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# On the dynamical breaking of chiral symmetry: a new mechanism

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## Abstract

We consider a  $U(1)$  gauge theory, minimally coupled to a massless Dirac field, where a higher-derivative term is added to the pure gauge sector, as in the Lee-Wick models. We find that this term can trigger chiral symmetry breaking at low energy in the weak coupling regime. Then, the fermion field acquires a mass that turns out to be a function of both the energy scale associated to the higher-derivative term and the gauge coupling. The dependence of the fermion mass on the gauge coupling is non-perturbative. Extensions to  $SU(N)$  gauge theories and fermion-scalar interactions are also analyzed, as well as to theories with massive gauge fields. A few implications of these results in the framework of quark-mass generation are discussed.

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## 1 Introduction

The appearance of indefinite metrics in quantum field theory is often cause of considerable concern. A theory with an indefinite-metric contains negative-norm states, namely “ghosts”. It is well known that, if ghosts belong to the asymptotic states of the S-matrix, and are also coupled to positive-norm states, unitarity is violated. However, not all theories with indefinite metric are ill-defined.

As shown long ago by Lee and Wick [1, 2], the introduction of a negative metric in quantum mechanics does not necessarily spoil unitarity. It can sometimes lead to a fully unitary S-matrix, provided all stable particles in the spectrum have positive square length (in the Hilbert probability-space). In other words, if negative norm states have a non-vanishing decay width, thus being unstable, they are not among the asymptotic states of the S-matrix, and unitarity can be restored. Problems with

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the violation of Lorentz invariance, that might in principle arise due to the indefinite metric [3], can also be circumvented [4]. A relativistic and unitary S-matrix can indeed be defined provided the prescription introduced by Cutkosky *et al.*, regarding the deformed energy contour in the Feynman integrals, is implemented [5]. Although the above prescription is not rigorously derived from a Lagrangian field theory approach, and might look rather *ad hoc* [7], it is well defined in perturbation theory [4, 5]. More recently, non-perturbative formulation of the Lee-Wick theories has been analyzed in [6].

The Lee-Wick approach to quantum field theories prompted the construction of more general theories in which the S-matrix is fully unitary, although the Lagrangian is not Hermitian. Moreover, the exchange of both negative and positive norm states in the quantum amplitudes turns out to be an advantage. Ultraviolet divergences may indeed cancel out in the loops due to the indefinite metric of the Hilbert space.

A model satisfying all these requirements was proposed by Lee and Wick in the framework of quantum electrodynamics (QED) [2]. In particular, by replacing the standard photon field  $A_\mu$  by a complex field  $\phi_\mu = A_\mu + iB_\mu$  in the electromagnetic interaction, where  $B_\mu$  is a massive boson field with negative norm, it is possible to remove all infinities from the electromagnetic mass differences between charged particles. This procedure is equivalent to the introduction of a higher (gauge-invariant) derivative term in the Lagrangian of a primary U(1) gauge field, as can be shown by the introduction of auxiliary fields [8]. Then, the mass of the ghost field turns out to be proportional to the new physics scale  $\Lambda$  connected to the higher-derivative term. In order to render charge renormalization finite, new higher-derivative terms should be introduced for the fermion fields as well.

Notice that an analogous mechanism is working in the standard Pauli-Villars regularization scheme. In this case, the mass of the ghost plays the role of the ultraviolet cut-off. This ghost decouples from the renormalized theory after its mass (or, analogously, the Pauli-Villars cut-off) is sent to infinity. In other words, the Lee-Wick approach is to promote the Pauli-Villars cut-off  $\Lambda$  to a physical mass of the theory.

The ultraviolet behavior of the Lee-Wick theories is similar to the one of supersymmetric models, as far as the cancellation of quadratic divergencies is concerned. However, while in supersymmetric models this is achieved by the mutual exchange of virtual particles with opposite statistics, in the Lee-Wick theories this is due to the indefinite metric of the Hilbert space.

Recently, the Lee-Wick model for QED (LWQED) has been reconsidered in view of its generalization to the Standard Model (SM) of electroweak and strong interac-

tions [8] (see also [9] for a few phenomenological applications). Indeed, this model leads to a theory which is naturally free of quadratic divergencies, thus providing an alternative way to the solution of the hierarchy problem [8]. However, contrary to the LWQED model, the Lee-Wick SM is not a finite theory, although it is still renormalizable. Higher dimensional operators, containing new interactions, naturally appear in higher-derivative theories with non-abelian gauge structure. This leads at most to logarithmic divergencies in the radiative corrections. Nevertheless, as can be easily understood by power-counting arguments, the new higher dimensional operators do not break renormalizability. This is due to the improved ultraviolet behavior of the bosonic propagator  $P(k)$  in the deep Euclidean region, which scales as  $P(k) \sim 1/k^4$ , instead of the usual  $P(k) \sim 1/k^2$  when  $k^2 \rightarrow \infty$ .

Note that, the presence of the energy scale  $\Lambda$ , associated to the higher-derivative term, manifestly breaks (at classical level) the conformal symmetry of the unbroken gauge sector. Therefore, one may wonder whether this term can also trigger (dynamically) chiral-symmetry breaking at low energy, or in other words, whether the fermion field could dynamically get a mass  $m$  satisfying the condition  $m < \Lambda$ . The aim of the present paper is to investigate this issue by analyzing a general class of renormalizable models containing higher-derivative terms.

We will show that a non-vanishing mass term for the fermion field can indeed be generated, depending on the kind of interaction, as a solution of the mass-gap equation. For a massive ghost field, the fermion mass can be predicted, and it turns out to be a function of the energy scale  $\Lambda$  and the gauge coupling constant. Moreover, we will see that the dependence of the fermion mass on the gauge coupling has a non-perturbative origin.

In section 2, we will consider a model where a higher-derivative term is added to an exact U(1) gauge theory, coupled to a massless fermion field. Then, we will extend the same approach to models including renormalizable scalar(pseudoscalar)-fermion interactions with massless scalar(pseudoscalar) fields. In section 3, we extend the analysis of section 2 to the case of SU( $N$ ) gauge interactions in the presence of a higher-derivative term for the non-abelian gauge fields. In section 4, the same mechanism is analyzed for the case of interactions mediated by a massive gauge field. Our conclusions are given in section 5.

## 2 Abelian gauge and scalar interactions

We start our analysis by considering a minimal version of the Lee-Wick extension of the U(1) gauge theory [2], where a (gauge-invariant) dimension-6 operator containing

higher-derivatives is added to the free Lagrangian of the U(1) gauge sector. The corresponding gauge field  $A_\mu$  is then minimally coupled to a Dirac field  $\psi$ .

In contrast to the original Lee-Wick model [2], we do not impose here finite charge renormalization, since this would require the introduction of extra higher-derivative terms in the fermion sector. Since we are interested in the dynamical fermion-mass generation, we will switch off the bare-mass of the fermion field. We consider the following gauge-invariant Lagrangian  $\mathcal{L}$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{\Lambda^2} (\partial^\alpha F_{\alpha\mu}) (\partial^\beta F_\beta^\mu) + i\bar{\psi}\gamma_\mu D^\mu\psi, \quad (1)$$

where the field strength and the covariant derivative are defined as  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  and  $D_\mu = \partial_\mu + igA_\mu$  respectively.

In order to make the theory consistent in perturbation theory, it is necessary to add a gauge-fixing term. We choose the covariant term  $L_\xi = -(\partial_\mu A^\mu)^2/(2\xi)$ , where  $\xi$ , as usual, plays the role of the gauge-fixing parameter. Due to the abelian structure of the theory, no Faddeev-Popov ghosts are required in this case.

According to the above gauge-fixing term, the propagator of the gauge field  $A_\mu$  in momentum space is given by

$$D_{\mu\nu}(k) = \frac{-i\Lambda^2}{k^2(\Lambda^2 - k^2)} \left( \eta_{\mu\nu} - (1 - \xi) \frac{k_\mu k_\nu}{k^2} - \xi \frac{k_\mu k_\nu}{\Lambda^2} \right). \quad (2)$$

This propagator has two poles: at  $k^2 = 0$  and  $k^2 = \Lambda^2$ . Indeed, the Lagrangian in Eq.(1) describes two independent (on-shell) spin-1 fields: massless one and massive one, with positive and negative norm respectively. As shown in [8], this can be easily shown by the help of an (on-shell) auxiliary field which can linearize the Lagrangian in Eq.(1). However, here it is more convenient to work in the representation of the gauge field  $A_\mu$  as given in Eq.(1), with the propagator as in Eq.(2). The Feynman rules for the coupling of the Dirac field  $\psi$  to the gauge field  $A_\mu$  are the same as in the standard U(1) gauge theory.

Since no mass term for the fermion field is present at tree-level, the Lagrangian in Eq.(1) is also invariant under global chiral transformations, namely  $\psi \rightarrow e^{i\gamma_5\varepsilon}\psi$ , where  $\varepsilon$  is a constant parameter. However, due to the presence of the scale  $\Lambda$ , the conformal symmetry is broken at classical level. Then, one might wonder whether  $\Lambda$  could also trigger chiral-symmetry breaking at low energy, hence dynamically generating a mass term for the fermion field lower than  $\Lambda$ .

In order to answer this question, we consider below the equation for the fermion mass-gap following the approach of the Nambu-Jona-Lasinio (NJL) model [10]. One has then to calculate the one-loop contribution to the fermion self-energy as a function

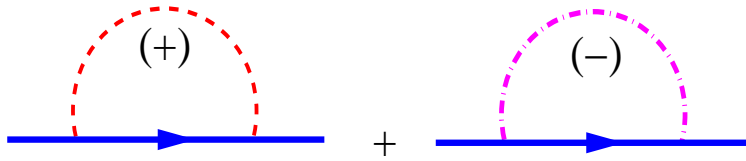


Figure 1: Feynman diagrams contribution to the fermion self-energy at one-loop in the U(1) gauge theory with a higher-derivative term. The symbols (+) and (-) indicate the usual gauge propagator (dashed line) and the “massive-ghost” propagator (dashed-dot line) respectively, while the continuous line stands for the fermion propagator.

of the fermion pole mass  $m$ . Due to the improved asymptotic behavior of the gauge propagator ( $\sim 1/k^4$  when  $k^2 \rightarrow \infty$ , as induced by the presence of the ghost field), this contribution turns out to be finite. Therefore, the fermion mass can be predicted as a function of the gauge coupling and  $\Lambda$ . Analogous results can be obtained by implementing the Pauli-Villars regularization, although their physical interpretation is different in that case.

The Feynman diagrams contribution to the one-loop self-energy are shown in Fig. 1. By assuming the propagator in Eq.(2), we obtain

$$\Sigma(\hat{p}, m) = \frac{\alpha}{2\pi} \left( \int_0^1 dx (2m - x \hat{p}) \log \left( \frac{x\Lambda^2 + (m^2 - p^2x)(1-x)}{(m^2 - p^2x)(1-x)} \right) + I(\xi) \right) \quad (3)$$

where  $\alpha = g^2/(4\pi)$  and  $\hat{p} \equiv \gamma^\mu p_\mu$ . The integral  $I(\xi)$  in Eq.(3) contains the pure gauge-dependent contribution to the self-energy due to terms proportional to  $k_\mu k_\nu$  in the gauge propagator. However,  $I(\xi)$  vanishes on shell ( $\hat{p} = m$ ), since the self-energy when evaluated on the physical pole is a gauge invariant quantity. As we will see later on, this property is not spoiled by an explicit mass term for the gauge field, since the (on-shell) external current is always conserved in an abelian gauge theory. Being interested in the self-energy evaluated on-shell, the explicit expression of  $I(\xi)$  is not needed here.

The exact resummation at any order of the self-energy contribution to the fermion propagator  $S(p)$  gives the well-known result

$$S(p) = \frac{i}{\hat{p} - m_0 - \Sigma(\hat{p}, m)}. \quad (4)$$

In the above formula  $m_0$  is the tree-level bare mass, while, as explained above,  $m$  is the physical pole mass. Since we are interested in the dynamical generation of fermion mass, following the approach of NJL [10], we set in Eq.(4) the bare mass  $m_0$  to zero. Then, one obtains the well known self-consistent equation for the mass-gap [10]

$$m = \Sigma(\hat{p}, m)|_{\hat{p}=m}. \quad (5)$$

Notice that, both Eq.(5) and its solutions are gauge invariant.

By substituting the on-shell condition  $\hat{p} = m$  in Eq.(3) we obtain

$$m = -\frac{\alpha}{2\pi} m \int_0^1 dx (2-x) \log \left( \frac{m^2 (1-x)^2}{\Lambda^2 x} \right) + \mathcal{O}(m^2/\Lambda^2) \quad (6)$$

Terms of the order  $\mathcal{O}(m^2/\Lambda^2)$  are neglected, since we are interested in finding non-trivial solutions of Eq.(3) corresponding to the breaking of chiral symmetry at low energy, that is at  $m \ll \Lambda$ . Notice that, due to the chiral symmetry of the original Lagrangian, there is always a trivial solution of Eq.(6) corresponding to the case  $m = 0$ . Remarkably, as in the NLJ model [10], Eq.(6) allows also a non-trivial solution with  $m \neq 0$ , satisfying the condition  $m < \Lambda$ . According to the interpretation of the NJL equation, the non-trivial solution should be identified with the physical one, since it corresponds to the non-perturbative (true) vacuum of the theory. On the other hand, the massless solution is always present, being connected to the false vacuum of perturbation theory, where chiral symmetry is unbroken.

Then, solving the self-consistent equation for the mass-gap

$$1 = -\frac{\alpha}{2\pi} \int_0^1 dx (2-x) \log \left( \frac{m^2 (1-x)^2}{\Lambda^2 x} \right) + \mathcal{O}(m^2/\Lambda^2), \quad (7)$$

we obtain

$$m = \Lambda \exp \left[ -\frac{2\pi}{3\alpha} + \frac{1}{4} \right]. \quad (8)$$

This solution corresponds to a dynamical breaking of the chiral symmetry. The  $1/\alpha$  dependence in Eq.(8) shows the non-perturbative origin of such effect. The fact that there exists a non-trivial solution to the mass-gap equation is a peculiar property of unbroken gauge theories. Indeed, as we will see in the following, there is no solution satisfying the condition  $m < \Lambda$  in the case of fermion-scalar(pseudoscalar) interactions.

On the other hand, Eq.(8) is not yet the complete solution, since the resummation of the leading-Log terms  $(\alpha \log(\Lambda/m))^n$ , arising from the inclusion of vacuum polarization diagrams, should be taken into account. From the renormalization group equation, we know that all these effects can be re-absorbed in the running coupling constant  $\alpha(Q)$  evaluated at the scale  $Q \sim m$ . This effect then can be taken into account by replacing  $\alpha \rightarrow \alpha(m)$  in Eq.(8)

$$m = \Lambda \exp \left[ -\frac{2\pi}{3\alpha(m)} + \frac{1}{4} \right], \quad (9)$$

where  $\alpha(m)$  is related to  $\alpha(\Lambda)$  by

$$\alpha(m) = \frac{\alpha(\Lambda)}{1 + b\alpha(\Lambda) \log(\Lambda/m)} \quad (10)$$

with  $b = 2/(3\pi)$  for a U(1) gauge theory.

Let's now consider the case of  $N_f$  fermions having the same charge  $e$ . Then, Eq.(9) can be easily generalized to a set of  $N_f$  self-consistent equations, as follows

$$m_f = \Lambda \exp \left[ -\frac{2\pi}{3\alpha(m_f)} + \frac{1}{4} \right]. \quad (11)$$

Although Eq.(11) depends on  $m_f$  through the running coupling constant  $\alpha(m_f)$ , its solution is consistent with a degenerate spectrum only. This is just a consequence of charge universality. In order to prove that, let's assume a non-degenerate spectrum for the solution of Eq.(11). Then, one can order the mass convention according to  $m_f > m_{f-1}$ , where  $f = 1, \dots, N_f$ . Now we rewrite the  $\alpha(m_f)$  dependence in terms of  $\alpha(m_{f-1})$  into Eq.(11), and we get  $\left(\frac{m_f}{m_{f-1}}\right)^{1-C} = 1$ , where  $C = 4/9N_{f-1}$ , which implies  $m_f = m_{f-1}$ . Iterating the same procedure for  $m_{f-1}$  and  $m_{f-2}$ , we find  $m_f = m_{f-1} = m_{f-2}$ , and hence a degenerate spectrum.

Since Eq.(11) has been derived within perturbation theory, we have to check that its solution is consistent with the perturbative regime. In particular, one can require that the coupling constant  $\alpha$  is perturbative up to the  $\Lambda$  scale and positive, that is  $0 < \alpha(\Lambda) < 1$ . To this end, we have to first express  $m_f$  in Eq.(11) in terms of  $\alpha(\Lambda)$ , obtaining

$$m_f = \Lambda \exp \left[ \left( -\frac{2\pi}{3\alpha(\Lambda)} + \frac{1}{4} \right) \left( \frac{9}{9 - 4N_f} \right) \right] \quad (12)$$

for any  $f$ . Remarkably, Eq.(12) is compatible with the condition  $m_f < \Lambda$  and the weak coupling regime ( $\alpha(\Lambda) \ll 1$ ), provided the number of charged fermions is  $N_f \leq 2$ . If  $N_f > 2$ , then the condition  $m_f < \Lambda$  is satisfied only for  $\alpha(\Lambda) < 0$ , which is clearly inconsistent. Nevertheless, we will see in the next section that the constraint  $N_f \leq 2$  can be relaxed for non-abelian gauge interactions.

Now we consider the most general case of  $N_f$  fermions  $f$  minimally coupled to a U(1) gauge theory as in Eq.(1), where all charges are different, namely  $Q_f$  in unity of  $e$ . Then, the non-universality of the charges can remove the degeneracy of the spectrum. Eq.(12) can be easily generalized to this case of non-universal charges. The corresponding set of  $N_f$  self-consistent equations is given by

$$m_f = \Lambda \exp \left[ \left( -\frac{2\pi}{3\alpha(\Lambda)} + \frac{Q_f^2}{4} \right) \left( \frac{9}{9Q_f^2 - 4\sum_f Q_f^2} \right) \right], \quad (13)$$

where  $\alpha = e^2/(4\pi)$ . We can see that, if  $Q_f > Q_{f-1}$ , then  $m_f > m_{f-1}$ , provided  $\alpha(\Lambda) < 8\pi/(3Q_{\max}^2)$ , where  $Q_{\max}$  is the largest charge. A closer inspection of Eq.(13) shows that, also in this case, the constraint  $N_f \leq 2$  cannot be avoided. Indeed,

let's consider the contribution to the lowest mass eigenvalue. In case we order the charges as  $|Q_i| < |Q_j|$  for  $i < j$  (where  $(i, j) = 1, \dots, N_f$ ), this would correspond to the fermion with charge  $Q_1$ . Moreover, the requirement of positivity of the second term in parenthesis in the exponential would imply that  $Q_1^2 > 4/5 \sum_{i=2}^{N_f} Q_i^2$ . This last condition cannot be satisfied if  $N_f > 2$ , since  $|Q_1| < |Q_i|$  for  $i \geq 2$ . Therefore, also in the general case of non-universal charges the restriction  $N_f \leq 2$  holds, and is supplemented by the additional constraint  $1 < Q_2^2/Q_1^2 < 5/4$ .

Let's now consider the case of a chiral model with massless scalar(pseudoscalar) fields coupled to a massless fermion field. In order to implement chiral symmetry, the minimal number of real scalar fields required is two, namely a scalar ( $\varphi$ ) and pseudoscalar ( $\bar{\varphi}$ ) field. In analogy with the U(1) gauge theory discussed above, we add a higher derivative term in the scalar sector. The corresponding Lagrangian is

$$\mathcal{L} = L_0(\varphi) + L_0(\bar{\varphi}) + L_0(\psi) - \frac{1}{2\Lambda^2} \left( (\partial_\mu \partial^\mu \varphi)^2 + (\partial_\mu \partial^\mu \bar{\varphi})^2 \right) + \lambda \left( \bar{\psi} \psi \varphi + i \bar{\psi} \gamma_5 \psi \bar{\varphi} \right) \quad (14)$$

where  $L_0(i)$  stands for the canonical kinetic term of the corresponding fields  $i = \varphi, \bar{\varphi}, \psi$ . The r.h.s of Eq.(14) is invariant under global chiral transformations, that, for an infinitesimal parameter  $\varepsilon$ , are defined as

$$\delta\psi = -i\varepsilon\gamma_5\psi, \quad \delta\bar{\psi} = -i\varepsilon\bar{\psi}\gamma_5, \quad \delta\varphi = -2\varepsilon\bar{\varphi}, \quad \delta\bar{\varphi} = 2\varepsilon\varphi. \quad (15)$$

The Feynman diagrams for the fermion self-energy are the same as in Fig. 1, where the gauge propagator is replaced by the corresponding scalar and pseudoscalar fields propagator. Then, one gets the following result for the self-consistent equation

$$1 = \frac{\alpha_\lambda}{2\pi} \int_0^1 dx x \log \left( \frac{m^2 (1-x)^2}{\Lambda^2 x} \right) + \mathcal{O}(m^2/\Lambda^2), \quad (16)$$

where  $\alpha_\lambda \equiv \lambda^2/(4\pi)$ . The resummation of the leading-Log terms  $(\alpha_\lambda \log(\Lambda/m))^n$ , as discussed above, can again be reabsorbed in the running coupling constant evaluated at the scale  $m$ . In particular, the replacement  $\alpha_\lambda \rightarrow \alpha_\lambda(m)$  should be done in Eq.(16). In this case, the sign of the term proportional to  $\log(m/\Lambda)$  in Eq.(16) is opposite with respect to Eq.(7). This has dramatic consequences for chiral symmetry breaking. The non-trivial solution of Eq.(16) would imply

$$m = \Lambda \exp \left[ \frac{2\pi}{\alpha_\lambda(m)} + \frac{5}{4} \right], \quad (17)$$

which is inconsistent with the requirement  $m < \Lambda$ . We conclude that, contrary to the gauge interactions case, chiral symmetry breaking cannot be triggered at low energy (that is for  $m < \Lambda$ ) by means of pure fermion-scalar(pseudoscalar) interactions as in Eq.(14).



We stress here that all these results hold under the assumption that the Lee-Wick theories can be well-defined at non-perturbative level. Although there is not yet any rigorous proof that the Lee-Wick extension could also work at non-perturbative level, there are studies in this direction leading to a consistent non-perturbative approach on the lattice [6].

### 3 SU( $N$ ) gauge interactions

In this section we generalize the U(1) gauge model, presented in section 2, to the non-abelian SU( $N$ ) gauge theory. We add to the standard Yang-Mills Lagrangian the corresponding (gauge-invariant) higher-derivative term for the non-abelian gauge fields. Following the Lee-Wick generalization of the pure SU( $N$ ) sector [8], we consider the following Lagrangian

$$\mathcal{L} = -\frac{1}{2}\text{Tr} [\hat{F}_{\mu\nu}\hat{F}^{\mu\nu}] + \frac{1}{\Lambda^2}\text{Tr} [(\hat{D}^\alpha\hat{F}_{\alpha\mu})(\hat{D}^\beta\hat{F}_\beta^\mu)] + i\bar{\psi}\gamma_\mu\hat{D}^\mu\psi, \quad (18)$$

where  $\hat{F}^{\mu\nu}$  is the standard field strength of Yang-Mills theories, and  $\hat{D}_\mu = \partial_\mu + ig\sum_a T^a A_\mu^a$ , with  $T^a$  the SU( $N$ ) generators in the fundamental representation. The trace Tr is understood acting on the fundamental representation of the  $T^a$  matrices. For the linear decomposition of the Lagrangian in Eq.(18) in terms of an auxiliary field, see [8].

If we add to the Lagrangian in Eq.(18) the covariant gauge-fixing term  $\mathcal{L}_{GF} = -\text{Tr}[(\partial_\mu\hat{A}^\mu)^2/2\xi]$ , the gauge propagator is given by

$$D_{\mu\nu}^{ab}(k) = \delta^{ab}\frac{-i\Lambda^2}{k^2(\Lambda^2 - k^2)}\left(\eta_{\mu\nu} - (1 - \xi)\frac{k_\mu k_\nu}{k^2} - \xi\frac{k_\mu k_\nu}{\Lambda^2}\right), \quad (19)$$

where the indices  $a, b$  run on the adjoint representation of SU( $N$ ). Notice that, apart from the  $\delta^{ab}$  term, this is the same propagator as in Eq.(2).

The calculation of the self-consistent equation for the mass-gap proceeds as in the case of the U(1) gauge theory. At one-loop, the contribution to the fermion self-energy is provided by the same kind of diagrams as in Fig. 1. As already mentioned, since the fermion self-energy evaluated on-shell is gauge invariant, the contribution of the gauge propagator proportional to terms  $k_\mu k_\nu$  vanishes. Therefore, the expression of the self-energy at one-loop is the same as in Eq.(3), apart from the SU( $N$ ) factor  $C_F = (N^2 - 1)/(2N)$ . Then, the corresponding solution to the self-consistent equation is

$$m = \Lambda \exp\left[-\frac{2\pi}{3\alpha(m)C_F} + \frac{1}{4}\right]. \quad (20)$$

As in the U(1) case,  $m$  has now to be expressed in terms of the running coupling  $\alpha(\Lambda)$  evaluated at the high-scale  $\Lambda$ . Apart from the factor  $C_F$ , there is now a crucial difference in the effective coupling constant. The opposite sign in the  $\beta$ -function for a SU( $N$ ) gauge theory, with respect to the abelian case, has dramatic impact on the solution. In particular, the constraint on the total number of charged fermions will be strongly relaxed. We stress that, since the scale  $\Lambda$  is assumed to be much larger than the characteristic intrinsic scale associated to SU( $N$ ), that we name  $\Lambda_{\text{SU}(N)}$  ( $\Lambda_{\text{SU}(3)} = \Lambda_{\text{QCD}}$  in the QCD case)\*, the massive ghost-field does not contribute to the SU( $N$ )  $\beta$ -function below the  $\Lambda$  scale. However, above the  $\Lambda$  scale, the one-loop  $\beta$ -function is modified due to the contribution of the massive ghost field. This effect has been recently evaluated in [11].

If we substitute in Eq.(20) the running coupling  $\alpha(m)$  in Eq.(10) with the corresponding coefficient  $b = -1/(6\pi)(11N - 2N_f)$  of the SU( $N$ )  $\beta$ -function, we get

$$m = \Lambda \exp \left[ \left( -\frac{6\pi}{\alpha(\Lambda)} + \frac{9C_F}{4} \right) \frac{1}{9C_F + 11N - 2N_f} \right]. \quad (21)$$

Now, one can rearrange Eq.(21) in a more compact expression. If we substitute  $\alpha(\Lambda) = 6\pi/[(11N - 2N_f) \log(\Lambda/\Lambda_{\text{SU}(N)})]$  in Eq.(21), where  $\Lambda > \Lambda_{\text{SU}(N)}$ , we obtain

$$m = \Lambda \left( \frac{\Lambda_{\text{SU}(N)}}{\Lambda} \right)^\beta e^\gamma, \quad (22)$$

where the coefficients  $\beta$  and  $\gamma$  are given by

$$\beta = \frac{11N - 2N_f}{9C_F + 11N - 2N_f}, \quad \gamma = \frac{9C_F}{4(9C_F + 11N - 2N_f)}. \quad (23)$$

In the QCD case with  $N_f = 6$ , we have  $\beta = 7/11$  and  $\gamma = 1/11$ .

In Fig. 2 we show the dependence of  $m$  versus  $\Lambda$  in the SU(3) case with  $N_f = 6$ , for a few  $\Lambda_{\text{SU}(3)}$  values,  $\Lambda_{\text{SU}(3)} = 0.1, 10, 100$  GeV, where the first one should roughly correspond to the QCD case. We can see that, a higher-dimensional term with a scale of the order of  $\Lambda = 1$  TeV would generate in QCD a fermion mass of the order of  $m \sim 3$  GeV.

We consider now some potential phenomenological application of this mechanism. The fact that scalar-fermion interactions act with a term of opposite sign in the argument of the exponential in Eq.(17) with respect to Eq.(21), suggests that the presence of both Yukawa and gauge couplings could induce a fermion mass-splitting. Remarkably, due to the non-perturbative dependence of the fermion mass on the

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\* Anyhow, one should keep in mind that for a generic SU( $N$ ) gauge theory the fundamental scale  $\Lambda_{\text{SU}(N)}$  is a free parameter.

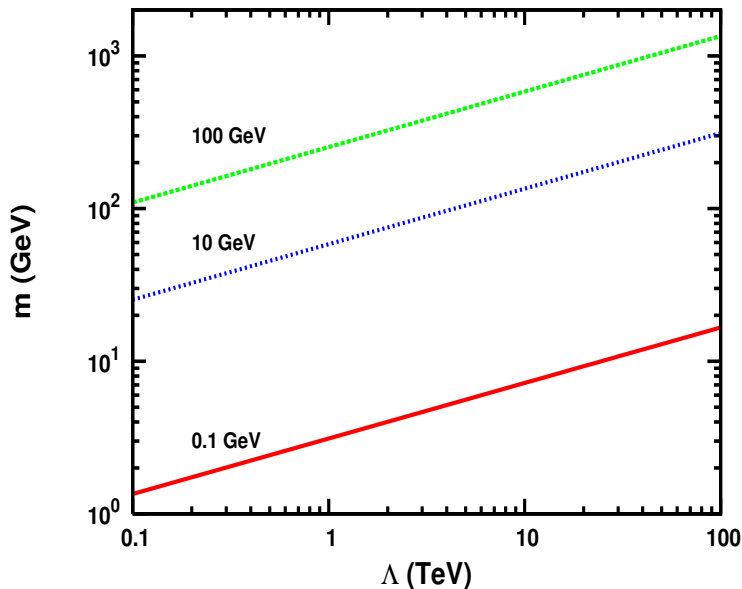


Figure 2: The fermion mass  $m$  (in GeV) versus the higher-derivative scale  $\Lambda$  (in TeV) in the SU(3) case, with fermions number  $N_f = 6$ , and for the values  $\Lambda_{\text{SU}(3)} = 0.1, 10, 100$  GeV.

Yukawa coupling (cf. Eq.(17)), a large mass-splitting could be achieved with Yukawa interactions of similar strength.

On the other hand, this mechanism requires scalar and pseudoscalar fields with positive norm that are massless. In order to have a realistic viable (and alternative to the standard Higgs mechanism) mechanism for quark mass generation, one needs *confined* massless scalar and pseudoscalar fields. Then, due to their Yukawa couplings with quarks, scalars and pseudoscalars must be in the adjoint representation of color, which is a quite intriguing phenomenological possibility.

Although attractive, we remind that, at this stage, this is just a speculative idea. A realistic model of fermion mass generation in the SM would require a careful analysis, which is beyond the purpose of the present paper.

## 4 Massive vector field

In this section, we study whether the proposed mechanism for chiral symmetry breaking can work for short-distance interactions, by considering a massive vector field coupled to a conserved fermion current. We start from the same model as in Eq.(1), with the addition of a mass term for the gauge field. The corresponding Lagrangian is then  $\mathcal{L}' = \mathcal{L} + \frac{1}{2}M^2 A_\mu A^\mu$ , where  $\mathcal{L}$  is given in Eq.(1). We restrict our analysis to the case of a single fermion field. The generalization to an arbitrary number of

fermions will be straightforward.

No matter which mechanism generates the gauge field mass, the gauge propagator, in the unitary gauge, is given by

$$D_{\mu\nu}(k) = \frac{-i\Lambda^2}{(k^2 - M^2)(\Lambda^2 - k^2)} \left( \eta_{\mu\nu} - \frac{k_\mu k_\nu}{M^2} \right), \quad (24)$$

where we assume that  $\Lambda$  is the highest scale in the theory, namely  $\Lambda \gg M$ . Notice that,  $\Lambda > 2M$  is necessary in order to have a non-vanishing ghost-field decay width. Indeed, in case fermions get a mass  $m > M$ , the ghost could anyway decay in two gauge bosons (i.e., by means of one-loop fermion diagram).

Then, Eq.(7) for the mass-gap becomes

$$1 = -\frac{\alpha}{2\pi} \int_0^1 dx (2-x) \log \left( \frac{xM^2 + m^2(1-x)^2}{x\Lambda^2 + m^2(1-x)^2} \right). \quad (25)$$

The r.h.s of Eq.(25) is insensitive, at one-loop, to the dynamics which generates the gauge boson mass. Indeed, as explained in section 2, due to the conservation of the on-shell fermion current, the  $k_\mu k_\nu$  contribution in the gauge propagator to the on-shell self-energy vanishes.

The new mass scale  $M$  in the one-loop self-energy makes the corresponding solution of Eq.(25) dependent on the ratio  $M/\Lambda$ . The exact analytical expression is quite difficult to obtain, due to the non-trivial dependence of Eq.(25) on  $m$ ,  $M$ , and  $\Lambda$ . Nevertheless, there are three particular scenarios where the problem can be easily solved by making use of some perturbative expansion: i)  $M \ll m \ll \Lambda$ , ii)  $m \simeq M \ll \Lambda$ , iii)  $m \ll M \ll \Lambda$ .

In the first case ( $m \gg M$ ), one can see that there exists always a non-trivial solution for  $m \neq 0$ . Indeed, the leading contribution to  $m$  would be the same as in Eq.(8), apart from small corrections of the order of  $\mathcal{O}(M/\Lambda)$ . The second and third cases admit different non-trivial solutions with respect to Eq.(8) and we are going to analyze them in the following.

Let's start with the case  $m \simeq M$ . Assuming  $m = M$  in the integral in Eq.(25), and solving the equation for  $m$ , we get

$$m = M = \Lambda \exp \left[ -\frac{2\pi}{3\alpha} + \frac{5 - 2\sqrt{3}\pi}{12} \right], \quad (26)$$

where the last term in the exponential can be well approximated by  $-1/2$  (indeed,  $\frac{5-2\sqrt{3}\pi}{12} \simeq -0.49$ ). As previously shown for U(1) and SU( $N$ ) gauge interactions, in order to resum the leading-Log terms,  $\alpha$  should be replaced by a running coupling constant evaluated at the characteristic renormalization scale of the problem. In the following we will omit this dependence in the notation.

Let's now consider the iii) scenario, where the fermion is the lightest state, namely  $m \ll M \ll \Lambda$ . By expanding the integral in Eq.(25) in terms of  $m/\Lambda \ll 1$ ,  $M/\Lambda \ll 1$ , and  $m/M \ll 1$ , and retaining only the leading-Log contributions, we find that there exists a non-trivial solution for  $m$  provided the following equation holds

$$\frac{2\pi}{\alpha} + 3 \log\left(\frac{M}{\Lambda}\right) - \frac{4m^2}{M^2} \log\left(\frac{m}{\Lambda}\right) = 0. \quad (27)$$

We see that Eq.(27) can be satisfied only if the term  $3 \log\left(\frac{M}{\Lambda}\right)$  can compensate the large term  $\frac{2\pi}{\alpha}$ . This, for  $m \neq 0$ , requires a critical  $M$  value. An approximate analytical solution for  $m$  can be obtained by using the following ansatz for  $M$

$$M = \Lambda \exp\left[-\frac{2\pi}{3\alpha} - \frac{\delta^2}{3}\right] \quad (28)$$

where  $\delta$  is a (real) free dimensionless parameter satisfying the condition  $0 < \delta \ll \sqrt{3/2}$ . Notice that, in order to satisfy Eq.(27) one must assume a considerable tuning between  $M$  and  $\Lambda$  as shown by the dependence of  $\delta$  in the argument of the exponential. By substituting Eq.(28) in Eq.(27), we find

$$m \simeq \sqrt{\frac{3\alpha}{8\pi}} \delta M. \quad (29)$$

Eq.(29) is quite different from the U(1) massless case in Eq.(8). The fermion mass here is suppressed by a term proportional to  $(\sqrt{\alpha}\delta) \ll 1$ . Indeed, as expected from Eq.(27), when  $M \gg \Lambda \exp\left[-\frac{2\pi}{3\alpha}\right]$ , no  $m \neq 0$  satisfying the condition  $m \ll M$  can be obtained.

Summarizing, we get the following results for  $m$ , as a function of  $M/\Lambda$ , in the three difference ranges

$$\begin{aligned} \text{i)} \quad & 0 < \frac{M}{\Lambda} \ll \exp\left[-\frac{2\pi}{3\alpha} - \frac{1}{2}\right] \Rightarrow m \simeq \Lambda \exp\left[-\frac{2\pi}{3\alpha} + \frac{1}{4}\right], \\ \text{ii)} \quad & \exp\left[-\frac{2\pi}{3\alpha} - \frac{\delta^2}{3}\right] \lesssim \frac{M}{\Lambda} \lesssim \exp\left[-\frac{2\pi}{3\alpha}\right] \Rightarrow m \simeq \sqrt{\frac{3\alpha}{8\pi}} \delta M, \\ \text{iii)} \quad & \exp\left[-\frac{2\pi}{3\alpha}\right] \ll \frac{M}{\Lambda} < 1 \Rightarrow m = 0, \end{aligned} \quad (30)$$

where  $0 < \delta \ll \sqrt{3/2}$ . These are approximate results. A precise  $m$  value at the thresholds would require the knowledge of the exact analytical expression of  $m$  in terms of  $M/\Lambda$ .

In conclusion, in order to trigger chiral symmetry breaking at low energy in the weak coupling regime, the gauge boson mass should satisfy a critical condition. In particular, in the case of U(1) gauge interactions, one needs  $M < \Lambda \exp\left[-\frac{2\pi}{3\alpha}\right]$ , where  $\alpha$  is understood to be evaluated at the scale  $M$ . However, a strong tuning between  $M$  and  $\Lambda$  is required in order to obtain a fermion mass  $m \ll M$ .

## 5 Conclusions

We have analyzed a new mechanism for chiral-symmetry-breaking which is based on renormalizable models with higher-derivatives. As an example, we considered chiral theories with gauge-fermion and scalar-fermion interactions, where a higher-derivative term is added to the free bosonic sector of the Lagrangian, as in the Lee-Wick models.

As order-parameter of chiral symmetry breaking, we considered the fermion mass-gap  $m$ . The corresponding self-consistent equation for the mass-gap has been derived by using the approach of the NJL model [10]. Remarkably, we find that a non-trivial solution  $m \neq 0$  exists in the weak coupling regime, satisfying the condition  $m \ll \Lambda$ . We show that this is a peculiar property of the fermion-gauge interactions, and does not hold in the case of (pure) scalar-fermion interactions. Moreover, due to the presence of the ghost field, the contribution to the fermion self-energy is finite, and the mass-gap can be predicted. Then,  $m$  turns out to be a function of the higher-derivative scale  $\Lambda$  and the gauge coupling constant. Although the self-consistent equation has been derived in a perturbative regime, the mass dependence on the gauge coupling is a pure non-perturbative effect.

We generalized this mechanism to the  $SU(N)$  gauge interactions by adding the corresponding higher-derivative term to the non-abelian gauge fields. We find that there exists a non-vanishing fermion mass-gap in the weak coupling regime, provided  $\Lambda > \Lambda_{SU(N)}$ . In particular, the mass-gap turns out to be a simple function of  $\Lambda$  and  $\Lambda_{SU(N)}$ , namely  $m = \Lambda \left( \frac{\Lambda_{SU(N)}}{\Lambda} \right)^\beta e^\gamma$ , where  $\beta$  and  $\gamma$  are some coefficients depending on  $N$  and fermions number  $N_f$ . In the  $SU(3)$  case, with  $N_f = 6$ , we get for these coefficients  $\beta = 7/11$  and  $\gamma = 1/11$ . We think that potential lattice studies of these theories could be very helpful in testing the above results on the fermion mass-gap.

We also considered the same mechanism in the presence of a massive gauge field. Then, we show that in order to trigger chiral symmetry breaking at low energy, the mass  $M$  of the gauge field needs to satisfy a critical condition. However, a strong tuning between  $M$  and  $\Lambda$  is required in order to obtain a fermion mass  $m \ll M$ .

In conclusion, we believe that further studies are necessary to assess the real potential of this mechanism. In particular, it would be interesting to analyze the interplay of  $SU(N)$  gauge-fermion and scalar-fermion interactions with higher-derivative terms. As discussed in section 3, this might help in explaining the observed quark spectrum in a natural way. This would require scalar fields in the adjoint representation of color, which is a quite intriguing phenomenological possibility.

We think that this mechanism for the fermion mass generation would deserve some consideration also in the framework of technicolor models.

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