Classes of f-Deformed Landau Operators: Nonlinear Noncommutative Coordinates from Algebraic Representations[1]

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We consider, in a superspace, new operator dependent noncommutative (NC) geometries of the nonlinear quantum Hall limit related to classes of f-deformed Landau operators in the spherical harmonic well. Different NC coordinate algebras are determined using unitary representation spaces of Fock-Heisenberg tensored algebras and of the Schwinger-Fock realisation of the $su(1,1)$ Lie algebra. A reduced model allowing an underlying $\mathcal{N}=2$ superalgebra is also discussed.

1 Introduction

Generalised Landau problems have recently been studied in the context of deformed quantum mechanics [1,2]. It has been shown that, beyond the well-known duality between interactions and noncommutative (NC) geometry [3], the deformed quantum Hall NC geometry becomes fully operator dependent.

In this contribution, we pursue our previous investigations [1] of generalised NC coordinate algebras stemming from effects of interactions and algebraic extensions of canonical quantum mechanics. Extending the deformed Landau model [1] to a superspace, we consider a f-deformed quantisation, à la Jannussis *et al.* [4], of a Landau operator in a spherical well with an additional odd Grassmann harmonic oscillator. It ought to be emphasized that the choice of the bosonic and fermionic harmonic potentials fits the exact solvability condition of the problem in the presence of an electromagnetic field. Furthermore, the NC harmonic strength has to be related to a nontrivial time reversal symmetry breaking [5] when the given system is only composed of constant piecewise potentials and of configurations with negative angular momentum values for some states.

The outline is as follows. Section 2, in which notations are specified, is devoted to the study of the classical model. In Section 3, the f-deformed theory is considered. Projecting the coordinates onto infinite dimensional sub-Hilbert spaces playing the rôle of specific types of algebra representation spaces, we derive NC geometries associated with the modified model. A reduced class of models allows us to deduce the existence of a $\mathcal{N}=2$ superalgebra. The paper ends with some remarks in Section 4.

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2 Classical Study

Consider the nonrelativistic confined motion of a particle of mass m and charge q with $\vec{r} = (x, y, 0)$ as position coordinate vector in some planar nanoscopic semiconductor sample subjected to static background fields, namely, a planar electric field $\vec{E}$ and a magnetic field $\vec{B}$ perpendicular to the (x, y)-plane. In addition, the particle is submitted to a spherical harmonic potential with angular frequency ω and stiffness constant $k = m\omega^2$. Finally, we introduce an odd Grassmann harmonic oscillator with angular frequency ω_0 and odd Grassmann degrees of freedom θ and $\theta^\dagger$. Units such that $c = 1$ are used implicitly.

The total Lagrange function L of the system consists of the sum of two functions, L_0 and L_1, given as follows in the symmetric gauge $\vec{A}(\vec{r}) = \vec{B} \times \vec{r}/2$,

$$\begin{aligned} L &= L_0 + L_1, \\ L_0 &= \tfrac{1}{2} m\, \dot{\vec{r}}^{\,2} + q\,(\vec{E}\cdot\vec{r} + \dot{\vec{r}}\cdot\vec{A}(\vec{r})) - \tfrac{1}{2} m\,\omega^{\,2}\vec{r}^{\,2}, \\ L_1 &= \tfrac{i}{2} m_0\,\omega_0(\theta^\dagger\,\dot{\theta} - \dot{\theta}^\dagger\theta) - \tfrac{1}{2} m_0\,\omega_0^2\theta^\dagger\theta. \end{aligned} \tag{1}$$

In terms of the shifted coordinates defined by $\vec{R}(t) = \vec{r}(t) - q\vec{E}/k$ which minimise the total potential energy in the radial sector, we have the momenta conjugate to $\vec{R}$, θ and $\theta^\dagger$, respectively (using left derivatives with respect to odd Grassmann variables),

$$\vec{P} = m\dot{\vec{R}} - \frac{1}{2}q(\frac{q}{k}\,\vec{E} + \vec{R}) \times \vec{B}, \qquad \pi_\theta = -\frac{i}{2}m_0\omega_0\,\theta^\dagger, \qquad \pi_{\theta^\dagger} = -\frac{i}{2}m_0\omega_0\,\theta.$$

Noting that L_1 is already in Hamiltonian form, the canonical Hamiltonian of the system is given as,

$$H = \tfrac{1}{2m}\left[\vec{\pi} - q\vec{A}(\vec{R})\right]^{2} + \tfrac{1}{2}k\,\vec{R}^{\,2} - \tfrac{q^2}{2k}\vec{E}^{\,2} + \tfrac{1}{2}k_0\,\theta^\dagger\theta, \tag{2}$$

where

$$\vec{\pi} := \vec{P} + \tfrac{q^2}{2k}\,\vec{E}\times\vec{B}, \qquad \vec{A}(\vec{R}) = \tfrac{1}{2}\vec{B}\times\vec{R}, \qquad k_0 := m_0\omega_0^2, \tag{3}$$

having introduced the parametrisation $\vec{R} = (X, Y, 0)$, $\vec{P} = (P_X, P_Y, 0)$ and $\vec{\pi} = (\pi_X, \pi_Y, 0)$. The reduced Poisson brackets for even (e) and odd (o) quantities are given by,

$$\{Z, P_Z\}_e = 1 = \{Z, \pi_Z\}_e, \quad Z = X, Y, \qquad \{\theta, \theta^\dagger\}_o = -i/(m_0\omega_0).$$

By changing variables to classical Fock modes, we obtain, with $\Omega^2 = \sqrt{\omega^2 + \varpi^2}$ and $\varpi = (qB)/(2m)$,

$$a_Z = \left(\tfrac{m\Omega}{2\hbar}\right)^{1/2}[Z + \tfrac{i}{m\Omega}\pi_Z], \qquad a_\pm = \tfrac{1}{\sqrt{2}}[a_X \mp i a_Y], \qquad \{a_\pm, a_\pm^*\} = -\tfrac{i}{\hbar}, \tag{4}$$

$$b = \left(\tfrac{m_0\omega_0}{2\hbar}\right)^{1/2}\theta, \qquad b^* = \left(\tfrac{m_0\omega_0}{2\hbar}\right)^{1/2}\theta^\dagger, \quad \{b, b^*\} = -\tfrac{i}{\hbar}, \tag{5}$$

$a_\pm^*$ and b^* being the complex conjugates of the variables $a_\pm$ and b, respectively. After some calculations from (2), one finds the reparametrised Hamiltonian,

$$\begin{aligned} H = \;& \tfrac{1}{2}\hbar\Omega(a_+a_+^* + a_+^*a_+ \,+\, a_-a_-^* + a_-^*a_-) \\ & -\tfrac{1}{2}\hbar\varpi(a_+a_+^* + a_+^*a_+ \,-\, (a_-a_-^* + a_-^*a_-)) \,+\, \tfrac{1}{2}\hbar\omega_0\, b^*b \,-\, \tfrac{(q\vec{E})^{\,2}}{2k}, \end{aligned} \tag{6}$$

where $\hbar$ is regarded simply at this stage as a numerical constant introduced for dimensional convenience.

3 f-Deformations of Landau Operators

In this Section, we consider f-deformed classes of the Landau Hamiltonian function (6).

3.1 Deformed models and spectra

Generalising the canonical helicity mode quantisation given by two usual commuting Fock algebras, $[a_\pm, a^\dagger_\pm] = 1$, with number operators $N_\pm = a^\dagger_\pm a_\pm$, we introduce two generalised deformed Fock algebras of the type constructed by Jannussis *et al.* [4]. These algebras are factorised with respect to helicity and are defined by two real functions $f_\pm(N_\pm) \neq 0$ and the generators $A_\pm$, $A^\dagger_\pm$ such that,

$$A_\pm = a_\pm f_\pm(N_\pm), \qquad A^\dagger_\pm = f_\pm(N_\pm) a^\dagger_\pm,$$

$$[A_\pm, A^\dagger_\pm] = (N_\pm + 1) f^2_\pm(N_\pm + 1) - (N_\pm) f^2_\pm(N_\pm). \tag{7}$$

Each of these $f_\pm(N_\pm)$-dependent algebras generalises the usual Fock algebra obtained for $f_\pm(N_\pm) = \mathbb{I}$. For notational convenience, we introduce

$$\{N\}_\pm := N_\pm f^2_\pm(N_\pm) = A^\dagger_\pm A_\pm, \qquad \{N+1\}_\pm := (N_\pm + 1) f^2_\pm(N_\pm + 1) = A_\pm A^\dagger_\pm.$$

For the odd Grassmann sector, we use the ordinary anticommuting algebra $\{b, b^\dagger\} = \mathbb{I}$, defining fermionic operators b and $b^\dagger$.

These algebras and operators have well defined representations on the Fock Hilbert space $\mathcal{F}$ of states defined by the tensor product of two copies, $F_{b,\pm}$, associated with the bosonic sector, and the two dimensional representation space, F_f, of the fermionic oscillator algebra provided by the spin orthonormalised states $|0\rangle$ and $|1\rangle$. We have,

$$\mathcal{F} = F_{b,+} \otimes F_{b,-} \otimes F_f = \{|n_+, n_-, s\rangle, \;\; n_\pm \in \mathbb{N}, \, s = 0, 1\}.$$

We restrict the study to the following $f_\pm$-deformed quantum Hamiltonians,

$$\begin{aligned} \mathcal{H} \;&=\; \tfrac{1}{2}\hbar\Omega \textstyle\sum_{\epsilon=\pm} \{A_\epsilon, A^\dagger_\epsilon\} - \tfrac{1}{2}\hbar\varpi \textstyle\sum_{\epsilon=\pm} \epsilon \{A_\epsilon, A^\dagger_\epsilon\} \\ &\quad + \tfrac{1}{4}\hbar\omega_0 \textstyle\sum_{\epsilon=\pm} [A_\epsilon, A^\dagger_\epsilon]\, b^\dagger b - \tfrac{q^2}{2k} \vec{E}^{\,2} + K_0, \end{aligned} \tag{8}$$

$$\begin{aligned} \mathcal{H}_0 \;&=\; \hbar\Omega \left[A^\dagger_+ A_+ + A^\dagger_- A_- + 1 \right] - \hbar\varpi \left[A^\dagger_+ A_+ - A^\dagger_- A_- \right] \\ &\quad + \tfrac{1}{2}\hbar\omega_0 \, [A_-, A^\dagger_-]\, b^\dagger b - \tfrac{q^2}{2k} \vec{E}^{\,2} + K_0. \end{aligned} \tag{9}$$

The Hamiltonian $\mathcal{H}$ has manifest rotational symmetry, while a reduced model which generates a superalgebra may be defined from $\mathcal{H}_0$. In the following, we focus our interest on $\mathcal{H}$ and deal with $\mathcal{H}_0$ in Subsection 3.3.

The Hamiltonian $\mathcal{H}$ is diagonal in the basis $\{\, |n_+, n_-, s\rangle \,\}$, $s = 0, 1$, with eigenvalues,

$$\begin{aligned} E(n_+, n_-, 0) \;&=\; \tfrac{1}{2}\hbar\Omega \textstyle\sum_{\epsilon=\pm} \left(\{n+1\}_\epsilon + \{n\}_\epsilon \right) \\ &\quad - \tfrac{1}{2}\hbar\varpi \textstyle\sum_{\epsilon=\pm} \epsilon \left(\{n+1\}_\epsilon + \{n\}_\epsilon \right) - \tfrac{q^2}{2k} \vec{E}^{\,2} + K_0, \end{aligned} \tag{10}$$

$$\begin{aligned} E(n_+, n_-, 1) \;&=\; \tfrac{1}{2}\hbar\Omega \textstyle\sum_{\epsilon=\pm} \left(\{n+1\}_\epsilon + \{n\}_\epsilon \right) \\ &\quad - \tfrac{1}{2}\hbar\varpi \textstyle\sum_{\epsilon=\pm} \epsilon \left(\{n+1\}_\epsilon + \{n\}_\epsilon \right) \\ &\quad + \tfrac{1}{4}\hbar\omega_0 \textstyle\sum_{\epsilon=\pm} \left(\{n+1\}_\epsilon - \{n\}_\epsilon \right) - \tfrac{q^2}{2k} \vec{E}^{\,2} + K_0, \end{aligned} \tag{11}$$

with $\{n\}_\epsilon = n_\epsilon f^2_\epsilon(n_\epsilon)$. The degeneracy can easily be determined for particular relevant reductions [2]. This is the case for the model reduced under $k = 0$ and $\vec{E} = \vec{0}$, leading to $\varpi = \Omega$, $\omega_0 = 0$, with the spectrum $E(n_+, n_-, s) = \hbar\varpi \left(\{n+1\}_- + \{n\}_- \right)$, $s = 0, 1$. Projecting onto one of the spin subsectors, the deformed Landau problem remains infinitely degenerate in the angular momentum $\ell = n_+ - n_- \geq -n_-$ for each of the Landau levels distinguished by $n_- \geq 0$. It is instructive [1] to consider the angular momentum operator as function of the ordinary numbers $N_\pm$ and not as the nonlinear combinations $a^\dagger_\pm a_\pm + a_\pm a^\dagger_\pm = \{N\}_\pm + \{N+1\}_\pm$, although this is actually a matter of choice. Thus the deformed angular momentum operator could be defined by $L = \hbar\,(N_+ - N_-)$, thence $\mathcal{H}$ and L form a complete set of commuting operators simultaneously diagonalisable.

3.2 Nonlinear noncommutative coordinates from algebraic representations

3.2.1 Projection onto deformed Landau levels

Due to the presence of spin degrees of freedom, a number of projectors onto the Landau level n_-^0 are available. The following projectors prove to be relevant,

$$\mathcal{P}(s,n_-^0) = \sum_{n_+=0}^{\infty} |n_+, n_-^0, s\rangle\langle n_+, n_-^0, s|, \qquad s = 0,1, \tag{12}$$

$$\mathcal{P}(n_-^0) = \mathcal{P}(0,n_-^0) + \mathcal{P}(1,n_-^0), \qquad \mathbb{P}(n_-^0) = \mathcal{P}(0,n_-^0) + \mathcal{P}(1,n_-^0 - 1). \tag{13}$$

For any operator T and $s = 0,1$, let us adopt the notations,

$$\bar{T}^{\,s} = \mathcal{P}(s,n_-^0)\; T\; \mathcal{P}(s,n_-^0), \qquad s = 0,1; \qquad \bar{T} = \mathcal{P}(n_-^0)\; T\; \mathcal{P}(n_-^0), \qquad \bar{\mathbf{T}} = \mathbb{P}(n_-^0)\; T\; \mathbb{P}(n_-^0). \tag{14}$$

On the other hand, the deformed realisations of the quantum cartesian plane coordinates can be cast into the form,

$$\mathcal{X} = \frac{1}{2}\left(\frac{\hbar}{m\Omega}\right)^{1/2}(A_+ + A_- + A_+^\dagger + A_-^\dagger), \tag{15}$$

$$\mathcal{Y} = \frac{i}{2}\left(\frac{\hbar}{m\Omega}\right)^{1/2}(A_+ - A_- - A_+^\dagger + A_-^\dagger). \tag{16}$$

Before projecting onto any given Landau level, the quantum cartesian plane coordinates $\mathcal{X}$ and $\mathcal{Y}$ do not commute as operators as expected from general deformed theories [1,2]. Indeed, from a direct evaluation the commutator

$$[\mathcal{X},\mathcal{Y}] = \frac{i}{2}\frac{\hbar}{m\Omega}\left(\{N\}_+ - \{N+1\}_+ + \{N+1\}_- - \{N\}_-\right) \tag{17}$$

is seen not to vanish, unless one effectively sets $f_\pm(N_\pm) = 1$.

Quantum geometry associated with the deformed quantum Hall effect can be derived from the commutator of the cartesian coordinate operators projected onto any of the Landau levels. After some calculations, we obtain, for $s = 0,1$,

$$\left[\bar{\mathcal{X}}^{\,s}, \bar{\mathcal{Y}}^{\,s}\right] = \frac{i\hbar}{2m\Omega}\left(\{N\}_+ - \{N+1\}_+\right)\mathcal{P}(s,n_-^0) \tag{18}$$

which radically differs from the unit operator and also from the general quantum plane noncommutativity (17). In the situation such that $f_\pm(N_\pm) = 1$, indeed the result characteristic of the quantum Hall effect in the plane is recovered, namely $\left[\bar{\mathcal{X}}^{\,s}, \bar{\mathcal{Y}}^{\,s}\right] = -(i\hbar)/(2m\Omega)\,\mathcal{P}(s,n_-^0)$. If an alternative is considered through the use of the total spin state projectors $\mathcal{P}(n_-^0)$ and $\mathbb{P}(n_-^0)$, one easily obtains the corresponding commutators. In the specific instance such that $\vec{E} = \vec{0}$, $k = 0$ and $\omega_0 = 0$, one then has $[x,y] = -(i\hbar)/(qB)$.

Actually, projection onto the lowest Landau level is not obligatory [1,3] in order to define a relevant NC geometry. The pertinent requirement is the separation of energy levels. In the following, we will consider the $su(1,1)$ algebraic representation spaces from which different NC geometries of the same model could be revealed.

3.2.2 Projection onto $su(1,1)$ representation spaces

The Schwinger realisation of the $su(1,1)$ algebra is defined by the two mode operators $J_+ = a_+^\dagger a_-^\dagger$, $J_- = a_+ a_-$ and $J_0 = (1/2)\,[N_+ + N_- + 1]$, which satisfy the relations

$$[J_0, J_\pm] = \pm J_\pm, \qquad [J_+, J_-] = -2J_0. \tag{19}$$

The Casimir operator takes the form $C_2 = -J_- J_+ + J_0(J_0+1)$ and the total number operator can be written as $L = N_+ - N_-$. These two last operators commute and can be diagonalised on a common vector basis. Denoting the eigenvalues of C_2 by $C_2 = \ell(\ell+1)$, then $L = -(2\ell+1)$ since one may show that $C_2 = (1/4)(L+1)(L-1)$. As a consequence, the $su(1,1)$ unitary representation spaces are characterised by half integers $\ell \in \mathbb{Z}/2$. We get the following spaces, given $k_\ell^0 = \max(-2\ell, 1)$,

$$\mathcal{E}_\ell = \left\{\, |k-1,\, 2\ell+k\rangle,\ k = k_\ell^0, k_\ell^0+1, k_\ell^0+2, \dots \right\}, \tag{20}$$

and the relations

$$J_0|k-1,\, 2\ell+k\rangle = (M=\ell+k)\,|k-1,\, 2\ell+k\rangle, \tag{21}$$

$$C_2|k-1,\, 2\ell+k\rangle = \ell(\ell+1)\,|k-1,\, 2\ell+k\rangle, \tag{22}$$

$$J_-|k-1,\, 2\ell+k\rangle = \sqrt{(k-1)(2\ell+k)}\,|k-2,\, 2\ell+k-1\rangle, \tag{23}$$

$$J_+|k-1,\, 2\ell+k\rangle = \sqrt{k(2\ell+k+1)}\,|k,\, 2\ell+k+1\rangle. \tag{24}$$

For $s \in \{0,1\}$ and $\ell \in \mathbb{Z}/2$, a family $\{P(s,\ell)\}_{s,\ell}$ of projectors onto the Hilbert subspaces $\mathcal{E}_\ell \otimes |s\rangle \subset \mathcal{F}$ is given by,

$$P(s,\ell) = \textstyle\sum_{k=k^0_\ell}^{\infty} |k-1, 2\ell+k, s\rangle\langle k-1, 2\ell+k, s|. \tag{25}$$

Let $\epsilon = \pm 1$ be a new parameter. We can form the linear combination of the spaces $\mathcal{E}_\ell$ and $\mathcal{E}_{\ell+\frac{\epsilon}{2}}$. Now, by further combining the operators (25), we get a new set of projectors onto the space $(\mathcal{E}_\ell \oplus \mathcal{E}_{\ell+\frac{\epsilon}{2}}) \otimes |s\rangle$, such that

$$\mathbb{P}(\epsilon,s,\ell) = P(s,\ell) + P(s,\ell+\frac{\epsilon}{2}), \qquad \epsilon = \pm 1, \qquad s = 0,1, \tag{26}$$

so that the projected coordinates onto this space are expressed by $\mathbb{P}(\epsilon,s,\ell)\, Z\, \mathbb{P}(\epsilon,s,\ell) = Z^{\epsilon,s,\ell},\ Z = \mathcal{X},\, \mathcal{Y}$. Finally, the noncommuting coordinate algebra associated with $\mathcal{E}_\ell$ can be written as,

$$\begin{aligned}
\left[\mathcal{X}^{\epsilon,s,\ell},\, \mathcal{Y}^{\epsilon,s,\ell}\right] = \frac{i\hbar}{2m\Omega}\ & \Big[\textstyle\sum_{k=0}^{\infty}\{k+1\}_+ \big[|k+1, 2\ell+k+\frac{\epsilon+3}{2}, s\rangle\langle k+1, 2\ell+k+\frac{\epsilon+3}{2}, s| \\
& -|k, 2\ell+k+\frac{\epsilon+3}{2}, s\rangle\langle k, 2\ell+k+\frac{\epsilon+3}{2}, s|\big] \\
& +\textstyle\sum_{k=0}^{\infty}\{2\ell+k+\frac{\epsilon+3}{2}\}_- \big[|k, 2\ell+k+\frac{\epsilon+1}{2}, s\rangle\langle k, 2\ell+k+\frac{\epsilon+1}{2}, s| \\
& -|k, 2\ell+k+\frac{\epsilon+3}{2}, s\rangle\langle k, 2\ell+k+\frac{\epsilon+3}{2}, s|\big] \\
& -\textstyle\sum_{k=0}^{\infty}\sqrt{\{k+1\}_+\{2\ell+k+\frac{\epsilon+5}{2}\}_-} \\
& \big[|k, 2\ell+k+\frac{\epsilon+3}{2}, s\rangle\langle k+1, 2\ell+k+\frac{\epsilon+5}{2}, s| \\
& +|k+1, 2\ell+k+\frac{\epsilon+5}{2}, s\rangle\langle k, 2\ell+k+\frac{\epsilon+3}{2}, s|\big] \\
& +\textstyle\sum_{k=0}^{\infty}\sqrt{\{k+1\}_+\{2\ell+k+\frac{\epsilon+3}{2}\}_-} \\
& \big[|k+1, 2\ell+k+\frac{\epsilon+3}{2}, s\rangle\langle k, 2\ell+k+\frac{\epsilon+1}{2}, s| \\
& +|k, 2\ell+k+\frac{\epsilon+1}{2}, s\rangle\langle k+1, 2\ell+k+\frac{\epsilon+3}{2}, s|\big] \quad \Big].
\end{aligned} \tag{27}$$

The NC geometry thus appears in a different fashion when the projection is done onto $\mathcal{E}_\ell$. Alternatively, a nontrivial and different NC coordinate structure could also be investigated onto the associated subspace of the $su(1,1)$ unitary representation space,

$$\mathcal{E}'_k = \left\{\ |k-1,\, 2\ell+k\rangle,\ \ell = -\tfrac{k}{2}, -\tfrac{k}{2}+\tfrac{1}{2}, \dots\ \right\},\ \ k \in \mathbb{N}/\{0\}. \tag{28}$$

3.3 Reduced deformed model and supersymmetries

Consider the Hamiltonian $\mathcal{H}_0$ in (9) under the set of conditions $k = 0 = \omega$, $\vec{E} = \vec{0}$, $\Omega = \varpi = \omega_0/4 = -K_0/\hbar$. It can be rewritten as,

$$\mathcal{H}'_0 = 2\hbar\varpi(A_-^\dagger A_- + [A_-, A_-^\dagger]\, b^\dagger b). \tag{29}$$

Therefore, defining $\mathcal{Q}_- = \kappa A_-^\dagger b$ and $\mathcal{Q}_-^\dagger = \kappa^* A_- b^\dagger$, with $\kappa = i\sqrt{2\hbar\varpi}$, one has

$$\{\mathcal{Q}_-, \mathcal{Q}_-^\dagger\} = \mathcal{H}'_0, \qquad [\mathcal{Q}_-, \mathcal{H}'_0] = 0, \qquad [\mathcal{Q}_-^\dagger, \mathcal{H}'_0] = 0. \tag{30}$$

Thus, for this reduction $\mathcal{Q}_-$ and $\mathcal{Q}_-^\dagger$ generate the well-known closed superalgebra $sl(1|1)$ [6]. These are the f-deformed supercharges of the model which generalise the ordinary charges for the canonical Pauli–Landau operator. Furthermore, according to the projectors (13) and still in the notation of (14), we have the following commutators

$$\left\{\overline{\boldsymbol{\mathcal{Q}}}_-, \overline{\boldsymbol{\mathcal{Q}}}_-^\dagger\right\} = \overline{\boldsymbol{\mathcal{H}}'}_0, \qquad \left[\overline{\boldsymbol{\mathcal{Q}}}_-, \overline{\boldsymbol{\mathcal{H}}'}_0\right] = 0 = \left[\overline{\boldsymbol{\mathcal{Q}}}_-^\dagger, \overline{\boldsymbol{\mathcal{H}}'}_0\right], \tag{31}$$

showing that the superalgebra (30) is conserved onto Landau levels for the projected deformed charges.

4 Concluding Remarks

Classes of f-deformations of Landau operator in a spherical harmonic well within a superspace have been considered. Even though the odd contribution of the total Hamiltonian appears decoupled from the even sector at the classical level, interesting features have emerged through f-deformation by coupling the bosonic and fermionic sectors. It has been shown that the deformed problem is still exactly solvable with more general underlying NC geometries by considering various coordinate projections onto two mode Fock and $su(1,1)$ infinite dimensional algebraic representations. NC geometries of the deformed quantum Hall limit have shown spin, potential and quantum algebra dependencies. Finally, a f-modified version of a $\mathcal{N}=2$ superalgebra has been discussed for a reduced model when the external electric field and the harmonic oscillator well have been removed.

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