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Neutrino-Charged Matter Interactions: a General Four-Fermion Effective Parametrization

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Abstract

Given the eventuality of neutrino and muon factories in the foreseeable future, all possible $2 \rightarrow 2$ processes involving two neutrinos, whether Dirac or Majorana ones, and two charged fermions are considered on the basis of the most general Lorentz invariant four-fermion effective interaction possible, in the limit of massless particles. Such a parametrization should enable the assessment of the sensitivity to physics beyond the Standard Model, including the eventual discrimination between the Dirac or Majorana character of neutrinos, of specific experimental beam and detector designs.

1 Introduction.

As is widely appreciated, the most general Lorentz invariant four-fermion effective parametrization of electroweak processes has played a central role in unravelling the basic structure and chiral properties of this fundamental interaction. Still to this day, the original analysis of Ref.[1] is used in precision studies of β-decay[2, 4], aiming at identifying at low energies tell-tale signs for physics beyond the Standard Model (SM). Likewise at intermediate and high energies in the purely muon and tau leptonic sectors, a similar parametrization[3] has become the standard[4] in terms of which to confine ever further parameter space, hoping to uncover a lack of overlap with that of the SM. A similar approach is also possible for precision studies of semi-leptonic processes which involve both the muon sector and the first (u, d) quark generation, for example at the intermediate energies of nuclear muon capture[5].

With the foreseen advent of neutrino factories and muon colliders, an analogous general analysis, involving in particular neutrino beams, appears to be of potential interest in the design of eventual experiments and detectors. For instance, the possibility of intersecting neutrino beams should not be dismissed, especially in the eventuality of very large intensities. Indeed, in spite of small rates, if only a single $\nu_a\nu_b \to \ell_i^-\ell_j^+$ event for instance—as opposed to $\nu_a\bar{\nu}_b \to \ell_i^-\ell_j^+$—were to be observed, lepton number violation would definitely have been established, which most likely would imply the Majorana character of neutrinos, one of the most pressing issues in neutrino physics today.

This note presents such an analysis, based on the most general four-fermion effective interaction possible of two neutrinos and two charged fermions (whether leptons or quarks) of fixed "flavours", or rather more correctly, of definite mass eigenstates, solely constrained by the requirements of Lorentz invariance and electric charge conservation. For instance, even though this might be realized only in small and peculiar classes of models beyond the SM, allowance is made for the possibility that both the neutrino fields and their charge conjugates couple in the effective Lagrangian density. Furthermore, the analysis is developed separately whether for Dirac or Majorana neutrinos, with the hope to identify circumstances under which scattering experiments involving neutrinos could help discriminate between these two cases through different angular correlations for differential cross sections, given the high rates to be expected at neutrino factories. As is well known, the "practical Dirac-Majorana confusion theorem"[6] states that within the SM, namely in the limit of massless neutrinos and $(V-A)$ interactions only, these two possibilities are physically totally equivalent, and hence cannot be distinguished. On the other hand, relaxing the purely $(V-A)$ structure of the electroweak interaction by including at least another interaction whose chirality structure is different should suffice to evade this conclusion, even in the limit of massless neutrinos.

The general classes of processes considered in this note comprise neutrino pair annihilation into charged leptons[1], the inverse process of neutrino pair production through lepton annihilation, and finally neutrino-lepton scattering. These processes will also be considered whether either one or both pairs of neutrino and lepton flavours[2], (a, b) and (i, j) respectively, are identical or not. The sole implicit assumption is that the energy available to the reaction is both sufficiently large in order to justify ignoring neutrino and lepton masses, and sufficiently small in order to justify the four-fermion parametrization of the boson exchanges responsible for the interactions. In other words, the calculations are all performed in the limit of zero mass for all external neutrino and lepton mass

[1]Henceforth, the charged fermions are referred to as leptons, even though exactly the same analysis and results apply to quark states, with due account then for the quark colour degree of freedom and the quark structure of the hadrons involved. Also, charged leptons will simply be called leptons, for short.

[2]In fact, our analysis considers specific mass eigenstates for the external neutrino and lepton states, in the massless limit, namely when all other energy scales are much larger than the masses of these particles. By abuse of language, we shall refer to these mass eigenstates as "flavour" ones, following a widespread usage.

eigenstates. Nonetheless, effects that distinguish Majorana from Dirac neutrinos should survive in this massless limit. In addition to being much larger than the neutrino and lepton mass scales, the energy scale available to the process must also be much smaller than the energy scales associated to the interactions modeled by the four-fermion interactions.

In terms of classes of processes, our analysis thus covers a wide variety of possibilities, and its results are presented in a manner which, it is hoped, will be found readily useful for implementation in numerical codes, whatever a specific model for physics beyond the SM, namely a specific set of effective couplings parametrizing the four-fermion interactions. In this note, no attempt is made at developing a systematic analysis to assess the physics potential of specific processes based on some particular beam and detector design, whether to look for physics beyond the SM, or discriminate between the Dirac or Majorana character of neutrinos. The main purpose of this work is to provide the general parametrization that is required for such a dedicated assessment, left for future analysis.

The note is organized as follows. The next section provides what might be called the "kinematics" of the analysis, by recalling some simple facts about Dirac and Majorana fermions. Sect.3 discusses the general four-fermion effective Lagrangian used in our analysis. Sects.4 to 6 then list the results for the three classes of processes mentioned above, with Sect.7 only superficially illustrating the potential reach of these types of processes at neutrino factories. Concluding remarks are presented in Sect.8.

2 A Compendium of Simple Properties

2.1 Dirac, Weyl and Majorana spinors

The purpose of this section is to recall a series of results relevant to Dirac, Weyl and Majorana quantum fermionic fields, and to specify our conventions. Since all processes are considered in the limit of massless neutrinos and leptons, the representation of the Clifford-Dirac algebra $\{\gamma^\mu,\gamma^\nu\}=2g^{\mu\nu}$ used througout is the chiral one, which we take to be

$$\gamma^0=\left(\begin{array}{cc} 0 & -\mathbb{1} \\ -\mathbb{1} & 0 \end{array}\right)\ \ ,\ \ \gamma^i=\left(\begin{array}{cc} 0 & \sigma^i \\ -\sigma^i & 0 \end{array}\right)\ ,\ i=1,2,3,\ \ ,\ \ \gamma_5=i\gamma^0\gamma^1\gamma^2\gamma^3=\left(\begin{array}{cc} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{array}\right)\ \ ,\tag{1}$$

σ^i $(i=1,2,3)$ being of course the usual Pauli matrices (our choice of Minkowski metric signature $g_{\mu\nu}$ is $(+---)$). The chiral projectors P_η $(\eta=\pm)$ are given by

$$P_\eta=\frac{1}{2}\left[1+\eta\gamma_5\right]\ \ ,\ \ P_\eta^2=P_\eta\ \ \ ,\ \ \ P_\eta P_{-\eta}=0\ \ ,\ \ \eta=+,-\ .\tag{2}$$

By definition, the charge conjugation matrix C is such that

$$C^{-1}\mathbb{1}C=\mathbb{1}^{\rm T}\ ,\ C^{-1}\gamma_5 C=\gamma_5^{\rm T}\ ,\ C^{-1}\gamma^\mu C=-\gamma^{\mu\,\rm T}\ ,\ C^{-1}\left(\gamma^\mu\gamma_5\right)C=\left(\gamma^\mu\gamma_5\right)^{\rm T}\ ,\tag{3}$$

$$C^{-1}\sigma_{\mu\nu}C=-\sigma_{\mu\nu}^{\rm T}\ ,\ C^{-1}\left(\sigma_{\mu\nu}\gamma_5\right)C=-\left(\sigma_{\mu\nu}\gamma_5\right)^{\rm T}\ \ ,\tag{4}$$

with

$$C^{\rm T}=C^\dagger=-C\ \ \ ,\ \ \ CC^\dagger=\mathbb{1}=C^\dagger C\ ,\tag{5}$$

and which, in the chiral representation, is given by

$$C=\left(\begin{array}{cc} -i\sigma^2 & 0 \\ 0 & i\sigma^2 \end{array}\right)\ .\tag{6}$$

Given a four component Dirac spinor ψ, our definition of the associated charge conjugate spinor is such that

$$\psi_c = \psi^c = \lambda\, C\overline{\psi}^{\rm T} \ , \tag{7}$$

where λ is some arbitrary unit phase factor, whose value would depend *a priori* on the choice of spinor field (*i.e.* on the neutrino or lepton flavour hereafter). This freedom in the choice of phase factor under charge conjugation is directly related to the "creation phase factor" of Ref.[7], as shown below.

Solutions to the free massless Dirac equation may be expanded in the helicity basis, in terms of the following mode representation of a Dirac quantum spinor $\psi_D(x)$,

$$\psi_D(x) = \int_{(\infty)} \frac{d^3\vec{k}}{(2\pi)^3 2|\vec{k}|} \sum_{\eta=\pm} \left[e^{-ik\cdot x} u(\vec{k},\eta) b(\vec{k},\eta) + e^{ik\cdot x} v(\vec{k},\eta) d^\dagger(\vec{k},\eta) \right] \ . \tag{8}$$

Here, the fermionic creation and annihilation operators have the Lorentz covariant normalization

$$\left\{ b(\vec{k},\eta), b^\dagger(\vec{k}',\eta') \right\} = (2\pi)^3 2|\vec{k}| \delta_{\eta,\eta'} \delta^{(3)}(\vec{k}-\vec{k}') = \left\{ d(\vec{k},\eta), d^\dagger(\vec{k}',\eta') \right\} \ , \tag{9}$$

while the plane wave spinors $u(\vec{k},\eta)$ and $v(\vec{k},\eta)$ are given by,

$$u(\vec{k},+) = v(\vec{k},-) = \sqrt{2|\vec{k}|} \begin{pmatrix} \chi_+(\hat{k}) \\ 0 \end{pmatrix} \quad , \quad u(\vec{k},-) = v(\vec{k},+) = \sqrt{2|\vec{k}|} \begin{pmatrix} 0 \\ \chi_-(\hat{k}) \end{pmatrix} \quad , \tag{10}$$

with the Pauli bi-spinors

$$\chi_+(\hat{k}) = \begin{pmatrix} e^{-i\varphi/2}\cos\theta/2 \\ e^{i\varphi/2}\sin\theta/2 \end{pmatrix} \quad , \quad \chi_-(\hat{k}) = \begin{pmatrix} -e^{-i\varphi/2}\sin\theta/2 \\ e^{i\varphi/2}\cos\theta/2 \end{pmatrix} \ , \tag{11}$$

such that

$$\hat{k}\cdot\vec{\sigma}\,\chi_\eta(\hat{k}) = \eta\,\chi_\eta(\hat{k}) \quad , \quad \chi_\eta(\hat{k})\chi^\dagger_\eta(\hat{k}) = \frac{1}{2}\left[\mathbb{1} + \eta\hat{k}\cdot\vec{\sigma} \right] \ , \tag{12}$$

φ and θ being of course the usual spherical angles for the unit vector $\hat{k} = \vec{k}/|\vec{k}|$ with respect to the axes $i = 1,2,3$, namely $\hat{k} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$.

The value of the index $\eta = \pm$ coincides with the helicity of the corresponding massless one-particle states, and coincides of course with the chirality of the associated quantum field. Namely, left- or right-handed four component Weyl spinors, with $\eta = -$ et $\eta = +$ respectively, have the following mode decompositions

$$\psi_\eta(x) = \int_{(\infty)} \frac{d^3\vec{k}}{(2\pi)^3 2|\vec{k}|} \left[e^{-ik\cdot x} u(\vec{k},\eta) b(\vec{k},\eta) + e^{ik\cdot x} v(\vec{k},-\eta) d^\dagger(\vec{k},-\eta) \right] \ , \tag{13}$$

as implied by the identification

$$\psi_\eta(x) = P_\eta\,\psi_D(x) \ . \tag{14}$$

Hence, $b^\dagger(\vec{k},\eta)$ and $d^\dagger(\vec{k},\eta)$ are the creation operators of a particle and of an antiparticule, respectively, both of helicity η and momentum $\vec{k}$.

This identification may also be established from the chiral properties of the plane wave spinors,

$$P_\eta\,u(\vec{k},\eta) = u(\vec{k},\eta) \quad , \quad P_\eta\,u(\vec{k},-\eta) = 0 \quad ; \quad P_\eta\,v(\vec{k},\eta) = 0 \quad , \quad P_\eta\,v(\vec{k},-\eta) = v(\vec{k},-\eta) \quad , \tag{15}$$

$$\overline{u}(\vec{k},\eta)\,P_\eta = 0 \quad , \quad \overline{u}(\vec{k},-\eta)\,P_\eta = \overline{u}(\vec{k},-\eta) \quad ; \quad \overline{v}(\vec{k},\eta)\,P_\eta = \overline{v}(\vec{k},\eta) \quad , \quad \overline{v}(\vec{k},-\eta)\,P_\eta = 0 \quad , \tag{16}$$

as well as

$$u(\vec{k},\eta)\overline{u}(\vec{k},\eta)=\frac{\mathbb{1}+\eta\gamma_5}{2}\not{k}\quad,\quad v(\vec{k},\eta)\overline{v}(\vec{k},\eta)=\frac{\mathbb{1}-\eta\gamma_5}{2}\not{k}\quad. \tag{17}$$

Likewise, their properties under charge conjugation are such that

$$C\overline{u}^{\rm T}(\vec{k},\eta)=v(\vec{k},\eta)\quad,\quad C\overline{v}^{\rm T}(\vec{k},\eta)=u(\vec{k},\eta)\quad;\quad \overline{v}(\vec{k},\eta)=u^{\rm T}(\vec{k},\eta)C\quad,\quad \overline{u}(\vec{k},\eta)=v^{\rm T}(\vec{k},\eta)C\ , \tag{18}$$

these results being specific to the helicity basis. Charge conjugates of spinors are then given by, say for a Dirac spinor $\psi_D(x)$,

$$\psi^c_D(x)=\int_{(\infty)}\frac{d^3\vec{k}}{(2\pi)^3 2|\vec{k}|}\sum_{\eta=\pm}\left[e^{-ik\cdot x}\lambda u(\vec{k},\eta)d(\vec{k},\eta)+e^{ik\cdot x}\lambda v(\vec{k},\eta)b^\dagger(\vec{k},\eta)\right]\ , \tag{19}$$

as one should expect of course.

Finally, let us turn to Majorana spinors. As opposed to a Dirac spinor which is comprised of two independent Weyl spinors of opposite chiralities, namely one of each of the two fundamental representations of the (covering group of the) Lorentz group,

$$\psi_D(x)=\psi_+(x)+\psi_-(x)\ , \tag{20}$$

a Majorana spinor $\psi_M(x)$ is a four component spinor which is covariant under Lorentz transformations but which is constructed this time from a single Weyl spinor, say of left-handed chirality[3] $\eta=-$, and which is invariant under charge conjugation[4]

$$\psi_M(x)=\psi_-(x)+\psi^c_-(x)\quad,\quad \psi^c_M(x)=\lambda_M C\overline{\psi}^{\rm T}=\psi_M(x)\ , \tag{21}$$

where it is now emphasized that the arbitrary phase factor λ_M arising in the definition of spinors which are self-conjugate under charge conjugation may *a priori* be different for each Majorana field. Consequently, the mode expansion of a Majorana spinor in the helicity basis is of the form,

$$\psi_M(x)=\int_{(\infty)}\frac{d^3\vec{k}}{(2\pi)^3 2|\vec{k}|}\sum_{\eta=\pm}\left[e^{-ik\cdot x}u(\vec{k},\eta)a(\vec{k},\eta)+e^{ik\cdot x}\lambda_M v(\vec{k},\eta)a^\dagger(\vec{k},\eta)\right]\ , \tag{22}$$

where the annihilation and creation operators $a(\vec{k},\eta)$ and $a^\dagger(\vec{k},\eta)$ obey the fermionic algebra

$$\left\{a(\vec{k},\eta),a^\dagger(\vec{k}',\eta')\right\}=(2\pi)^3 2|\vec{k}|\delta_{\eta,\eta'}\delta^{(3)}(\vec{k}-\vec{k}')\ . \tag{23}$$

In terms of the quanta of the basic Weyl spinor used in the construction, we thus have the following correspondence[5],

$$\begin{array}{lllllll} a(\vec{k},-) & : & b(\vec{k},-) & ; & a^\dagger(\vec{k},-) & : & b^\dagger(\vec{k},-)\ , \\ a(\vec{k},+) & : & \lambda_M d(\vec{k},+) & ; & a^\dagger(\vec{k},+) & : & \lambda^*_M d^\dagger(\vec{k},+)\ , \end{array} \tag{24}$$

[3]Since charge conjugation exchanges left- and right-handed chiralities, the chirality of the basic Weyl spinor used in this construction is irrelevant to the definition of a Majorana spinor.

[4]Note that a similar definition starting from a Dirac rather than a Weyl spinor might be contemplated, then leading however to two independent Majorana spinors, each of which is obtained in the manner just described from a single distinct Weyl spinor, namely $\psi^{(1)}_M=(\psi_D+\psi^c_D)/\sqrt{2}$ and $\psi^{(2)}_M=-i(\psi_D-\psi^c_D)/\sqrt{2}$, in complete analogy with the real and imaginary parts of a single complex scalar field as well as the physical interpretation of the associated quanta as being particles which are or not their own antiparticles. Specifically, we have $\psi^{(1)}_M=\psi^{(1)}_-+\psi^{(1)\,c}_-$, $\psi^{(2)}_M=\psi^{(2)}_-+\psi^{(2)\,c}_-$ with $\psi^{(1)}_-=(\psi_-+\psi^c_+)/\sqrt{2}$, $\psi^{(2)}_-=-i(\psi_--\psi^c_+)/\sqrt{2}$, where $\psi_D=\psi_-+\psi_+$. Setting either ψ_- or ψ_+ to zero, the Weyl spinors $\psi^{(1)}_-$, $\psi^{(2)}_-$ hence also the Majorana ones $\psi^{(1)}_M$, $\psi^{(2)}_M$ are then no longer independent, leading back to the construction above.

[5]The complex conjugate of a complex number z is denoted z^* throughout.

which once again shows that $a^\dagger(\vec{k},\eta)$ is the creation operator of a particle of momentum $\vec{k}$ and helicity η, which furthermore is in the present case also its own antiparticle.

Note the charge conjugation phase factor λ_M multiplying the creation operator contribution to the mode expansion of the Majorana quantum field $\psi_M(x)$. This phase factor corresponds exactly to the "creation phase factor" whose role has been emphasized already in Ref.[7] on different grounds, and again more recently[8].

2.2 Differential cross sections

All $2 \to 2$ processes of interest in this note are directly considered in the center-of-mass (CM) frame of the reaction, with a kinematics of the form

$$p_1 + p_2 \to q_1 + q_2 \ , \tag{25}$$

the quantities $p_{1,2}$, $q_{1,2}$ standing of course for the four-momenta of the respective in-coming and out-going massless particles. Given rotation invariance, and the fact that all particles are of spin 1/2 and of zero mass, hence of definite helicity, the sole angle of relevance is the CM scattering angle θ between, say, the momenta $\vec{p}_1$ and $\vec{q}_1$. For all the reactions listed hereafter, the same order is used for the pairs (p_1, p_2) and (q_1, q_2) of the initial and final particles involved, hence leading always to the same interpretation for this angle θ as being the scattering angle between the first particles in these two pairs of in-coming and out-going particles.

For specific external particles of definite helicity, the differential CM cross section of all such processes is given by

$$\frac{d\sigma}{d\Omega_{\hat{q}_1}} = \frac{1}{S_f}\frac{1}{64\pi^2\, s}\,|\mathcal{M}|^2 \quad , \quad \frac{d\sigma}{d\cos\theta} = \frac{1}{S_f}\frac{1}{32\pi\, s}\,|\mathcal{M}|^2 \ . \tag{26}$$

Here, $\sqrt{s}$ stands for the total invariant energy of the reaction, with

$$s = (p_1 + p_2)^2 = (q_1 + q_2)^2 \ , \tag{27}$$

$d\Omega_{\hat{q}_1}$ is the solid angle associated to the outgoing particle of normalized momentum $\hat{q}_1 = \vec{q}_1/|\vec{q}_1|$, $S_f = 2$ or $S_f = 1$ depending on whether the two particles—including their helicity—in the final state are identical or not, respectively, and $\mathcal{M}$ is Feynman's scattering matrix element. Thus, it is only through $|\mathcal{M}|^2$ that the differential cross section depends on[6] the scattering angle θ. Furthermore, this expression also shows that it is sufficient for our purposes to simply determine the amplitude $\mathcal{M}$ for each of the relevant processes, a single complex quantity function of θ. All our results are thus listed in terms of the amplitude $\mathcal{M}$ for each process given an arbitrary combination of helicities for the external states.

3 The Effective Lagrangian

Given a choice of external states including their helicities, Feynman's amplitude $\mathcal{M}$ is determined from the interaction Lagrangian for these particles. Assuming that the energy $\sqrt{s}$ remains much

[6]Note that this fact implies that relative angular dependencies of cross sections are energy independent (within the regime to which the four-fermion parametrization applies), while of course reaction rates are directly energy dependent with their usual linear dependency in s.

smaller than any of the mass scales of the fundamental interactions at work, an effective four-fermion parametrization of this interaction is warranted, constrained by the sole requirements of Lorentz invariance and electric charge conservation. Since fermion number is not necessarily conserved in interactions involving neutrinos, *a priori* one may couple equally well the neutrino fields and their charge conjugates to the charged fermionic fields. For the latter, the Dirac fields that will be used represent the usual charged leptons (or quarks), rather than their antiparticules. It is relative to this choice for the charged fields that the neutrino fields and their charge conjugates are thus specified.

With this understanding in mind, we shall consider all processes involving neutrinos (or their antineutrinos) of definite flavours a and b as well as leptons (or their antileptons) of flavours i and j, all denoted as ν_a, ν_b, ℓ_i^- and ℓ_j^-, respectively. The same notation is used for the associated spinor fields, except for the indication of the lepton charge. Hence, the total four-fermion effective Lagrangian that is considered throughout in the case of Dirac neutrino fields is of the form[7]

$$\mathcal{L}_{\rm eff} = 4\frac{g^2}{8M^2}\left[\mathcal{L}_D + \mathcal{L}_D^\dagger\right] \ , \tag{28}$$

where

$$\mathcal{L}_D = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 \ , \tag{29}$$

while each of the separate contributions is given by

$$\mathcal{L}_1 = S_1^{\eta_a,\eta_b}\overline{\nu_a}P_{-\eta_a}\ell_i\,\overline{\ell_j}P_{\eta_b}\nu_b \ + \ V_1^{\eta_a,\eta_b}\overline{\nu_a}\gamma^\mu P_{\eta_a}\ell_i\,\overline{\ell_j}\gamma_\mu P_{\eta_b}\nu_b \ + \ \frac{1}{2}T_1^{\eta_a,\eta_b}\overline{\nu_a}\sigma^{\mu\nu}P_{-\eta_a}\ell_i\,\overline{\ell_j}\sigma_{\mu\nu}P_{\eta_b}\nu_b \ , \tag{30}$$

$$\mathcal{L}_2 = S_2^{\eta_a,\eta_b}\overline{\nu_a^c}P_{\eta_a}\ell_i\,\overline{\ell_j}P_{\eta_b}\nu_b \ + \ V_2^{\eta_a,\eta_b}\overline{\nu_a^c}\gamma^\mu P_{-\eta_a}\ell_i\,\overline{\ell_j}\gamma_\mu P_{\eta_b}\nu_b \ + \ \frac{1}{2}T_2^{\eta_a,\eta_b}\overline{\nu_a^c}\sigma^{\mu\nu}P_{\eta_a}\ell_i\,\overline{\ell_j}\sigma_{\mu\nu}P_{\eta_b}\nu_b \ , \tag{31}$$

$$\mathcal{L}_3 = S_3^{\eta_a,\eta_b}\overline{\nu_a}P_{-\eta_a}\ell_i\,\overline{\ell_j}P_{-\eta_b}\nu_b^c \ + \ V_3^{\eta_a,\eta_b}\overline{\nu_a}\gamma^\mu P_{\eta_a}\ell_i\,\overline{\ell_j}\gamma_\mu P_{-\eta_b}\nu_b^c \ + \ \frac{1}{2}T_3^{\eta_a,\eta_b}\overline{\nu_a}\sigma^{\mu\nu}P_{-\eta_a}\ell_i\,\overline{\ell_j}\sigma_{\mu\nu}P_{-\eta_b}\nu_b^c \ , \tag{32}$$

$$\mathcal{L}_4 = S_4^{\eta_a,\eta_b}\overline{\nu_a^c}P_{\eta_a}\ell_i\,\overline{\ell_j}P_{-\eta_b}\nu_b^c + V_4^{\eta_a,\eta_b}\overline{\nu_a^c}\gamma^\mu P_{-\eta_a}\ell_i\,\overline{\ell_j}\gamma_\mu P_{-\eta_b}\nu_b^c + \frac{1}{2}T_4^{\eta_a,\eta_b}\overline{\nu_a^c}\sigma^{\mu\nu}P_{\eta_a}\ell_i\,\overline{\ell_j}\sigma_{\mu\nu}P_{-\eta_b}\nu_b^c \ , \tag{33}$$

an implicit summation over the chiralities η_a and η_b being understood of course. On the other hand, it is important to keep in mind that no summation over the flavour indices a and b, nor i and j is implied; all four of these values are fixed at the outset, keeping open still the possibility that a and b might be equal or not, and likewise for i and j.

The overall normalization factor $4g^2/8M^2$ involves a dimensionless coupling constant g as well as a mass scale M, while the factor 4 cancels the two 1/2 factors present in the definition of the chiral projection operators $P_{\pm\eta_a}$ and $P_{\pm\eta_b}$ which appear in the effective interactions. The motivation for this choice of normalization is that in the specific limit of the SM, the effective interaction is normalized precisely in this manner with g then being the $SU(2)_L$ gauge coupling constant g_L and M the massive $W^\pm$ gauge boson mass M_W, with in particular their tree-level relation to Fermi's constant, $G_F/\sqrt{2} = g_L^2/(8M_W^2)$ (see further details below in the case of the SM).

In the above definitions, the complex coupling coefficients $\{S,V,T\}_{1,2,3,4}^{\eta_a,\eta_b}$ parametrize the most general four-fermion interactions possible, including the eventuality of CP violation whenever at least one of these coefficients is complex. The choice for the indices η_a and η_b is made such that they each correspond to the chirality and helicity η_a or η_b of the neutrino spinor fields and associated particles involved in the effective coupling, with the chiralities of the leptonic fields being then determined

[7]The charge exchange form of these interactions is used here, but a charge conserving one could likewise be contemplated, one being related to the other through a Fierz transformation.

according to the selection rules governing scalar, vector or tensor couplings, namely these chiralities are those of the associated neutrino for vector couplings, and opposite to it for scalar an tensor couplings. The situation with regards to tensor couplings is particular, in that the relation $\sigma^{\mu\nu}\gamma_5 = i\epsilon^{\mu\nu\rho\sigma}\sigma_{\rho\sigma}/2$ implies that the couplings $T_1^{\eta_a,\eta_b}$ and $T_4^{\eta_a,\eta_b}$ contribute to the effective interaction only if the chiralities η_a and η_b are opposite, $\eta_a = -\eta_b$, while the couplings $T_2^{\eta_a,\eta_b}$ and $T_3^{\eta_a,\eta_b}$ contribute only if $\eta_a = \eta_b$. These conventions and remarks are also those that apply to the by now standard four-fermion parametrization used in the $\mu - e$ sector[4].

In the case of Majorana neutrino fields, a similar parametrization is of application, namely

$$\mathcal{L}_{\rm eff} = 4\frac{g^2}{8M^2}\left[\mathcal{L}_M + \mathcal{L}_M^\dagger\right] \ , \tag{34}$$

where

$$\mathcal{L}_M = S^{\eta_a,\eta_b}\overline{\nu_a}P_{-\eta_a}\ell_i\ \overline{\ell_j}P_{\eta_b}\nu_b \ + \ V^{\eta_a,\eta_b}\overline{\nu_a}\gamma^\mu P_{\eta_a}\ell_i\ \overline{\ell_j}\gamma_\mu P_{\eta_b}\nu_b \ + \ \frac{1}{2}T^{\eta_a,\eta_b}\overline{\nu_a}\sigma^{\mu\nu}P_{-\eta_a}\ell_i\ \overline{\ell_j}\sigma_{\mu\nu}P_{\eta_b}\nu_b \ . \tag{35}$$

Compared to the definitions above, and upon using the property $\psi_M^c = \psi_M$ characterizing Majorana spinors, we thus have the following correspondence between the effective coupling coefficients in the Majorana and Dirac cases,

$$S^{\eta_a,\eta_b}\ :\quad S_1^{\eta_a,\eta_b} + S_2^{-\eta_a,\eta_b} + S_3^{\eta_a,-\eta_b} + S_4^{-\eta_a,-\eta_b}\ , \tag{36}$$

$$V^{\eta_a,\eta_b}\ :\quad V_1^{\eta_a,\eta_b} + V_2^{-\eta_a,\eta_b} + V_3^{\eta_a,-\eta_b} + V_4^{-\eta_a,-\eta_b}\ , \tag{37}$$

$$T^{\eta_a,\eta_b}\ :\quad T_1^{\eta_a,\eta_b} + T_2^{-\eta_a,\eta_b} + T_3^{\eta_a,-\eta_b} + T_4^{-\eta_a,-\eta_b}\ . \tag{38}$$

These effective Lagrangians are still not yet the most general ones possible, when either $a \neq b$ or $i \neq j$, or both. In the above, it is implicitly assumed that the flavours a and i, on the one hand, and b and j on the other, couple to one another in the "current×current" representation of these interactions. One could still add other similar terms in which the roles of the flavours a and b, say, are exchanged, providing still futher interactions whenever $a \neq b$ or $i \neq j$. Nonetheless, such a possibility may easily be included in the results hereafter, since the explicit expressions for the matrix elements $\mathcal{M}$, rather than the cross sections, which are provided, are linear in the coupling coefficients.

As a final remark, let us also note that the total neutrino fermionic number is conserved in these effective interactions only for couplings of type 1 and 4, $\{S,V,T\}_{1,4}^{\eta_a,\eta_b}$, whereas those associated to the couplings of type 2 and 3, $\{S,V,T\}_{2,3}^{\eta_a,\eta_b}$, violate that quantum number by two units.

It is of interest to determine the effective coupling coefficients in the specific case of the electroweak Standard Model, for which the normalization factor $4g_L^2/(8M_W^2)$ was discussed previously already. Due to flavour conservation rules in that instance, different situations must be distinguished, depending on whether only $W^\pm$ or only Z_0 exchanges are involved, or both.

Purely $W^\pm$ exchange processes arise when $a = i$ and $b = j$ but also $a \neq b$ and $i \neq j$, in which case the only nonvanishing effectif coupling is

$$\mathrm{SM}:\quad a = i\ ;\ b = j\ ;\ a \neq b\ ;\ i \neq j\ :\quad V_1^{-,-} = -1\ . \tag{39}$$

In the case of purely Z_0 neutral current processes, we have for the only nonvanishing couplings,

$$\mathrm{SM}:\quad (a = b) \neq (i = j)\ :\quad S_1^{-,-} = \sin^2\theta_W \quad , \quad V_1^{-,-} = \frac{1}{4}\left(1 - 2\sin^2\theta_W\right)\ , \tag{40}$$

θ_W being the usual electroweak gauge mixing angle. Note that in this situation, the Lagrangians $\mathcal{L}_D$ and $\mathcal{L}_D^\dagger$, or $\mathcal{L}_M$ and $\mathcal{L}_M^\dagger$, are equal.

Finally, charged and neutral exchanges both contribute only when $a=b=i=j$, in which case the only nonvanishing couplings are

$$\mathrm{SM}: \quad a=b=i=j \ : \quad S_1^{-,-}=\sin^2\theta_W \quad , \quad V_1^{-,-}=-\frac{1}{2}+\frac{1}{4}\left(1-2\sin^2\theta_W\right) \ . \qquad (41)$$

In this case as well, the Lagrangians $\mathcal{L}_D$ and $\mathcal{L}_D^\dagger$, or $\mathcal{L}_M$ and $\mathcal{L}_M^\dagger$, are equal.

Any extra coupling coefficient introduced beyond these ones thus corresponds to some new physics beyond the Standard Model. Any particular model beyond the Standard Model predicts specific values for a subclass of the effective couplings parametrizing the general expression being used here, to which the general results to be presented hereafter may thus readily be applied.

The remainder of the calculation proceeds straightforwardly. Given any choice of external states for the in-coming and out-going particles with their specific helicities, the substitution of the effective Lagrangian operator enables the direct evaluation of the associated matrix element $\mathcal{M}$ using the Fock algebra of the creation and annihilation operators that appear in the mode expansions of the fermion fields. Rather than working out the quantity $|\mathcal{M}|^2$ through the usual trace techniques, it proves much more efficient to simply substitute for the explicit expressions of the $u(\vec{k},\eta)$ and $v(\vec{k},\eta)$ spinors solving the free Dirac equation in the helicity basis and in the chiral representation, given in Sect.2.1. Choosing a specific CM kinematics configuration in which only the scattering angle θ is involved for the reasons of rotational invariance advocated previously, one then readily obtains a single complex quantity, namely simply the value for the amplitude $\mathcal{M}$ as a function of θ. This is the procedure that has been applied to each of the processes, leading to the results listed herafter.

4 Neutrino Pair Annihilation

The first general class of processes to be considered is that of neutrino annihilations into charged lepton (or quark) pairs. In the Dirac case, these reactions are labelled as follows,

<u>$(ab)(ij)$ Dirac neutrino annihilations</u>

$$\begin{array}{llll}
\mathrm{ab1:}\ \nu_a+\nu_b\rightarrow\ell_i^-+\ell_j^+ & , & \mathrm{ab2:}\ \nu_a+\nu_b\rightarrow\ell_i^++\ell_j^- & , \\
\mathrm{ab3:}\ \nu_a+\bar\nu_b\rightarrow\ell_i^-+\ell_j^+ & , & \mathrm{ab4:}\ \nu_a+\bar\nu_b\rightarrow\ell_i^++\ell_j^- & , \\
\mathrm{ab5:}\ \bar\nu_a+\nu_b\rightarrow\ell_i^-+\ell_j^+ & , & \mathrm{ab6:}\ \bar\nu_a+\nu_b\rightarrow\ell_i^++\ell_j^- & , \\
\mathrm{ab7:}\ \bar\nu_a+\bar\nu_b\rightarrow\ell_i^-+\ell_j^+ & , & \mathrm{ab8:}\ \bar\nu_a+\bar\nu_b\rightarrow\ell_i^++\ell_j^- & ,
\end{array}$$

while in the Majorana case, this list reduces to

<u>$(ab)(ij)$ Majorana neutrino annihilations</u>

$$\mathrm{Mab1:}\ \nu_a+\nu_b\rightarrow\ell_i^-+\ell_j^+ \quad , \quad \mathrm{Mab2:}\ \nu_a+\nu_b\rightarrow\ell_i^++\ell_j^- \ .$$

Due to identical angular momentum selection rules for all these processes, the associated matrix element $\mathcal{M}$ is, for all these ten processes, of the form

$$\begin{aligned}\mathcal{M}_{(ab)(ij)} \;=\; & -4s\left(\frac{g^2}{8M^2}\right) N_1\,\delta_{ij} & \Big\{ & \delta_{ab}\delta^{-\eta_a}_{\eta_i}\delta^{-\eta_b}_{\eta_j}\left[A_{11}\sin^2\theta/2+2\delta_{\eta_a,\eta_b}B_{11}(1+\cos^2\theta/2)\right] \\ & & & +\delta_{ab}\delta^{\eta_a}_{\eta_i}\delta^{\eta_b}_{\eta_j}C_{11}\left[(1+\eta_a\eta_b)-(1-\eta_a\eta_b)\cos^2\theta/2\right] \\ & & & +\delta^{-\eta_b}_{\eta_i}\delta^{-\eta_a}_{\eta_j}\eta_a\eta_b D_1\left[A_{12}\cos^2\theta/2+2\delta_{\eta_a,\eta_b}B_{12}(1+\sin^2\theta/2)\right] \\ & & & +\,\delta^{\eta_b}_{\eta_i}\delta^{\eta_a}_{\eta_j}\eta_a\eta_b D_1 C_{12}\left[(1+\eta_a\eta_b)-(1-\eta_a\eta_b)\sin^2\theta/2\right]\Big\} \\ & -4s\left(\frac{g^2}{8M^2}\right) N_2 & \Big\{ & \delta_{ab}\delta^{-\eta_b}_{\eta_i}\delta^{-\eta_a}_{\eta_j}\left[A_{21}\cos^2\theta/2+2\delta_{\eta_a,\eta_b}B_{21}(1+\sin^2\theta/2)\right] \\ & & & +\delta_{ab}\delta^{\eta_b}_{\eta_i}\delta^{\eta_a}_{\eta_j}C_{21}\left[(1+\eta_a\eta_b)-(1-\eta_a\eta_b)\sin^2\theta/2\right] \\ & & & +\delta^{-\eta_a}_{\eta_i}\delta^{-\eta_b}_{\eta_j}\eta_a\eta_b D_2\left[A_{22}\sin^2\theta/2+2\delta_{\eta_a,\eta_b}B_{22}(1+\cos^2\theta/2)\right] \\ & & & +\,\delta^{\eta_a}_{\eta_i}\delta^{\eta_b}_{\eta_j}\eta_a\eta_b D_2 C_{22}\left[(1+\eta_a\eta_b)-(1-\eta_a\eta_b)\cos^2\theta/2\right]\Big\}\ ,\end{aligned} \tag{42}$$

where, in agreement with our conventions, θ is the scattering angle between the incoming neutrino of flavour a and the produced charged lepton of flavour i. The particle helicities are η_a, η_b, η_i and η_j, respectively. Table 1 lists the values for the constant phase factors $N_{1,2}$ and $D_{1,2}$ and the subsets of the scalar, tensor and vector effective couplings constants, in that order, which define the quantities $A_{11,12,21,22}$, $B_{11,12,21,22}$ and $C_{11,12,21,22}$, whether in the case of Dirac of Majorana neutrinos.

The overall phase and sign of this amplitude is of course irrelevant physically, and is function of the phase convention adopted for the external $|\mathrm{In}>$ and $|\mathrm{Out}>$ states. The latter were defined by having the associated creation operators acting on the vacuum state $|0>$ in the same order as that in which the corresponding particles are given in the above lists of processes. For example in the case of the process "ab1", we have thus taken

$$|\mathrm{In}>=b^\dagger_a(\vec{k}_a,\eta_a)\,b^\dagger_b(\vec{k}_b,\eta_b)\,|0> \quad,\quad |\mathrm{Out}>=b^\dagger_i(\vec{\ell}_i,\eta_i)\,d^\dagger_j(\vec{\ell}_j,\eta_j)\,|0> \quad, \tag{43}$$

in a notation that should be self-explanatory. Similarly in the case "Mab2" for instance,

$$|\mathrm{In}>=a^\dagger_a(\vec{k}_a,\eta_a)\,a^\dagger_b(\vec{k}_b,\eta_b)\,|0> \quad,\quad |\mathrm{Out}>=d^\dagger_i(\vec{\ell}_i,\eta_i)\,b^\dagger_j(\vec{\ell}_j,\eta_j)\,|0> \ . \tag{44}$$

Obviously, exactly all the same conventions have been used throughout this work.

5 Neutrino Pair Production

Although neutrino pair production processes as such pose a genuine experimental challenge for their detection, as opposed to processes in which they are accompanied for instance by a photon in the final state[9], $\ell^-\ell^+ \to \nu\bar{\nu}\gamma$, the corresponding list of results is provided here for completeness. All $2\to 2$ neutrino pair production processes are labelled according to the following list when both neutrinos are of the Dirac type,

<u>$(ij)(ab)$ Dirac processes</u>

ij1: $\ell^-_i+\ell^+_j\to\nu_a+\nu_b$, ij2: $\ell^+_i+\ell^-_j\to\nu_a+\nu_b$,
ij3: $\ell^-_i+\ell^+_j\to\nu_a+\bar{\nu}_b$, ij4: $\ell^+_i+\ell^-_j\to\nu_a+\bar{\nu}_b$,
ij5: $\ell^-_i+\ell^+_j\to\bar{\nu}_a+\nu_b$, ij6: $\ell^+_i+\ell^-_j\to\bar{\nu}_a+\nu_b$,
ij7: $\ell^-_i+\ell^+_j\to\bar{\nu}_a+\bar{\nu}_b$, ij8: $\ell^+_i+\ell^-_j\to\bar{\nu}_a+\bar{\nu}_b$,

while in the Majorana case

$\underline{(ij)(ab)}$ Majorana processes

$$\text{Mij1: } \ell_i^- + \ell_j^+ \to \nu_a + \nu_b \quad , \quad \text{Mij2: } \ell_i^+ + \ell_j^- \to \nu_a + \nu_b \ .$$

For all these ten processes, the amplitude $\mathcal{M}$ is always of the following form

$$\begin{aligned}
\mathcal{M}_{(ij)(ab)} \;=\; & 4s\left(\frac{g^2}{8M^2}\right) N_1\,\delta_{ij} \quad \Big\{\delta_{ab}\delta_{\eta_i}^{-\eta_a}\delta_{\eta_j}^{-\eta_b}\left[A_{11}\sin^2\theta/2 + 2\delta_{\eta_a,\eta_b}B_{11}(1+\cos^2\theta/2)\right] \\
& \qquad -\delta_{ab}\delta_{\eta_i}^{\eta_a}\delta_{\eta_j}^{\eta_b}C_{11}\left[(1+\eta_a\eta_b)-(1-\eta_a\eta_b)\cos^2\theta/2\right] \\
& \qquad +\delta_{\eta_i}^{-\eta_b}\delta_{\eta_j}^{-\eta_a}D_1\left[A_{12}\cos^2\theta/2 + 2\delta_{\eta_a,\eta_b}B_{12}(1+\sin^2\theta/2)\right] \\
& \qquad -\,\delta_{\eta_i}^{\eta_b}\delta_{\eta_j}^{\eta_a}D_1C_{12}\left[(1+\eta_a\eta_b)-(1-\eta_a\eta_b)\sin^2\theta/2\right]\Big\} \\
& +4s\left(\frac{g^2}{8M^2}\right) N_2 \quad \Big\{\delta_{ab}\delta_{\eta_i}^{-\eta_b}\delta_{\eta_j}^{-\eta_a}\left[A_{21}\cos^2\theta/2 + 2\delta_{\eta_a,\eta_b}B_{21}(1+\sin^2\theta/2)\right] \\
& \qquad -\delta_{ab}\delta_{\eta_i}^{\eta_b}\delta_{\eta_j}^{\eta_a}C_{21}\left[(1+\eta_a\eta_b)-(1-\eta_a\eta_b)\sin^2\theta/2\right] \\
& \qquad +\delta_{\eta_i}^{-\eta_a}\delta_{\eta_j}^{-\eta_b}D_2\left[A_{22}\sin^2\theta/2 + 2\delta_{\eta_a,\eta_b}B_{22}(1+\cos^2\theta/2)\right] \\
& \qquad -\,\delta_{\eta_i}^{\eta_a}\delta_{\eta_j}^{\eta_b}D_2C_{22}\left[(1+\eta_a\eta_b)-(1-\eta_a\eta_b)\cos^2\theta/2\right]\Big\}\ ,
\end{aligned} \tag{45}$$

with the same conventions as previously, in particular that θ is the angle between the first lepton of flavour i and the first produced neutrino of flavour a. The different factors and coefficients appearing in this expression are detailed in Table 2, whether in the case of Dirac or Majorana neutrinos.

6 Neutrino Scattering

Even though it would suffice in the case of neutrino scattering onto a charged lepton to give only two classes of processes, for instance $(ai)(bj)$ and $(aj)(bi)$, since the two other classes could be obtained by appropriate permutations of indices and of the coupling coefficients with their complex conjugates, the results for all four classes of processes are listed nonetheless, for ease of practical use, and for explicit check of expressions through their symmetry properties under such permutations.

6.1 $(ai)(bj)$ neutrino scattering processes

In the case of neutrinos of Dirac character, the list of processes is labelled according to

$\underline{(ai)(bj)}$ Dirac processes

$$\begin{array}{llll}
\text{ai1: } \nu_a + \ell_i^- \to \nu_b + \ell_j^- & , & \text{ai2: } \nu_a + \ell_i^+ \to \nu_b + \ell_j^+ & , \\
\text{ai3: } \nu_a + \ell_i^- \to \bar{\nu}_b + \ell_j^- & , & \text{ai4: } \nu_a + \ell_i^+ \to \bar{\nu}_b + \ell_j^+ & , \\
\text{ai5: } \bar{\nu}_a + \ell_i^- \to \nu_b + \ell_j^- & , & \text{ai6: } \bar{\nu}_a + \ell_i^+ \to \nu_b + \ell_j^+ & , \\
\text{ai7: } \bar{\nu}_a + \ell_i^- \to \bar{\nu}_b + \ell_j^- & , & \text{ai8: } \bar{\nu}_a + \ell_i^+ \to \bar{\nu}_b + \ell_j^+ & ,
\end{array}$$

while in the Majorana case

$\underline{(ai)(bj)}$ Majorana processes

$$\text{Mai1: } \nu_a + \ell_i^- \to \nu_b + \ell_j^- \quad , \quad \text{Mai2: } \nu_a + \ell_i^+ \to \nu_b + \ell_j^+ \ .$$

The general amplitude $\mathcal{M}$ then reads in all ten cases as follows

$$\begin{aligned}
\mathcal{M}_{(ai)(bj)} \quad = \quad & 4s\left(\frac{g^2}{8M^2}\right) N_1\, \delta_{ij} \quad \Big\{\delta_{ab}\delta^{\eta_a}_{\eta_i}\delta^{\eta_b}_{\eta_j}\left[A_{11} - 2\delta_{\eta_a,-\eta_b}B_{11}(\cos^2\theta/2 - \sin^2\theta/2)\right] \\
& +\delta_{ab}\delta^{-\eta_a}_{\eta_i}\delta^{-\eta_b}_{\eta_j}C_{11}\left[1 + \eta_a\eta_b(\cos^2\theta/2 - \sin^2\theta/2)\right] \\
& +\delta^{-\eta_b}_{\eta_i}\delta^{-\eta_a}_{\eta_j}\eta_a\eta_b D_1\left[A_{12}\cos^2\theta/2 - 2\delta_{\eta_a,-\eta_b}B_{12}(1+\sin^2\theta/2)\right] \\
& + \delta^{\eta_b}_{\eta_i}\delta^{\eta_a}_{\eta_j}\eta_a\eta_b D_1 C_{12}\left[(1+\eta_a\eta_b) - (1-\eta_a\eta_b)\sin^2\theta/2)\right]\Big\} \\
& +4s\left(\frac{g^2}{8M^2}\right) N_2 \quad \Big\{\delta_{ab}\delta^{-\eta_b}_{\eta_i}\delta^{-\eta_a}_{\eta_j}\left[A_{21}\cos^2\theta/2 - 2\delta_{\eta_a,-\eta_b}B_{21}(1+\sin^2\theta/2)\right] \\
& +\delta_{ab}\delta^{\eta_b}_{\eta_i}\delta^{\eta_a}_{\eta_j}C_{21}\left[(1+\eta_a\eta_b) - (1-\eta_a\eta_b)\sin^2\theta/2\right] \\
& +\delta^{\eta_a}_{\eta_i}\delta^{\eta_b}_{\eta_j}\eta_a\eta_b D_2\left[A_{22} - 2\delta_{\eta_a,-\eta_b}B_{22}(\cos^2\theta/2 - \sin^2\theta/2)\right] \\
& + \delta^{-\eta_a}_{\eta_i}\delta^{-\eta_b}_{\eta_j}\eta_a\eta_b D_2 C_{22}\left[1 + \eta_a\eta_b(\cos^2\theta/2 - \sin^2\theta/2)\right]\Big\}\ ,
\end{aligned} \tag{46}$$

θ being the neutrino scattering angle. The list of factors and coefficients appearing in this expression is detailed in Table 3, both in the Dirac and in the Majorana case.

6.2 $(aj)(bi)$ neutrino scattering processes

The list of processes in the Dirac case is labelled according to

$(aj)(bi)$ Dirac processes

aj1: $\nu_a + \ell^-_j \to \nu_b + \ell^-_i$,	aj2: $\nu_a + \ell^+_j \to \nu_b + \ell^+_i$,
aj3: $\nu_a + \ell^-_j \to \bar{\nu}_b + \ell^-_i$,	aj4: $\nu_a + \ell^+_j \to \bar{\nu}_b + \ell^+_i$,
aj5: $\bar{\nu}_a + \ell^-_j \to \nu_b + \ell^-_i$,	aj6: $\bar{\nu}_a + \ell^+_j \to \nu_b + \ell^+_i$,
aj7: $\bar{\nu}_a + \ell^-_j \to \bar{\nu}_b + \ell^-_i$,	aj8: $\bar{\nu}_a + \ell^+_j \to \bar{\nu}_b + \ell^+_i$,

while in the Majorana case

$(aj)(bi)$ Majorana processes

Maj1: $\nu_a + \ell^-_j \to \nu_b + \ell^-_i$, Maj2: $\nu_a + \ell^+_j \to \nu_b + \ell^+_i$.

The general scattering amplitude $\mathcal{M}$ is of the form

$$\begin{aligned}
\mathcal{M}_{(aj)(bi)} \quad = \quad & 4s\left(\frac{g^2}{8M^2}\right) N_1\, \delta_{ij} \quad \Big\{\delta_{ab}\delta^{-\eta_a}_{\eta_i}\delta^{-\eta_b}_{\eta_j}\left[A_{11}\cos^2\theta/2 - 2\delta_{\eta_a,-\eta_b}B_{11}(1+\sin^2\theta/2)\right] \\
& +\delta_{ab}\delta^{\eta_a}_{\eta_i}\delta^{\eta_b}_{\eta_j}C_{11}\left[(1+\eta_a\eta_b) - (1-\eta_a\eta_b)\sin^2\theta/2\right] \\
& +\delta^{\eta_b}_{\eta_i}\delta^{\eta_a}_{\eta_j}\eta_a\eta_b D_1\left[A_{12} - 2\delta_{\eta_a,-\eta_b}B_{12}(\cos^2\theta/2 - \sin^2\theta/2)\right] \\
& + \delta^{-\eta_b}_{\eta_i}\delta^{-\eta_a}_{\eta_j}\eta_a\eta_b D_1 C_{12}\left[1 + \eta_a\eta_b(\cos^2\theta/2 - \sin^2\theta/2)\right]\Big\} \\
& +4s\left(\frac{g^2}{8M^2}\right) N_2 \quad \Big\{\delta_{ab}\delta^{\eta_b}_{\eta_i}\delta^{\eta_a}_{\eta_j}\left[A_{21} - 2\delta_{\eta_a,-\eta_b}B_{21}(\cos^2\theta/2 - \sin^2\theta/2)\right] \\
& +\delta_{ab}\delta^{-\eta_b}_{\eta_i}\delta^{-\eta_a}_{\eta_j}C_{21}\left[1 + \eta_a\eta_b(\cos^2\theta/2 - \sin^2\theta/2)\right] \\
& +\delta^{-\eta_a}_{\eta_i}\delta^{-\eta_b}_{\eta_j}\eta_a\eta_b D_2\left[A_{22}\cos^2\theta/2 - 2\delta_{\eta_a,-\eta_b}B_{22}(1+\sin^2\theta/2)\right] \\
& + \delta^{\eta_a}_{\eta_i}\delta^{\eta_b}_{\eta_j}\eta_a\eta_b D_2 C_{22}\left[(1+\eta_a\eta_b) - (1-\eta_a\eta_b)\sin^2\theta/2)\right]\Big\}\ ,
\end{aligned} \tag{47}$$

the angle θ being that of the scattered neutrino. Table 4 lists the relevant factors and coefficients both in the Dirac and in the Majorana case.

6.3 $(bi)(aj)$ neutrino scattering processes

In the Dirac case, we have the following labelling of processes

$\underline{(bi)(aj)\text{ Dirac processes}}$

$$
\begin{array}{llll}
\text{bi1: } \nu_b + \ell_i^- \to \nu_a + \ell_j^- & , & \text{bi2: } \nu_b + \ell_i^+ \to \nu_a + \ell_j^+ & , \\
\text{bi3: } \bar{\nu}_b + \ell_i^- \to \nu_a + \ell_j^- & , & \text{bi4: } \bar{\nu}_b + \ell_i^+ \to \nu_a + \ell_j^+ & , \\
\text{bi5: } \nu_b + \ell_i^- \to \bar{\nu}_a + \ell_j^- & , & \text{bi6: } \nu_b + \ell_i^+ \to \bar{\nu}_a + \ell_j^+ & , \\
\text{bi7: } \bar{\nu}_b + \ell_i^- \to \bar{\nu}_a + \ell_j^- & , & \text{bi8: } \bar{\nu}_b + \ell_i^+ \to \bar{\nu}_a + \ell_j^+ & ,
\end{array}
$$

while in the Majorana case

$\underline{(bi)(aj)\text{ Majorana processes}}$

$$
\text{Mbi1: } \nu_b + \ell_i^- \to \nu_a + \ell_j^- \quad , \quad \text{Mbi2: } \nu_b + \ell_i^+ \to \nu_a + \ell_j^+ \ .
$$

The general scattering amplitude $\mathcal{M}$ reads

$$
\begin{aligned}
\mathcal{M}_{(bi)(aj)} \;=\; & 4s\left(\frac{g^2}{8M^2}\right) N_1\, \delta_{ij} \Big\{ \delta_{ab}\delta_{\eta_i}^{-\eta_a}\delta_{\eta_j}^{-\eta_b}\left[A_{11}\cos^2\theta/2 - 2\delta_{\eta_a,-\eta_b}B_{11}(1+\sin^2\theta/2)\right] \\
& \quad +\delta_{ab}\delta_{\eta_i}^{\eta_a}\delta_{\eta_j}^{\eta_b}C_{11}\left[(1+\eta_a\eta_b)-(1-\eta_a\eta_b)\sin^2\theta/2\right] \\
& \quad +\delta_{\eta_i}^{\eta_b}\delta_{\eta_j}^{\eta_a}\eta_a\eta_b D_1\left[A_{12} - 2\delta_{\eta_a,-\eta_b}B_{12}(\cos^2\theta/2 - \sin^2\theta/2)\right] \\
& \quad +\delta_{\eta_i}^{-\eta_b}\delta_{\eta_j}^{-\eta_a}\eta_a\eta_b D_1 C_{12}\left[1+\eta_a\eta_b(\cos^2\theta/2-\sin^2\theta/2)\right]\Big\} \\
& +4s\left(\frac{g^2}{8M^2}\right) N_2 \Big\{ \delta_{ab}\delta_{\eta_i}^{\eta_b}\delta_{\eta_j}^{\eta_a}\left[A_{21} - 2\delta_{\eta_a,-\eta_b}B_{21}(\cos^2\theta/2-\sin^2\theta/2)\right] \\
& \quad +\delta_{ab}\delta_{\eta_i}^{-\eta_b}\delta_{\eta_j}^{-\eta_a}C_{21}\left[1+\eta_a\eta_b(\cos^2\theta/2-\sin^2\theta/2)\right] \\
& \quad +\delta_{\eta_i}^{-\eta_a}\delta_{\eta_j}^{-\eta_b}\eta_a\eta_b D_2\left[A_{22}\cos^2\theta/2 - 2\delta_{\eta_a,-\eta_b}B_{22}(1+\sin^2\theta/2)\right] \\
& \quad +\delta_{\eta_i}^{\eta_a}\delta_{\eta_j}^{\eta_b}\eta_a\eta_b D_2 C_{22}\left[(1+\eta_a\eta_b)-(1-\eta_a\eta_b)\sin^2\theta/2)\right]\Big\}\ ,
\end{aligned}
\tag{48}
$$

θ being of course the neutrino scattering angle. The factors and coefficients appearing in this representation are detailed in Table 5.

6.4 $(bj)(ai)$ neutrino scattering processes

Processes in the Dirac case are labelled according to

$\underline{(bj)(ai)\text{ Dirac processes}}$

$$
\begin{array}{llll}
\text{bj1: } \nu_b + \ell_j^- \to \nu_a + \ell_i^- & , & \text{bj2: } \nu_b + \ell_j^+ \to \nu_a + \ell_i^+ & , \\
\text{bj3: } \bar{\nu}_b + \ell_j^- \to \nu_a + \ell_i^- & , & \text{bj4: } \bar{\nu}_b + \ell_j^+ \to \nu_a + \ell_i^+ & , \\
\text{bj5: } \nu_b + \ell_j^- \to \bar{\nu}_a + \ell_i^- & , & \text{bj6: } \nu_b + \ell_j^+ \to \bar{\nu}_a + \ell_i^+ & , \\
\text{bj7: } \bar{\nu}_b + \ell_j^- \to \bar{\nu}_a + \ell_i^- & , & \text{bj8: } \bar{\nu}_b + \ell_j^+ \to \bar{\nu}_a + \ell_i^+ & ,
\end{array}
$$

while in the Majorana case

$\underline{(bj)(ai)\text{ Majorana processes}}$

$$
\text{Mbj1: } \nu_b + \ell_j^- \to \nu_a + \ell_i^- \quad , \quad \text{Mbj2: } \nu_b + \ell_j^+ \to \nu_a + \ell_i^+ \ .
$$

The general scattering amplitude $\mathcal{M}$ is given by

$$\begin{aligned}
\mathcal{M}_{(bj)(ai)} &= 4s\left(\frac{g^2}{8M^2}\right) N_1\,\delta_{ij} \quad \Big\{\delta_{ab}\delta^{\eta_a}_{\eta_i}\delta^{\eta_b}_{\eta_j}\left[A_{11}-2\delta_{\eta_a,-\eta_b}B_{11}(\cos^2\theta/2-\sin^2\theta/2)\right] \\
&\qquad +\delta_{ab}\delta^{-\eta_a}_{\eta_i}\delta^{-\eta_b}_{\eta_j}C_{11}\left[1+\eta_a\eta_b(\cos^2\theta/2-\sin^2\theta/2)\right] \\
&\qquad +\delta^{-\eta_b}_{\eta_i}\delta^{-\eta_a}_{\eta_j}\eta_a\eta_b D_1\left[A_{12}\cos^2\theta/2-2\delta_{\eta_a,-\eta_b}B_{12}(1+\sin^2\theta/2)\right] \\
&\qquad +\,\delta^{\eta_b}_{\eta_i}\delta^{\eta_a}_{\eta_j}\eta_a\eta_b D_1 C_{12}\left[(1+\eta_a\eta_b)-(1-\eta_a\eta_b)\sin^2\theta/2)\right]\Big\} \\
&\quad +4s\left(\frac{g^2}{8M^2}\right) N_2 \quad \Big\{\delta_{ab}\delta^{-\eta_b}_{\eta_i}\delta^{-\eta_a}_{\eta_j}\left[A_{21}\cos^2\theta/2-2\delta_{\eta_a,-\eta_b}B_{21}(1+\sin^2\theta/2)\right] \\
&\qquad +\delta_{ab}\delta^{\eta_b}_{\eta_i}\delta^{\eta_a}_{\eta_j}C_{21}\left[(1+\eta_a\eta_b)-(1-\eta_a\eta_b)\sin^2\theta/2\right] \\
&\qquad +\delta^{\eta_a}_{\eta_i}\delta^{\eta_b}_{\eta_j}\eta_a\eta_b D_2\left[A_{22}-2\delta_{\eta_a,-\eta_b}B_{22}(\cos^2\theta/2-\sin^2\theta/2)\right] \\
&\qquad +\,\delta^{-\eta_a}_{\eta_i}\delta^{-\eta_b}_{\eta_j}\eta_a\eta_b D_2 C_{22}\left[1+\eta_a\eta_b(\cos^2\theta/2-\sin^2\theta/2)\right]\Big\}\ ,
\end{aligned} \tag{49}$$

with θ being the neutrino scattering angle. Table 6 lists the relevant factors and coefficients both in the Dirac and Majorana cases.

7 Exploratory Examples

Before turning to some simple illustrative examples of the potential physics reach of these $2 \to 2$ processes involving neutrinos, let us point out the following simple fact. For each of the six classes of ten processes above, when comparing the cases with Dirac or with Majorana neutrinos, one notices that Dirac neutrino processes labelled "xy$n$" ("x" and "y" each being one of the neutrino or lepton flavour symbols and n being an integer) must be in correspondence with the Majorana process "Mxy1" for $n = 1, 3, 5, 7$, and with the Majorana process "Mxy2" for $n = 2, 4, 6, 8$. Namely, given the correspondence (24), and for a specific choice of the external particle helicities, in each case the sum of the corresponding four Dirac amplitudes $\mathcal{M}$ must coincide with the amplitude $\mathcal{M}$ for the Majorana process through the association of coupling coefficients described in (36)-(38). It is straightforward to check that this correspondence is indeed obtained.

This fact, with in particular an identical angular parametrization of the differential cross section for all ten processes belonging to each one of the above six $2 \to 2$ classes, also implies that a model independent discrimination between the Dirac of Majorana character of neutrinos would require stringent precision requirements for systematic measurements in which different helicity combinations are compared, in order to isolate the relevant coefficients $A_{\alpha\beta}$, $B_{\alpha\beta}$ and $C_{\alpha\beta}$ ($\alpha, \beta = 1, 2$), and eventually determine their possible relations as applicable either to the Dirac or to the Majorana case.

However, the four-fermion parametrization used is far more general than what is usually achieved in any specific model beyond the SM, and as a general rule, couplings of type 2, 3 and 4, namely $S^{\eta_a,\eta_b}_{2,3,4}$, $V^{\eta_a,\eta_b}_{2,3,4}$ and $T^{\eta_a,\eta_b}_{2,3,4}$, do not arise. Under such a situation which remains quite model independent, it is clear that in principle there exist large classes of processes which, simply through the angular dependency of their differential cross sections, should enable the experimental discrimination between the Dirac or Majorana neutrino character. However, without any prior knowledge of the order of magnitude of the coupling coefficients which determine the relative strengths of different angular dependencies, such a discrimination in a model independent manner requires that at least one of the flavour pairs (ab) or (ij) be identical. Indeed, one gets different angular contributions to the Dirac or Majorana amplitudes $\mathcal{M}$ provided only at least one of the set of terms proportional either to δ_{ij} or to δ_{ab}, or both, contributes to the differential cross section. Nevertheless, this still leaves open *a priori* quite some possibilities, contingent onto the properties of the neutrino beams that would become available in the future, in particular their flavour and helicity contents.

In fact, in the latter respect, any helicity content of a neutrino beam other than left-handed for what is thought to be a neutrino and right-handed for what is thought to an antineutrino in the Dirac case, depends on possible interactions beyond the SM that might contribute to the neutrino beam production mechanism. In any event, such a helicity "contamination" of a beam must be expected not to exceed, say, one percent, given present limits on neutrino helicities[4]. Depending on the intensities of beams to become available at neutrino factories, this might provide an additional aspect of physical interest nonetheless. For the time being however, let us conservatively assume that available beams would only be purely left-handed for would-be Dirac neutrinos and purely right-handed for would-be Dirac antineutrinos.

The experimental possibility to eventually discriminate through neutrino annihilation and scattering processes between their Dirac or Majorana character using the difference in the angular dependency of the associated cross sections, is also contingent on the strength of any new interaction beyond those of the SM—as is also the neutrinoless double β-decay process for that matter[10]—, since in the latter model such a possibility simply evaporates in the massless limit[6]. Hence, even though the possibility exists in principle, its actual experimental realization hinges, on the one hand, on sufficiently intense beams at neutrino factories to allow for reasonably precise angular measurements, and on the other hand, on the physical existence of a new interaction different from $(V-A)$ that couples sufficiently strongly to neutrinos as compared to those of the SM. Clearly, a definite assessment of the physics potential of such an approach to the Dirac-Majorana neutrino issue requires a systematic and dedicated analysis which is not attempted here, based on actual neutrino factory designs as presently foreseen, and the general low-energy parametrization developed here.

Besides the potential resolution of the Dirac-Majorana neutrino issue, intense neutrino beams should also help turn into reality the systematic determination of the neutrino electroweak interactions, in a manner similar to what has been done in the leptonic $(\nu_\mu\mu)(e\nu_e)$ and semi-leptonic $(ud)(e\nu_e)$ sectors[2, 4]. Through detailed precisions measurements in different combinations of flavour and helicity channels, which is also part of the neutrino oscillation programmes at neutrino factories, it should become possible to set ever more stringent experimental bounds on the different coupling constants that parametrize the general effective four-fermion interaction, and search for a lack of overlap with those of the SM. In the same way that Refs.[1, 3, 4] provide the expressions of all possible observables in terms of such coupling coefficients for the above leptonic and semi-leptonic channels, the results listed in this note provide those for all $2 \to 2$ processes in which two neutrinos and two charged fermions take part, whatever their mass eigenstates.

In the remainder of this section, simple illustrative examples are briefly considered, whose sole purpose is a first exploration of some of the above issues. The emphasis here is only on the Dirac-Majorana neutrino issue.

First, let us consider the elastic scattering[8]

$$\nu_\mu + e^- \to \nu_\mu + e^- \ , \tag{50}$$

which is thus a reaction of type "ai1" belonging to the $(ai)(bj)$ class, with $a = b \neq i = j$. Having in mind for instance the left-right symmetric extensions[11] of the SM, based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, let us assume for utmost simplicity that besides the $S_1^{-,-}$ and $V_1^{-,-}$ couplings of the SM, the only nonvanishing extra couplings are $S_1^{+,+}$ and $V_1^{+,+}$. In the simplest minded situation where the two chiral sectors are identical in as far as is possible in all their aspects and do

[8]The ν_μ component is indeed dominant in neutrino beams.

not mix, we thus have,

$$S_1^{-,-} = \sin^2\theta_W \quad , \quad V_1^{-,-} = \frac{1}{4}[1 - 2\sin^2\theta_W] \quad , \quad S_1^{+,+} = \delta\sin^2\theta_W \quad , \quad V_1^{+,+} = \delta\frac{1}{4}[1 - 2\sin^2\theta_W] \ , \quad (51)$$

where

$$\delta = \frac{M_1^2}{M_2^2} \ , \quad (52)$$

is the physical light $W_1^{\pm}$ to heavy $W_2^{\pm}$ squared gauge boson masses ratio, with $M_1 \simeq 80$ GeV and $M_2 > 720$ GeV[4], and θ_W is the usual electroweak gauge mixing angle, $\sin^2\theta_W \simeq 0.231$[4]. Assuming then that the in-coming ν_μ neutrino is necessarily left-handed, only two of the eight possible scattering amplitudes $\mathcal{M}$ are nonvanishing due to helicity selection rules, namely those with $(\eta_a, \eta_i; \eta_b, \eta_j) = (-,-;-,-), (-,+;-,+)$. Since the angular dependency of each of these two amplitudes is identical whether the neutrinos are Dirac or Majorana, the sole difference being in their absolute normalization, and given the difficulty in performing absolute cross section measurements, let us consider the summation over all processes irrespective of the particle helicities, except of course for that of the in-coming ν_μ, thus corresponding to an unpolarized measurement. In the Dirac case, one then finds

$$|\mathcal{M}_{-,-;-,-}|^2 + |\mathcal{M}_{-,+;-,+}|^2 = (4s)^2 \left(\frac{g^2}{8M^2}\right)^2 \left[4\mathrm{Re}\,V_1^{-,-}\right]^2 \left\{1 + \left[\frac{\mathrm{Re}\,S_1^{-,-}}{4\mathrm{Re}\,V_1^{-,-}}\right]^2 (1+\cos\theta)^2\right\} \ , \quad (53)$$

while in the Majorana case,

$$\begin{aligned} |\mathcal{M}_{-,-;-,-}|^2 + |\mathcal{M}_{-,+;-,+}|^2 = \ & (4s)^2 \left(\tfrac{g^2}{8M^2}\right)^2 [4\mathrm{Re}\,V^{-,-} + 2\mathrm{Re}\,S^{+,+}]^2 \\ & \times \left\{1 + \left[\tfrac{\mathrm{Re}\,S^{-,-} + 2\mathrm{Re}\,V^{+,+}}{4\mathrm{Re}\,V^{-,-} + 2\mathrm{Re}\,S^{+,+}}\right]^2 (1+\cos\theta)^2\right\} \ . \end{aligned} \quad (54)$$

Consequently, if there are indeed interactions whose chirality structure is different from those of the SM, processes for Dirac or Majorana neutrinos do possess different angular properties, enabling in principle the discrimination between the two cases through precision measurements of the angular dependency of the cross section, in the present case by comparing the strength of the $(1+\cos\theta)^2$ term to its value predicted in the SM. Unfortunately, in this specific case and under the very restrictive form of the coefficients $S_1^{+,+}$ and $V_1^{+,+}$ considered above, limits on the possible extra interaction are already such that it is too weak to render any deviation observable, the relative variation in the relevant factor being less than a percent given the small value for $\delta \le (80 \text{ GeV}/720 \text{ GeV})^2 \simeq 1.23 \times 10^{-2}$.

Scalar or tensor couplings being typically less well constrained than vector ones, let us now consider the possibility of an extra scalar interaction, for either of the following two elastic scattering reactions,

$$\nu_\mu + e^- \to \nu_\mu + e^- \quad , \quad \nu_\mu + \mu^- \to \nu_\mu + \mu^- \ . \quad (55)$$

Both these reactions are of the "ai1" type in the $(ai)(bj)$ class, with $a = b \neq i = j$ in the first case, and $a = b = i = j$ in the second. Within the SM, the corresponding nonvanishing couplings are thus

$$S_1^{-,-} = \sin^2\theta_W \quad , \quad V_1^{-,-} = \pm\frac{1}{4}\left[1 \mp 2\sin^2\theta_W\right] \ , \quad (56)$$

where the upper (resp. lower) sign is for the $(\nu_\mu e)$ (resp. $(\nu_\mu \mu)$) reaction. Assuming that $S_1^{+,-}$ is the sole nonvanishing extra interaction, and given a left-handed in-coming ν_μ, one finds that whether in the Dirac or Majorana case the only nonvanishing amplitudes $\mathcal{M}$ correspond to the following helicity combinations: $(\eta_a, \eta_i; \eta_b, \eta_j) = (-,-;-,-), (-,-;+,+), (-,+;-,+)$. Considering again the situation of an unpolarized measurement, the sum over these polarization states reads,

$$\sum |\mathcal{M}|^2 = (4s)^2 \left(\frac{g^2}{8M^2}\right)^2 \left\{ \left[4\mathrm{Re}\, V_1^{-,-}\right]^2 + \left[\mathrm{Re}\, S_1^{-,-}\right]^2 (1+\cos\theta)^2 + \frac{1}{4}|S_1^{+,-}|^2 (1 \pm \cos\theta)^2 \right\} \ , \quad (57)$$

where in the last term the upper sign corresponds to the Dirac case, and the lower sign to the Majorana case. Thus once again, we see that any new interaction whose chirality structure differs from those of the SM leads to processes in which the angular dependency discriminates between Dirac and Majorana neutrinos. Taking as an illustration a value $|S_1^{+,-}| = 0.10$ which is a typical upper-bound on such a coupling in the leptonic $(e\mu)$ sector[4], one finds a 10% sensitivity in the forward-backward asymmetry. A reasonably precise measurement of the differential cross section, and in particular a fit to the expected distributions in either case, thus offers the prospect to resolve the Dirac-Majorana neutrino issue at neutrino factories. A dedicated study should hopefully confirm the present exploratory assessment.

To also highlight the potentiel interest of intersecting neutrino beams, as a final example let us consider the following two annihilation reactions,

$$\nu_\mu + \nu_\mu \rightarrow e^- + e^+ \quad , \quad \nu_\mu + \nu_\mu \rightarrow \mu^- + \mu^+ \ . \quad (58)$$

These two processes are of the type "ab1" in the $(ab)(ij)$ class, with $a = b \neq i = j$ in the first case, and $a = b = i = j$ in the second case, and thus again with the following interactions in the SM,

$$S_1^{-,-} = \sin^2\theta_W \quad , \quad V_1^{-,-} = \pm\frac{1}{4}\left[1 \mp 2\sin^2\theta_W\right] \ , \quad (59)$$

where the upper (resp. lower) sign is associated to the first (resp. second) reaction. Assuming now that none of the other possible coupling coefficients belongs to the classes $S_{2,3,4}^{\eta_a,\eta_b}$, $V_{2,3,4}^{\eta_a,\eta_b}$ and $T_{2,3,4}^{\eta_a,\eta_b}$, one readily finds that given left-handed initial ν_μ neutrinos only, the amplitude $\mathcal{M}$ for these processes vanishes identically in the Dirac case, but not in the Majorana case, with then a specific angular dependency in the latter case which is function of the interactions that might contribute. Even though the detection of the final state products should be straightforward, the difficulty lies of course in the density of the initial neutrino beams even for very intense ones, implying thus an extremely low rate. Nonetheless, even if only through a single event, the observation of either of the above processes, or similar ones for other neutrino mass eigenstates, would definitely help settle the Dirac-Majorana neutrino puzzle through accelerator experiments.

8 Conclusions

In the present work, all possible $2 \rightarrow 2$ processes involving definite mass eigenstates of two neutrinos and two charged fermions have been considered in the massless limit, on the basis of the most general four-fermion effective Lagrangian possible. All interactions, whether preserving the neutrino fermion number or not, and for whatever helicities of the external particles, have been included. The Feynman amplitudes in the center-of-mass frame have been listed for all these processes, from which the relevant differential cross sections readily follow. Since any particular model for physics beyond the Standard Model predicts specific values for the four-fermion nonderivative effective coupling coefficients, this analysis should be of value to assess the low energy merits of any such model.

In the same way as has been done for β- and μ-decay and μ-capture through analogous four-fermion effective parametrizations[4, 5], the advent in the foreseeable future of neutrino factories with their intense beams is the main motivation for the considerations developed here. Of direct interest is the systematic study of the electroweak interactions in the neutrino sector, by setting ever more stringent limits on the effective interactions of these particles through precision measurements. Another physics issue of great topical interest that could be addressed through such experiments is that of the Dirac-Majorana discrimination of the character of neutrinos. In agreement with the Dirac-Majorana confusion theorem[6], as soon as interactions with a chirality structure different from the $(V - A)$ one of the Standard Model are introduced, there exist processes which in principle distinguish between these two possible characters of the neutrino through the angular dependency of differential cross sections, even in the massless limit. The sensitivity of such reactions however, is contingent of course on the relative strength of these new interactions beyond the Standard Model. Nonetheless, some simple examples of such a situation were briefly described, albeit not following any systematic investigation.

The main purpose of this work is to provide the general results for the Feynman amplitudes for all possible $2 \to 2$ processes with two neutrinos. On that basis, it should now be possible to develop a detailed and dedicated analysis of the potential reach of different such reactions towards the above physics issues, given a specific design both of neutrino beams and their intensities, and of detector set-ups. Besides the great interest to be found in neutrino scattering experiments, the possibilities offered by intersecting neutrino beams should not be dismissed offhand without first a dedicated assessment as well, the more so since they could possibly run in parasitic mode in conjunction with other experiments given the proper neutrino beam geometrical layout.

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	ab1	ab2	ab3	ab4	ab5	ab6	ab7	ab8	Mab1	Mab2
N_1	$\eta_a\lambda_a^*$	$\eta_a\lambda_b^*$	η_a	η_a	η_a	η_a	$\eta_a\lambda_b$	$\eta_a\lambda_a$	$\eta_a\lambda_a^*$	$\eta_a\lambda_b^*$
A_{11}	$S_2^{\eta_b,\eta_a}$	$S_3^{\eta_b,\eta_a*}$	$S_1^{-\eta_b,\eta_a}$	$S_4^{-\eta_b,\eta_a*}$	$S_4^{\eta_b,-\eta_a}$	$S_1^{\eta_b,-\eta_a*}$	$S_3^{-\eta_b,-\eta_a}$	$S_2^{-\eta_b,-\eta_a*}$	$S^{-\eta_b,\eta_a}$	$S^{\eta_b,-\eta_a*}$
B_{11}	$T_2^{\eta_b,\eta_a}$	$T_3^{\eta_b,\eta_a*}$	$T_1^{-\eta_b,\eta_a}$	$T_4^{-\eta_b,\eta_a*}$	$T_4^{\eta_b,-\eta_a}$	$T_1^{\eta_b,-\eta_a*}$	$T_3^{-\eta_b,-\eta_a}$	$T_2^{-\eta_b,-\eta_a*}$	$T^{-\eta_b,\eta_a}$	$T^{\eta_b,-\eta_a*}$
C_{11}	$V_2^{\eta_b,\eta_a}$	$V_3^{\eta_b,\eta_a*}$	$V_1^{-\eta_b,\eta_a}$	$V_4^{-\eta_b,\eta_a*}$	$V_4^{\eta_b,-\eta_a}$	$V_1^{\eta_b,-\eta_a*}$	$V_3^{-\eta_b,-\eta_a}$	$V_2^{-\eta_b,-\eta_a*}$	$V^{-\eta_b,\eta_a}$	$V^{\eta_b,-\eta_a*}$
D_1	1	1	$\lambda_a^*\lambda_b$	1	1	$\lambda_a\lambda_b^*$	1	1	1	1
A_{12}	$S_2^{\eta_a,\eta_b}$	$S_3^{\eta_a,\eta_b*}$	$S_4^{\eta_a,-\eta_b}$	$S_1^{\eta_a,-\eta_b*}$	$S_1^{-\eta_a,\eta_b}$	$S_4^{-\eta_a,\eta_b*}$	$S_3^{-\eta_a,-\eta_b}$	$S_2^{-\eta_a,-\eta_b*}$	$S^{-\eta_a,\eta_b}$	$S^{\eta_a,-\eta_b*}$
B_{12}	$T_2^{\eta_a,\eta_b}$	$T_3^{\eta_a,\eta_b*}$	$T_4^{\eta_a,-\eta_b}$	$T_1^{\eta_a,-\eta_b*}$	$T_1^{-\eta_a,\eta_b}$	$T_4^{-\eta_a,\eta_b*}$	$T_3^{-\eta_a,-\eta_b}$	$T_2^{-\eta_a,-\eta_b*}$	$T^{-\eta_a,\eta_b}$	$T^{\eta_a,-\eta_b*}$
C_{12}	$V_2^{\eta_a,\eta_b}$	$V_3^{\eta_a,\eta_b*}$	$V_4^{\eta_a,-\eta_b}$	$V_1^{\eta_a,-\eta_b*}$	$V_1^{-\eta_a,\eta_b}$	$V_4^{-\eta_a,\eta_b*}$	$V_3^{-\eta_a,-\eta_b}$	$V_2^{-\eta_a,-\eta_b*}$	$V^{-\eta_a,\eta_b}$	$V^{\eta_a,-\eta_b*}$
N_2	$\eta_b\lambda_b^*$	$\eta_b\lambda_a^*$	η_b	η_b	η_b	η_b	$\eta_b\lambda_a$	$\eta_b\lambda_b$	$\eta_b\lambda_b^*$	$\eta_b\lambda_a^*$
A_{21}	$S_3^{\eta_b,\eta_a*}$	$S_2^{\eta_b,\eta_a}$	$S_4^{-\eta_b,\eta_a*}$	$S_1^{-\eta_b,\eta_a}$	$S_1^{\eta_b,-\eta_a*}$	$S_4^{\eta_b,-\eta_a}$	$S_2^{-\eta_b,-\eta_a*}$	$S_3^{-\eta_b,-\eta_a}$	$S^{\eta_b,-\eta_a*}$	$S^{-\eta_b,\eta_a}$
B_{21}	$T_3^{\eta_b,\eta_a*}$	$T_2^{\eta_b,\eta_a}$	$T_4^{-\eta_b,\eta_a*}$	$T_1^{-\eta_b,\eta_a}$	$T_1^{\eta_b,-\eta_a*}$	$T_4^{\eta_b,-\eta_a}$	$T_2^{-\eta_b,-\eta_a*}$	$T_3^{-\eta_b,-\eta_a}$	$T^{\eta_b,-\eta_a*}$	$T^{-\eta_b,\eta_a}$
C_{21}	$V_3^{\eta_b,\eta_a*}$	$V_2^{\eta_b,\eta_a}$	$V_4^{-\eta_b,\eta_a*}$	$V_1^{-\eta_b,\eta_a}$	$V_1^{\eta_b,-\eta_a*}$	$V_4^{\eta_b,-\eta_a}$	$V_2^{-\eta_b,-\eta_a*}$	$V_3^{-\eta_b,-\eta_a}$	$V^{\eta_b,-\eta_a*}$	$V^{-\eta_b,\eta_a}$
D_2	1	1	1	$\lambda_a^*\lambda_b$	$\lambda_a\lambda_b^*$	1	1	1	1	1
A_{22}	$S_3^{\eta_a,\eta_b*}$	$S_2^{\eta_a,\eta_b}$	$S_1^{\eta_a,-\eta_b*}$	$S_4^{\eta_a,-\eta_b}$	$S_4^{-\eta_a,\eta_b*}$	$S_1^{-\eta_a,\eta_b}$	$S_2^{-\eta_a,-\eta_b*}$	$S_3^{-\eta_a,-\eta_b}$	$S^{\eta_a,-\eta_b*}$	$S^{-\eta_a,\eta_b}$
B_{22}	$T_3^{\eta_a,\eta_b*}$	$T_2^{\eta_a,\eta_b}$	$T_1^{\eta_a,-\eta_b*}$	$T_4^{\eta_a,-\eta_b}$	$T_4^{-\eta_a,\eta_b*}$	$T_1^{-\eta_a,\eta_b}$	$T_2^{-\eta_a,-\eta_b*}$	$T_3^{-\eta_a,-\eta_b}$	$T^{\eta_a,-\eta_b*}$	$T^{-\eta_a,\eta_b}$
C_{22}	$V_3^{\eta_a,\eta_b*}$	$V_2^{\eta_a,\eta_b}$	$V_1^{\eta_a,-\eta_b*}$	$V_4^{\eta_a,-\eta_b}$	$V_4^{-\eta_a,\eta_b*}$	$V_1^{-\eta_a,\eta_b}$	$V_2^{-\eta_a,-\eta_b*}$	$V_3^{-\eta_a,-\eta_b}$	$V^{\eta_a,-\eta_b*}$	$V^{-\eta_a,\eta_b}$

Table 1: List of the constant factors appearing in the amplitude (42) for all $(ab)(ij)$ neutrino annihilation processes according to their labelling defined in Sect.4.

	ij1	ij2	ij3	ij4	ij5	ij6	ij7	ij8	Mij1	Mij2
N_1	$\eta_a\lambda_a$	$\eta_a\lambda_b$	η_a	η_a	η_a	η_a	$\eta_a\lambda_b^*$	$\eta_a\lambda_a^*$	$\eta_a\lambda_a$	$\eta_a\lambda_b$
A_{11}	$S_2^{\eta_b,\eta_a*}$	$S_3^{\eta_b,\eta_a}$	$S_1^{-\eta_b,\eta_a*}$	$S_4^{-\eta_b,\eta_a}$	$S_4^{\eta_b,-\eta_a*}$	$S_1^{\eta_b,-\eta_a}$	$S_3^{-\eta_b,-\eta_a*}$	$S_2^{-\eta_b,-\eta_a}$	$S^{-\eta_b,\eta_a*}$	$S^{\eta_b,-\eta_a}$
B_{11}	$T_2^{\eta_b,\eta_a*}$	$T_3^{\eta_b,\eta_a}$	$T_1^{-\eta_b,\eta_a*}$	$T_4^{-\eta_b,\eta_a}$	$T_4^{\eta_b,-\eta_a*}$	$T_1^{\eta_b,-\eta_a}$	$T_3^{-\eta_b,-\eta_a*}$	$T_2^{-\eta_b,-\eta_a}$	$T^{-\eta_b,\eta_a*}$	$T^{\eta_b,-\eta_a}$
C_{11}	$V_2^{\eta_b,\eta_a*}$	$V_3^{\eta_b,\eta_a}$	$V_1^{-\eta_b,\eta_a*}$	$V_4^{-\eta_b,\eta_a}$	$V_4^{\eta_b,-\eta_a*}$	$V_1^{\eta_b,-\eta_a}$	$V_3^{-\eta_b,-\eta_a*}$	$V_2^{-\eta_b,-\eta_a}$	$V^{-\eta_b,\eta_a*}$	$V^{\eta_b,-\eta_a}$
D_1	1	1	$\lambda_a\lambda_b^*$	1	1	$\lambda_a^*\lambda_b$	1	1	1	1
A_{12}	$S_2^{\eta_a,\eta_b*}$	$S_3^{\eta_a,\eta_b}$	$S_4^{\eta_a,-\eta_b*}$	$S_1^{\eta_a,-\eta_b}$	$S_1^{-\eta_a,\eta_b*}$	$S_4^{-\eta_a,\eta_b}$	$S_3^{-\eta_a,-\eta_b*}$	$S_2^{-\eta_a,-\eta_b}$	$S^{-\eta_a,\eta_b*}$	$S^{\eta_a,-\eta_b}$
B_{12}	$T_2^{\eta_a,\eta_b*}$	$T_3^{\eta_a,\eta_b}$	$T_4^{\eta_a,-\eta_b*}$	$T_1^{\eta_a,-\eta_b}$	$T_1^{-\eta_a,\eta_b*}$	$T_4^{-\eta_a,\eta_b}$	$T_3^{-\eta_a,-\eta_b*}$	$T_2^{-\eta_a,-\eta_b}$	$T^{-\eta_a,\eta_b*}$	$T^{\eta_a,-\eta_b}$
C_{12}	$V_2^{\eta_a,\eta_b*}$	$V_3^{\eta_a,\eta_b}$	$V_4^{\eta_a,-\eta_b*}$	$V_1^{\eta_a,-\eta_b}$	$V_1^{-\eta_a,\eta_b*}$	$V_4^{-\eta_a,\eta_b}$	$V_3^{-\eta_a,-\eta_b*}$	$V_2^{-\eta_a,-\eta_b}$	$V^{-\eta_a,\eta_b*}$	$V^{\eta_a,-\eta_b}$
N_2	$\eta_a\lambda_b$	$\eta_a\lambda_a$	η_a	η_a	η_a	η_a	$\eta_a\lambda_a^*$	$\eta_a\lambda_b^*$	$\eta_a\lambda_b$	$\eta_a\lambda_a$
A_{21}	$S_3^{\eta_b,\eta_a}$	$S_2^{\eta_b,\eta_a*}$	$S_4^{-\eta_b,\eta_a}$	$S_1^{-\eta_b,\eta_a*}$	$S_1^{\eta_b,-\eta_a}$	$S_4^{\eta_b,-\eta_a*}$	$S_2^{-\eta_b,-\eta_a}$	$S_3^{-\eta_b,-\eta_a*}$	$S^{\eta_b,-\eta_a}$	$S^{-\eta_b,\eta_a*}$
B_{21}	$T_3^{\eta_b,\eta_a}$	$T_2^{\eta_b,\eta_a*}$	$T_4^{-\eta_b,\eta_a}$	$T_1^{-\eta_b,\eta_a*}$	$T_1^{\eta_b,-\eta_a}$	$T_4^{\eta_b,-\eta_a*}$	$T_2^{-\eta_b,-\eta_a}$	$T_3^{-\eta_b,-\eta_a*}$	$T^{\eta_b,-\eta_a}$	$T^{-\eta_b,\eta_a*}$
C_{21}	$V_3^{\eta_b,\eta_a}$	$V_2^{\eta_b,\eta_a*}$	$V_4^{-\eta_b,\eta_a}$	$V_1^{-\eta_b,\eta_a*}$	$V_1^{\eta_b,-\eta_a}$	$V_4^{\eta_b,-\eta_a*}$	$V_2^{-\eta_b,-\eta_a}$	$V_3^{-\eta_b,-\eta_a*}$	$V^{\eta_b,-\eta_a}$	$V^{-\eta_b,\eta_a*}$
D_2	1	1	1	$\lambda_a\lambda_b^*$	$\lambda_a^*\lambda_b$	1	1	1	1	1
A_{22}	$S_3^{\eta_a,\eta_b}$	$S_2^{\eta_a,\eta_b*}$	$S_1^{\eta_a,-\eta_b}$	$S_4^{\eta_a,-\eta_b*}$	$S_4^{-\eta_a,\eta_b}$	$S_1^{-\eta_a,\eta_b*}$	$S_2^{-\eta_a,-\eta_b}$	$S_3^{-\eta_a,-\eta_b*}$	$S^{\eta_a,-\eta_b}$	$S^{-\eta_a,\eta_b*}$
B_{22}	$T_3^{\eta_a,\eta_b}$	$T_2^{\eta_a,\eta_b*}$	$T_1^{\eta_a,-\eta_b}$	$T_4^{\eta_a,-\eta_b*}$	$T_4^{-\eta_a,\eta_b}$	$T_1^{-\eta_a,\eta_b*}$	$T_2^{-\eta_a,-\eta_b}$	$T_3^{-\eta_a,-\eta_b*}$	$T^{\eta_a,-\eta_b}$	$T^{-\eta_a,\eta_b*}$
C_{22}	$V_3^{\eta_a,\eta_b}$	$V_2^{\eta_a,\eta_b*}$	$V_1^{\eta_a,-\eta_b}$	$V_4^{\eta_a,-\eta_b*}$	$V_4^{-\eta_a,\eta_b}$	$V_1^{-\eta_a,\eta_b*}$	$V_2^{-\eta_a,-\eta_b}$	$V_3^{-\eta_a,-\eta_b*}$	$V^{\eta_a,-\eta_b}$	$V^{-\eta_a,\eta_b*}$

Table 2: List of the constant factors appearing in the amplitude (45) for all $(ij)(ab)$ neutrino pair production processes according to their labelling defined in Sect.5.

	ai1	ai2	ai3	ai4	ai5	ai6	ai7	ai8	Mai1	Mai2
N_1	η_a	η_a	$\eta_a\lambda_b^*$	$\eta_a\lambda_a^*$	$\eta_a\lambda_a$	$\eta_a\lambda_b$	η_a	η_a	η_a	η_a
A_{11}	$S_4^{\eta_b,\eta_a *}$	$S_1^{\eta_b,\eta_a}$	$S_3^{-\eta_b,\eta_a *}$	$S_2^{-\eta_b,\eta_a}$	$S_2^{\eta_b,-\eta_a *}$	$S_3^{\eta_b,-\eta_a}$	$S_1^{-\eta_b,-\eta_a *}$	$S_4^{-\eta_b,-\eta_a}$	$S^{-\eta_b,-\eta_a *}$	S^{η_b,η_a}
B_{11}	$T_4^{\eta_b,\eta_a *}$	$T_1^{\eta_b,\eta_a}$	$T_3^{-\eta_b,\eta_a *}$	$T_2^{-\eta_b,\eta_a}$	$T_2^{\eta_b,-\eta_a *}$	$T_3^{\eta_b,-\eta_a}$	$T_1^{-\eta_b,-\eta_a *}$	$T_4^{-\eta_b,-\eta_a}$	$T^{-\eta_b,-\eta_a *}$	T^{η_b,η_a}
C_{11}	$V_4^{\eta_b,\eta_a *}$	$V_1^{\eta_b,\eta_a}$	$V_3^{-\eta_b,\eta_a *}$	$V_2^{-\eta_b,\eta_a}$	$V_2^{\eta_b,-\eta_a *}$	$V_3^{\eta_b,-\eta_a}$	$V_1^{-\eta_b,-\eta_a *}$	$V_4^{-\eta_b,-\eta_a}$	$V^{-\eta_b,-\eta_a *}$	V^{η_b,η_a}
D_1	1	$\lambda_a^*\lambda_b$	1	1	1	1	$\lambda_a\lambda_b^*$	1	1	$\lambda_a^*\lambda_b$
A_{12}	$S_1^{\eta_a,\eta_b *}$	$S_4^{\eta_a,\eta_b}$	$S_3^{\eta_a,-\eta_b *}$	$S_2^{\eta_a,-\eta_b}$	$S_2^{-\eta_a,\eta_b *}$	$S_3^{-\eta_a,\eta_b}$	$S_4^{-\eta_a,-\eta_b *}$	$S_1^{-\eta_a,-\eta_b}$	$S^{\eta_a,\eta_b *}$	$S^{-\eta_a,-\eta_b}$
B_{12}	$T_1^{\eta_a,\eta_b *}$	$T_4^{\eta_a,\eta_b}$	$T_3^{\eta_a,-\eta_b *}$	$T_2^{\eta_a,-\eta_b}$	$T_2^{-\eta_a,\eta_b *}$	$T_3^{-\eta_a,\eta_b}$	$T_4^{-\eta_a,-\eta_b *}$	$T_1^{-\eta_a,-\eta_b}$	$T^{\eta_a,\eta_b *}$	$T^{-\eta_a,-\eta_b}$
C_{12}	$V_1^{\eta_a,\eta_b *}$	$V_4^{\eta_a,\eta_b}$	$V_3^{\eta_a,-\eta_b *}$	$V_2^{\eta_a,-\eta_b}$	$V_2^{-\eta_a,\eta_b *}$	$V_3^{-\eta_a,\eta_b}$	$V_4^{-\eta_a,-\eta_b *}$	$V_1^{-\eta_a,-\eta_b}$	$V^{\eta_a,\eta_b *}$	$V^{-\eta_a,-\eta_b}$
N_2	η_b	η_b	$\eta_b\lambda_a^*$	$\eta_b\lambda_b^*$	$\eta_b\lambda_b$	$\eta_b\lambda_a$	η_b	η_b	η_b	η_b
A_{21}	$S_1^{\eta_b,\eta_a}$	$S_4^{\eta_b,\eta_a *}$	$S_2^{-\eta_b,\eta_a}$	$S_3^{-\eta_b,\eta_a *}$	$S_3^{\eta_b,-\eta_a}$	$S_2^{\eta_b,-\eta_a *}$	$S_4^{-\eta_b,-\eta_a}$	$S_1^{-\eta_b,-\eta_a *}$	S^{η_b,η_a}	$S^{-\eta_b,-\eta_a *}$
B_{21}	$T_1^{\eta_b,\eta_a}$	$T_4^{\eta_b,\eta_a *}$	$T_2^{-\eta_b,\eta_a}$	$T_3^{-\eta_b,\eta_a *}$	$T_3^{\eta_b,-\eta_a}$	$T_2^{\eta_b,-\eta_a *}$	$T_4^{-\eta_b,-\eta_a}$	$T_1^{-\eta_b,-\eta_a *}$	T^{η_b,η_a}	$T^{-\eta_b,-\eta_a *}$
C_{21}	$V_1^{\eta_b,\eta_a}$	$V_4^{\eta_b,\eta_a *}$	$V_2^{-\eta_b,\eta_a}$	$V_3^{-\eta_b,\eta_a *}$	$V_3^{\eta_b,-\eta_a}$	$V_2^{\eta_b,-\eta_a *}$	$V_4^{-\eta_b,-\eta_a}$	$V_1^{-\eta_b,-\eta_a *}$	V^{η_b,η_a}	$V^{-\eta_b,-\eta_a *}$
D_2	$\lambda_a^*\lambda_b$	1	1	1	1	1	1	$\lambda_a\lambda_b^*$	$\lambda_a^*\lambda_b$	1
A_{22}	$S_4^{\eta_a,\eta_b}$	$S_1^{\eta_a,\eta_b *}$	$S_2^{\eta_a,-\eta_b}$	$S_3^{\eta_a,-\eta_b *}$	$S_3^{-\eta_a,\eta_b}$	$S_2^{-\eta_a,\eta_b *}$	$S_1^{-\eta_a,-\eta_b}$	$S_4^{-\eta_a,-\eta_b *}$	$S^{-\eta_a,-\eta_b}$	S^{η_a,η_b}
B_{22}	$T_4^{\eta_a,\eta_b}$	$T_1^{\eta_a,\eta_b *}$	$T_2^{\eta_a,-\eta_b}$	$T_3^{\eta_a,-\eta_b *}$	$T_3^{-\eta_a,\eta_b}$	$T_2^{-\eta_a,\eta_b *}$	$T_1^{-\eta_a,-\eta_b}$	$T_4^{-\eta_a,-\eta_b *}$	$T^{-\eta_a,-\eta_b}$	T^{η_a,η_b}
C_{22}	$V_4^{\eta_a,\eta_b}$	$V_1^{\eta_a,\eta_b *}$	$V_2^{\eta_a,-\eta_b}$	$V_3^{\eta_a,-\eta_b *}$	$V_3^{-\eta_a,\eta_b}$	$V_2^{-\eta_a,\eta_b *}$	$V_1^{-\eta_a,-\eta_b}$	$V_4^{-\eta_a,-\eta_b *}$	$V^{-\eta_a,-\eta_b}$	V^{η_a,η_b}

Table 3: List of the constant factors appearing in the amplitude (46) for all $(ai)(bj)$ neutrino scattering processes according to their labelling defined in Sect.6.1.

	aj1	aj2	aj3	aj4	aj5	aj6	aj7	aj8	Maj1	Maj2
N_1	η_b	η_b	$\eta_b\lambda_a^*$	$\eta_b\lambda_b^*$	$\eta_b\lambda_b$	$\eta_b\lambda_a$	η_b	η_b	η_b	η_b
A_{11}	$S_1^{\eta_b,\eta_a}$	$S_4^{\eta_b,\eta_a *}$	$S_2^{-\eta_b,\eta_a}$	$S_3^{-\eta_b,\eta_a *}$	$S_3^{\eta_b,-\eta_a}$	$S_2^{\eta_b,-\eta_a *}$	$S_4^{-\eta_b,-\eta_a}$	$S_1^{-\eta_b,-\eta_a *}$	S^{η_b,η_a}	$S^{-\eta_b,-\eta_a *}$
B_{11}	$T_1^{\eta_b,\eta_a}$	$T_4^{\eta_b,\eta_a *}$	$T_2^{-\eta_b,\eta_a}$	$T_3^{-\eta_b,\eta_a *}$	$T_3^{\eta_b,-\eta_a}$	$T_2^{\eta_b,-\eta_a *}$	$T_4^{-\eta_b,-\eta_a}$	$T_1^{-\eta_b,-\eta_a *}$	T^{η_b,η_a}	$T^{-\eta_b,-\eta_a *}$
C_{11}	$V_1^{\eta_b,\eta_a}$	$V_4^{\eta_b,\eta_a *}$	$V_2^{-\eta_b,\eta_a}$	$V_3^{-\eta_b,\eta_a *}$	$V_3^{\eta_b,-\eta_a}$	$V_2^{\eta_b,-\eta_a *}$	$V_4^{-\eta_b,-\eta_a}$	$V_1^{-\eta_b,-\eta_a *}$	V^{η_b,η_a}	$V^{-\eta_b,-\eta_a *}$
D_1	$\lambda_a^*\lambda_b$	1	1	1	1	1	1	$\lambda_a\lambda_b^*$	$\lambda_a^*\lambda_b$	1
A_{12}	$S_4^{\eta_a,\eta_b}$	$S_1^{\eta_a,\eta_b *}$	$S_2^{\eta_a,-\eta_b}$	$S_3^{\eta_a,-\eta_b *}$	$S_3^{-\eta_a,\eta_b}$	$S_2^{-\eta_a,\eta_b *}$	$S_1^{-\eta_a,-\eta_b}$	$S_4^{-\eta_a,-\eta_b *}$	$S^{-\eta_a,-\eta_b}$	$S^{\eta_a,\eta_b *}$
B_{12}	$T_4^{\eta_a,\eta_b}$	$T_1^{\eta_a,\eta_b *}$	$T_2^{\eta_a,-\eta_b}$	$T_3^{\eta_a,-\eta_b *}$	$T_3^{-\eta_a,\eta_b}$	$T_2^{-\eta_a,\eta_b *}$	$T_1^{-\eta_a,-\eta_b}$	$T_4^{-\eta_a,-\eta_b *}$	$T^{-\eta_a,-\eta_b}$	$T^{\eta_a,\eta_b *}$
C_{12}	$V_4^{\eta_a,\eta_b}$	$V_1^{\eta_a,\eta_b *}$	$V_2^{\eta_a,-\eta_b}$	$V_3^{\eta_a,-\eta_b *}$	$V_3^{-\eta_a,\eta_b}$	$V_2^{-\eta_a,\eta_b *}$	$V_1^{-\eta_a,-\eta_b}$	$V_4^{-\eta_a,-\eta_b *}$	$V^{-\eta_a,-\eta_b}$	$V^{\eta_a,\eta_b *}$
N_2	η_a	η_a	$\eta_a\lambda_b^*$	$\eta_a\lambda_a^*$	$\eta_a\lambda_a$	$\eta_a\lambda_b$	η_a	η_a	η_a	η_a
A_{21}	$S_4^{\eta_b,\eta_a *}$	$S_1^{\eta_b,\eta_a}$	$S_3^{-\eta_b,\eta_a *}$	$S_2^{-\eta_b,\eta_a}$	$S_2^{\eta_b,-\eta_a *}$	$S_3^{\eta_b,-\eta_a}$	$S_1^{-\eta_b,-\eta_a *}$	$S_4^{-\eta_b,-\eta_a}$	$S^{-\eta_b,-\eta_a *}$	S^{η_b,η_a}
B_{21}	$T_4^{\eta_b,\eta_a *}$	$T_1^{\eta_b,\eta_a}$	$T_3^{-\eta_b,\eta_a *}$	$T_2^{-\eta_b,\eta_a}$	$T_2^{\eta_b,-\eta_a *}$	$T_3^{\eta_b,-\eta_a}$	$T_1^{-\eta_b,-\eta_a *}$	$T_4^{-\eta_b,-\eta_a}$	$T^{-\eta_b,-\eta_a *}$	T^{η_b,η_a}
C_{21}	$V_4^{\eta_b,\eta_a *}$	$V_1^{\eta_b,\eta_a}$	$V_3^{-\eta_b,\eta_a *}$	$V_2^{-\eta_b,\eta_a}$	$V_2^{\eta_b,-\eta_a *}$	$V_3^{\eta_b,-\eta_a}$	$V_1^{-\eta_b,-\eta_a *}$	$V_4^{-\eta_b,-\eta_a}$	$V^{-\eta_b,-\eta_a *}$	V^{η_b,η_a}
D_2	1	$\lambda_a^*\lambda_b$	1	1	1	1	$\lambda_a\lambda_b^*$	1	1	$\lambda_a^*\lambda_b$
A_{22}	$S_1^{\eta_a,\eta_b *}$	$S_4^{\eta_a,\eta_b}$	$S_3^{\eta_a,-\eta_b *}$	$S_2^{\eta_a,-\eta_b}$	$S_2^{-\eta_a,\eta_b *}$	$S_3^{-\eta_a,\eta_b}$	$S_4^{-\eta_a,-\eta_b *}$	$S_1^{-\eta_a,-\eta_b}$	$S^{\eta_a,\eta_b *}$	$S^{-\eta_a,-\eta_b}$
B_{22}	$T_1^{\eta_a,\eta_b *}$	$T_4^{\eta_a,\eta_b}$	$T_3^{\eta_a,-\eta_b *}$	$T_2^{\eta_a,-\eta_b}$	$T_2^{-\eta_a,\eta_b *}$	$T_3^{-\eta_a,\eta_b}$	$T_4^{-\eta_a,-\eta_b *}$	$T_1^{-\eta_a,-\eta_b}$	$T^{\eta_a,\eta_b *}$	$T^{-\eta_a,-\eta_b}$
C_{22}	$V_1^{\eta_a,\eta_b *}$	$V_4^{\eta_a,\eta_b}$	$V_3^{\eta_a,-\eta_b *}$	$V_2^{\eta_a,-\eta_b}$	$V_2^{-\eta_a,\eta_b *}$	$V_3^{-\eta_a,\eta_b}$	$V_4^{-\eta_a,-\eta_b *}$	$V_1^{-\eta_a,-\eta_b}$	$V^{\eta_a,\eta_b *}$	$V^{-\eta_a,-\eta_b}$

Table 4: List of the constant factors appearing in the amplitude (47) for all $(aj)(bi)$ neutrino scattering processes according to their labelling defined in Sect.6.2.

	bi1	bi2	bi3	bi4	bi5	bi6	bi7	bi8	Mbi1	Mbi2
N_1	η_a	η_a	$\eta_a\lambda_a$	$\eta_a\lambda_b$	$\eta_a\lambda_b^*$	$\eta_a\lambda_a^*$	η_a	η_a	η_a	η_a
A_{11}	$S_1^{\eta_b,\eta_a*}$	$S_4^{\eta_b,\eta_a}$	$S_2^{-\eta_b,\eta_a*}$	$S_3^{-\eta_b,\eta_a}$	$S_3^{\eta_b,-\eta_a*}$	$S_2^{\eta_b,-\eta_a}$	$S_4^{-\eta_b,-\eta_a*}$	$S_1^{-\eta_b,-\eta_a}$	S^{η_b,η_a*}	$S^{-\eta_b,-\eta_a}$
B_{11}	$T_1^{\eta_b,\eta_a*}$	$T_4^{\eta_b,\eta_a}$	$T_2^{-\eta_b,\eta_a*}$	$T_3^{-\eta_b,\eta_a}$	$T_3^{\eta_b,-\eta_a*}$	$T_2^{\eta_b,-\eta_a}$	$T_4^{-\eta_b,-\eta_a*}$	$T_1^{-\eta_b,-\eta_a}$	T^{η_b,η_a*}	$T^{-\eta_b,-\eta_a}$
C_{11}	$V_1^{\eta_b,\eta_a*}$	$V_4^{\eta_b,\eta_a}$	$V_2^{-\eta_b,\eta_a*}$	$V_3^{-\eta_b,\eta_a}$	$V_3^{\eta_b,-\eta_a*}$	$V_2^{\eta_b,-\eta_a}$	$V_4^{-\eta_b,-\eta_a*}$	$V_1^{-\eta_b,-\eta_a}$	V^{η_b,η_a*}	$V^{-\eta_b,-\eta_a}$
D_1	$\lambda_a\lambda_b^*$	1	1	1	1	1	1	$\lambda_a^*\lambda_b$	$\lambda_a\lambda_b^*$	1
A_{12}	$S_4^{\eta_a,\eta_b*}$	$S_1^{\eta_a,\eta_b}$	$S_2^{\eta_a,-\eta_b*}$	$S_3^{\eta_a,-\eta_b}$	$S_3^{-\eta_a,\eta_b*}$	$S_2^{-\eta_a,\eta_b}$	$S_1^{-\eta_a,-\eta_b*}$	$S_4^{-\eta_a,-\eta_b}$	$S^{-\eta_a,-\eta_b*}$	S^{η_a,η_b}
B_{12}	$T_4^{\eta_a,\eta_b*}$	$T_1^{\eta_a,\eta_b}$	$T_2^{\eta_a,-\eta_b*}$	$T_3^{\eta_a,-\eta_b}$	$T_3^{-\eta_a,\eta_b*}$	$T_2^{-\eta_a,\eta_b}$	$T_1^{-\eta_a,-\eta_b*}$	$T_4^{-\eta_a,-\eta_b}$	$T^{-\eta_a,-\eta_b*}$	T^{η_a,η_b}
C_{12}	$V_4^{\eta_a,\eta_b*}$	$V_1^{\eta_a,\eta_b}$	$V_2^{\eta_a,-\eta_b*}$	$V_3^{\eta_a,-\eta_b}$	$V_3^{-\eta_a,\eta_b*}$	$V_2^{-\eta_a,\eta_b}$	$V_1^{-\eta_a,-\eta_b*}$	$V_4^{-\eta_a,-\eta_b}$	$V^{-\eta_a,-\eta_b*}$	V^{η_a,η_b}
N_2	η_b	η_b	$\eta_b\lambda_b$	$\eta_b\lambda_a$	$\eta_b\lambda_a^*$	$\eta_b\lambda_b^*$	η_b	η_b	η_b	η_b
A_{21}	$S_4^{\eta_b,\eta_a}$	$S_1^{\eta_b,\eta_a*}$	$S_3^{-\eta_b,\eta_a}$	$S_2^{-\eta_b,\eta_a*}$	$S_2^{\eta_b,-\eta_a}$	$S_3^{\eta_b,-\eta_a*}$	$S_1^{-\eta_b,-\eta_a}$	$S_4^{-\eta_b,-\eta_a*}$	$S^{-\eta_b,-\eta_a}$	S^{η_b,η_a*}
B_{21}	$T_4^{\eta_b,\eta_a}$	$T_1^{\eta_b,\eta_a*}$	$T_3^{-\eta_b,\eta_a}$	$T_2^{-\eta_b,\eta_a*}$	$T_2^{\eta_b,-\eta_a}$	$T_3^{\eta_b,-\eta_a*}$	$T_1^{-\eta_b,-\eta_a}$	$T_4^{-\eta_b,-\eta_a*}$	$T^{-\eta_b,-\eta_a}$	T^{η_b,η_a*}
C_{21}	$V_4^{\eta_b,\eta_a}$	$V_1^{\eta_b,\eta_a*}$	$V_3^{-\eta_b,\eta_a}$	$V_2^{-\eta_b,\eta_a*}$	$V_2^{\eta_b,-\eta_a}$	$V_3^{\eta_b,-\eta_a*}$	$V_1^{-\eta_b,-\eta_a}$	$V_4^{-\eta_b,-\eta_a*}$	$V^{-\eta_b,-\eta_a}$	V^{η_b,η_a*}
D_2	1	$\lambda_a\lambda_b^*$	1	1	1	1	$\lambda_a^*\lambda_b$	1	1	$\lambda_a\lambda_b^*$
A_{22}	$S_1^{\eta_a,\eta_b}$	$S_4^{\eta_a,\eta_b*}$	$S_3^{\eta_a,-\eta_b}$	$S_2^{\eta_a,-\eta_b*}$	$S_2^{-\eta_a,\eta_b}$	$S_3^{-\eta_a,\eta_b*}$	$S_4^{-\eta_a,-\eta_b}$	$S_1^{-\eta_a,-\eta_b*}$	S^{η_a,η_b}	$S^{-\eta_a,-\eta_b*}$
B_{22}	$T_1^{\eta_a,\eta_b}$	$T_4^{\eta_a,\eta_b*}$	$T_3^{\eta_a,-\eta_b}$	$T_2^{\eta_a,-\eta_b*}$	$T_2^{-\eta_a,\eta_b}$	$T_3^{-\eta_a,\eta_b*}$	$T_4^{-\eta_a,-\eta_b}$	$T_1^{-\eta_a,-\eta_b*}$	T^{η_a,η_b}	$T^{-\eta_a,-\eta_b*}$
C_{22}	$V_1^{\eta_a,\eta_b}$	$V_4^{\eta_a,\eta_b*}$	$V_3^{\eta_a,-\eta_b}$	$V_2^{\eta_a,-\eta_b*}$	$V_2^{-\eta_a,\eta_b}$	$V_3^{-\eta_a,\eta_b*}$	$V_4^{-\eta_a,-\eta_b}$	$V_1^{-\eta_a,-\eta_b*}$	V^{η_a,η_b}	$V^{-\eta_a,-\eta_b*}$

Table 5: List of the constant factors appearing in the amplitude (48) for all $(bi)(aj)$ neutrino scattering processes according to their labelling defined in Sect.6.3.

	bj1	bj2	bj3	bj4	bj5	bj6	bj7	bj8	Mbj1	Mbj2
N_1	η_b	η_b	$\eta_b\lambda_b$	$\eta_b\lambda_a$	$\eta_b\lambda_a^*$	$\eta_b\lambda_b^*$	η_b	η_b	η_b	η_b
A_{11}	$S_4^{\eta_b,\eta_a}$	$S_1^{\eta_b,\eta_a*}$	$S_3^{-\eta_b,\eta_a}$	$S_2^{-\eta_b,\eta_a*}$	$S_2^{\eta_b,-\eta_a}$	$S_3^{\eta_b,-\eta_a*}$	$S_1^{-\eta_b,-\eta_a}$	$S_4^{-\eta_b,-\eta_a*}$	$S^{-\eta_b,-\eta_a}$	S^{η_b,η_a*}
B_{11}	$T_4^{\eta_b,\eta_a}$	$T_1^{\eta_b,\eta_a*}$	$T_3^{-\eta_b,\eta_a}$	$T_2^{-\eta_b,\eta_a*}$	$T_2^{\eta_b,-\eta_a}$	$T_3^{\eta_b,-\eta_a*}$	$T_1^{-\eta_b,-\eta_a}$	$T_4^{-\eta_b,-\eta_a*}$	$T^{-\eta_b,-\eta_a}$	T^{η_b,η_a*}
C_{11}	$V_4^{\eta_b,\eta_a}$	$V_1^{\eta_b,\eta_a*}$	$V_3^{-\eta_b,\eta_a}$	$V_2^{-\eta_b,\eta_a*}$	$V_2^{\eta_b,-\eta_a}$	$V_3^{\eta_b,-\eta_a*}$	$V_1^{-\eta_b,-\eta_a}$	$V_4^{-\eta_b,-\eta_a*}$	$V^{-\eta_b,-\eta_a}$	V^{η_b,η_a*}
D_1	1	$\lambda_a\lambda_b^*$	1	1	1	1	$\lambda_a^*\lambda_b$	1	1	$\lambda_a\lambda_b^*$
A_{12}	$S_1^{\eta_a,\eta_b}$	$S_4^{\eta_a,\eta_b*}$	$S_3^{\eta_a,-\eta_b}$	$S_2^{\eta_a,-\eta_b*}$	$S_2^{-\eta_a,\eta_b}$	$S_3^{-\eta_a,\eta_b*}$	$S_4^{-\eta_a,-\eta_b}$	$S_1^{-\eta_a,-\eta_b*}$	S^{η_a,η_b}	$S^{-\eta_a,-\eta_b*}$
B_{12}	$T_1^{\eta_a,\eta_b}$	$T_4^{\eta_a,\eta_b*}$	$T_3^{\eta_a,-\eta_b}$	$T_2^{\eta_a,-\eta_b*}$	$T_2^{-\eta_a,\eta_b}$	$T_3^{-\eta_a,\eta_b*}$	$T_4^{-\eta_a,-\eta_b}$	$T_1^{-\eta_a,-\eta_b*}$	T^{η_a,η_b}	$T^{-\eta_a,-\eta_b*}$
C_{12}	$V_1^{\eta_a,\eta_b}$	$V_4^{\eta_a,\eta_b*}$	$V_3^{\eta_a,-\eta_b}$	$V_2^{\eta_a,-\eta_b*}$	$V_2^{-\eta_a,\eta_b}$	$V_3^{-\eta_a,\eta_b*}$	$V_4^{-\eta_a,-\eta_b}$	$V_1^{-\eta_a,-\eta_b*}$	V^{η_a,η_b}	$V^{-\eta_a,-\eta_b*}$
N_2	η_a	η_a	$\eta_a\lambda_a$	$\eta_a\lambda_b$	$\eta_a\lambda_b^*$	$\eta_a\lambda_a^*$	η_a	η_a	η_a	η_a
A_{21}	$S_1^{\eta_b,\eta_a*}$	$S_4^{\eta_b,\eta_a}$	$S_2^{-\eta_b,\eta_a*}$	$S_3^{-\eta_b,\eta_a}$	$S_3^{\eta_b,-\eta_a*}$	$S_2^{\eta_b,-\eta_a}$	$S_4^{-\eta_b,-\eta_a*}$	$S_1^{-\eta_b,-\eta_a}$	S^{η_b,η_a*}	$S^{-\eta_b,-\eta_a}$
B_{21}	$T_1^{\eta_b,\eta_a*}$	$T_4^{\eta_b,\eta_a}$	$T_2^{-\eta_b,\eta_a*}$	$T_3^{-\eta_b,\eta_a}$	$T_3^{\eta_b,-\eta_a*}$	$T_2^{\eta_b,-\eta_a}$	$T_4^{-\eta_b,-\eta_a*}$	$T_1^{-\eta_b,-\eta_a}$	T^{η_b,η_a*}	$T^{-\eta_b,-\eta_a}$
C_{21}	$V_1^{\eta_b,\eta_a*}$	$V_4^{\eta_b,\eta_a}$	$V_2^{-\eta_b,\eta_a*}$	$V_3^{-\eta_b,\eta_a}$	$V_3^{\eta_b,-\eta_a*}$	$V_2^{\eta_b,-\eta_a}$	$V_4^{-\eta_b,-\eta_a*}$	$V_1^{-\eta_b,-\eta_a}$	V^{η_b,η_a*}	$V^{-\eta_b,-\eta_a}$
D_2	$\lambda_a\lambda_b^*$	1	1	1	1	1	1	$\lambda_a^*\lambda_b$	$\lambda_a\lambda_b^*$	1
A_{22}	$S_4^{\eta_a,\eta_b*}$	$S_1^{\eta_a,\eta_b}$	$S_2^{\eta_a,-\eta_b*}$	$S_3^{\eta_a,-\eta_b}$	$S_3^{-\eta_a,\eta_b*}$	$S_2^{-\eta_a,\eta_b}$	$S_1^{-\eta_a,-\eta_b*}$	$S_4^{-\eta_a,-\eta_b}$	$S^{-\eta_a,-\eta_b*}$	S^{η_a,η_b}
B_{22}	$T_4^{\eta_a,\eta_b*}$	$T_1^{\eta_a,\eta_b}$	$T_2^{\eta_a,-\eta_b*}$	$T_3^{\eta_a,-\eta_b}$	$T_3^{-\eta_a,\eta_b*}$	$T_2^{-\eta_a,\eta_b}$	$T_1^{-\eta_a,-\eta_b*}$	$T_4^{-\eta_a,-\eta_b}$	$T^{-\eta_a,-\eta_b*}$	T^{η_a,η_b}
C_{22}	$V_4^{\eta_a,\eta_b*}$	$V_1^{\eta_a,\eta_b}$	$V_2^{\eta_a,-\eta_b*}$	$V_3^{\eta_a,-\eta_b}$	$V_3^{-\eta_a,\eta_b*}$	$V_2^{-\eta_a,\eta_b}$	$V_1^{-\eta_a,-\eta_b*}$	$V_4^{-\eta_a,-\eta_b}$	$V^{-\eta_a,-\eta_b*}$	V^{η_a,η_b}

Table 6: List of the constant factors appearing in the amplitude (49) for all $(bj)(ai)$ neutrino scattering processes according to their labelling defined in Sect.6.4.