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Influence of compressor degradation on optimal operation of a compressor station

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ABSTRACT

Normal practice in a compressor station with compressors in parallel is to allocate the mass flows equally. However, this strategy is not optimal if the compressors are not identical. A common reason why compressors become non-identical is because their performance degrades over time. Degradation increases the power necessary to run the compressor station and changes the optimal allocation of mass flows.

This paper presents a framework for optimal operation in a compressor station with degrading compressors. The optimisation framework proposed in this work explicitly includes a model of degradation in the optimisation problem and analyses how the optimal load-sharing changes when the compressors are degrading.

The optimisation framework was applied in an industrial case study of a compressor station in which three parallel compressors are subject to degradation. The case study confirms that it is possible to minimise the extra power consumption due to degradation by adjusting the operating conditions of the compressor station. The analysis also gives insights into the impact of degradation on the optimal solution when compressors work at their limits.

1. Introduction

A typical lifetime of a compressor is 30 to 50 years. During this time, the compressor is subject to varying operating conditions. Time and use of a compressor result in its *degradation*, defined as a 'detrimental change in physical condition' (BSI, 2017). Degradation then increases the power necessary to run the compressor station.

A compressor station is typically a part of a gas transport network and provides a boost for transporting the gas to the receivers (BSI, 2014). Improving the operation of a compressor station would thus improve the performance of the whole network. The primary objective of load-sharing is to allocate the loads to compressors in a compressor station to minimise the operating cost. The term *load* refers to the amount of gas processed by a compressor. The goal of this work is to model the influence of degradation on the power necessary to run the compressors and analyse how the optimal load-sharing changes when the compressors are degrading.

Several approaches have been used to share the load among compressors working in parallel. If the compressors are identical, optimal load sharing is obtained by assigning the same load to each unit, as indicated by Liptak (2005). However, if the compressors are dissimilar, the equal load approach is not optimal and should be adjusted to the nonlinear characteristics of the units, such as the compressor map and the compressor power curve.

Nonlinear characteristics of compressors were used by Carter (1996), Wu (1998), Wu et al. (2000), as well as Osiadacz (1980), Osiadacz and Bell (1981), and Jenicek and Kralik (1995) who took into account individual characteristics of each compressor when allocating the loads. A similar optimisation approach was employed by Zhang et al. (2014), Zuo et al. (2016), and Xenos et al. (2016b) who included the demand for the flow required from the compressors. A review of the approaches to the optimal operation of a compressor station was presented by Rios-Mercado and Borraz-Sanchez (2015). However, the existing approaches do not take degradation into account.

The main source of degradation of compressors is related to *fouling*. Fouling is caused by deposits forming on the surfaces inside the equipment. It affects the performance of a compressor and leads to increased costs of operation, as indicated by Syverud and Bakken (2006). A common way is to describe degra-

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dation simply as a function of time as suggested, among others, by Tarabrin et al. (1996). At the same time, Verheyleweghen and Jäschke (2017) suggested that the speed of the compressor and the composition of the gas are main factors contributing to the degradation. The composition of the gas as a source of degradation in turbomachinery was also indicated by Brekke et al. (2009), Syverud and Bakken (2006), and Syverud et al. (2005). As the composition of the gas might be unknown, the current paper analyses degradation as a function of the mass flow rate through the compressor.

The influence of degradation on the power consumption was simulated by Foss (2013). However, Foss did not analyse the relationships between the compressors, which is discussed in the current work. An optimisation problem with varying characteristics of a compressor was analysed by Paparella et al. (2013), Cortinovis et al. (2016), Milosavljevic et al. (2016), Milosavljevic et al. (2018), and Kumar and Cortinovis (2017). The variations of characteristics were caused by measuring errors or changes to the compressors, but the mechanisms underlying the changes were not analysed. The authors selected operating conditions that maximised the efficiency of a compressor with no need for exact degradation models.

The compressor characteristics relevant to this work relate power consumption at a given pressure ratio to the mass flow rate. This work assumes that the characteristics of the individual compressors change with degradation. The load-sharing problem is focused on minimisation of the power to run a compressor station in which compressors are subject to degradation. The degradation is explicitly included in the objective function to analyse the influence of degradation on the power necessary to run the system. It is also assumed that the dynamics of a compressor are much faster than the changes due to degradation. Industrial data presented in Zagorowska et al. (2020a) show that degradation of a compressor affects the system over multiple days or even months. Hence, a steady-state analysis has been used. A similar approach based on steady-state analysis has already been considered by Jäschke and Skogestad (2014) and Xenos (2015), and proved sufficient for applications related to operation of parallel systems.

The current work analyses the effects of the changes of the compressor characteristics due to degradation on the solution of the optimal load-sharing problem. The objectives of this paper are:

- To present a framework to minimise the increase in power consumption in a compressor station with degrading compressors,
- To analyse how various models of degradation affect the solution of the optimisation problem.

The rest of the paper is structured as follows. Section 2 introduces the load sharing problem for a compressor station with parallel compressors, and describes the characteristics of the compressors. Subsequently, Section 3 introduces degradation of a compressor and presents its effects on the compressor characteristics. The optimisation problem for a compressor station with degradation is then formulated in Section 4 and solved in Section 5. The effects of degradation on the solution and the relationships between the compressors are discussed in Section 6. Section 7 applies the optimisation to a realistic case study and presents recommendations regarding minimizing the increase in power consumption when compressors are degrading. Finally, the paper ends with a discussion and conclusions in Section 8.

2. Compressor station with multiple compressors

The load-sharing problem is analysed in a compressor station with three units, as shown in Fig. 1. A compressor can be described by a compressor map or by the power necessary to run the compressor station at the desired operating point on the compressor map.

It is assumed that all the compressors are connected to the same suction and discharge pipelines and thus work at the same suction and discharge pressures p_s and p_d. The controllers FC1, FC2, and FC3 ensure that the flows m_1, m_2, and m_3 are equal to their set-points. The calculation of these set-points taking compressor degradation into account is the objective of this work. M1, M2, M3 are electric motors, and SC1, SC2 and SC3 are speed controllers

2.1. Compressor map

A compressor is characterised by the relationships between pressure head, compressor speed, and mass flow rate. Compressor head captures the thermodynamics of a compression process and depends on the pressure ratio and the properties of the gas. The text book by Lüdtke (2013) gives details on the thermodynamics and provides formulas for calculation of compressor head.

Figure 2 shows the compressor map used in this work. The data for the compressor map were obtained from Nørstebø (2008) using the software from Rohatgi (2018). The thick horizontal line in Fig. 2 denotes the operating line for a constant value of head 119 kJ kg^{-1}. The circle denotes the minimal flow $m_i^{\min}$ and the asterisk indicates the maximal flow $m_i^{\max}$. The values of the minimal and maximal flow define the operating range for a given head and provide constraints for the optimisation problem in this paper.

It is assumed that a compressor map can be obtained from a manufacturer or derived from first principles. Moreover, it is assumed that the compressor map accurately describes the behaviour of a compressor. The validity of this assumption is discussed in Section 8.

2.2. Compressor power

If a compressor operates on the red line from Fig. 2, the power consumption W is a second order polynomial function of the mass flow m:

$$W = am^2 + bm + c \quad (1)$$

where a, b, and c are constant parameters obtained for a given head. A discussion on the parameters a, b, c will follow in Section 8.

Each of the compressors from Fig. 1 has the power curves given in Fig. 3. The black lines in Fig. 3 suggest that the power consumption is an increasing and convex function of the head and the flow for the whole operating range of a compressor.

3. Compressor degradation

The standard BSI (2017) distinguishes between the equipment in *up state*, when there is no degradation, and in *degraded state*, when the equipment is affected by degradation. The influence of degradation on a compressor station will be presented from the perspective of how the degradation influences the power necessary to run a compressor.

3.1. Influence of degradation on power

The power to run a compressor in degraded state can be written as:

$$W^{\mathrm{D}} = h_1(d)(am^2 + bm + c) + h_2(d) \quad (2)$$

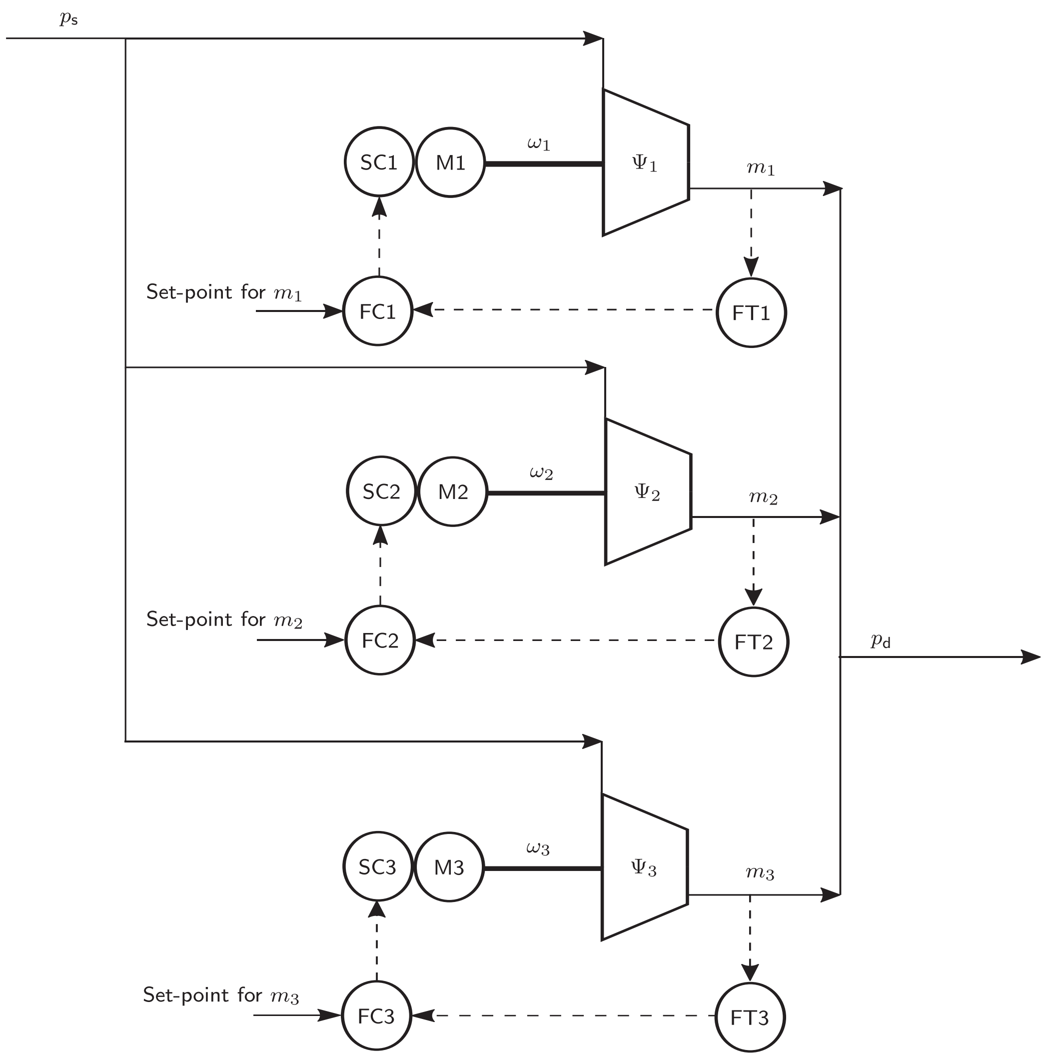


Fig. 1. Compressor station with three compressors connected to the same pipeline.

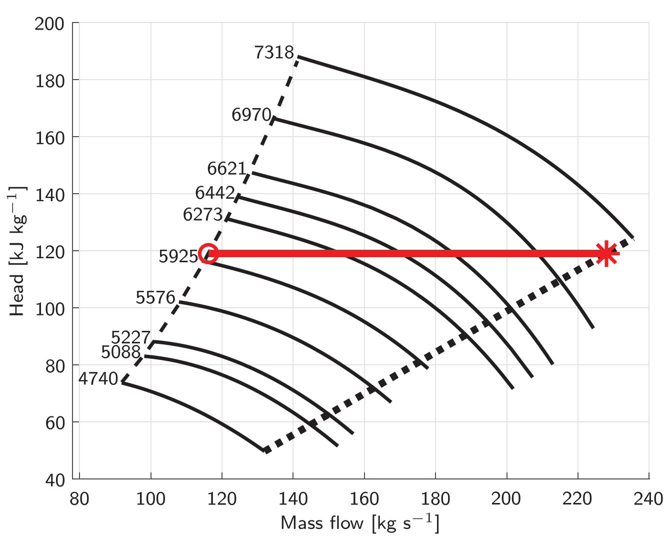


Fig. 2. Compressor head as a function of flow and speed. The speed curves (▬) are labelled with the corresponding speed in rpm. The red line (▬) shows the constant head $H = 119$ kJ kg^{-1} with the operating limits for the mass flow marked with a circle (**O**, lower limit m^{min}, on the surge line (▬ ▬)) and an asterisk (✱, upper limit m^{max}, on the choke line (...)). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

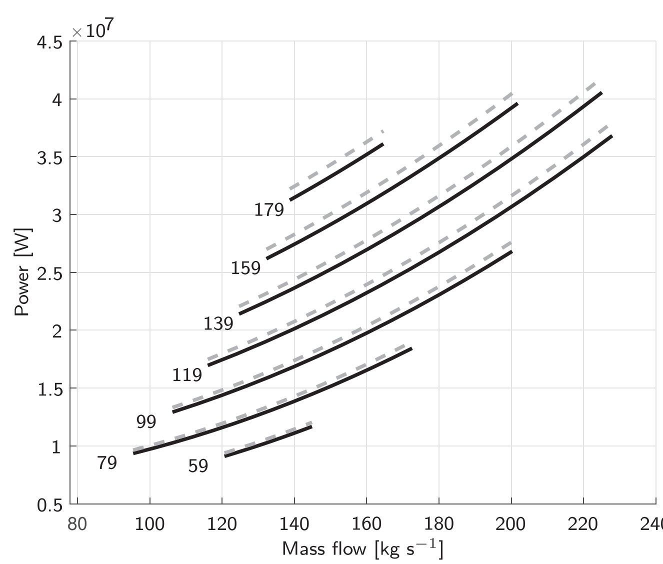


Fig. 3. Compressor power in up state (▬) and in degraded state (- -), as a function of the mass flow for various head in kJ kg^{-1}. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

where $h_1(d)$ and $h_2(d)$ represent a multiplicative and an additive degradation function, respectively. The coefficients *a*, *b*, and *c* describe the power consumption for the undegraded compressor. Additive degradation represents a bias, whereas multiplicative degradation shows a loss of effectiveness of a piece of equipment (Noura et al., 2009). As the degradation results in increased power to run the compressors, it is assumed that $h_1(d) > 1$ and $h_2(d) > 0$.

The degradation indicator *d* is typically obtained from a condition monitoring system. For instance, *d* may represent fouling, or it can correspond to an overall increase of power to run the compressor. The grey dotted lines in Fig. 3 show the power consumption of a compressor with degradation such that the degradation function $h_1(d) = 1.03$.

3.2. Factors influencing degradation

An *influencing factor* is an 'observable qualitative or measurable quantitative item that affects a system property' (BSI, 2016). The mathematical models of the degradation functions from Section 3.1 can be classified into two groups according to their dependence on influencing factors:

- Factor-based models of degradation, for degradation that depends on influencing factors

$$h_i(d) = h_i(d(t, \nu)) \tag{3}$$

- Factor-free models of degradation, for degradation that is independent of any influencing factors

$$h_i(d) = h_i(d(t)) \tag{4}$$

where $i \in \{1, 2\}$ indicates the kind of degradation from Eq. (2).

Examples of influencing factors ν include the way in which the system is operated, and external factors such as ambient temperature or humidity. Factor-free models of degradation typically depend only on time. Further details of the classification of degradation models can be found in the work by Zagorowska et al. (2020b).

Equation (2) enables the modelling of multiple types of degradation. For instance, additive degradation that depends linearly on the mass flow can be formulated using Eq. (3) taking $\nu = m$ as

$$h_2(d) = h_2(d(m)) = \alpha_0 m \tag{5}$$

where $\alpha_0 > 0$. Assuming $h_1(d) = 1$ and inserting Eq. (5) into Eq. (2) yields:

$$W^{\mathrm{D}} = am^2 + (b + \alpha_0)m + c \tag{6}$$

Further analysis and other types of degradation are considered in Section 6.

4. Optimal load-sharing

Osiadacz (1980) indicated that the problem of optimal load-sharing in a compressor station can be formulated as a nonlinear convex optimisation problem. A convex nonlinear problem has a unique solution, which can be obtained using Karush-Kuhn-Tucker conditions. This paper uses the formulation of the conditions provided by Nocedal, Wright, 1999.

The objective of this paper is to propose an optimisation framework to minimise the power necessary to run a compressor station with *N* compressors over a time horizon *T* (Wu et al., 2000). The goal of the optimisation is to find the flows m_{it}, $i \in \{1, \ldots, N\}$, $t \in \{1, \ldots, T\}$ which minimise the objective function:

$$\min_{m_{it}} \sum_{t=1}^{T} \sum_{i=1}^{N} W_{it}(m_{it}) \tag{7}$$

where W_{it} for the *i*-th compressor in period *t* is calculated according to Eq. (2). The coefficients of the objective function change from period *t* to $t+1$ because degradation develops over time and with use. The *t*-th period corresponds to one day of operation and the flows m_{it} are constant in each day. This is typical for a natural gas compressor station, as indicated by Xenos et al. (2016c).

The compressor station must satisfy the demand in each period $t = 1, 2, \ldots, T$:

$$\sum_{i=1}^{N} m_{it} = M_t \tag{8}$$

which provides an *equality constraint*. The load assigned to each compressor must be within the operating range defined by the compressor map in each period $t = 1, 2, \ldots, T$:

$$m_{it}^{\min} \le m_{it} \le m_{it}^{\max} \tag{9}$$

which provides *inequality constraints* for the optimisation problem. The values $m_{it}^{\min}$ and $m_{it}^{\max}$ in the inequality (9) are obtained for a fixed value of head from the compressor map bounded by the surge line m_i^{s}, minimal speed line m_i^{smin}, maximal speed line m_i^{smax} and choke line m_i^{c}:

$$m_i^{\min} = \max\{m_i^{\mathrm{s}}, m_i^{\mathrm{smin}}\} \tag{10a}$$

$$m_i^{\max} = \min\{m_i^{\mathrm{c}}, m_i^{\mathrm{smax}}\} \tag{10b}$$

The values of minimal and maximal flows for each compressor $m_{it}^{\min}$, $m_{it}^{\max}$, as well as the demand M_t in *t*-th period can change from period to period. Both the demand M_t and the minimal and maximal flows change due to operational requirements, in particular due to the amount of gas required from the station and the suction and discharge pressures.

The objective function (7) is a convex function of the flows, because the power to run each compressor is assumed convex. The demand constraint from Eq. (8) and the mass flow constraints from Eq. (9) are linear. The optimisation problem with the objective function given by (7) and the constraints (8) and (9) is therefore a convex quadratic optimisation problem. Thus, the Karush-Kuhn-Tucker conditions can be used to find the optimal solution.

The optimisation problem described with Eqs. (7)-(10) is a steady-state optimisation problem. Moreover, since compressor dynamics have been omitted from the problem in the current paper, the objective function from Eq. (7) is equivalent to solving *N* individual optimisation problems each with a time horizon of one day.

5. Solution of the optimisation problem

A nonlinear and convex optimisation problem such as described with Eq. (7)-(10) can be solved using a Lagrangian function. Following the formulation proposed by Nocedal, Wright, 1999, this paper applies a Lagrangian function to solve the problem with the objective function (7) and constraints (8) and (9). This section presents selected parts of the solution, whereas Appendix A presents the full solution.

5.1. Lagrangian function

The Lagrangian function for the problem with the objective function (7) and constraints (8) and (9) with $T > 1$ and *N* compressors is:

$$\mathcal{L} = \sum_{i=1}^{N} \left(\sum_{t=1}^{T} \left(\bar{a}_{it} m_{it}^2 + \bar{b}_{it} m_{it} + \bar{c}_{it} \right) \right) + \sum_{t=1}^{T} \lambda_t \left(\sum_{i=1}^{N} m_{it} - M_t \right) + \sum_{i=1}^{N} \left(\sum_{t=1}^{T} \mu_{it} \left(m_{it}^{\min} - m_{it} \right) \right) + \sum_{i=1}^{N} \left(\sum_{t=1}^{T} \gamma_{it} \left(m_{it} - m_{it}^{\max} \right) \right) \tag{11}$$

where γ_{it}, μ_{it}, λ_t are called *dual variables*. The values of the optimal flows and the dual variables are found by calculating derivatives with respect to the mass flows m_{it} of the Lagrangian function from Eq. (11).

The quantity W_i' is the gradient of the power W_{it} with respect to mass flow m_{it}:

$$W_{it}'(m_{it}) = \frac{\mathrm{d}W_{it}}{\mathrm{d}m_{it}} \tag{12}$$

The power to run a compressor is assumed to be approximated by a quadratic function of the flow

$$W_{it} = \bar{a}_{it} m_{it}^2 + \bar{b}_{it} m_{it} + \bar{c}_{it} \tag{13}$$

where $\bar{a}$, $\bar{b}$, and $\bar{c}$ capture the corresponding value of parameters in Eq. (1) and Eq. (2):

$$\bar{a}_{it} = a,\ \ \bar{b}_{it} = b,\ \ \bar{c}_{it} = c$$

For instance, if $h_1(d)$ and $h_2(d)$ are constant, the parameters $\bar{a}_{it}$, $\bar{b}_{it}$, and $\bar{c}_{it}$ will take the form:

$$\bar{a}_{it} = h_1(d)a,\ \ \bar{b}_{it} = h_1(d)b,\ \ \bar{c}_{it} = h_1(d)c + h_2(d)$$

The parameters $\bar{a}_{it}$, $\bar{b}_{it}$, and $\bar{c}_{it}$ are affected by the functional form of the degradation functions $h_1(d)$ and $h_2(d)$, and will be the basis for analysing the influence of degradation on the optimisation problem in Section 6.

The quadratic functional form of the power consumption from Eq. (13) indicates that:

$$W_{it}'(m_{it}) = 2\bar{a}_{it} m_{it} + \bar{b}_{it} \tag{14}$$

5.2. None of the compressors work at the limits

To solve the problem with the objective function (7) and constraints (8) and (9), it is first assumed that the inequality constraints are not active, i.e. none of the compressors work at their minimal or maximal limit. As none of the compressors are assumed at their limits, $m_{it}^* \neq m_{it}^{\min}$ and $m_{it}^* \neq m_{it}^{\max}$ which means that the corresponding dual variables are zero, $\mu_{it}^* = 0$ and $\gamma_{it}^* = 0$. Then:

$$m_{it}^* = \frac{-\lambda^* - \bar{b}_{it}}{2\bar{a}_{it}} \tag{15}$$

where

$$\lambda^* = -\left(\sum_{i=1}^{N} \frac{W_{it}'\left(\frac{2M_t}{N}\right)}{\bar{a}_{it}} \right) \Big/ \left(\sum_{i=1}^{N} \frac{1}{\bar{a}_{it}} \right) \tag{16}$$

5.3. Compressors work at the limits

If the solution obtained in Eq. (15) violates the inequality constraints (9) for k compressors, formula (16) is reformulated as:

$$\lambda^* = -\left(\sum_{i=k+1}^{N} \frac{W_{it}'\left(\frac{2\overline{M}_t}{N-k}\right)}{\bar{a}_{it}} \right) \Big/ \left(\sum_{i=k+1}^{N} \frac{1}{\bar{a}_{it}} \right) \tag{17}$$

where

$$\overline{M}_t = M_t - \sum_{i=1}^{k} m_{it} \tag{18}$$

with m_{it} taking either the maximal or minimal value, $m_{it} \in \{m_{it}^{\min}, m_{it}^{\max}\}$. The compressors that do not work at their limits are then assigned the flows according to Eq. (15) using Eq. (17) for λ^*.

The conditions for when the compressor k works at its limit are based on the dual variables μ_{it} and γ_{it} corresponding to inequality constraints (9) and are derived in Appendix A.

5.4. Comment on the solution

The expressions for the optimum mass flow rates derived in this section have an analytic form. This is because the power curves are quadratic functions of the mass flow rate. The derivation of the solution has made use of the gradient (Eq. (14)), but the final analytic form enables a one-step calculation of the optimum mass flows without the need to estimate the gradient. The optimum mass flow rates can be calculated directly from the total mass flow rate and the coefficients of the power curves.

6. Influence of degradation on optimisation

To analyse the influence of degradation on the optimal load sharing, the objective function from Eq. (7) is reformulated using Eq. (2):

$$\min_{m_{it}} \sum_{t=1}^{T} \sum_{i=1}^{N} (h_1(d) W_{it} + h_2(d)) \tag{19}$$

where W_{it} is calculated according to Eq. (1) and d is the values of degradation over the horizon T. The constraints regarding the minimum and maximum flow are assumed independent of the degradation and the compressor operates in the same range as in up state. Similarly, the demand constraint is independent of degradation.

The influence of degradation on the solution of the optimisation problem with the objective function (19) will depend on how the degradation functions $h_1(d)$ and $h_2(d)$ affect Eq. (2). In principle, they have a two-fold influence on the power consumption, because they can change the values of the parameters $\bar{a}_{it}$, $\bar{b}_{it}$, and $\bar{c}_{it}$ in Eq. (13), as well as modify the functional form. Their role will depend on whether degradation is factor-free of factor-based as well as on whether the compressors have identical characteristics.

6.1. Optimisation of identical compressors

Equation (15) confirms that equal load assignment is optimal if compressors are identical, as it yields:

$$m_{it}^* = \frac{M}{N} \tag{20}$$

In particular, this means that if identical compressors are affected in the same way by degradation, the equal load approach will remain optimal. In the rest of this paper, it is assumed that the compressors are identical if there is no degradation, but degradation affects each compressor to a different extent.

6.2. Factor-free degradation

Additive and multiplicative degradation functions $h_2(d)$ and $h_1(d)$ affect the power consumption in two different ways and therefore will have a different influence on the optimisation problem with the objective function from Eq. (7), and constraints from Eq. (8) and Eq. (9). A summary of how the optimal flow changes if the compressor is subject to degradation is presented in Table 1.

6.2.1. Additive factor-free degradation

Additive factor-free degradation is modelled with $h_1(d) = 1$ and $h_2(d)$ expressed as:

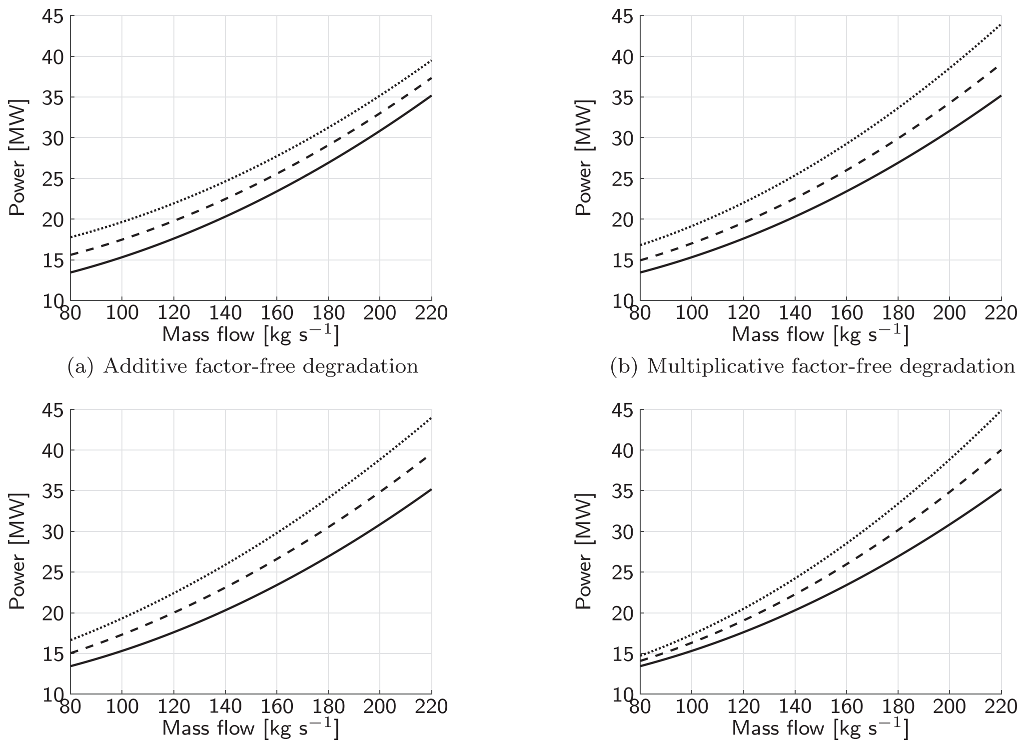
$$h_2(d) = \alpha \tag{21}$$

with $\alpha > 0$. This corresponds to a uniform degradation of i-th compressor across the whole operating range:

$$W_{it}^{\mathrm{D}} = W_{it} + \alpha \tag{22}$$

Table 1
The optimal value of the flow through the i-th compressor in period t if the compressor is subject to degradation and degradation does not affect the quadratic functional form of the power characteristics.

Type of degradation	$m^*_{it} = \frac{-\lambda^* - \bar{b}_{it}}{2\bar{a}_{it}}$ with λ^* from Eq. (16) or (17)
No degradation	$\bar{a}_{it} = a_{it},\ \bar{b}_{it} = b_{it}$
Additive independent of the flow	$\bar{a}_{it} = a_{it},\ \bar{b}_{it} = b_{it}$
Multiplicative independent of the flow	$\bar{a}_{it} = \beta a_{it},\ \bar{b}_{it} = \beta b_{it}$
Additive and linear in the flow	$\bar{a}_{it} = a_{it},\ \bar{b}_{it} = \alpha_0 + b_{it}$
Additive and quadratic in the flow	$\bar{a}_{it} = \alpha_1 + a_{it},\ \bar{b}_{it} = b_{it}$



(a) Additive factor-free degradation
(b) Multiplicative factor-free degradation
(c) Additive factor-based degradation - linear function of the flow
(d) Additive factor-based degradation - quadratic function of the flow

Fig. 4. Effect of medium (- -) and high (...) value of degradation on the power to run a compressor compared with the case without degradation (—) for four various types of degradation.

where W^{D}_{it} is the power in degraded state. Additive degradation is equivalent to shifting the objective function vertically, so α must be expressed in Watts. An example is shown in Fig. 4a. This type of degradation represents an increase of power consumption over the whole operating range. This model was used by Xenos (2015) for maintenance planning in a Norwegian compressor station.

The power consumption of a compressor is a quadratic function of the flow and additive degradation will change the value of parameter $\bar{c}_{it}$:

$$W^{D}_{it} = \bar{a}_{it} m^2_{it} + \bar{b}_{it} m_{it} + \bar{c}_{it} \tag{23}$$

where

$$\bar{c}_{it} = c_{it} + \alpha \tag{24}$$

The optimal solution given by Eq. (15) to the load-sharing problems is independent of the value of $\bar{c}_{it}$. Thus, additive factor-free degradation $h_2(d)$ will not change the solution of the load-sharing problem. This means that the solution from Eq. (15) obtained in Section 5 for a case without degradation remains valid if degradation affects the power in an additive way and is independent of the mass flow.

6.2.2. Multiplicative factor-free degradation

Factor-free multiplicative degradation has the form:

$$h_1(d) = \beta \tag{25}$$

with $h_2(d) = 0$. The parameter $\beta > 1$ corresponds to scaling the power to run the i-th compressor in up-state, W_{it} by a certain value $\beta > 1$ to obtain the power in degraded state, W^{D}_{it}:

$$W^{D}_{it} = \beta a_{it} m^2_{it} + \beta b_{it} m_{it} + \beta c_{it} \tag{26}$$

The multiplicative degradation function $h_1(d)$ is dimensionless.

The solution from Eqs. (16) and (15) changes if one the compressor is affected by the degradation function from Eq. (25). The loads assigned in the multiplicative case are different than the loads obtained in the additive case. The new values can be obtained from Eq. (16) or (17) and (15) by setting $\bar{a}_{it} = \beta a_{it}$ and $\bar{b}_{it} = \beta b_{it}$.

The reason for this difference is that the multiplicative degradation affects parameters $\bar{a}_{it}, \bar{b}_{it}$ of the power curve W_{it} from Eq. (13), and thus the gradient of the degraded power curve $(W^{D}_{it})'$ will change:

$$(W^{D}_{it})' = \beta W'_{it} \tag{27}$$

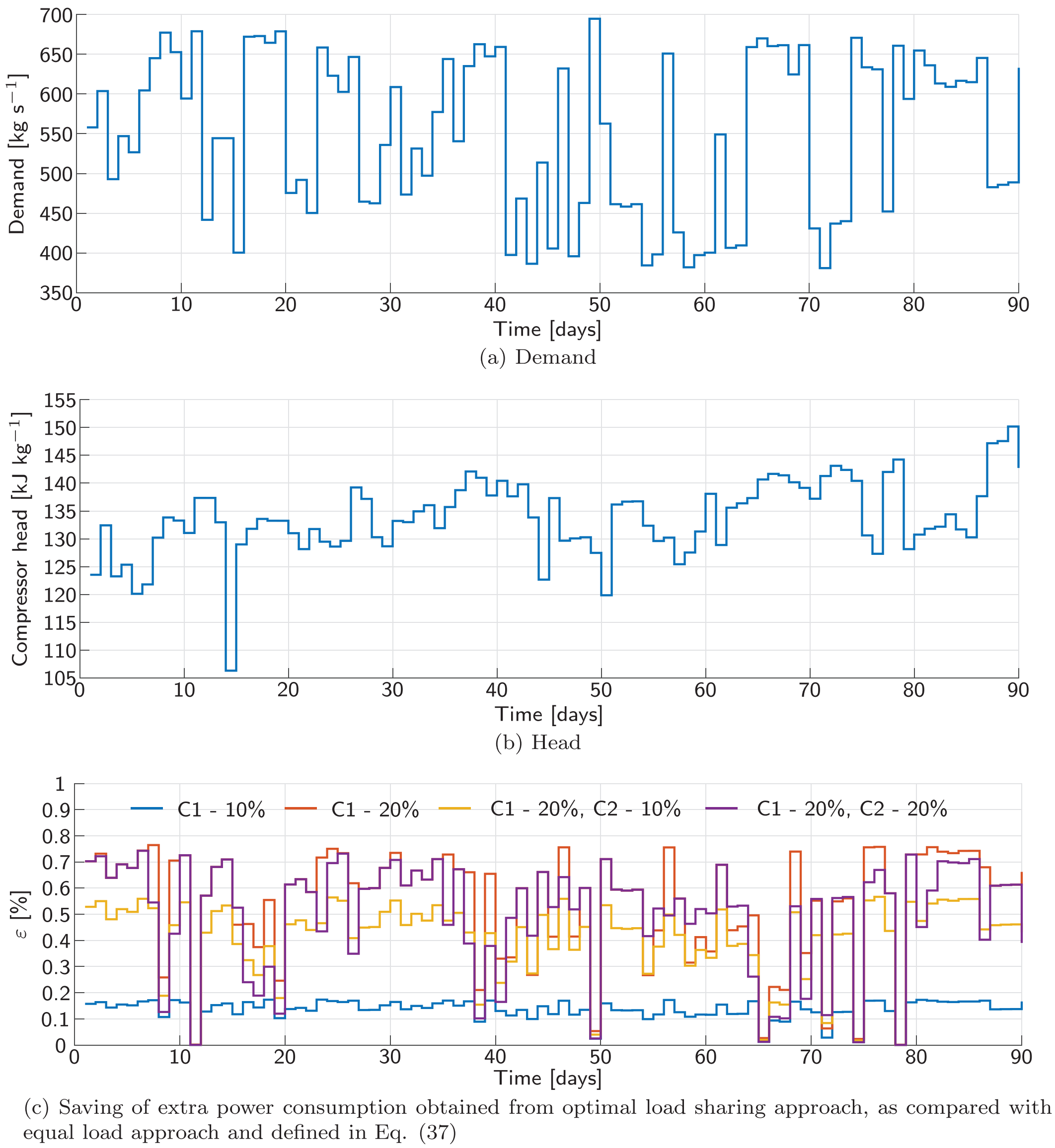


Fig. 5. Demand (Fig. 5a) for the whole station and compressor head (Fig. 5b) over the period of 90 days, obtained from Xenos (2015) using software from Rohatgi (2018), and the corresponding saving of extra power consumption (Fig. 5c). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

The gradient of the power consumption is used to calculate the values of the dual variables as shown in Eqs. (16) and (17), and in detail in Appendix A. An example is shown in Fig. 4b. However, multiplicative and factor-free degradation does not affect the quadratic character of the power from Eq. (1). Therefore, the formulas (16) and (15) obtained in Section 5 can be used to find the flows if multiplicative factor-free degradation affects the compressor.

6.2.3. Additive and multiplicative factor free degradation

Degradation might also have a two-fold influence on the power and include both the additive and multiplicative degradation functions:

$$W_{it}^{\mathrm{D}} = h_1(d)W_{it} + h_2(d) \quad (28)$$

A combination of the additive and multiplicative degradation can be seen as first scaling the power due to multiplicative degradation function $h_1(d)$, and then shifting it up the vertical axis as per the additive degradation $h_2(d)$. Shifting the power curves up does not influence the solution, because it does not affect the derivative W'. Thus, from the perspective of obtaining a solution, a compressor subject to both additive and multiplicative factor-free degradation can be treated as a compressor subject to multiplicative degradation.

6.3. Factor-based degradation

Additive factor-based degradation will shift the power curve up the vertical axis similarly to the additive factor-free degradation.

Multiplicative factor-based degradation changes the optimisation problem, regardless of the form of the degradation function $h_1(d)$.

6.3.1. Linear additive factor-based degradation

If the additive degradation function $h_2(d)$ is a linear function of the flow, then $h_1(d) = 1$ and:

$$h_2(d) = \alpha_0 m_1 \quad (29)$$

with $\alpha_0 > 0$. The parameter α_0 has dimensions of $m^2\ s^{-2}$. In this case, the power in degraded state becomes:

$$W_{it}^{D} = a_{it}m_{it}^2 + (b_{it} + \alpha_0)m_{it} + c_{it} \tag{30}$$

with derivative with respect to the mass flow m_{it}:

$$(W_{it}^{D})' = 2a_{it}m_{it} + \alpha_0 + b_{it} \tag{31}$$

Thus the solution in Eqs. (16) and (15) will be influenced, because the coefficient of m_{it} has changed. The new solution can be obtained from Eq. (16) or (17) and (15) by setting $\bar{b}_{it} = \alpha_0 + b_{it}$. An example is shown in Fig. 4c.

However, additive degradation that is linear in the flow does not affect the quadratic character of the power from Eq. (1). Therefore, the formulas Eq. (16) or (17) and (15) obtained in Section 5 can be used to find the flows if additive linear in the flow degradation affects the compressor.

6.3.2. Quadratic additive factor-based degradation

If additive degradation is a quadratic function of the flow, then $h_1(d) = 1$ and:

$$h_2(d) = \alpha_1 m_{it}^2 \tag{32}$$

with $\alpha_1 > 0$ in $m^2\ kg^{-1}\ s^{-1}$. In this case, the power in the degraded state becomes:

$$W_{it}^{D} = (a_{it} + \alpha_1)m_{it}^2 + b_{it}m_{it} + c_{it} \tag{33}$$

with derivative with respect to the mass flow m_{it}:

$$(W_{it}^{D})' = 2(a_{it} + \alpha_1)m_{it} + b_{it} \tag{34}$$

Thus the solution in Eqs. (16) and (15) will be influenced, because the coefficient of m_{it} changes. An example is shown in Fig. 4d.

However, additive degradation that is quadratic in the flow does not affect the quadratic character of the power from Eq. (1). Therefore, the formulas obtained in Section 5 can be used to find the flows if additive quadratic in the flow degradation affects the compressor by substituting (W_{it})' in Eq. (16) by Eq. (34). The new values can be obtained from Eq. (16) or (17) and (15) by setting $\bar{a}_{it} = a_{it} + \alpha_1$ and $\bar{b}_{it} = b_{it}$.

6.3.3. Multiplicative factor-based degradation

Similarly to Section 6.3.1, if the influencing factors do not depend on the flow, multiplicative factor-based degradation is equivalent to multiplicative factor-free degradation. A degradation function $h_1(d)$ of any other form that depends on the flow would necessitate a new solution to the optimisation problem. For instance, if the degradation of the i-th compressor on day t is a function of the flow up to that day, $h_1(d) = f(m_{i1}, \ldots, m_{it})$, the problem from Eq. (7) would not be quadratic any more. The Lagrangian function from Eq. (11) would change and a new solution would have to be calculated. For the future, it would be useful to have more industrial data to examine the relationship between degradation and mass flow.

A particular type of multiplicative factor based degradation is given by:

$$h_1(d, m) = \frac{f_1(m, d)}{W(m)} \tag{35}$$

where $f_1(m, d)$ can be a function of both mass flow and degradation, including other influencing factors. The formulation from Eq. (35) makes it possible to take into account degradation that modifies the functional form of the power.

The multiplicative degradation from Eq. (35) might also introduce a case when the scaling of the power curve is not uniform with respect to the parameters *a, b, c*. Setting

$$h_1(d) = \frac{\bar{a}_{it}m_1^2 + \bar{b}_{it}m_1 + \bar{c}_{it}}{W(m)} \tag{36}$$

where $\bar{a}_{it} = k^a a,\ \bar{b}_{it} = k^b b,\ \bar{c}_{it} = k^c c,\ k^a \neq k^b \neq k^c$ rewrites the objective function from Eq. (2), but preserves the properties of a quadratic optimisation.

In particular, the formulation from Eq. (36) corresponds to a power curve which can be obtained in each period separately. Cortinovis et al. (2016), Milosavljevic et al. (2016), and Milosavljevic et al. (2020) implicitly used this type of degradation functions to optimise the operation of a compressor station with compressors in parallel.

6.3.4. Other types of degradation

Degradation that is a function of the flow such as a polynomial of order higher than two, or a non-polynomial function, changes the optimisation problem. In particular, the minimised function from Eq. (7) might be no longer quadratic. If the objective function from Eq. (7) is not quadratic any more, it is still possible to use the KKT conditions to solve the problem. However, the new objective function might be non-convex and the solution might be non-unique. In particular, this means that there are several assignments of the flows to the compressors and the power consumption in all these assignments would be identical. An investigation of these matters is recommended for future work.

Furthermore, if a model of degradation is entirely unavailable, other approaches, such as modifier adaptation proposed by Milosavljevic et al. (2020), can be used.

6.4. Synopsis of the influence of degradation

The solutions obtained in Section 5 indicate that the values of the flows depend on the parameters $\bar{a}_{it}$ and $\bar{b}_{it}$ of the power approximation from Eq. (13). At the same time, the solutions are independent of parameter $\bar{c}_{it}$ which indicates that the solution remains the same if the value of $\bar{c}_{it}$ changes. This is reflected in Fig. 4a where both medium and high value of degradation shift the power curve, but without distorting its shape. The solutions are summarised in Table 1.

Degradation will affect the solution if it changes the parameters $\bar{a}_{it}$ and $\bar{b}_{it}$. The equal load approach will not be optimal any more, because the compressors will not have the same parameters of the power characteristics and the flows obtained from the formulas from Table 1 will depend on individual characteristics. If none of the compressors is at its limit, the compressors with more significant degradation are allocated lower flows to avoid an increase of power. Conversely, undegraded compressors will get a larger value of the flow to satisfy the overall demand, because their power consumption is not affected by degradation.

The constraints regarding the minimum and maximum flow will also affect the solutions. Therefore it is necessary to consider whether degradation can force the compressors to work at their limits. As presented in Fig. 4c-4b, degradation can also change the steepness of the power curves, as measured by the gradient of power with respect to the flow through a compressor. A change of gradient can force a compressor to work on the limits, as indicated in Section 5. A detailed analysis of the effects of gradients on when the compressors work at the limits is done in Appendix A.

The optimisation assigns the flows in such a way that minimises the overall power consumption. This means that the extra power consumption due to degradation will not be more than the extra power consumption if loads are allocated equally. The actual extra power will depend on the degradation of the compressors, as well as on whether the compressors work at their limits.

The formulas obtained in Section 5 can be used to find a solution to a problem with degrading compressors, if degradation does not change the quadratic functional form of the power curves. If the power curves are not quadratic, the problem from Section 4 has to be solved again.

Table 2
Cases considered for analysis of the influence of degradation on the objective function and the solution.

	Independent of the flow		Dependent on the flow			
	Additive	Multiplicative	Additive and linear in the flow	Additive and quadratic in the flow	Additive and neither quadratic, nor linear in the flow	Multiplicative
Affects the value of the objective function, and may or may not affect the solution	✓	✓	✓	✓	✓	✓
Affects the solution, but keeps the quadratic character of the optimisation problem		✓	✓	✓		
Affects the solution, changes the character of the optimisation problem					✓	✓

Table 2 summarizes the findings of this section classifying degradation with respect to its dependence on the flow.

7. Case study

The optimisation results were applied to a realistic case study with a compressor station with three compressors. The case study used real industrial compressor maps and demand requirements. This section demonstrates optimal load sharing for a compressor station with three units when the compressors are subject to degradation. To solve the optimisation problem with the objective function from Eq. (19) it is necessary to obtain the power curves for each head and degradation of each compressor. Each of the compressors has the characteristics from Fig. 2 and thus its power consumption is given by Fig. 3. Figure 5b shows the varying head. It is assumed that all three compressors work at the same head. Fig. 5a shows the demand, which is used to formulate the constraint from Eq. (8). The data for the demand and the head were adapted from Xenos (2015) using software from Rohatgi (2018). The current value of compressor degradation can be obtained using the approach proposed by Zagorowska et al. (2020a). The values of degradation used for the case study are described in Section 7.1.

The results from the optimisation are presented in relation to the equal load approach using the formula:

$$\varepsilon = 100\frac{W_{\mathrm{EL}} - W_{\mathrm{OL}}}{W_{\mathrm{EL}}} \tag{37}$$

where W_{EL} is the power in the equal load approach, and W_{OL} is the power consumption in the optimisation approach. The indicator ε measures the relative difference between the power consumption if equal load is used and if the optimal load sharing is applied. Therefore, it quantifies the ability of the optimal load sharing to mitigate the increase of power consumption due to degradation.

7.1. Degradation

It is typically assumed that degradation affects compressors in a multiplicative way and is independent of the flow. Examples can be found in works by Tarabrin et al. (1996), Li and Nilkitsaranont (2009), Tsoutsanis and Meskin (2017), and Zagorowska et al. (2020a) who presented degradation as a function of time. In particular, Tarabrin et al. (1996) and Zagorowska et al. (2020a) observed that degradation of compressors usually increases until it reaches a certain value and then remains constant. Meher-Homji and Bromley (2004) indicated that the maximal value of compressors degradation can reach up to 20% in unfavourable settings. This case study analyses the influence of multiplicative and factor-free degradation on the optimal load-sharing if degradation has already reached a threshold and remains constant. Four scenarios are considered:

- Compressor 1 has degradation of 10%, Compressors 2 and 3 are not degraded
- Compressor 1 has degradation of 20%, Compressors 2 and 3 are not degraded
- Compressors 1 and 2 have degradation of 20%, Compressor 3 is not degraded
- Compressor 1 has degradation of 20%, Compressor 2 has degradation of 10%, Compressor 3 is not degraded

7.2. Results - power consumption

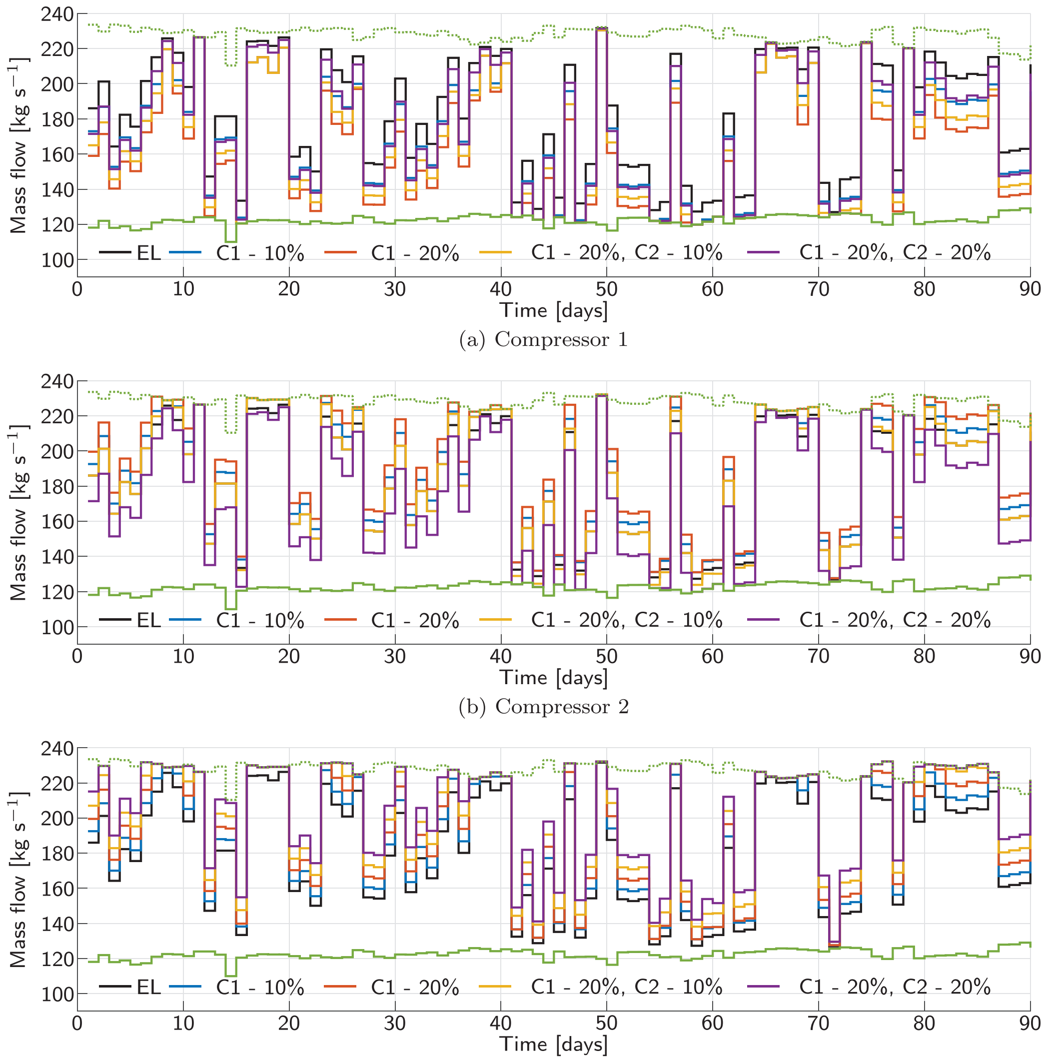
Figure 5c shows that the increase in power consumption can be reduced by up to 0.8% if the optimal load sharing is used instead of the equal load approach. This means that it is possible to minimise the extra power consumption of the compressors station by adjusting the operating conditions only, without the need to perform maintenance activities. As indicated by Xenos et al. (2016c), the power to run one compressor can be up to 40 MW, so a compressor station with three compressors can reach 120 MW. In this case, the optimal load sharing enables a saving of the extra power of approximately 1 MW. A compressor optimization study done by Xenos et al. (2016a) in an industrial air separation plant evaluated savings in power consumption in the region of 0.8% as commercially valuable and measurable.

7.2.1. Impact of the value of degradation

In particular, Fig. 5c shows that power saving of the optimal load allocation relative to equal load allocation increases if the degradation is large. For instance, the value of ε obtained if Compressor 1 has 20% degradation (orange) is larger than the value of ε obtained if Compressor 1 has only 10% degradation (blue). This indicates that by using the optimal load sharing, it is possible to mitigate the extra power consumption of significantly degraded compressors.

7.2.2. Impact of differences in degradation between compressors

Figure 5c indicates also that the differences between degraded compressors and the undegraded compressors are of importance. In particular, if Compressors 1 and 2 have both 20% degradation (purple) the value of ε are larger than if Compressor 2 has only 10% degradation (yellow). This is because the flow assigned to a compressor with 10% degradation is closer to the flow obtained in the equal load approach than the flow assigned to a compressors with 20% degradation. Therefore, the relative difference obtained for one compressor will be smaller for 10% degradation.



(a) Compressor 1

(b) Compressor 2

(c) Compressor 3

Fig. 6. Mass flows through Compressor 1 (C1), Compressor 2 (C2), Compressors 3 (C3) obtained from the optimal load-sharing approach, compared with the equal load approach (EL) and minimal (—) and maximal (...) values in each day. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

However, on same days, for example on day 19 or 26, the value of ε obtained if Compressors 1 and 2 both have 20% degradation (purple) is smaller than if Compressor 2 has only 10% degradation (yellow). This is explained by analysis of the flows assigned to the compressors (Fig. 6) in relation to the boundaries of the operating range for each compressor. Degradation of Compressor 2 means that Compressor 3 is assigned a larger load, which can lead to Compressor 3 hitting the upper limit. Therefore, Compressors 1 and 2 get a larger load than in the case without hitting the limit by Compressor 3. In consequence, a larger degradation of Compressor 2 leads to larger power consumption than if Compressor 2 has smaller degradation.

7.3. Results - flows through compressors

Figure 6 shows that degraded compressors are assigned a lower load if optimal load sharing is used than they would in the equal load approach. This is because degradation increases the power to run the degraded compressors (Fig. 4b), so it is better to assign lower loads to prevent a large increase of power. Conversely, Compressor 3 is assigned a larger value of the flow than in the equal load approach.

7.3.1. Impact of compressors working at limits

However, the assigned flows are bounded from above (dotted green) and from below (solid green). If a compressor works on the limit, the value of ε presented in Fig. 5c are small. This is visible in Fig. 5c in days 11, 49, 65, 74, and 78. In particular, ε is close to zero on days 11 and 78, because all the compressors work at their limits due to a high value of the demand on these days. A value of ε close to zero indicates that the power to run the compressors is close to the equal load assignment when a compressor is working at its limits. This suggests that the savings are the largest if the compressors are not working at their limits.

If the compressors are often run at their limits, this might prompt a decision to modify the structure of the compressor station. For instance, to increase the load on the compressor least prone to degradation, the compressor can be replaced by a machine with a larger throughput. Increasing the maximal flow $m^{\max}$ on the healthiest compressor would then make it possible to assign a larger load to the healthy compressor, and in consequence, a smaller load to the degraded compressor. Thus, the degraded compressor would move away from its upper limit, even if the demand is high.

8. Discussion and conclusions

Compressors are subject to degradation due to time, use and varying operating conditions, which results in an increase of power necessary to run a compressor station. The paper proposed an optimisation model for load-sharing in a compressor station taking degradation of compressors into account.

8.1. Discussion

The purpose of this paper has been to provide an integrated framework to analyse the influence of degradation on the optimal operation of a compressor station with compressors in parallel. This section discusses some potential practical issues.

8.1.1. Measurements needed

The optimisation framework in this work requires a model of degradation and knowledge about the power consumption of the undegraded compressors. The map of power consumption as a function of mass flow rate can be kept up to date based on historical data, as shown by Xenos et al. (2016a). Typically, the power curves will be recalibrated using operating measurements of power consumption and mass flow rate when the compressor is in good condition after major maintenance.

The novelty of the framework proposed in the paper consists in explicitly including degradation in the objective function. Therefore, in addition the power curves, it is necessary to estimate the current value of degradation. This value can be estimated using the approach proposed by Zagorowska et al. (2020a), if degradation is assumed to be factor-free and dependent solely on time. Conversely, if a model of degradation is not available, the power curves of a system subject to degradation can be updated online using the approach proposed by Milosavljevic et al. (2020).

8.1.2. Impact of measurement errors

If the power versus mass flow rate curves were to be calibrated using process measurements, then measurement errors could potentially affect the predicted power. Moreover, the measurement of degradation may also have errors, as discussed in Zagorowska et al. (2020a). The power consumption predicted by the optimizer might therefore be different from the actual power consumption measured from the process. In a practical trial of the proposed optimizer, it would be important when assessing the outcome of the trial to show that the calculated power savings are consistently greater than the error between the measured power and the predicted power.

The influence of measuring errors can be analysed by reformulating Eq. (13) as a linear function of the parameters *a*, *b*, and *c* for a given mass flow *m*. Equation (13) can be rewritten as:

$$W(V) = A^{\mathrm{T}} V \tag{38}$$

where $V = [a, b, c]^{\mathrm{T}}$ and $A^{\mathrm{T}} = [m^2, m, 1]$. After calibration, there could be an error $\Delta V = [\Delta a, \Delta b, \Delta c]^{\mathrm{T}}$ between the true and estimated values of the parameters *a*, *b*, and *c*, the mismatch $\Delta W(V)$ between the measured and the actual value, if there is an error in the *i*-th parameter, can be calculated as:

$$\Delta W(V) = W(V + \Delta V) - W(V) = A^{\mathrm{T}} \Delta V \tag{39}$$

In particular, if only one parameter is affected, the mismatch at a given points *m* can be obtained from Table (3). Conversely, if all the parameters are affected in the same way relatively to the original value, $\frac{\Delta a}{a} = \frac{\Delta b}{b} = \frac{\Delta c}{c} = q$, the error ΔV is given as $q[a, b, c]$. The differences arise from errors in measurement of power consumption and mass flow rate. The mismatch from Eq. (39) can be obtained as:

$$\Delta W = qW \tag{40}$$

Table 3
Error values.

	$a + \Delta a$	$b + \Delta b$	$c + \Delta c$
$W + \Delta W$	$W + \Delta a m^2$	$W + \Delta b m$	$W + \Delta c$

For instance, if each parameter *a, b*, and *c* has a possible error of $\pm$ 0.1%, i.e. $q = \pm 0.001$, the mismatch can be obtained as:

$$\Delta W = \pm 0.001 W \tag{41}$$

which is equivalent of a 0.1% mismatch in the power consumption.

8.1.3. Effect of degradation on the operating range of the compressor

The formulation in this paper assumed that the maximum and minimum mass flow rates through the compressors depend on the head, but do not depend on the severity of degradation. However, as indicated by Bakken et al. (2002), degradation can affect the operating range of a compressor. A change in the operating range would then affect the constraints in Eq. (10), and in turn change the solution. Bakken et al. (2002) show the surge line moves to the left. Hence the lower constraint lines in Fig. 6 will be lower than the undegraded case, and potentially there may be fewer instances when the compressors work at their limits. A detailed examination remains a topic for future work.

8.2. Conclusions

The optimisation model proposed in this work mitigates the loss of performance of the compressor station by explicitly including the degradation in the objective function and analysing its influence on the optimal solution.

The effects of degradation on the power consumption and the optimal load-sharing depend on the type of degradation. In particular, degradation due to fouling, which is a typical phenomenon in industrial turbomachinery, will change the optimal load-assignment.

The optimisation results were applied to a realistic case study with a compressor station with three compressors. The case study used real industrial compressor maps and demand requirements. The mitigation of extra power consumption is largest if the compressors work away from the limits. If the compressors are often run at their limits, this might prompt a decision to modify the structure of the compressor station, which makes the proposed framework useful as an operating strategy as well as a decision support tool.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

Marta Zagorowska: Conceptualization, Formal analysis, Methodology, Writing - original draft, Visualization. **Charlotte Skourup:** Supervision, Resources. **Nina F. Thornhill:** Conceptualization, Writing - review & editing, Supervision, Funding acquisition.

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Appendix A. Solution of the optimisation problem

The Lagrangian function for the problem with the objective function (7) and constraints (8) and (9) with $T > 1$ is:

$$\begin{aligned}\mathcal{L} = &\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\bar{a}_{it}m_{it}^2 + \bar{b}_{it}m_{it} + \bar{c}_{it}\right) + \sum_{t=1}^{T}\lambda_t\left(\sum_{i=1}^{N}m_{it} - M_t\right)\\ + &\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\mu_{it}\left(m_{it}^{\min} - m_{it}\right)\right) + \sum_{i=1}^{N}\left(\sum_{t=1}^{T}\gamma_{it}\left(m_{it} - m_{it}^{\max}\right)\right)\end{aligned} \tag{A.1}$$

where γ_{it}, μ_{it}, λ_t are called *dual variables* (Nocedal, Wright, 1999). The value W_i' is the gradient of the power W_{it} with respect to mass flow m_{it}:

$$W_{it}'(m_{it}) = \frac{\mathrm{d}W_{it}}{\mathrm{d}m_{it}} \tag{A.2}$$

The power to run a compressor in this work is assumed to be approximated by a quadratic function of the flow from Eq. (13), and then:

$$W_{it}'(m_{it}) = 2\bar{a}_{it}m_{it} + \bar{b}_{it} \tag{A.3}$$

The solution of Eq. (A.1) is considered from the perspective of how the degradation affects the power from Eq. (2). In particular, the relationship between the operating range, the demand, and the load-sharing in degraded state is considered.

Two cases are considered:

- None of the compressors works at their limits which gives $\mu_{it}^* = 0$ and $\gamma_{it}^* = 0$
- A number k of compressors work at their limits which gives $\mu_{jt}^* \neq 0$ and $\gamma_{jt}^* \neq 0$ for $j \in \{1, 2, \dots, k\}$

It is also assumed that the demand does not require all the compressors to work at their minimal and maximal limits. If $M_t = \sum_{i=1}^{N} m_{it}^{\min}$, there is no flexibility in assigning the loads m_{it}. Similarly, there is no flexibility if $M_t = \sum_{i=1}^{N} m_{it}^{\max}$, so these cases are not considered.

A1. None of the compressors works at the limits

As none of the compressors are assumed at their limits, $m_{it}^* \neq m_{it}^{\min}$ and $m_{it}^* \neq m_{it}^{\max}$ which means that the corresponding dual variables are zero, $\mu_{it}^* = 0$ and $\gamma_{it}^* = 0$. The KKT conditions for Eq. (A.1) become a system of $NT + 1$ equations with $NT + 1$ variables:

$$W_i'(m_{it}^*) + \lambda^* = 0 \tag{A.4a}$$

$$\sum_{i=1}^{N} m_{it}^* - M_t = 0 \tag{A.4b}$$

The system of Eq. (A.4) can be solved by expressing all m_{it}^* as functions of λ^* from Eq. (A.4a) and substituting them in (A.4b). Then:

$$\lambda^* = -\left(\sum_{i=1}^{N}\frac{W_i'\left(\frac{2M_t}{N}\right)}{a_{it}}\right)\Bigg/\left(\sum_{i=1}^{N}\frac{1}{a_{it}}\right) \tag{A.5}$$

and

$$m_{it}^* = \frac{-\lambda^* - b_{it}}{2a_{it}} \tag{A.6}$$

A2. Compressors work at the limits

If there is a number k of compressors working at their limits, the KKT conditions can be written:

$$W_{jt}'(m_{jt}) + \lambda - \mu_{jt} + \gamma_{jt} = 0, \text{ for } j \in \{1, 2, \dots, k-1\} \tag{A.7a}$$

$$W_{kt}'(m_{kt}) + \lambda - \mu_{kt} + \gamma_{kt} = 0 \tag{A.7b}$$

$$W_{vt}'(m_{vt}) + \lambda = 0, \text{ for } v \in \{k+1, \dots, V\} \tag{A.7c}$$

$$\sum_{i=1}^{V} m_{it} - M_t = 0 \tag{A.7d}$$

$$\mu_{it}(m_{it}^{\min} - m_{it}) = 0 \tag{A.7e}$$

$$\gamma_{it}(m_{it} - m_{it}^{\max}) = 0 \tag{A.7f}$$

where the index j indicates the compressors working at the limits, the index k indicates the compressor for which the conditions will be derived, and the index v indicates unrestrained compressors, i.e. compressors which work not at their limits. It is assumed for now that $k \leq N - 1$, so that there exist at least two compressors that are not at their limits. The case with one compressor not at limits is a special case and will be considered separately. The conditions when the compressor k works at its limit can be derived as follows. Since a compressor is able to work only at one limit at the time, the indicator y_μ shows which limit is active for which compressor:

$$y_{\mu_i} = \begin{cases} 1 \text{ if } m_{it}^* = m_{it}^{\min} \\ 0 \text{ if } m_{it}^* = m_{it}^{\max} \end{cases} \tag{A.8}$$

Introducing:

$$\xi_{it} = -y_{\mu_i}\mu_{it} + (1 - y_{\mu_i})\gamma_{it} \tag{A.9}$$

in Eq. (A.7b) yields:

$$W_j'(m_{jt}^*) + \lambda^* + \xi_{jt} = 0, \text{ for } j \in \{1, 2, \dots, k-1\} \tag{A.10a}$$

$$W_k'(m_{kt}^*) + \lambda^* + \xi_{kt} = 0 \tag{A.10b}$$

Equation (A.7d) can be rewritten as:

$$\sum_{j=1}^{N-k} m_{jt}^* + \sum_{v=1}^{N-k+1} m_{vt}^* - M_t = 0 \tag{A.11}$$

which is an equation with $N - k + 1$ unknown variables m_{vt}^*. Combining Eq. (A.11) with Eq. (A.7c) gives $N - k + 1$ equations with $N - k + 1$ variables that can be used to find the values of m_{vt}^* and λ^*. In particular, Eq. (A.7d) yields:

$$\lambda^* = -W_v'(m_{vt}^*) \tag{A.12}$$

Inserting Eq. (A.12) in (A.10b) and solving for ξ_{kt} yields:

$$\xi_{kt} = -W_k'(m_{kt}^*) + W_v'(m_{vt}^*) \tag{A.13}$$

Equation (A.13) gives:

$$\mu_{kt}^* = W_k'(m_{kt}^*) - W_v'(m_{vt}^*) > 0 \tag{A.14}$$

if the compressor k works at its minimal limit $m_{kt}^* = m_{kt}^{\min}$ and

$$\gamma_{kt}^* = -W_k'(m_{kt}) + W_v'(m_{vt}^*) > 0 \tag{A.15}$$

if the compressor k works at its maximal limit $m^*_{kt} = m^{\max}_{kt}$. Therefore, the compressor k will work at its boundary depending on the relationship between the gradients. For instance, for Eq. (A.14) to be a valid solution for μ^*_{kt}, the inequality must be fulfilled:

$$W'_k(m^*_{kt}) \geq W'_v(m^*_{vt}) \tag{A.16}$$

Therefore, the compressor k will work at its minimal bound if the gradient of its power curve at $m^{\min}_k$ is larger than the gradient of the power curve of an unrestrained compressor v at its value obtained from Eq. (A.11) with Eq. (A.7c).

From Eq. (A.7c), it is obtained that for any $v_1, v_2 \in \{k+1, \ldots, N\}$

$$W'_{v_1}(m^*_{v_1 t}) = W'_{v_2}(m^*_{v_2 t}) \tag{A.17}$$

Therefore, the condition from Eq. (A.16) holds for all $v \in \{k+1, \ldots, N\}$.

The case with one compressor not at limits is a special case for certain values of demand, depending on Eq. (A.11):

$$m^*_{Nt} = M_t - \sum_{j=1}^{N-1} m^*_{jt} \tag{A.18}$$

If $m^{\min}_{Nt} < m^*_{Nt} < m^{\max}_{Nt}$, then the corresponding μ^*_{Nt} and γ^*_{Nt} are zero and the problem can be solved to obtain Eq. (A.19) and (A.20):

$$\mu^*_{kt} = W'_{kt}(m^*_{kt}) - W'_{vt}(m^*_{vt}) > 0 \tag{A.19}$$

if the compressor k works at its minimal limit $m^*_{kt} = m^{\min}_{kt}$ and

$$\gamma^*_{kt} = -W'_{kt}(m_{kt}) + W'_{vt}(m^*_{vt}) > 0 \tag{A.20}$$

if the compressor k works at its maximal limit $m^*_{kt} = m^{\max}_{kt}$. If the compressor N is forced to work at its limits for M_t, then the corresponding parameter μ^*_{Nt} or γ^*_{Nt} will be different than zero. Nocedal, Wright, 1999 indicates that in that case there will be infinitely many values of λ^* that satisfy Eq. (A.7).

Equation (A.7) gives N equations with $N+1$ variables

$$W'_j(m^*_{jt}) + \lambda^* + \xi_{jt} = 0, \text{ for } j \in \{1, 2, \ldots, N\} \setminus \{k\} \tag{A.21a}$$

$$W'_k(m^*_{kt}) + \lambda^* + \xi_{kt} = 0 \tag{A.21b}$$

Equation (A.21) has infinitely many solutions of form:

$$\lambda^* = -W'_k(m^*_{kt}) - \xi_{kt} \tag{A.22}$$

and

$$\xi_{jt} = W'_k(m^*_{kt}) - W'_j(m^*_{jt}) + \xi_{kt} \tag{A.23}$$

where ξ_{kt} is treated as a parameter. Taking into account that $\mu^*_{jt} > 0$, $\gamma^*_{jt} > 0$, $\mu^*_{kt} > 0$, $\gamma^*_{kt} > 0$ yields the conditions for μ^*_{kt} and γ^*_{kt}:

$$\mu^*_{kt} \in \left(W'_k(m^*_{kt}) - W'_u(m^*_{ut}), W'_k(m^*_{kt}) - W'_w(m^*_{wt})\right) \tag{A.24}$$

and

$$\gamma^*_{kt} \in \left(W'_w(m^*_{wt}) - W'_k(m^*_{kt}), W'_u(m^*_{ut}) - W'_k(m^*_{kt})\right) \tag{A.25}$$

where index w denotes compressors working at their maximal limits and index u denotes compressors working at their minimal limits.

References

Bakken, L.E., Bjørge, T., Bradley, T.M., Smith, N., 2002. Validation of compressor transient behavior. In: Proceedings of the ASME Turbo Expo 2002: Power for Land, Sea, and Air. Volume 2: Turbo Expo 2002, Parts a and b. In: Turbo Expo: Power for Land, Sea, and Air, vol. 2, pp. 1111–1121. doi:10.1115/GT2002-30279 June 3-6, 2002, Amsterdam, The Netherlands.

Brekke, O., Bakken, L.E., Syverud, E., 2009. Compressor fouling in gas turbines offshore: composition and sources from site data. In: Proceedings of ASME Turbo Expo 2009: Power for Land, Sea and Air, June 08-12, 2009, Orlando, FL. ASME; Int Gas Turbine Inst, pp. 381–391. doi:10.1115/GT2009-59203.

BSI (2014). BS EN 12583:2014. Gas infrastructure. Compressor stations - functional requirements. Standard, British Standards Institution URL https://bsol.bsigroup.com/.

BSI (2016). BS EN 61069-1:2016. Industrial-process measurement, control and automation. Evaluation of system properties for the purpose of system assessment. Terminology and basic concepts. Standard, British Standards Institution.URL https://bsol.bsigroup.com/.

BSI (2017). BS EN 13306:2017. Maintenance - Maintenance terminology. Standard, British Standards Institution.URL https://bsol.bsigroup.com/.

Carter, R.G., 1996. Compressor station optimization: computational accuracy and speed. PSIG Annual Meeting. Pipeline Simulation Interest Group, Pipeline Simulation Interest Group. 23-25 October, San Francisco, California

Cortinovis, A., Mercangöz, M., Zovadelli, M., Pareschi, D., De Marco, A., Bittanti, S., 2016. Online performance tracking and load sharing optimization for parallel operation of gas compressors. Comput. Chem. Eng. 88, 145–156. doi:10.1016/j.compchemeng.2016.01.012.

Foss, T., 2013. Optimal parallellkjøring av eksportkompressorer. Norwegian University of Science and Technology (NTNU). (in Norwegian) Master's thesis..

Jäschke, J., Skogestad, S., 2014. Optimal operation of heat exchanger networks with stream split: only temperature measurements are required. Comput. Chem. Eng. 70, 35–49. doi:10.1016/j.compchemeng.2014.03.020.

Jenicek, T., Kralik, J., et al., 1995. Optimized control of generalized compressor station. PSIG Annual Meeting, 18-20 October, Albuquerque, New Mexico.

Kumar, S., Cortinovis, A., 2017. Load sharing optimization for parallel and serial compressor stations. In: 2017 IEEE Conference on Control Technology and Applications (CCTA). IEEE, pp. 499–504. doi:10.1109/CCTA.2017.8062511.

Li, Y.G., Nilkitsaranont, P., 2009. Gas turbine performance prognostic for condition-based maintenance. Appl. Energy 86, 2152–2161. doi:10.1016/j.apenergy.2009.02.011.

Liptak, B.G., 2005. Instrument Engineers' Handbook, Volume Two: Process Control and Optimization, 2. CRC Press.

Lüdtke, K.H., 2013. Process centrifugal compressors: Basics, function, operation, design, application, 2nd Springer Science & Business Media.

Meher-Homji, C.B., Bromley, A., 2004. Gas turbine axial compressor fouling and washing. In: Proceedings of the 33rd Turbomachinery Symposium doi:10.21423/R1S66R. Texas A&M University. Turbomachinery Laboratories

Milosavljevic, P., Cortinovis, A., Marchetti, A.G., Faulwasser, T., Mercangöz, M., Bonvin, D., 2016. Optimal load sharing of parallel compressors via modifier adaptation. In: 2016 IEEE Conference on Control Applications (CCA), pp. 1488–1493. doi:10.1109/cca.2016.7588011.

Milosavljevic, P., Marchetti, A.G., Cortinovis, A., Faulwasser, T., Mercangöz, M., Bonvin, D., 2020. Real-time optimization of load sharing for gas compressors in the presence of uncertainty. Appl. Energy 272, 114883. doi:10.1016/j.apenergy.2020.114883.

Milosavljevic, P., Schneider, R., Cortinovis, A., Faulwasser, T., Bonvin, D., 2018. A distributed feasible-side convergent modifier-adaptation scheme for interconnected systems, with application to gas-compressor stations. Comput. Chem. Eng. 115, 474–486. doi:10.1016/j.compchemeng.2018.05.006.

Nocedal, J., Wright, S.J., 1999. Numerical optimation, 1st Springer-Verlag New York doi:10.1007/b98874.

Nørstebø, V.S., 2008. Optimum Operation of Gas Export Systems Ph.D. thesis.. Norwegian University of Science and Technology (NTNU)

Noura, H., Theilliol, D., Ponsart, J.C., Chamseddine, A., 2009. Actuator and sensor fault-tolerant control design. In: Fault-Tolerant Control Systems. Springer, pp. 7–40. doi:10.1007/978-1-84882-653-3_2.

Osiadacz, A., 1980. Non-linear programming applied to the optimum control of a gas-compressor station. Int. J. Numer. Methods Eng. 15, 1287–1301. doi:10.1002/nme.1620150902.

Osiadacz, A., Bell, D.J., 1981. A local optimization procedure for a gas-compressor station. Opt. Control Appl. Methods 2, 239–250. doi:10.1002/oca.4660020304.

Paparella, F., Domínguez, L., Cortinovis, A., Mercangöz, M., Pareschi, D., Bittanti, S., 2013. Load sharing optimization of parallel compressors. In: European Control Conference (ECC) 2013, pp. 17–19. doi:10.23919/ECC.2013.6669697.

Rios-Mercado, R.Z., Borraz-Sanchez, C., 2015. Optimization problems in natural gas transportation systems: a state-of-the-art review. Appl. Energy 147, 536–555. doi:10.1016/j.apenergy.2015.03.017.

Rohatgi, A. (2018). Webplotdigitizer, v. 4.1. https://automeris.io/WebPlotDigitizer. automeris.io/WebPlotDigitizer, accessed: 10 AUG 2018.

Syverud, E., Bakken, L.E., 2006. The impact of surface roughness on axial compressor performance deterioration. In: 51st ASME Turbo Expo 2006: Power for Land, Sea, and Air, May 06-11, Barcelona, Spain, pp. 491–501. doi:10.1115/GT2006-90004.

Syverud, E., Brekke, O., Bakken, L.E., 2005. Axial compressor deterioration caused by saltwater ingestion. In: ASME Turbo Expo 2005: Power for Land, Sea, and Air, pp. 327–337. doi:10.1115/1.2219763. June 6–9, 2005, Reno, Nevada, USA

Tarabrin, A.P., Schurovsky, V.A., Bodrov, A.I., Stalder, J.P., 1996. An analysis of axial compressors fouling and a cleaning method of their blading. ASME 1996 International Gas Turbine and Aeroengine Congress and Exhibition V001T01A093 doi:10.1115/96-GT-363.

Tsoutsanis, E., Meskin, N., 2017. Derivative-driven window-based regression method for gas turbine performance prognostics. Energy 128, 302–311. doi:10.1016/j.energy.2017.04.006.

Verheyleweghen, A., Jäschke, J., 2017. Health-aware operation of a subsea gas compression station under uncertainty. Foundations of Computer Aided Process Operations/chemical Process Control. Paper ID F26, FOCAPO/CPC; 8–12 Jan 2017 Tucson, AZ.

Wu, S., 1998. Steady-state simulation and fuel cost minimization of gas pipeline networks. University of Houston Ph.D. thesis.
Wu, S., Rios-Mercado, R.Z., Boyd, E.A., Scott, L.R., 2000. Model relaxations for the fuel cost minimization of steady-state gas pipeline networks. Math. Comput. Model. 31, 197–220. doi:10.1016/S0895-7177(99)00232-0.
Xenos, D.P., 2015. Optimal operation of industrial compressor stations in systems with large energy consumption Ph.D. thesis, Imperial College London.
Xenos, D.P., Kahrs, O., Cicciotti, M., Leira, F.M., Thornhill, N.F., 2016a. Challenges of the application of data-driven models for the real-time optimization of an industrial air separation plant. In: 2016 European Control Conference (ECC). IEEE, pp. 1025–1030. doi:10.1109/ecc.2016.7810424.
Xenos, D.P., Kopanos, G.M., Cicciotti, M., Thornhill, N.F., 2016b. Operational optimization of networks of compressors considering condition-based maintenance. Comput. Chem. Eng. 84, 117–131. doi:10.1016/j.compchemeng.2015.08.008.
Xenos, D.P., Lunde, E., Thornhill, N.F., 2016c. Optimal operation and maintenance of gas compressor stations: An integrated framework applied to a large-scale industrial case. J. Eng. Gas Turb. Power – Trans. ASME 138. doi:10.1115/1.4031307, (042401)1–10.
Zagorowska, M., Schulze Spüntrup, F., Ditlefsen, A.M., Imsland, L., Lunde, E., Thornhill, N.F., 2020a. Adaptive detection and prediction of performance degradation in off-shore turbomachinery. Appl. Energy 268, 114934. doi:10.1016/j.apenergy.2020.114934.
Zagorowska, M., Wu, O., Ottewill, J.R., Reble, M., Thornhill, N.F., 2020b. A survey of models of degradation for control applications. Annual Reviews in Control (in press).
Zhang, X., Wu, C., Zuo, L., Meng, X., 2014. Dynamic programming based algorithm for compressor station optimization. ASME 2014 International Mechanical Engineering Congress and Exposition. American Society of Mechanical Engineers doi:10.1115/IMECE2014-36289. V011T14A028–V011T14A028
Zuo, L., Zhang, X., Wu, C., Yu, Y., 2016. Unit commitment for a compressor station by mixed integer linear programming. J. Nat. Gas Sci. Eng. 30, 338–342. doi:10.1016/j.jngse.2016.02.038.