Charm-pair Rescattering Mechanism
for Charmonium Production in High-energy Collisions

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Abstract

A new mechanism for heavy quarkonium production in high-energy collisions called the "s–channel cut" was proposed in 2005 by Lansberg, Cudell, and Kalinovsky. We identify this mechanism physically as the production of a heavy quark and anti-quark that are on-shell followed by their rescattering to produce heavy quarkonium. We point out that in the NRQCD factorization formalism this rescattering mechanism is a contribution to the color-singlet model term at next-to-next-to-leading order in perturbation theory. Its leading contribution to the production rate can be calculated without introducing any additional phenomenological parameters. We calculate the charm-pair rescattering (or s–channel cut) contribution to the production of J/ψ at the Tevatron and compare it to estimates by Lansberg et al. using phenomenological models. This contribution competes with the leading-order term in the color-singlet model at large transverse momentum but is significantly smaller than the next-to-leading-order term. We conclude that charm-pair rescattering is not a dominant mechanism for charmonium production in high-energy collisions.

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I. INTRODUCTION

The production of heavy quarkonium in high energy collisions remains a challenging problem in QCD. This problem has some aspects that are inherently perturbative, because it requires the creation of a heavy quark-antiquark pair. It also has aspects that are inherently nonperturbative, because the heavy quark Q and the antiquark $\bar{Q}$ are required to form a bound state. The nonrelativistic QCD (NRQCD) factorization approach uses an effective field theory to systematically separate perturbative short-distance aspects from nonperturbative long-distance aspects [1]. The inclusive production cross section is expressed as the sum of products of parton scattering factors that can be calculated using perturbative QCD and NRQCD matrix elements that can be treated as phenomenological parameters. The NRQCD factorization formulas have not yet been proven rigorously, but they are well-motivated by proofs of factorization to all orders of perturbation theory for other processes in high energy physics. The NRQCD factorization formula for gluon fragmentation into quarkonium has survived a nontrivial test at next-to-next-to-leading order, although it required a modification of the formal definition of the NRQCD matrix elements [2].

NRQCD predicts a definite hierarchy in the size of the NRQCD matrix elements according to how they scale with the typical relative velocity v of the heavy quark in quarkonium [1]. However, the NRQCD factorization framework is sufficiently flexible that any model for quarkonium production that is compatible with perturbative QCD at short distances should be expressible in terms of assumptions about the NRQCD matrix elements. The color-singlet model (CSM) can be expressed as the vanishing of all NRQCD matrix elements except one. The CSM matrix element corresponds to formation of the quarkonium from $Q\bar{Q}$ in a color-singlet state with the same angular-momentum quantum numbers as the quarkonium in the potential model. The color-evaporation model can be expressed as the NRQCD matrix elements having a hierarchy according to the orbital angular momentum of the $Q\bar{Q}$ pair without regard to their color and spin quantum numbers [3].

In 2005, Lansberg, Cudell, and Kalinovsky (LCK) proposed a new mechanism for heavy quarkonium production that they called the "s–channel" cut [4]. In the case of the production of quarkonium from gluon-gluon collisions, they claimed it was leading order in the QCD coupling constant but had not been taken into account in any previous analyses. The name "s–channel cut" refers to a mathematical description of the mechanism. It can be identified as a contribution to the imaginary part of the amplitude for quarkonium production in gluon-gluon collisions that corresponds to a Cutkosky cut through Q and $\bar{Q}$ lines only. It is thus associated with a discontinuity of the amplitude in the square s of the center-of-mass energy of the colliding gluons. This mechanism corresponds physically to the creation of the heavy quark and antiquark through the parton scattering process $gg \to Q\bar{Q}$ followed by the formation of quarkonium through the rescattering of the $Q\bar{Q}$ pair. A physical description of this mechanism is $Q\bar{Q}$ rescattering into quarkonium plus a recoiling gluon.

LCK applied their s–channel cut mechanism to J/ψ production in hadron-hadron collisions. They expressed the s–channel cut contribution to the production amplitude as the sum of diagrams with a vertex Γ_μ for $c\bar{c} \to J/\psi$ and diagrams with another vertex $\Gamma_{\mu\nu}$ for $c\bar{c} \to J/\psi\, g$ that was introduced in order to restore a Ward identity. They developed simple phenomenological models for these vertices and used them to estimate the s–channel cut contribution to the inclusive J/ψ cross section in $p\bar{p}$ and pp collisions [4, 5].

Since the $Q\bar{Q}$ rescattering (or s–channel cut) mechanism involves the formation of quarkonium from a $Q\bar{Q}$ pair that is created by a perturbative parton process, it must be expressible
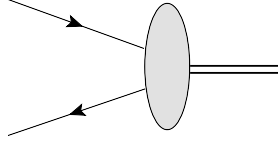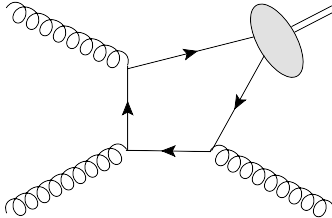
FIG. 1: Vertex Γ_μ for c and $\bar{c}$ to form J/ψ.

FIG. 2: One of the 6 Feynman diagrams for $g\,g \to J/\psi\,g$ at order g_s^3 with vertex Γ_μ.

in terms of the conventional NRQCD factorization formula. In this paper, we identify the charm-pair rescattering contribution to the process $gg \to J/\psi\,g$ as a next-to-next-to-leading order (NNLO) contribution to the color-singlet model term in the NRQCD factorization formula. It can therefore be calculated without introducing any new phenomenological parameters.

In Section II, we summarize the results of Lansberg et al. on the s–channel cut mechanism for J/ψ production in hadron-hadron collisions. In Section III, we show that this mechanism, which can be described physically as charm-pair rescattering, is an NNLO contribution to the CSM term in the NRQCD factorization formula. In Section IV, we explain how the charm-pair rescattering contribution can be calculated without introducing any new phenomenological parameters. In Section V, we compare our numerical results for J/ψ production at the Tevatron with those from the phenomenological models introduced by LCK [4] and by Lansberg and Haberzettl (LH) in Ref. [5]. We also compare our results to other contributions to the CSM term at leading order (LO) and next-to-leading order (NLO) in the QCD coupling constant.

II. s–CHANNEL CUT

We begin by summarizing the s–channel cut mechanism for charmonium production proposed by Lansberg, Cudell, and Kalinovsky (LCK) [4]. Their starting point was the amplitude $\Gamma_\mu(q, P)\epsilon^\mu$ for a color-singlet $c\bar{c}$ pair with total 4-momentum P and relative 4-momentum q to form J/ψ with 4-momentum P and polarization vector ϵ. This vertex is illustrated in Fig. 1. It can be defined diagrammatically by starting with the sum of all diagrams with two incoming c and $\bar{c}$ lines and two outgoing c and $\bar{c}$ lines. This sum defines an amplitude with a pole in the $c\bar{c}$ invariant mass at the mass of the J/ψ. In the residue of the pole, the desired amplitude $\Gamma_\mu\epsilon^\mu$ is the factor associated with the incoming c and $\bar{c}$. Any amplitude for the production of J/ψ can be expressed as a sum of diagrams with a vertex Γ_μ and additional QCD vertices. For example, the amplitude for $g\,g \to J/\psi\,g$ at order g_s^3 has contributions from 6 one-loop box diagrams, one of which is shown in Fig. 2. The other five diagrams are obtained by permuting the attachments of the three gluons.
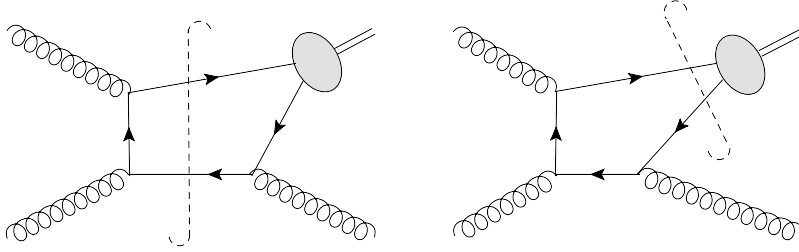
FIG. 3: The s–channel $c\bar{c}$ cut (a) and the meson $c\bar{c}$ cut (b) for the Feynman diagram in Fig. 2.

For general quarkonium states, there are also one-loop triangle diagrams involving a 3-gluon vertex. By setting the incoming and outgoing gluons on their mass shells, we obtain the T-matrix element for this process.

A Feynman diagram has a discontinuity in the invariant mass of any set of lines that are kinematically allowed to be on shell. This discontinuity can be represented by a cut diagram in which the cut passes through that set of lines. There are two possible cuts through the parton lines in the diagram in Fig. 2:

- the *meson $c\bar{c}$ cut*, in which the c and $\bar{c}$ lines that attach to the Γ_μ vertex are cut, as illustrated in Fig. 3(b). The discontinuity in the amplitude is in the invariant mass P^2 of the $c\bar{c}$ pair. The cut diagram is the product of diagrams for $g\,g \to c\,\bar{c}\,g$ and $c\,\bar{c} \to J/\psi$ integrated over the relative momentum of the c and $\bar{c}$, which are on shell. All 6 diagrams of order g_s^3 have a meson cut if the charm quark mass satisfies $m_c < M_{J/\psi}/2$.

- the *s–channel $c\bar{c}$ cut*, in which the c and $\bar{c}$ lines that are attached to the incoming gluons but do not join them are cut, as illustrated in Fig. 3(a). The discontinuity in the amplitude is in the invariant mass s of the colliding gluons. The cut diagram is the product of diagrams for $g\,g \to c\,\bar{c}$ and $c\,\bar{c} \to J/\psi\,g$ integrated over the relative momentum of the c and $\bar{c}$, which are on shell. Four of the 6 diagrams of order g_s^3 have an s–channel cut if $s > M_{J/\psi}^2$.

In the color-singlet model for quarkonium production, the production amplitude for J/ψ is a product of a wavefunction factor and the amplitude for producing a c and $\bar{c}$ that are on their mass shell in a color-singlet $^3S_1$ state with relative momentum 0 and total momentum equal to that of the J/ψ. In an amplitude for J/ψ production with vertex Γ_μ, the meson $c\bar{c}$ cut puts the c and $\bar{c}$ on shell and therefore gives a contribution that resembles the amplitude in the color-singlet model. In the amplitude for $g\,g \to J/\psi\,g$ at order g_s^3 with vertex Γ_μ, LCK identified the meson cut with the amplitude in the color-singlet model and they argued that the s–channel cut gives an additional leading-order contribution that had not previously been taken into account. As will be explained in Section III, the meson $c\bar{c}$ cut is subleading compared to the conventional color-singlet model amplitude and the s–channel cut is actually a contribution to the color-singlet model amplitude at next-to-next-to-leading order in the running coupling constant of QCD.

We now describe the formalism that LCK used to calculate the s–channel cut contribution to J/ψ production. They assumed that the amplitude Γ_μ was dominated by the contribution from the $c\bar{c}$ Fock state and has the form $\Gamma(q_{\rm rel})\gamma_\mu$, where Γ is a scalar function of the relative 3-momentum $q_{\rm rel}$ of c and $\bar{c}$ in the J/ψ rest frame. This reduces Γ_μ to a single function
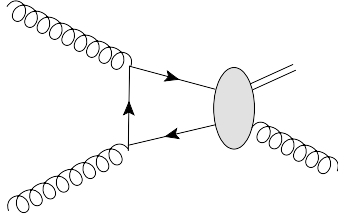
FIG. 4: One of the 2 Feynman diagrams for $g\, g \to J/\psi\, g$ at order g_s^2 with vertex $\Gamma_{\mu\nu}$ in the model for the s–channel cut contribution introduced in Ref. [4].

of one variable. They parameterized Γ_μ as a Gaussian with normalization factor N and a momentum scale Λ. This reduces Γ_μ to two phenomenological parameters. They used results from relativistic potential models to determine Λ to be approximately 1.8 GeV and they used the electronic width of J/ψ to determine N.

LCK recognized that the vertex Γ_μ is not gauge-invariant when the incoming c and $\bar{c}$ are not on their mass shells. One consequence is that the s–channel cut to the amplitude for $g\, g \to J/\psi\, g$ at order g_s^3 with vertex Γ_μ does not satisfy the Ward identity associated with the current that creates the outgoing gluon. They proposed that the Ward identity could be restored by adding diagrams with a 4-point vertex $\Gamma_{\mu\nu}(p_1, p_2, q, P)$ for $c\bar{c} \to J/\psi\, g$. They did not give any diagrammatic definition of this vertex, but it was assumed to be of order g_s relative to Γ_μ. There are two diagrams for $g\, g \to J/\psi\, g$ at order g_s^2 with a vertex $\Gamma_{\mu\nu}$, both of which have an s–channel cut. One of the diagrams is shown in Fig. 4 and the other is obtained by reversing the arrows on the c and $\bar{c}$ lines.

The complete model of LCK for the s–channel cut contribution to the T-matrix element for $g\, g \to J/\psi\, g$ at order g_s^3 is the sum of the 4 cut diagrams with vertex Γ_μ and the 2 cut diagrams with vertex $\Gamma_{\mu\nu}$. They took the charm quark mass to be $m_c = 1.87$ GeV, which is greater than $M_{J/\psi}/2$ so there is no meson $c\bar{c}$ cut. In Ref. [4], LCK proposed an expression for the vertex $\Gamma_{\mu\nu}$ with no additional parameters that respects the Ward identity. Thus this model for the s–channel cut contribution has 2 parameters, N and Λ, in addition to m_c and the QCD coupling constant α_s. LCK showed that in this model the cross section for J/ψ production at the Tevatron at large transverse momentum (p_T) could be as large as the leading-order color-singlet model contribution. In Ref. [5], LH proposed a more general expression for $\Gamma_{\mu\nu}$ that depends on a phenomenological function. They introduced a simple parameterization for this function that depends on two parameters. Thus this model for the s–channel cut contribution has 4 parameters in addition to m_c and α_s. LH showed that if these parameters are adjusted so that the s–channel cut contribution alone fits the Tevatron data for J/ψ production at low p_T, the s–channel cut contribution alone also fits the RHIC data on J/ψ production.

III.   CHARM-PAIR RESCATTERING

As pointed out in the Introduction, the s–channel $c\bar{c}$ cut mechanism for J/ψ production can be interpreted physically as charm-pair rescattering. The s–channel cut to the one-loop amplitude with vertex Γ_μ for $g\, g \to J/\psi\, g$ separates the incoming gluons from the outgoing J/ψ and gluon. By the Cutkosky cutting rules, the imaginary part generated by the discontinuity of the amplitude in the variable s is the sum over all such cuts. All the
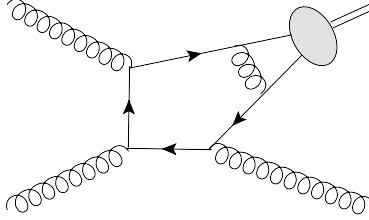
FIG. 5: Two-loop Feynman diagrams for $g\,g \to J/\psi\,g$ with a soft vertex Γ_μ. This diagram of order $g_s(m_c)^5$ is also a contribution to the diagram in Fig. 2 with a hard vertex Γ_μ.

other cuts that contribute to that imaginary part must cut through the vertex Γ_μ. (The meson $c\bar{c}$ cut is not one of these cuts, because it does not separate the incoming lines from the outgoing lines.) A Cutkosky cut through a set of parton lines corresponds to the scattering of $g\,g$ into this set of on-shell partons followed by the rescattering of those partons into $J/\psi\,g$. The s–channel cut discussed by LCK is through c and $\bar{c}$ lines, so it corresponds to the scattering of $g\,g$ into $c\,\bar{c}$ followed by the rescattering of $c\,\bar{c}$ into $J/\psi\,g$.

One misconception in Ref. [4] (LCK) is that the meson $c\bar{c}$ cut of the amplitude for J/ψ production should be identified with the color-singlet model contribution. At first glance, this identification may seem plausible, because the meson cut puts the c and $\bar{c}$ on their mass shells and the color-singlet model amplitude is usually calculated by taking the c and $\bar{c}$ lines to be on their mass shells with 0 relative momentum. However this identification implies incorrectly that if the arbitrary phase of Γ_μ is chosen so that its dominant contribution is real, the leading color-singlet model contribution is pure imaginary. The color-singlet model amplitude should instead be identified with the contribution from the region of phase space that gives the dominant contributions to Γ_μ. Since the J/ψ is a nonrelativistic bound state, its primary constituents c and $\bar{c}$ have typical relative velocity v that is significantly smaller than 1. The largest contributions to Γ_μ come from the region in which the c and $\bar{c}$ have relative momentum of order v in the J/ψ rest frame and energies that differ by m_c by amounts of order $m_c v^2$. Calculations in the color-singlet model in which c and $\bar{c}$ are on their mass shells with 0 relative momentum should be understood simply as a technical device for calculating the contributions from this dominant region. If the arbitrary phase of Γ_μ is chosen so that its dominant contribution is real, the leading color-singlet model contribution will also be real.

By presenting the s-channel $c\bar{c}$ cut as a leading order contribution, LCK implicitly assumed that the vertex Γ_μ for a $c\bar{c}$ pair to form J/ψ can be treated as if it has a definite order in the QCD coupling constant g_s. However its order in g_s actually depends on the relative momentum of the $c\bar{c}$ pair. The running coupling constant $g_s(\mu)$ is small at momentum scales μ of order m_c and becomes large at small momentum scales. It is convenient to treat $g_s(\mu)$ as being of order 1 at small momentum scales $\mu \ll m_c$. We then need only count powers of the small running coupling constant $g_s(m_c)$ from QCD vertices with momentum scale μ of order m_c and larger. If the relative momentum of the c and $\bar{c}$ is small compared to m_c, then Γ_μ is of order $g_s(m_c)^0 = 1$. We will refer to the vertex Γ_μ in this momentum region as a "soft vertex". If the relative momentum of c and $\bar{c}$ is of order m_c, large momentum must be transferred between the c and $\bar{c}$ by the exchange of hard gluons with coupling constant $g_s(m_c)$ in order for the c and $\bar{c}$ to bind to form J/ψ. The lowest order contribution comes from one-gluon exchange, so Γ_μ is of order $g_s(m_c)^2$. We will refer to Γ_μ in this momentum region as a "hard vertex".
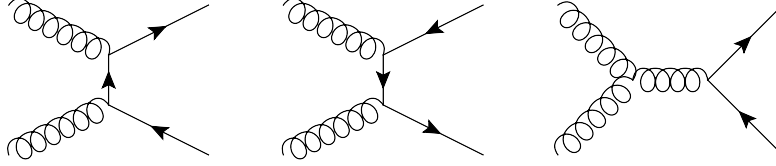
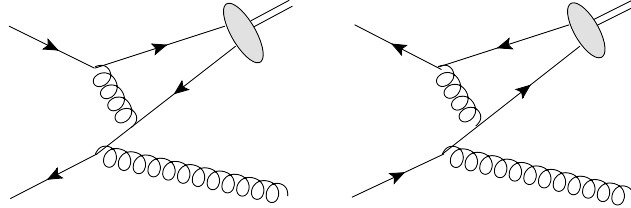FIG. 6: Feynman diagrams for $g\,g \to c\,\bar{c}$.

FIG. 7: Two of the 5 one-loop Feynman diagrams for $c\,\bar{c} \to J/\psi\,g$ with a soft-vertex Γ_μ.

In a one-loop diagram for $g\,g \to J/\psi\,g$ with vertex Γ_μ, such as the diagram in Fig. 2, the s–channel cut forces either the c or $\bar{c}$ that enters the vertex to be off-shell by an amount of order m_c or larger. Thus Γ_μ is a hard vertex of order $g_s^2(m_c)$ and the diagram is of order $g_s(m_c)^5$. The leading contribution to the hard vertex comes from the exchange of a single hard gluon, which can be pulled out of the hard vertex as illustrated in Fig. 5, changing it into a soft vertex. One-loop diagrams with a hard vertex, like the one in Fig. 2, can therefore be expressed as two-loop diagrams with a soft vertex, like the one in Fig. 5. The cancellations that restore the Ward identity must come from other diagrams of order $g_s(m_c)^5$. Since the complete amplitude for J/ψ production can be expressed as a sum of diagrams with a soft vertex Γ_μ, the cancellations must come specifically from other two-loop diagrams with a soft vertex Γ_μ. There are hundreds of two-loop diagrams with a vertex Γ_μ, but only 15 of them have an s–channel $c\bar{c}$ cut. They can be obtained by connecting the c lines and the $\bar{c}$ lines in the products of the three diagrams for $g\,g \to c\,\bar{c}$ in Fig. 6 and the five diagrams for $c\,\bar{c} \to c\bar{c}_1(^3S_1)\,g$ in Figs. 7 and 8.

If we retain only this leading s-channel $c\bar{c}$ cut contribution, the phenomenological model of LCK has a straightforward interpretation in terms of the diagrams in Figs. 6, 7, and 8. The 4 cut diagrams of LCK with vertex Γ_μ can be identified with the products of the first two diagrams for $gg \to c\bar{c}$ in Fig. 6 and the two diagrams for $c\,\bar{c} \to J/\psi\,g$ in Fig. 7. The third diagram for $gg \to c\bar{c}$ in Fig. 6, which involves a 3-gluon vertex, does not contribute at leading order because of the $^3S_1$ quantum numbers of the J/ψ. The vertex Γ_μ used by LCK is a phenomenological model for the hard vertex obtained by absorbing the exchanged gluon into the soft vertex in the two diagrams of Fig. 7. The 2 cut diagrams of LCK with vertex $\Gamma_{\mu\nu}$ can be identified with the products of the first two diagrams for $gg \to c\bar{c}$ in Fig. 6 and the 3 diagrams for $c\,\bar{c} \to J/\psi\,g$ in Fig. 8. Thus, at leading order, LCK's vertex $\Gamma_{\mu\nu}$ is just an ad-hoc model for the sum of the 3 diagrams in Fig. 8.

This argument shows that the s-channel $c\bar{c}$ cut is a contribution to the color-singlet amplitude that is subleading in the strong coupling constant. Indeed, the leading contribution to the amplitude for $g\,g \to J/\psi\,g$ for a J/ψ with transverse momentum of order m_c comes from one-loop diagrams with a soft vertex Γ_μ, like the one in Fig. 2. The amplitude is therefore of order $g_s^3(m_c)$, so the cross section is of order $\alpha_s^3(m_c)$. With the appropriate choice of the phase of Γ_μ, the leading contribution to the amplitude is real-valued and can be identified
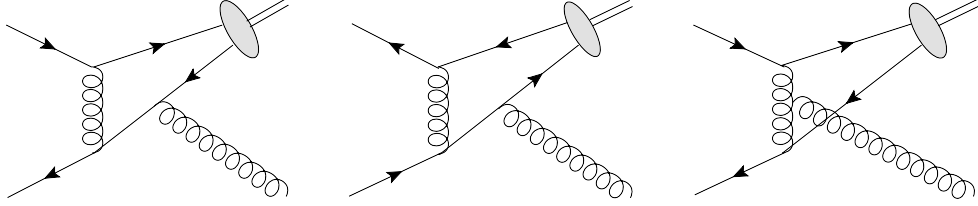
FIG. 8: Three of the 5 one-loop Feynman diagrams for $c\bar{c} \to J/\psi\, g$ with a soft-vertex Γ_μ.

with the CSM amplitude. The charm-pair rescattering amplitude from the s–channel cut for $g\,g \to J/\psi\, g$ is of order $g_s^5(m_c)$ and is pure imaginary. There is no interference between this charm-pair rescattering amplitude and the leading term in the CSM amplitude. The cross section from charm-pair rescattering is therefore of order $\alpha_s^5(m_c)$, which is NNLO in the QCD coupling constant.

IV.   NRQCD CALCULATION

We now consider the inclusive cross section for J/ψ production in $p\bar{p}$ collisions. At leading order in α_s, the only parton process that contributes to the CSM term is $gg \to c\bar{c}_1(^3S_1)\, g$, where $c\bar{c}_1(^3S_1)$ represents a c and $\bar{c}$ in a color-singlet spin-triplet state whose momenta are both equal to half that of the J/ψ. The contribution of this parton process to the cross section is

$$\sigma_{\text{CSM}}[p\bar{p} \to J/\psi + X] = \int dx_1 dx_2 f_{g/p}(x_1) f_{g/\bar{p}}(x_2)\hat{\sigma}[g\,g \to c\bar{c}_1(^3S_1)\, g]\langle\mathcal{O}_1^{J/\psi}(^3S_1)\rangle. \quad (1)$$

The parton cross section $\hat{\sigma}$ can be expressed as an integral over the solid angle $\Omega_{c\bar{c}}$ for the momentum of the $c\bar{c}$ pair:

$$\hat{\sigma}\left[g\,g \to c\bar{c}_1(^3S_1)\, g\right] = \frac{\hat{s} - 4m_c^2}{64\pi^2\hat{s}^2} \int d\Omega_{c\bar{c}} \left|\mathcal{M}[g\,g \to c\bar{c}_1(^3S_1)\, g]\right|^2. \quad (2)$$

The prefactor comes from the 2-body phase space for a $c\bar{c}$ pair and a gluon multiplied by the flux factor $1/(2\hat{s})$, where $\hat{s}$ is the square of the invariant mass of the colliding gluons (which was denoted by s in the previous sections). The leading order term in the invariant matrix element $\mathcal{M}$ comes from tree diagrams, such as the diagram in Fig. 2 with the blob omitted, and is order g_s^3. This term is real-valued provided a real-valued basis for the polarization vector of the gluons is used. The leading imaginary terms in the invariant matrix element are of order g_s^5. Using the Cutkosky cutting rules, the imaginary part can be expressed as a sum over the s–channel cuts through the diagrams, which separate the incoming gluons from the $c\bar{c}$ pair and the gluon in the final state. At order g_s^5, the only s–channel cuts are gg cuts and $c\bar{c}$ cuts. The contribution from the s–channel $c\bar{c}$ cut, which is the charm-pair rescattering term, can be expressed as in integral over the solid angle Ω_c of the charm quark in the intermediate state:

$$\text{Im}\mathcal{M}_{c\bar{c}\text{ cut}}[g\,g \to c\bar{c}_1(^3S_1)\, g] = \frac{(\hat{s} - 4m_c^2)^{1/2}}{64\pi^2\hat{s}^{1/2}} \int d\Omega_c \sum \mathcal{M}[g\,g \to c\,\bar{c}]\mathcal{M}[c\,\bar{c} \to c\bar{c}_1(^3S_1)\, g]. \quad (3)$$

The prefactor comes from multiplying the 2-body phase space for c and $\bar{c}$ by a factor of $1/2$ from the optical theorem. The sum is over the helicities and the color indices of the intermediate c and $\bar{c}$. At leading order, the first invariant matrix element $\mathcal{M}$ on the right side of Eq. (3) is the sum of the three Feynman diagrams for $g\,g \to c\,\bar{c}$ in Fig. 6, whereas the second invariant matrix element $\mathcal{M}$ on the right side of Eq. (3) is the sum of the five Feynman diagrams for $c\,\bar{c} \to c\bar{c}_1(^3S_1)\,g$ in Figs. 7 and 8 with the blobs omitted. In the expression for $\mathrm{Im}\mathcal{M}_{c\bar{c}\,\mathrm{cut}}$ that comes directly from the optical theorem, the last factor in Eq. (3) is the complex conjugate of the invariant matrix element for $c\bar{c}_1(^3S_1)\,g \to c\,\bar{c}$. The time reversal symmetry of QCD has been used to express this as the invariant matrix element for $c\,\bar{c} \to c\bar{c}_1(^3S_1)\,g$.

The charm-pair rescattering contribution to the CSM cross section is obtained by replacing the invariant matrix element $\mathcal{M}$ in Eq. (2) by the contribution to its imaginary part from the $c\bar{c}$ cut given in Eq. (3), and then inserting the resulting expression for the parton cross section $\hat{\sigma}$ into the J/ψ production cross section in Eq. (1). We proceed to explain how we calculate this cross section numerically. We use a helicity basis for the polarization vectors of the gluons and the $c\bar{c}$ pair. The projection of the matrix element for $g\,g \to c\bar{c}\,g$ onto the appropriate $c\bar{c}$ helicity state is described in Ref. [6]. We use MadGraph [7] to evaluate the helicity amplitudes $\mathcal{M}[g\,g \to c\,\bar{c}]$ and $\mathcal{M}[c\,\bar{c} \to c\bar{c}_1(^3S_1)\,g]$ numerically. The sum over the color indices of the intermediate $c\bar{c}$ pair is carried out by calculating the color factors by hand. The replacement of $\mathcal{M}$ in Eq. (2) by the expression for $\mathrm{Im}\mathcal{M}$ in Eq. (3) gives a 3-fold angular integral. Together with the integration over parton momentum fractions in Eq. (1), we have an 8-dimensional integral that is calculated numerically using the adaptive integration program VEGAS [8].

V. NUMERICAL RESULTS

In this Section, we calculate the charm-pair rescattering contribution to the CSM term in the inclusive cross section for direct J/ψ production in $p\bar{p}$ collisions at the Tevatron. We compare our results, which have no additional parameters, to those from models for the s–channel cut contributions proposed by Lansberg et al. [4, 5]. We also compare our charm-pair rescattering contribution, which is a contribution to the CSM term at NNLO, to the CSM terms at LO and NLO [9, 10].

The parton cross section for charm-pair rescattering in the color-singlet model depends only on α_s, m_c, and the NRQCD matrix element $\langle \mathcal{O}_1^{J/\psi}(^3S_1) \rangle$. To compare our results with those of Lansberg et al., we use parameters that are as close to theirs as possible. They used a large value of the charm quark mass: $m_c = 1.87$ GeV. They determined the normalization factor in their vertex Γ_μ by fitting the partial width for $J/\psi \to e^+ e^-$. The corresponding factor in the color-singlet model is $\langle \mathcal{O}_1^{J/\psi}(^3S_1) \rangle$. At leading order in α_s, the NRQCD factorization formula for the partial width for $J/\psi \to e^+ e^-$ is

$$\Gamma[J/\psi \to e^+ e^-] = \frac{8\pi\alpha^2}{81m_c^2} \langle \mathcal{O}_1^{J/\psi}(^3S_1) \rangle \, , \tag{4}$$

where we have used the relation $\langle \mathcal{O}_1^{J/\psi}(^3S_1) \rangle = 3\langle \mathcal{O}_1(^3S_1) \rangle_{J/\psi}$ between the standard NRQCD production and decay matrix elements [1]. Setting $m_c = 1.87$ GeV and $\alpha = 1/129.6$, we determine the NRQCD matrix element to be $\langle \mathcal{O}_1^{J/\psi}(^3S_1) \rangle = 1.05$ GeV^3. In Refs. [4, 5], the other parameter in the vertex Γ_μ is a momentum scale $\Lambda = 1.8$ GeV. In Ref. [5], there
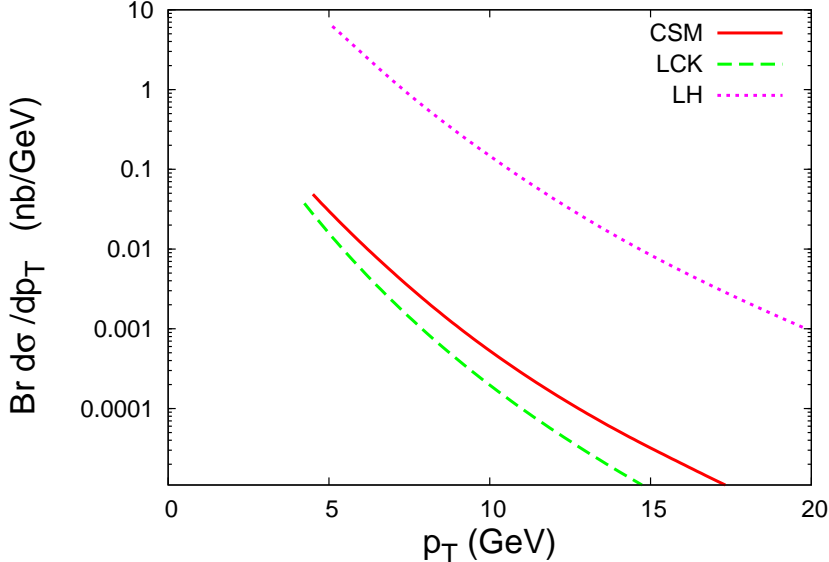
FIG. 9: Differential cross section for direct J/ψ production at the Tevatron from charm-pair rescattering (s–channel cut). The differential cross section $d\sigma/dp_T$ integrated over the pseudorapidity interval $|\eta| < 0.6$ and multiplied by the branching fraction for $J/\psi \to \mu^+\mu^-$ is shown as a function of p_T. The curves are the CSM result (solid line), the result from the model of Ref. [4] (dashed line), and the result from the model of Ref. [5] (dotted line).

were also two additional parameters associated with the vertex $\Gamma_{\mu\nu}$. They were adjusted to fit the inclusive cross sections for direct J/ψ production at transverse momenta up to 10 GeV that were measured by the CDF Collaboration. Following Lansberg et al., we set the center-of-mass energy of the colliding p and $\bar{p}$ to $\sqrt{s} = 1.8$ TeV, we impose a pseudo-rapidity cut $|\eta| < 0.6$ on the J/ψ, and we use the Martin-Roberts-Stirling-Thorne (MRST) parton distribution function parton distributions at leading order [11]. We take both the renormalization and factorization scales to be $\sqrt{4m_c^2 + p_T^2}$. In Fig. 9, our result for the charm-pair rescattering (s–channel cut) contribution is compared to the results from Refs. [4] and [5]. Our result is larger than that from Ref. [4] by a factor that increases from about 2 at $p_T = 4$ GeV to about 4 at $p_T = 15$ GeV. Our result is more than two orders of magnitude smaller than that from Ref. [5]. Since the primary difference between Refs. [4] and [5] is the model for the vertex $\Gamma_{\mu\nu}$, our results suggest that the model for the vertex $\Gamma_{\mu\nu}$ used in Ref. [5] gives an unrealistically large contribution from the s-channel $c\bar{c}$ cut mechanism.

We now proceed to assess the phenomenological importance of charm-pair rescattering by comparing it to experimental data and to other CSM contributions. The data on inclusive J/ψ production at the Tevatron published by the CDF collaboration are shown in Fig 10. The differential cross section for prompt J/ψ measured in Ref. [14], which includes a factor Br$= 0.0588$ for the branching fraction of J/ψ into $\mu^+\mu^-$, has been converted into a differential cross section for direct J/ψ by multiplying by the fraction of J/ψ's that are produced directly, which was measured in Ref. [15]. For a consistent comparison with these data, we set $\sqrt{s} = 1.96$ TeV and we impose a rapidity cut $|y| < 0.6$ in our calculations of CSM contributions. We set $m_c = 1.5$ GeV, which is natural as it is close to half the mass of
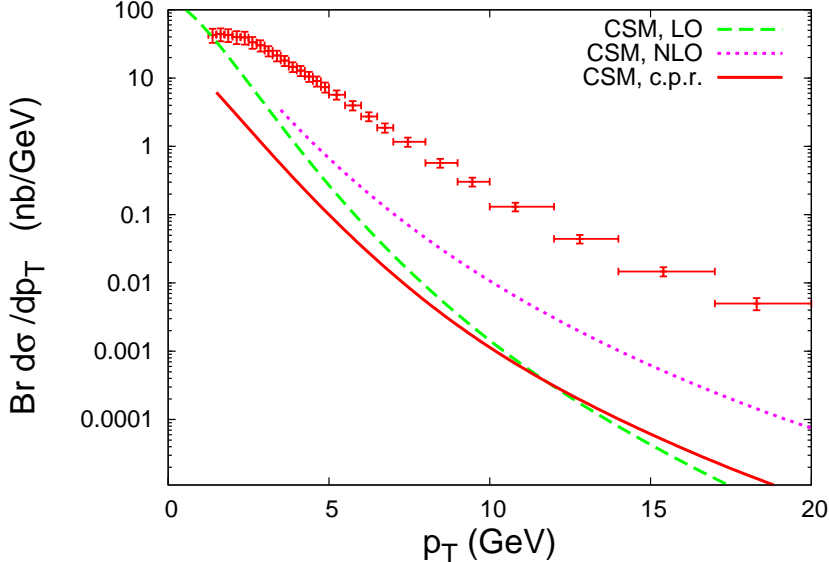
FIG. 10: Color-singlet contributions to the differential cross section for direct J/ψ production at the Tevatron compared to data from the CDF Collaboration [14, 15]. The differential cross section $d\sigma/dp_T$ integrated over the rapidity interval $|y| < 0.6$ and multiplied by the branching fraction for $J/\psi \to \mu^+\mu^-$ is shown as a function of p_T. The curves are the color-singlet model terms at LO (dashed line), through NLO (dotted line), and at NNLO from charm-pair rescattering (solid line).

the J/ψ. We set $\langle \mathcal{O}_1^{J/\psi}(^3S_1) \rangle = 1.16$ GeV^3, which is the value obtained from the wave function at the origin in the Buchmüller-Tye potential [12]. We also use a more recent set of parton distributions [13]. We choose the factorization and renormalization scales to be $\sqrt{4m_c^2 + p_T^2}$. In Fig. 10, the NNLO contribution from charm-pair rescattering to the CSM term in the differential cross section is compared to the LO and NLO terms. The overall charm-pair rescattering contribution is a subdominant fraction of the CSM term. At $p_T = 4$ GeV, it is smaller than the LO term by about a factor of 3. As it decreases more slowly with p_T, it is larger than the LO term for $p_T > 12$ GeV. This is expected since the differential cross section associated to the leading order CSM production mechanism shows a well-known strong kinematic suppression at large p_T. Compared to the NLO term, the charm-pair rescattering contribution is negligible in the whole p_T range accessible at the Tevatron: it is suppressed by a factor that increases from about 7 at $p_T = 4$ GeV to about 10 at $p_T = 20$ GeV. We also note that the CSM yield at NLO substantially underestimates the Tevatron data. We conclude that charm-pair rescattering can account only for a small fraction of the cross section measured by the CDF collaboration.

VI. SUMMARY

We have identified the s–channel cut mechanism for quarkonium production proposed by Lansberg, Cudell, and Kalinovsky [4] as $Q\bar{Q}$ rescattering. In the NRQCD factorization approach, this mechanism gives a contribution to the color-singlet model (CSM) term at NNLO and it can be calculated without introducing any additional phenomenological parameters beyond the usual NRQCD matrix elements. In the case of J/ψ production at the

Tevatron, the NNLO contribution from charm-pair rescattering is comparable to the LO CSM term but is much smaller than the NLO term. The charm-pair rescattering contribution is substantially smaller than the CDF data in the whole p_T range accessible at the Tevatron. We conclude that charm-pair rescattering (the s–channel cut mechanism) is not a dominant mechanism for charmonium production in high-energy collisions.

Acknowledgments

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