark masses. Their naturalness condition is based on the requirement that none of the small observables at low energies are derived by the approximate cancellation of large quantities. As a result they have obtained several possible scenarios for the GU scale Yukawa couplings.

## 2 Quark masses and mixings at 1 GeV

As already mentioned, the hierarchy observed in low energy values of quark masses and mixings can be used as a guide to search for the structure of quark Yukawa couplings. This hierarchy can be better appreciated by using the small parameter $\lambda \approx 0.22$. The measured quark masses below the top mass scale can be written as:

$$\begin{aligned} M_u &= m_t \cdot \mathrm{Diag}\left(\alpha_1^u \lambda^8, \alpha_2^u \lambda^4, 1\right) \\ M_d &= m_b \cdot \mathrm{Diag}\left(\alpha_1^d \lambda^4, \alpha_2^d \lambda^2, 1\right) \end{aligned} \tag{3}$$

where the coefficients $\alpha_i^u$ and $\alpha_i^d$ are of the order 1 (see for example [8]).

On the other hand, the hierarchical structure of the CKM matrix is better seen in Wolfenstein's parameterization Ref. [10] (which is unitary up to order

$\lambda^4$):

$$[V_{ckm}]_W = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & \lambda^3 A(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & \lambda^2 A \\ \lambda^3 A(1-\rho-i\eta) & -\lambda^2 A & 1 \end{pmatrix}. \qquad (4)$$

It is worth noting the following properties from Eqs. (3) and (4):

1. The up and down quark mass matrices $M_{u,d}$ exhibit the scaling

$$M_u \propto M_d^2. \qquad (5)$$

2. With high precision the CKM matrix is almost symmetrical. In fact

$$|V_{12}|^2 - |V_{21}|^2 = |V_{23}|^2 - |V_{32}|^2 = |V_{31}|^2 - |V_{13}|^2 \sim \lambda^6. \qquad (6)$$

   where the last equality follows from Eq. (4) and the other two from unitarity of the CKM matrix.

The properties given in Eqs. (5) and (6) will be the basis of our further discussion. In our formalism we will generate the symmetrical CKM matrix at the scale of GU and we will show that upon the evolution with the method of the Renormalization Group (RG) there will appear terms that break the symmetry of the CKM matrix to the correct order.

# 3 Eigenvalue and eigenvector parameterization of the CKM matrix

Our formalism of the generation of the CKM matrix will be guided by the eigenvalue and eigenvector (EE) parameterization of the CKM matrix Ref. [11]. In this parameterization the CKM matrix is written in the following form:

$$V_{ckm} = \hat{A} D \hat{A}^\dagger. \qquad (7)$$

Here $D$ is the diagonal matrix

$$D = \mathrm{Diag}\left(e^{-2\pi i/3}, \quad e^{2\pi i/3}, \quad 1\right) \qquad (8)$$

and the matrix $\hat{A}$ is unitary. Because of the rephasing freedom of the quark fields, $\hat{A}$ has only 4 parameters as the original CKM matrix. In practice, it turns out that a two-angle parameterization of the real matrix $\hat{A}$ , namely

$$\hat{A} = \begin{pmatrix} c_1 & -s_1 & 0 \\ s_1 c_2 & c_1 c_2 & -s_2 \\ s_1 s_2 & c_1 s_2 & c_2 \end{pmatrix} \tag{9}$$

where $c_i = \cos\beta_i$, $s_i = \sin\beta_i$, is good enough to have a reasonable agreement with data (the fit to entries of the CKM matrix gives: $\beta_1 = 0.1293 \pm 0.0010$, $\beta_2 = 0.0239 \pm 0.0017$, $\chi^2 = 8.73$. See also Ref. [12]).

The matrix $\hat{A}$ in Eq. (7) can be interpreted as a universal matrix that diagonalizes the mass matrices of up and down quarks. For this to be true a rephasing of quark fields according to Eq. (12) (below) is required before diagonalization of mass matrices (otherwise the CKM matrix would become the unit matrix). After this rephasing of quark fields, the mass matrices in weak ($\widetilde{M_i}$) and mass eigenstates ($M_i$) are related by

$$M_u = \hat{A}\widetilde{M_u}\hat{A}^\dagger, \quad M_d = \hat{A}\widetilde{M_d}\hat{A}^\dagger. \tag{10}$$

Since $\widetilde{M_i}$ are proportional to Yukawa coupling matrices $Y_i$, Eqs. (10) and (5) would lead to the relation

$$Y_u = C\ Y_d^2 \tag{11}$$

where $C$ is some proportionality constant (see Eq. (20) below). This means that the Yukawa couplings of up quarks are simple functions of the Yukawa couplings of down quarks.

To conclude this section let us observe that in our scenario the $\hat{A}$ matrix given in Eq. (9) is real. The corresponding CKM matrix given in Eq. (7) is symmetric, a property that may not be fulfilled by present experimental data if we assume the unitarity of the CKM matrix. For this reason we will assume that the matrix $\hat{A}$ has the form given in Eq. (9) only at the scale of GU. The CKM matrix at the scale of 1 GeV will then be obtained by using the method of the RG and is not symmetric.

# 4 Scheme for the generation of the CKM matrix

Using the ideas outlined in the previous sections we will present here the explicit construction of the CKM matrix. As we discussed the matrices of the Yukawa couplings have a especially simple form after the rephasing of the quark fields. The transformation of the rephasing that we will use is the following

$$\begin{pmatrix} u \\ c \\ t \end{pmatrix}_L \rightarrow \begin{pmatrix} e^{-\frac{\pi}{3}i} & 0 & 0 \\ 0 & e^{\frac{\pi}{3}i} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_L, \quad \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \rightarrow \begin{pmatrix} e^{\frac{\pi}{3}i} & 0 & 0 \\ 0 & e^{-\frac{\pi}{3}i} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L . \tag{12}$$

Upon this transformation only the charged current and the Yukawa couplings are changed. The matrix of the charged weak current becomes $D$ as given in Eq. (8), and we choose the Yukawa couplings at the GU scale to be

$$\frac{v}{m_b\sqrt{2}}\, Y_d \sim \begin{pmatrix} \alpha_{11}\lambda^4 & \alpha_{12}\lambda^3 & \frac{A\lambda^3}{3} \\ \alpha_{21}\lambda^3 & \alpha_{22}\lambda^2 & \frac{A\lambda^2}{\sqrt{3}} \\ \frac{A\lambda^3}{3} & \frac{A\lambda^2}{\sqrt{3}} & 1 \end{pmatrix}, \tag{13}$$

where $v$ is the vacuum expectation value of the Higgs field. The corresponding structure of $Y_u$ is determined by Eq. (11). The parameters $A$ and $\lambda$ are the same as those of Wolfenstein's parameterization and only the leading powers of $\lambda$ are displayed. $\alpha_{ij}$ are numerical constants of $\mathcal{O}(1)$ which specific values are not essential for our discussion. Indeed, the structure of the CKM matrix will emerge only from the hierarchy of the third row and column of $Y_{u,d}$.

The matrix $\hat{A}$ that diagonalizes the matrices $Y_u$ and $Y_d$ has the form

$$\hat{A} = \begin{pmatrix} n_1 & -\frac{\lambda}{\sqrt{3}}n_1 & 0 \\ \frac{\lambda}{\sqrt{3}}n_2 & n_2 & -\frac{A\lambda^2}{\sqrt{3}}n_2 \\ \frac{A\lambda^3}{3}n_3 & \frac{A\lambda^2}{\sqrt{3}}n_3 & \left(1+\frac{\lambda^2}{3}\right)n_3 \end{pmatrix} \tag{14}$$

where $\frac{1}{n_1} = \sqrt{1+\frac{|\lambda|^2}{3}}$, $\frac{1}{n_2} = \sqrt{1+\frac{|\lambda|^2}{3}+\frac{|A|^2|\lambda|^4}{3}}$, and $n_3 = n_1 n_2$. The matrix $\hat{A}$ given in Eq. (14) is unitary and real and leads to the symmetrical CKM matrix

$$V_{ckm} = \hat{A} D \hat{A}^T. \tag{15}$$

with entries given by

$$|V_{11}| = n_1^2 \left| e^{-\frac{2\pi}{3}i}\left(1+\frac{\lambda^2}{3}e^{\frac{4\pi}{3}i}\right)\right| \approx 1$$

$$|V_{12}| = n_1 n_2 \left| \frac{\lambda}{\sqrt{3}} e^{-\frac{2\pi}{3}i} \left(1 - e^{\frac{4\pi}{3}i}\right) \right| \approx \lambda$$

$$|V_{13}| = n_1 n_3 \frac{A\lambda^3}{3} \left| e^{-\frac{2\pi}{3}i} \left(1 - e^{\frac{4\pi}{3}i}\right) \right| \approx \frac{A\lambda^3}{\sqrt{3}}$$

$$|V_{22}| = n_2^2 \left| e^{\frac{2\pi}{3}i} + \frac{\lambda^2}{3} e^{-\frac{2\pi}{3}i} + \frac{A^2\lambda^4}{3} \right| \approx 1$$

$$|V_{23}| = n_2 n_3 \frac{A\lambda^2}{\sqrt{3}} \left| e^{\frac{2\pi}{3}i} + \frac{\lambda^2}{3} e^{-\frac{2\pi}{3}i} - \left(1 + \frac{\lambda^2}{3}\right) \right| \approx A\lambda^2$$

so it reproduces the Wolfenstein's hierarchy of the CKM matrix with fixed values of $\rho$ and $\eta$. Moreover the matrix (15) already includes the CP violating phase and the "plaquette"

$$J(GU) = \mathrm{Im}\left(V_{11} V_{22} V_{12}^* V_{21}^*\right) = \frac{A^2\lambda^6}{2\sqrt{3}} \approx (2 \sim 3) \cdot 10^{-5}, \tag{16}$$

has the right order of magnitude (see Table 1).

Now we perform the RG evolution of the matrices $Y_u$ and $Y_d$ from the GU energy to the scale of 1 GeV. The RG equations become modified [13] by the quark field rephasing transformation given in Eq. (12) and the evolution of the matrices $Y_u$ and $Y_d$ are different.

At the GU scale, the matrices $Y_u$ and $Y_d$ depend on the parameters $A$ and $\lambda$ that also define the common diagonalization matrix, Eq. (14). Upon the RG evolution, the matrices that diagonalize the Yukawa couplings $Y_u$ and $Y_d$ evolve in such a way that the values of $A$ and $\lambda$ do not change for $Y_u$ and they become complex for $Y_d$. At the scale of 1 GeV $\tilde{\lambda}$ and $\tilde{A}$ are equal to

$$\begin{aligned} \tilde{\lambda} &= \lambda\left(1\ GeV\right) = \frac{\left(1 + Re^{-2i\pi/3}\right)}{\left(1 + Re^{2i\pi/3}\right)} \lambda\left(GU_{scale}\right), \\ \tilde{A} &= A\left(1\ GeV\right) = \frac{a\left(1 + Re^{2i\pi/3}\right)^3}{\left(1 + Re^{-2i\pi/3}\right)^2} A\left(GU_{scale}\right). \end{aligned} \tag{17}$$

Here $R$ and $a$ are the coefficients that depend on the model that is used. Their form follows from the RG equations and is given in Ref. [13]. The matrix that diagonalizes $Y_u$ at the scale of 1 GeV is thus equal to $\hat{A}$ given in

Eq. (14) and the matrix that diagonalizes $Y_d$ is

$$\hat{A}_d = \begin{pmatrix} n_1 & -\frac{\tilde{\lambda}^*}{\sqrt{3}} n_1 & 0 \\ \frac{\tilde{\lambda}}{\sqrt{3}} n_2 & n_2 & -\frac{\tilde{A}^* \left(\tilde{\lambda}^*\right)^2}{\sqrt{3}} n_2 \\ \frac{\tilde{A}\tilde{\lambda}^3}{3} n_3 & \frac{\tilde{A}\tilde{\lambda}^2}{\sqrt{3}} n_3 & \left(1 + \frac{\left|\tilde{\lambda}\right|^2}{3}\right) n_3 \end{pmatrix}. \tag{18}$$

Using the matrices $\hat{A}$ and $\hat{A}_d$ the CKM matrix at the scale of 1 GeV becomes

$$\hat{V}_{ckm} = \hat{A} D \hat{A}_d^\dagger. \tag{19}$$

The matrix (19) is not symmetric and the pattern of non-symmetry of the CKM matrix is the same as in Wolfenstein's parameterization, Eq. (6).

In what follows, the parameters $A$ and $\lambda$ defined at the GU scale are adjusted so as to reproduce the CKM matrix at low energies. In order to illustrate their evolution to low energies, we have used the SM and its two Higgs (DHM) and minimal supersymmetric (MSSM) extensions. The experimental data on the CKM matrix are taken from Ref. [2] and the summary of our results is given in Table 1. From Table 1 we observe that the asymmetry of CKM matrix elements are of the expected order of magnitude (see Eq. (6)). The main difference in the results lies in the fact that $|V_{ub}| < |V_{td}|$ in the standard model, while $|V_{ub}| > |V_{td}|$ for the other two models. Since the values of $A$ and $\lambda$ are essentially fixed from $|V_{us}|$ and $|V_{cb}|$, the rather large value for $|V_{ud}|$ arises from unitarity of the CKM matrix. Although neither of the considered models can be really excluded, observe that the ratio $|V_{ub}/V_{cb}|$ is closest to its experimental value in the case of the SM and that a better fit[1], reflected in the lowest $\chi^2$ value, is also obtained in this case.

From Eq. (19) we can also compute the value of Jarlskog's parameter (Eq. (16)) at low energies. The change in the value of $J$ is, as expected [14], negligibly small (at most 3 % in the case of the SM) when evolving from GU to low energy scales.

[1] There is a large contribution to $\chi^2$ that has its origin in the fact that the values of $|V_{ub}|$ calculated from unitarity ($|V_{ub}| = 0.059 \pm 0.018$) and from $|V_{ub}| = |V_{cb}| \frac{|V_{ub}|}{|V_{cb}|} = 0.0033 \pm 0.0009$ differ by 3 standard deviations. The data are thus incompatible at this level with unitarity and any model that preserves the unitarity of CKM matrix will show a corresponding discrepancy.

|  | Experiment | MSSM | SM | DHM |
|---|---|---|---|---|
| $\chi^2$ |  | 11.47 | 5.60 | 10.34 |
| $A \pm \Delta A$ |  | 0.607±0.045 | 1.397±0.104 | 0.682±0.051 |
| $\lambda \pm \Delta\lambda$ |  | 0.2433±0.0019 | 0.1984±0.0015 | 0.2363±0.0018 |
| $\|V_{ud}\|$ | 0.9736±0.0010 | 0.97517 | 0.97514 | 0.97517 |
| $\|V_{us}\|$ | 0.2205±0.0018 | 0.22138 | 0.22153 | 0.22141 |
| $\|V_{cd}\|$ | 0.224±0.016 | 0.22141 | 0.22146 | 0.22142 |
| $\|V_{ub}\|$ | – | 0.00571 | 0.00467 | 0.00554 |
| $\|V_{td}\|$ | – | 0.00455 | 0.00699 | 0.00482 |
| $\frac{\|V_{ub}\|}{\|V_{cb}\|}$ | 0.08±0.02 | 0.1391 | 0.1138 | 0.1352 |
| $\|V_{cb}\|$ | 0.041±0.0030 | 0.0410 | 0.0410 | 0.0410 |

Table 1: Comparison of the CKM matrix results for various models. Notice that the fitted values of $A$ and $\lambda$ are at the GU scale while $|V_{ij}|$ correspond to their low energy values.

# 5 Discussion of the results and conclusions

We have presented a new kind of symmetry for the up and down quark mass matrices that generates a CKM matrix in excellent agreement with the experimental data. Our method is not based on textures (or zeros) for the Yukawa couplings at GUT scales, but on the assumption that the symmetry of the CKM matrix is intimately related to a simple relation between Yukawa couplings of up and down quarks (see Eq. (11)). Thus, we propose that at GUT scales $Y_u = CY_d^2$ (which is valid for a special choice of quark fields phases) where $C$ can be estimated to be

$$C^{-1} = \frac{\sqrt{2}m_b^2}{vm_t} \approx 5.4 \cdot 10^{-4} \approx \lambda^5, \tag{20}$$

Unfortunately we cannot say what kind of physics might be behind the relation given in Eq. (11) but its simplicity and excellent predictions make it a very attractive scenario for the generation of the CKM matrix.

It might seem that our method does not depend on the values of the quark masses. This is true only to some extent. Our method only works if there is a hierarchy for the quark masses and in such a case the values of the quark masses are irrelevant. The form of the CKM matrix is entirely determined only by the third row and column of the $Y_u$ and $Y_d$ matrices, which suggests the essential role played by the heaviness of the third generation of quarks. We will discuss the problem of the quark masses elsewhere [13]. Let us only mention that we can also impose textures and, in such a way, reduce the number of parameters still further.

One might ask if the method based on textures is equivalent to ours. To examine this one has to check the relation (11) for the matrices $Y_u$ and $Y_d$ with given textures. We found that Eq. (11) is not fulfilled in any of the known schemes with textures.

The advantages of our method are the following

1. It is based on a new symmetry between the $Y_u$ and $Y_d$ matrices which manifests itself that they are real and diagonalizable by the same orthogonal matrix at the GU scale.

2. CP violation appears thanks to the phase factors that are included in the charged weak current already at the GU scale. These factors are not fitted and are obtained from the condition $\hat{V}^3_{ckm} = 1$.

3. We reproduce with high accuracy the CKM matrix including the small asymmetry of the order $\lambda^6$. This last asymmetry is the consequence of the RG evolution from the GU scale to 1 GeV.

4. Our method is very stable with respect to small changes in the values of the initial data.

For all these reasons this new symmetry of the $Y_u$ and $Y_d$ matrices may be a very important piece of information about physics that lies at the basis of the standard model.

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# References

[1] H. Fritzsch, Phys. Lett. **B70**, 436 (1977); **B73**, 317 (1978).

[2] R.M. Barnett *et al.*, *Particle Data Group* Phys. Rev. **D54**, Part I, (1996).

[3] S. Dimopoulos, L.J. Hall, S. Raby, Phys. Rev. Lett. **68**, 1984 (1992); Phys. Rev. **D45**, 4192 (1992); G. Anderson, S. Raby, S. Dimopoulos and L.J. Hall, Phys. Rev. **D47**, 3702 (1993).

[4] P. Ramond, R.G. Roberts and G.G. Ross, Nucl. Phys. **B406**, 19 (1993).

[5] P. Binetruy and P. Ramond, Phys. Lett. **B350**, 49 (1995).

[6] E. Dudas, S. Pokorski and C.A. Savoy, Phys. Lett. **B356**, 45 (1995).

[7] P.H. Frampton and C.W. Kong, Phys. Rev. Lett. **75**, 78 (1995).

[8] R.D. Peccei and K. Wang, Phys. Rev. **D53**, 2712 (1996).

[9] K. Wang, Phys. Rev. **D54**, 5750 (1996).

[10] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).

[11] P. Kielanowski, Phys. Rev. Lett. **63**, 2189 (1989).

[12] T. Kobayashi and Z.-Z. Xing, Mod. Phys. Lett. **A11**, 2531 (1996).

[13] H. González, S.R. Juárez W., P. Kielanowski and G. López Castro, to be published.

[14] G.C. Athanasiu, S. Dimopoulos and F.J. Gilman, Phys. Rev. Lett. **57**, 1982 (1986).