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Research Article

Group Decision Algorithm for Aged Healthcare Product Purchase Under q-Rung Picture Normal Fuzzy Environment Using Heronian Mean Operator

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ABSTRACT

With the intensification of the aging, the health issue of the elderly is arousing public concern increasingly. Various healthcare products for the elderly are emerging from the market, thus how to select suitable aged healthcare product is critical to the well-being of the elderly. In the literature, nonetheless, a comprehensive and standardized evaluation framework to support healthcare product purchase decision for the aged is currently lacking. This paper proposes a novel group decision-making method to aid the decision-making of aged healthcare product purchase based on q-rung picture normal fuzzy Heronian mean (q-RPtNoFHM) operators. In it, firstly, a new fuzzy variable called the q-rung picture normal fuzzy set (q-RPtNoFS) is defined to reasonably describe different responses to healthcare product evaluation, for which, some definitions including operational laws, a score function, and an accuracy function of q-RPtNoFSs are introduced. Then, two q-RPtNoFHM operators are presented to aggregate group decision information. In addition, some properties of q-RPtNoFHM operators, such as monotonicity, commutativity, and idempotency, are discussed. Finally, an example on antihypertensive drugs purchase is gave to illustrate the practicality of the proposed method, and conduct sensitivity analysis to analyze the effectiveness and flexibility of proposed methods.

1. INTRODUCTION

The global population is aging and the size of the elderly population in is increasing. In 2019, one out of 11 will be 65 years old (9%), however, by 2050, one out of every six people in the world will be above 65 years old (16%) [1]. In developed countries and some developing countries, the improvement of living standards gives older people more capital to pay their healthcare. As a result, huge consumer markets of healthcare products for the elderly have been developed in some countries, such as China. In 2018, the business opportunities of China's pension market were about 4 trillion yuan. By 2050, the agedness consumption market in China will reach 106 trillion yuan, and its share of GDP will increase from about 8% to about 33%, of which the expenditure will be on healthcare products [2]. As the elderly are more eager for healthcare than young people, they become the main profit-makers of healthcare product manufacturers and their retailers. Some healthcare product enterprises often use various marketing methods to con the elderly into purchasing their products. According to the survey on the market of healthcare products for the elderly in China, some healthcare product enterprises have even formed a marketing model specifically for the elderly, such as inviting the elderly to participate in health lectures under the guise of "experts," giving gifts free of charge, greetings, returning cash, even organizing free tourism, free physical examination, etc. [3]. Most of these marketing are gaining the trust of a substantial amount of elderly people, who then buy the products, resulting in many elderly people being deceived. In recent years, there are many cases of swindle in the purchase of healthcare products by the elderly. In 2018, public security organs in China cracked more than 3,000 such cases, arrested more than 1,900 suspects and recovered 140 million yuan of booties [4]. Faced with the phenomenon and problem that the elderly people are deceived in purchasing health care products, it is urgent to put forward a method to help the elderly identify and purchase effective healthcare products correctly.

Since there are many healthcare products in the market, each product involves many evaluation factors, and the elderly maybe influenced by subjective suggestions or word-of-mouth of various groups of family members, doctors, and friends. Furthermore, their own judgments on their performance, price level, and other factors in the process of purchasing products are also subjective and vague. Thus, it is clear that the purchasing process of healthcare products for the elderly is essentially a multi-attribute group decision-making (MAGDM) problem

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based on multiple heterogeneous groups and attributes. As such, this paper proposes a MAGDM for the purchase of healthcare products by the elderly and it integrates multi-group and multi-attribute evaluation information.

The main contributions of this paper are as follows:

1. It defines the q-rung picture normal fuzzy set (q-RPtNoFS) and their operational rule, obtain useful properties, and then defines the scoring function and accuracy function in the q-rung picture normal fuzzy (q-RPtNoF) environment.
2. It proposes some information aggregators in the q-RPtNoF environment, including the q-RPtNoF Heronian mean (q-RPtNoFHM) operator and the q-RPtNoF weighted Heronian mean (q-RPtNoFGHM) operator. It also gives the properties of the information aggregation operators.
3. It proposes a MAGDM method based on the q-RPtNoFGHM aggregator under the q-RPtNoF environment.

Other contents of this are organized as follows: Section 2 reviews literature on product purchase decision-making and information aggregator operator. Section 3 introduces some basic theoretical concepts about normal fuzzy numbers (NFNs) and q-rung orthopair fuzzy numbers (q-ROFNs). Section 4 presents the concept of q-RPtNoF and q-RPtNoFHM operators along with their desirable properties. Section 5 establishes a new MAGDM process, based on q-RPtNoFWHM operators, to solve the problems and illustrate it with a numerical example related to purchasing products. Finally, Section 6 concludes the paper.

2. LITERATURE REVIEW

2.1. Product Purchase Decision-Making

Many authors have paid attention to consumer's product purchase decision-making, focusing on the motivation or intention of their decision-making by collecting e-commerce data or investigating primary data. Kim *et al.* [5] constructed a model for consumer decision-making in e-business to analyze how trust and risk affect an Internet consumer's purchasing decision. Karimi *et al.* [6] explored the influence path of individual decision-making style and prior product knowledge on the consumers' purchase process. Kim and Krishnan [7] found that online shopping experience could affect whether consumers purchase more of the cheaper products by using individual-level transaction data. Through the empirical study, Gu *et al.* [8] found that the systematic provision of information on online products could have a significant impact on consumers' purchase decision process. Ren and Nickerson [9] discovered that product type could affect the relationship between multi opinions of online review and consumer purchase decision. Li and Meshkova [10] examined the rich media can significantly affect online purchase intentions and willingness. Von *et al.* [11] investigated how average consumer ratings, and consumer reviews influenced online purchasing decisions invention of younger and older adults. Furthermore, many scholars found some other factors which affect the consumer's online purchase motivations or intentions, such as online social ties and product-related risks [12], media channels [13], product review balance and volume [14], etc.

Based on product word-of-mouth or online reviews, the above studies analyze the influencing factors of consumers' decision-making and explore their consumption behavior, while ignoring the impact of multiple attributes of products on consumers' purchase decision-making.

2.2. Product Purchase Decision-Making Based on Multi-Attribute Perspective

Some scholars have constructed a multi-attribute decision-making (MADM) model for product purchase from the perspective of product attributes. For instance, considering the multi-dimensional emotional tendency of product attributes, Liu *et al.* [15] established an online product sorting method by using intuitionistic fuzzy (IF) set theory and sentiment computing. Liu *et al.* [16] constructed a product-sorting model by combing emotional classification and IF -TOPSIS. In addition, Fan *et al.* [17] established a comprehensive product sorting model to support consumers' online purchasing decisions, taking into account factors such as online product ratings and product attributes. Yang and Zhu [18] proposed a normal stochastic multiple attribute decision-making method for product sorting considering the normal random distribution of online comment information. Ji *et al.* [19] presented a fuzzy purchase decision model with combining probability multivalued neutrosophic linguistic numbers and sentiment analysis. Liang and Wang [20] developed a purchase decision method for online consumer using linguistic Intuitionistic Cloud theory and sentiment analysis technique. Yang *et al.* [21] proposed a purchase decision model by using the sentiment computing and dynamic IF operator with consumer's dynamic information preferences. Cali and Balaman [22] presented a decision support model for product ranking based on MADM and aspect level sentiment analysis method. Bi *et al.* [23] established a product ranking method for product purchase decision by integrating sentiment analysis and interval type-2 fuzzy numbers. Furthermore, also some scholars developed product purchase decision by combining fuzzy sets theory and MADM, such as IF-based sentiment word framework and MADM method [24], combining Sentiment Analysis With a Fuzzy Kano Model [25], hesitant fuzzy set and sentiment word framework [26], etc.

Although the above literature further optimized the product sorting method, the data considered are from a single online review. Moreover, due to the existence of false comments, it is easy to mislead consumers' decision, especially the elderly people are vulnerable to the influence of a single group of word-of-mouth.

2.3. Fuzzy Information Aggregation Operator

Since product purchase decision-making is essentially a MADM problem, the key to which lies in the expression of attribute information and its information aggregator. In addition, the decision-making process of product purchasing is mainly influenced by the decision-makers judgment with preference. Therefore, by using various types of fuzzy set theory, many scholars have carried out research on fuzzy sets and the aggregators with their application to MADM. Especially since Atanassov [27] put forward intuitionistic fuzzy sets (IFSs) based on Zadeh's fuzzy sets [28], many scholars have developed MADM methods based on extended IFS [29–31], etc. However, the sum of membership degree (MBD, u) and nonmembership degree (NMBD, v) of IFSs is less than or equal to 1, which further restricts its practical application. If the decision-maker gives the MBD and NMBD of the attribute value independently, the sum of the two will be greater than 1, e.g., $u = 0.6$, $v = 0.7$, and the sum of squares is less than or equal to 1, or the sum of their squares is more than 1, while the sum of q-th power will be less than 1. We can easily find the IFS cannot address the information environment. For which, Atanassov [32] initially developed a theoretical concept of orthopair fuzzy sets, based on that, Yager proposed the concepts of Pythagorean fuzzy sets (PFSs) [33] and q-rung orthopair fuzzy sets (q-ROFSs) [34], and pointed out that the characteristics of q-ROFSs is that the sum of q-th power of MBD and NMBD is not greater than 1 ($q > 1$). By using the q-ROFSs, many scholars proposed a MADM method by q-ROFSs information aggregator. Ju *et al.* [35] presented the family of the q-ROF power operators for MADM. Wang *et al.* [36] proposed a series of q-rung orthopair fuzzy linguistic (q-ROFL) operator for MADM. Chen and Luo [37] developed the q-ROFL weighted Muirhead mean. Wei *et al.* [38] developed the family of q-rung orthopair maclaurin symmetric mean operators (q-ROFMSM) operators. Gao *et al.* [39] developed the q-RIVOF weighted Archimedean Muirhead mean (IVq-ROFWAMM) operator. Yang *et al.* [40] combined the q-ROFSs and deep learning to online shopping decision-making problems.

In real life, many natural phenomena and human activities are also normally distributed [41,42], such as information related to product attributes: "product life span," "customer experience score," "treatment effect," "price level," etc. In view of these phenomena, Yang and Ko [43] put forward a NFN to describe them. Compared with TINFSs and TIFSs, NFNs have higher-order derivative continuity, which can describe natural and social phenomena, science and technology and human production activities more extensively. Besides, their membership functions are closer to human thinking. Wang *et al.* [42] found that the expansion of IF numbers based on NFNs is better than that of other types of IF numbers through empirical analysis. Therefore, Wang *et al.* [30,42] defined intuitionistic normal fuzzy (INF) numbers and their operation rules and some information aggregators. On this basis, some scholars have studied the INF numbers, including the extension of basic theory of INF set [44], some information aggregation under INF environment [45,46].

Because the answer given by decision-makers in IFS or q-ROFS environment consists of MBD and NMBD, and the hesitation degree (HED) is determined by the former two, the IFS or q-ROFS cannot express some complicated decision information which is made up of multiple answer. In addition, the sum of MBD, NMBD, and HED, or the sum of the q-power of them is required to be equal to 1, they have certain limitations in dealing with practical decision-making problems. For instance, when a decision-maker evaluates a specific target attribute, there are many types of answers: the MBD of negative is 0.4, that of positive answers is 0.2, and that of hesitant answers is 0.3. The sum of the three or the sum of the q-th power of them is less than 1. Therefore, similar information cannot be processed using IFS and q-ROFS. Hence, motivated by the extensions of FIS proposed by Vassilev and Atanassov [47], Cuong and Kreinovich [48] and Cuong [49] presented picture fuzzy set (PtFS), which is characterized by three functions expressing the degree of positive membership, the degree of neutral membership, and the degree of the negative membership. Due to the superiority of PtFS, PtFS is widely applied to evaluating energy performance [50], selecting the location of power station [51], ranking electric vehicle charging station [52], selecting alternative on end-of-life vehicle [53], etc. What's more, Akram *et al.* [54] presented the edge-regular q-rung picture fuzzy graphs. Li *et al.* [55] developed a MADM method based on q-Rung Picture Linguistic sets (q-RPtLS). He *et al.* [56] presented q-rung picture fuzzy Dombi Hamy mean operators, Liu *et al.* [57] proposed T-Spherical fuzzy Power Muirhead mean operator by combining IFSs, PFSs, q-ROFSs, and PtFSs for MADM.

In conclusion, PFS and IFS are the special cases of q-ROFS. The fuzzy information described by q-ROFs is broader and more comprehensive, but the types of answers given by q-ROFs and IFS and PFS in describing fuzzy information are fewer, and PtFS can break their limitations and have stronger ability of describing fuzzy information. In addition, NFN is closer to human decision-making thinking than TINFSs and TIFSs. PFS and IFS based on TINFSs and TIFSs have been reported successively. However, PtFS and q-ROFs based on NFN have not been proposed. Therefore, focusing on the decision-making of healthcare products purchase by the elderly, a MAGDM based on Heronian mean operator and q-RPtNoFSs is proposed. In the proposed method, considering that the elderly listen to the different opinions of multiple heterogeneous groups, a new fuzzy set named q-RPtNoFS is presented to describe evaluation information. Moreover, according to the correlation between different groups and product attributes, a new q-RPtNoFHM operator is used to aggregate information from different opinions of multiple heterogeneous groups.

3. PRELIMINARIES

Definition 1. [43] Let R be a real number set, the fuzzy number of membership function of

$$\tilde{A}(x) = e^{-\left(\frac{x-\alpha}{\sigma}\right)^2} (\sigma > 0) \tag{1}$$

is called as a NFN $\tilde{A} = (\alpha, \sigma)$, and the NFN set is denoted by $\tilde{N}$.

Definition 2. [58] Let $\tilde{A} = (\alpha, \sigma)$, $\tilde{B} = (\beta, \tau)$ $\tilde{A}, \tilde{B} \in \tilde{N}$, $\tilde{A}, \tilde{B} \in \tilde{N}$, and λ be a nonnegative real number. We define

1. $\lambda \otimes \tilde{A} = \lambda(\alpha, \sigma) = (\lambda\alpha, \lambda\sigma), \lambda > 0$, and
2. $\tilde{A} \oplus \tilde{B} = (\alpha, \sigma) + (\beta, \tau) = (\alpha + \beta, \sigma + \tau)$.

Definition 3. [32,34] A q-ROFS A in a finite universe of discourse X is defined by

$$A = \{\langle x, u_A(x), \nu_A(x)\rangle \,|x \in X\}$$

where $u_A(x)$ and $\nu_A(x)$ represent the membership and NMBD respectively, $u_A(x) \in [0,1]$, $\nu_A(x) \in [0,1]$, and $0 \leq u_A(x)^q + \nu_A(x)^q \leq 1\left(q \geq 1\right)$. The degree of indeterminacy is given as $\pi_A(x) = (u_A(x)^q + \nu_A(x)^q - u_A(x)^q \nu_A(x)^q)^{1/q}$. For convenience, we call $A = (u_A, \nu_A)$ a q-ROFN. Let $A_1 = (u_1, \nu_1)$ and $A_2 = (u_2, \nu_2)$ be two q-ROFNs, and λ be a nonnegative real number, we define

1. $A_1 \oplus A_2 = \left(\left(u_1^q + u_2^q - u_1^q u_2^q\right)^{1/q}, \nu_1 \nu_2\right)$,
2. $A_1 \otimes A_2 = \left(u_1 u_2, \left(\nu_1^q + \nu_2^q - \nu_1^q \nu_2^q\right)^{1/q}\right)$,
3. $\lambda A_1 = \left(\left(1 - (1 - u_1^q)^\lambda\right)^{1/q}, \nu_1^\lambda\right)$, and
4. $A_1^\lambda = \left(u_1^\lambda, \left(1 - (1 - \nu_1^q)^\lambda\right)^{1/q}\right)$.

Definition 4. [59] Let $A = (u_A, \nu_A)$ be a q-ROFN. The score function of A is defined as $S(A) = u_A^q - \nu_A^q$, and the accuracy function of A is defined as $H(A) = u_A^q + \nu_A^q$. For any two q-ROFNs, $A_1 = (u_1, \nu_1)$ and $A_2 = (u_2, \nu_2)$, we define

1. If $S(A_1) > S(A_2)$, then $A_1 > A_2$;
2. If $S(A_1) = S(A_2)$, then
   If $H(A_1) > H(A_2)$, then $A_1 > A_2$, and
   If $H(A_1) = H(A_2)$, then $A_1 = A_2$.

4. THE q-RPtNoFN AND ITS OPERATIONS

In this section, we introduce the concept of q-rung picture normal fuzzy number (q-RPtNoFN) and state its operations. Based on its, we also define the aggregation operators for the collection of q-RPtNoFNs.

4.1. A Concept of q-RPtNoFN

This section introduces the q-RPtNoFN and its operations.

Definition 5. [56] Let X be an ordinary fixed set. A q-rung picture fuzzy set (q-RPtFS) A defined on X is given by:

$$A = \{\langle x, u_A(x), \eta_A(x), \nu_A(x)\rangle \,|x \in X\} \tag{2}$$

where $u_A(x), \eta_A(x), \nu_A(x)$ are the degree of positive membership, the degree of neutral membership, and the degree of negative membership, respectively, and $u_A(x), \eta_A(x), \nu_A(x) \in [0,1]$, and $0 \leq u_A(x)^q + \eta_A(x)^q + \nu_A(x)^q \leq 1, \forall x \in X$. Then $\pi_A(x) = (1 - (u_A(x)^q + \eta_A(x)^q + \nu_A(x)^q))^{1/q}$ is the degree of refusal membership of A to X. $A = (u, \eta, \nu)$ is referred to as A q-rung picture fuzzy number (q-RPFN).

Definition 6. Let X be an ordinary fixed non-empty set and $(\alpha, \sigma) \in \tilde{N}$, $A = \langle(\alpha, \sigma), (u_A, \eta_A, \nu_A)\rangle$ is a q-RPtNoFS when its positive membership function is defined as

$$\zeta_A(x) = u_A e^{-\left(\frac{x-\alpha}{\sigma}\right)^2},\ x \in X \tag{3}$$

its negative membership function is defined as

$$\vartheta_A(x) = 1 - (1 - \nu_A) e^{-\left(\frac{x-\alpha}{\sigma}\right)^2},\ x \in X \tag{4}$$

and its neutral membership function is defined as

$$\varphi_A(x) = 1 - (1 - \eta_A) e^{-\left(\frac{x-\alpha}{\sigma}\right)^2},\ x \in X \tag{5}$$

where $\alpha, \sigma, u_A, \eta_A, v_A$ are known numbers, $0 \leq u_A^q + v_A^q + \eta_A^q \leq 1$ and $q \geq 1$ is integer. For convenience, a q-RPtNoFN is denoted as $A = \langle (\alpha, \sigma), (u_A, \eta_A, v_A) \rangle$.

Remark: When $u_A = 1$, $v_A = 0$, and $\eta_A = 0$, the q-RPtNoFS will be transformed into a NFN.

Definition 7. Let $A_1 = \langle (\alpha_1, \sigma_1), (u_1, \eta_1, v_1) \rangle$ and $A_2 = \langle (\alpha_2, \sigma_2), (u_2, \eta_2, v_2) \rangle$ be any two q-RPtNoFNs, and λ be a nonnegative real number, we define

1. $A_1 \oplus A_2 = \left\langle (\alpha_1 + \alpha_2, \sigma_1 + \sigma_2), \left(u_1^q + u_2^q - u_1^q u_2^q\right)^{1/q}, \eta_1 \eta_2, v_1 v_2 \right\rangle$,
2. $A_1 \otimes A_2 = \left\langle \left(\alpha_1 \alpha_2, \alpha_1 \alpha_2 \sqrt{\frac{\sigma_1^2}{\alpha_1^2} + \frac{\sigma_2^2}{\alpha_2^2}}\right), u_1 u_2, \left(\eta_1^q + \eta_2^q - \eta_1^q \eta_2^q\right)^{1/q}, \left(v_1^q + v_2^q - v_1^q v_2^q\right)^{1/q} \right\rangle$,
3. $\lambda A_1 = \left\langle (\lambda \alpha_1, \lambda \sigma_1), \left(\left(1 - (1 - u_1^q)^\lambda\right)^{1/q}, \eta_1^\lambda, v_1^\lambda\right) \right\rangle$, and
4. $A_1^\lambda = \left\langle \left(\alpha_1^\lambda, \lambda^{\frac{1}{2}} \alpha_1^{\lambda - 1} \sigma_1\right), u_1^\lambda, \left(1 - (1 - \eta_1^q)^\lambda\right)^{1/q}, \left(1 - (1 - v_1^q)^\lambda\right)^{1/q} \right\rangle$.

Proposition 1. *Let $A_1 = \langle (\alpha_1, \sigma_1), (u_1, \eta_1, v_1) \rangle$, $A_2 = \langle (\alpha_2, \sigma_2), (u_2, \eta_2, v_2) \rangle$, $A_3 = \langle (\alpha_3, \sigma_3), (u_3, \eta_3, v_3) \rangle$ be any three q-RPtNoFNs, and $\lambda, \lambda_1, \lambda_2$ be nonnegative real numbers, we can obtain that*

(1) $A_1 \oplus A_2 = A_2 \oplus A_1$,

(2) $(A_1 \oplus A_2) \oplus A_3 = A_1 \oplus (A_2 \oplus A_3)$,

(3) $A_1 \otimes A_2 = A_2 \otimes A_1$,

(4) $(A_1 \otimes A_2) \otimes A_3 = A_1 \otimes (A_2 \otimes A_3)$,

(5) $\lambda_1 A_1 \oplus \lambda_2 A_1 = (\lambda_1 \oplus \lambda_2) A_1$,

(6) $\lambda (A_1 \oplus A_2) = \lambda A_1 \oplus \lambda A_2$,

(7) $\left(A_1^{\lambda_1}\right)^{\lambda_2} = A_1^{\lambda_1 \lambda_2}$, *and*

(8) $A_1^{\lambda_1} \otimes A_1^{\lambda_2} = A_1^{\lambda_1 + \lambda_2}$.

Proof. According to Definition 7, we can easily infer that (1), (3), (5), (6) and (7) are obviously established, respectively. The parts (2), (4) and (8) need to be proved as follows:

For (2) $(A_1 \oplus A_2) \oplus A_3 = A_1 \oplus (A_2 \oplus A_3)$

Let the NFN of q-RPtNoFN r be $\tilde{N}_r$, the degree of positive membership of $(A_1 \oplus A_2) \oplus A_3$ and $A_1 \oplus (A_2 \oplus A_3)$ be $u_{(A_1 \oplus A_2) \oplus A_3}$ and $u_{A_1 \oplus (A_2 \oplus A_3)}$, respectively. Let the degree of neutral membership of $(A_1 \oplus A_2) \oplus A_3$, $A_1 \oplus (A_2 \oplus A_3)$ be $\eta_{(A_1 \oplus A_2) \oplus A_3}$ and $\eta_{A_1 \oplus (A_2 \oplus A_3)}$, the degree of negative membership of $(A_1 \oplus A_2) \oplus A_3$ and $A_1 \oplus (A_2 \oplus A_3)$ be $v_{(A_1 \oplus A_2) \oplus A_3}$, and $v_{A_1 \oplus (A_2 \oplus A_3)}$. We can obtain that

$$\tilde{N}_{(r_1 \oplus r_2) \oplus r_3} = \tilde{N}_{r_1 \oplus (r_2 \oplus r_3)} = (\alpha_1 + \alpha_2 + \alpha_3, \sigma_1 + \sigma_2 + \sigma_3),$$

$$\begin{aligned} u_{(A_1 \oplus A_2) \oplus A_3} &= \left(u_1^q + u_2^q - u_1^q u_2^q + u_3^q - (u_1^q + u_2^q - u_1^q u_2^q) u_3^q\right)^{1/q} \quad, \\ &= \left(u_1^q + u_2^q + u_3^q - u_1^q u_2^q - u_1^q u_3^q - u_2^q u_3^q + u_1^q u_2^q u_3^q\right)^{1/q} \end{aligned}$$

$$\begin{aligned} u_{A_1 \oplus (A_2 \oplus A_3)} &= \left(u_2^q + u_3^q - u_2^q u_3^q + u_1^q - (u_2^q + u_3^q - u_2^q u_3^q) u_1^q\right)^{1/q} \quad, \\ &= \left(u_1^q + u_2^q + u_3^q - u_1^q u_2^q - u_1^q u_3^q - u_2^q u_3^q + u_1^q u_2^q u_3^q\right)^{1/q} \end{aligned}$$

and

$$u_{(A_1 \oplus A_2) \oplus A_3} = u_{A_1 \oplus (A_2 \oplus A_3)},$$

Similarly, we can obtain that $\eta_{(A_1 \oplus A_2) \oplus A_3} = \eta_{A_1 \oplus (A_2 \oplus A_3)}$, and $v_{(A_1 \oplus A_2) \oplus A_3} = v_{A_1 + (A_2 + A_3)}$.

Therefore, $(A_1 \oplus A_2) \oplus A_3 = A_1 \oplus (A_2 \oplus A_3)$.

Now we prove (4), i.e., $(A_1 \otimes A_2) \otimes A_3 = A_1 \otimes (A_2 \otimes A_3)$.

Let the NFN of q-RPtNoFNs r be $\tilde{N}_r$, the degree of membership of $(A_1 \otimes A_2) \otimes A_3$ and $A_1 \otimes (A_2 \otimes A_3)$ be $u_{(A_1 \otimes A_2) \otimes A_3}$ and $u_{A_1 \otimes (A_2 \otimes A_3)}$, respectively. Let the degree of neutral membership of $(A_1 \otimes A_2) \otimes A_3$ and $A_1 \otimes (A_2 \otimes A_3)$ be $\eta_{(A_1 \otimes A_2) \otimes A_3}$ and $\eta_{A_1 \otimes (A_2 \otimes A_3)}$, respectively, and let the degree of non-membership of $(A_1 \otimes A_2) \otimes A_3$ and $A_1 \otimes (A_2 \otimes A_3)$ be $v_{(A_1 \otimes A_2) \otimes A_3}$ and $v_{A_1 \otimes (A_2 \otimes A_3)}$, respectively. We can obtain that

$$\begin{aligned}\tilde{N}_{(r_1 \otimes r_2) \otimes r_3} &= \tilde{N}_{r_1 \otimes (r_2 \otimes r_3)} \\ &= \left(\alpha_1\alpha_2\alpha_3, \alpha_1\alpha_2\alpha_3 \sqrt{\left(\frac{\sigma_1^2}{\alpha_1^2} + \frac{\sigma_1^2}{\alpha_2^2}\right) + \frac{\sigma_3^2}{\alpha_3^2}} \right) \\ &= \left(\alpha_1\alpha_2\alpha_3, \alpha_1\alpha_2\alpha_3 \sqrt{\frac{\sigma_1^2}{\alpha_1^2} + \left(\frac{\sigma_1^2}{\alpha_2^2} + \frac{\sigma_3^2}{\alpha_3^2}\right)} \right)\end{aligned}$$

$$\begin{aligned}\eta_{(A_1 \otimes A_2) \otimes A_3} &= \left(\eta_1^q + \eta_2^q - \eta_1^q\eta_2^q + \eta_3^q - (\eta_1^q + \eta_2^q - \eta_1^q\eta_2^q)\eta_3^q\right)^{1/q} \quad , \\ &= \left(\eta_1^q + \eta_2^q + \eta_3^q - \eta_1^q\eta_2^q - \eta_1^q\eta_3^q - \eta_2^q\eta_3^q + \eta_1^q\eta_2^q\eta_3^q\right)^{1/q}\end{aligned}$$

$$\begin{aligned}\eta_{A_1 \otimes (A_2 \otimes A_3)} &= \left(\eta_2^q + \eta_3^q - \eta_2^q\eta_3^q + \eta_1^q - (\eta_2^q + \eta_3^q - \eta_2^q\eta_3^q)\eta_1^q\right)^{1/q} \quad , \\ &= \left(\eta_1^q + \eta_2^q + \eta_3^q - \eta_1^q\eta_2^q - \eta_1^q\eta_3^q - \eta_2^q\eta_3^q + \eta_1^q\eta_2^q\eta_3^q\right)^{1/q}\end{aligned}$$

and

$$\eta_{(A_1 \otimes A_2) \otimes A_3} = \eta_{A_1 \otimes (A_2 \otimes A_3)}.$$

Similarly, we can get that $v_{(A_1 \otimes A_2) \otimes A_3} = v_{A_1 \otimes (A_2 \otimes A_3)}$, and $u_{(A_1 \otimes A_2) \otimes A_3} = u_{A_1 \otimes (A_2 \otimes A_3)}$. This establishes item (4), i.e., $(A_1 \otimes A_2) \otimes A_3 = A_1 \otimes (A_2 \otimes A_3)$.

We now prove item (8), i.e., $A_1^{\lambda_1} \otimes A_1^{\lambda_2} = A_1^{\lambda_1 + \lambda_2}$.

Let $A_1 = \langle(\alpha_1, \sigma_1), (u_1, \eta_1, v_1)\rangle$ be a q-RPtNoFN, λ_1 and λ_2 be nonnegative real numbers, and the NFN of q-RPtNoFN r be $\tilde{N}_r$. We can obtain that

$$\begin{aligned}\tilde{N}_{r_1}^{\lambda_1} \otimes \tilde{N}_{r_1}^{\lambda_2} &= \tilde{N}_{r_1}^{(\lambda_1+\lambda_2)} \quad , \\ &= \left(\alpha_1^{\lambda_1}\alpha_1^{\lambda_2}, \alpha_1^{\lambda_1}\alpha_1^{\lambda_2} \sqrt{\lambda_1 \alpha_1^{-2}\sigma_1^2 + \lambda_2 \alpha_1^{-2}\sigma_1^2} \right) \\ &= \left(\alpha_1^{(\lambda_1+\lambda)_2}, \alpha_1^{(\lambda_1+\lambda)_2} \frac{\sigma_1}{\alpha_1} \sqrt{(\lambda_1 + \lambda_2)} \right)\end{aligned}$$

and

$$\begin{aligned}\eta_1^{\lambda_1} \otimes \eta_1^{\lambda_2} &= \left(1 - (1-\eta_1^q)^{\lambda_1} + 1 - (1-\eta_1^q)^{\lambda_2} - \left(\left(1-(1-\eta_1^q)^{\lambda_1}\right)\left(1-(1-\eta_1^q)^{\lambda_2}\right)\right)\right)^{1/q}. \\ &= \left(1 - (1-\eta_1^q)^{\lambda_1} + 1 - (1-\eta_1^q)^{\lambda_2} - \left(\left(1-(1-\eta_1^q)^{\lambda_1}\right)\left(1-(1-\eta_1^q)^{\lambda_2}\right)\right)\right)^{1/q} \\ &= \left(1 - (1-\eta_1^q)^{\lambda_1} + \left(1-(1-\eta_1^q)^{\lambda_2}\right)(1-\eta_1^q)^{\lambda_1}\right)^{1/q} \\ &= \left(1 - (1-\eta_1^q)^{\lambda_1+\lambda_2}\right)^{1/q} = \eta_1^{(\lambda_1+\lambda_2)}\end{aligned}$$

Similarly, we can get that $u_1^{\lambda_1} \otimes u_1^{\lambda_2} = u_1^{(\lambda_1+\lambda_2)}$, and $v_1^{\lambda_1} \otimes v_1^{\lambda_2} = v_1^{(\lambda_1+\lambda_2)}$. As such, $A_1^{\lambda_1} \otimes A_1^{\lambda_2} = A_1^{\lambda_1+\lambda_2}$ is established.

Definition 8. Let $A = \langle(\alpha, \sigma), (u, \eta, v)\rangle$ be a q-RPtNoFN, whose score function is defined as $S_1(A) = \alpha\left(u_A^q - \eta_A^q - v_A^q\right)$, $S_2(A) = \sigma\left(u_A^q - \eta_A^q - v_A^q\right)$ and its accuracy function is defined as $H_1(A) = \alpha\left(u_A^q + \eta_A^q + v_A^q\right)$, $H_2(A) = \sigma\left(u_A^q + \eta_A^q + v_A^q\right)$.

Definition 9. Let $A_1 = \langle(\alpha_1, \sigma_1), (u_1, \eta_1, v_1)\rangle$ and $A_2 = \langle(\alpha_2, \sigma_2), (u_2, \eta_2, v_2)\rangle$ be any two q-RPtNoFNs. If their score functions are $S_1(A)$ and $S_2(A)$, respectively, and their accuracy functions are $H_1(A)$ and $H_2(A)$, respectively, then we can obtain

1. If $S_1(A_1) > S_1(A_2)$, then $A_1 > A_2$,

2. If $S_1(A_1) = S_1(A_2)$ and $H_1(A_1) > H_1(A_2)$, then $A_1 > A_2$,
3. If $S_1(A_1) = S_1(A_2)$ and $H_1(A_1) = H_1(A_2)$, then
   If $S_2(A_1) < S_2(A_2)$, then $A_1 > A_2$,
   If $S_2(A_1) = S_2(A_2)$ and $H_2(A_1) < H_2(A_2)$, then $A_1 > A_2$,
   If $S_2(A_1) = S_2(A_2)$ and $H_2(A_1) = H_2(A_2)$, then $A_1 = A_2$.

Definition 10. [60] Let $g > 0, l > 0$, and $g + l > 0$, a_i $(i = 1, 2, \cdots, n)$ be any nonnegative real number, then

$$HM(A_1, A_2, \cdots, A_n) = \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=1}^{n} a_i^g a_j^l\right)^{\frac{1}{g+l}}. \tag{6}$$

is called Heronian mean operator.

4.2. q-RPtNoFHM Weighed Averaging Operators

Based on the operational rules of q-RPtNoFNs and Heronian mean operator, the Heronian mean weighed averaging operators for q-RPtNoFN are presented as follows:

Definition 11. Let $A_i = \langle(\alpha_i, \sigma_i), (u_i, \eta_i, v_i)\rangle$ $(i = 1, 2, \cdots, n)$ be a collection of q-RPtNoFN. Then the q-RPtNoFHM operator can be defined as

$$q - RPtNoFHM(A_1, A_2, \cdots, A_n) = \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=1}^{n} (a_i)^g \otimes \left(a_j\right)^l\right)^{\frac{1}{g+l}}. \tag{7}$$

Theorem 1. *Let $A_i = \langle(\alpha_i, \sigma_i), (u_i, \eta_i, v_i)\rangle$ $(i = 1, 2, \cdots, n)$ be a collection of q-RPtNoFN, then the aggregated value using q-RPtNoFHM operator is still a q-RPtNoFN, i.e.,*

$$\begin{aligned} & q - RPtNoFHM(A_1, A_2, \cdots, A_n) \\ & = \left\langle \begin{array}{l} \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i^g \alpha_j^l, \sqrt{\frac{1}{g+l}} \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i^g \alpha_j^l\right)^{\frac{1}{g+l}-1} \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i^g \alpha_j^l \sqrt{\frac{g\sigma_i^2}{\alpha_i^2} + \frac{l\sigma_j^2}{\alpha_j^2}}\right)\right), \\ \left(\left(1 - \left(\prod_{i=1}^{n} \prod_{j=1}^{n} \left(1 - (\mu_i^q)^g (\mu_j^q)^l\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q}}\right)^{\frac{1}{g+l}}, \left(1 - \left(1 - \left(\prod_{i=1}^{n} \prod_{j=1}^{n} \left(1 - (1-\eta_i^q)^g (1-\eta_j^q)^l\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{g+l}}\right)^{\frac{1}{q}}, \\ \left(1 - \left(1 - \left(\prod_{i=1}^{n} \prod_{j=1}^{n} \left(1 - (1-v_i^q)^g (1-v_j^q)^l\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{g+l}}\right)^{\frac{1}{q}} \end{array} \right\rangle. \end{aligned} \tag{8}$$

Proof. Based on the operations of q-RPtNoFNs, we can get:

$$A_i^g = \left(\left(\alpha_i^g, g^{\frac{1}{2}} \alpha_i^{g-1} \sigma_i\right), u_i^g, \left(1 - (1-\eta_i^q)^g\right)^{1/q}, \left(1 - (1-v_i^q)^g\right)^{1/q}\right),$$

$$A_j^l = \left(\left(\alpha_j^l, l^{\frac{1}{2}} \alpha_i^{l-1} \sigma_i\right), u_j^l, \left(1 - (1-\eta_i^q)^l\right)^{1/q}, \left(1 - (1-v_i^q)^l\right)^{1/q}\right),$$

and

$$
\begin{aligned}
&(A_i)^g \otimes (A_j)^l \\
&= \left\langle \begin{bmatrix} \alpha_i^g \alpha_j^l, \alpha_i^g \alpha_j^l \sqrt{\dfrac{g\alpha_i^{2(g-1)}\sigma_i^2}{\alpha_i^{2g}} + \dfrac{l\alpha_j^{2(l-1)}\sigma_j^2}{\alpha_j^{2l}}}, \\ \mu_i^g \mu_j^l, \\ \left(\left(1-(1-\eta_i^q)^g\right) + \left(1-(1-\eta_j^q)^l\right) - \left(1-(1-\eta_i^q)^g\right)\left(1-(1-\eta_j^q)^l\right)\right)^{\frac{1}{q}}, \\ \left(\left(1-(1-\nu_i^q)^g\right) + \left(1-(1-\nu_j^q)^l\right) - \left(1-(1-\nu_i^q)^g\right)\left(1-(1-\nu_j^q)^l\right)\right)^{\frac{1}{q}} \end{bmatrix} \right\rangle \\
&= \left\langle \begin{bmatrix} \alpha_i^g \alpha_j^l, \alpha_i^g \alpha_j^l \sqrt{\dfrac{g\sigma_i^2}{\alpha_i^2} + \dfrac{l\sigma_j^2}{\alpha_j^2}}, \\ \mu_i^g \mu_j^l, \left(1-(1-\eta_i^q)^g (1-\eta_j^q)^l\right)^{\frac{1}{q}}, \left(1-(1-\nu_i^q)^g (1-\nu_j^q)^l\right)^{\frac{1}{q}} \end{bmatrix} \right\rangle.
\end{aligned}
$$

Then we use the mathematical induction method to get

$$
\sum_{i=1}^{n} (A_i)^g \otimes (A_j)^l = \left\langle \begin{matrix} \sum_{i=1}^{n} \alpha_i^g \alpha_j^l, \sum_{i=1}^{n} \alpha_i^g \alpha_j^l \sqrt{\dfrac{g\sigma_i^2}{\alpha_i^2} + \dfrac{l\sigma_j^2}{\alpha_j^2}}, \\ \left(1-\prod_{i=1}^{n}\left(1-(\mu_i^g \mu_j^l)^q\right)\right)^{\frac{1}{q}}, \prod_{i=1}^{n}\left(1-(1-\eta_i^q)^g (1-\eta_j^q)^l\right)^{\frac{1}{q}}, \prod_{i=1}^{n}\left(1-(1-\nu_i^q)^g (1-\nu_j^q)^l\right)^{\frac{1}{q}} \end{matrix} \right\rangle,
$$

and

$$
\sum_{i=1}^{n}\sum_{j=1}^{n} (A_i)^g \otimes (A_j)^l = \left\langle \begin{matrix} \sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_i^g \alpha_j^l, \sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_i^g \alpha_j^l \sqrt{\dfrac{g\sigma_i^2}{\alpha_i^2} + \dfrac{l\sigma_j^2}{\alpha_j^2}} \\ \left(1-\prod_{i=1}^{n}\prod_{j=1}^{n}\left(1-(\mu_i^g \mu_j^l)^q\right)\right)^{\frac{1}{q}}, \prod_{i=1}^{n}\prod_{j=1}^{n}\left(1-(1-\eta_i^q)^g (1-\eta_j^q)^l\right)^{\frac{1}{q}}, \prod_{i=1}^{n}\prod_{j=1}^{n}\left(1-(1-\nu_i^q)^g (1-\nu_j^q)^l\right)^{\frac{1}{q}} \end{matrix} \right\rangle.
$$

Furthermore, the following result can be derived:

$$
\begin{aligned}
&\frac{2}{n(n+1)} \sum_{i=1}^{n}\sum_{j=1}^{n} (A_i)^g \otimes (A_j)^l \\
&= \left\langle \begin{matrix} \dfrac{2}{n(n+1)} \sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_i^g \alpha_j^l, \dfrac{2}{n(n+1)} \sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_i^g \alpha_j^l \sqrt{\dfrac{g\sigma_i^2}{\alpha_i^2} + \dfrac{l\sigma_j^2}{\alpha_j^2}} \\ \left(1-\left(\prod_{i=1}^{n}\prod_{j=1}^{n}\left(1-(\mu_i^g \mu_j^l)^q\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q}}, \left(\prod_{i=1}^{n}\prod_{j=1}^{n}\left(1-(1-\eta_i^q)^g (1-\eta_j^q)^l\right)^{\frac{1}{q}}\right)^{\frac{2}{n(n+1)}}, \left(\prod_{i=1}^{n}\prod_{j=1}^{n}\left(1-(1-\nu_i^q)^g (1-\nu_j^q)^l\right)^{\frac{1}{q}}\right)^{\frac{2}{n(n+1)}} \end{matrix} \right\rangle,
\end{aligned}
$$

and

$$
\begin{aligned}
&\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}(A_i)^g\otimes\left(A_j\right)^l\right)^{\frac{1}{g+l}}\\
&\quad\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i^g\alpha_j^l\right)^{\frac{1}{g+l}},\sqrt{\frac{1}{g+l}}\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i^g\alpha_j^l\right)^{\frac{1}{g+l}-1}\\
&\quad\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i^g\alpha_j^l\sqrt{\frac{g\sigma_i^2}{\alpha_i^2}+\frac{l\sigma_j^2}{\alpha_j^2}}\right)\\
&=\left\langle\begin{array}{l}
\left(\left(1-\left(\prod_{i=1}^{n}\prod_{j=1}^{n}\left(1-\left(\mu_i^g\mu_j^l\right)^q\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q}}\right)^{\frac{1}{g+l}},\\
\left(1-\left(1-\left(\prod_{i=1}^{n}\prod_{j=1}^{n}\left(1-\left(1-\eta_i^q\right)^g\left(1-\eta_j^q\right)^l\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{g+l}}\right)^{\frac{1}{q}},\\
\left(1-\left(1-\left(\prod_{i=1}^{n}\prod_{j=1}^{n}\left(1-\left(1-\nu_i^q\right)^g\left(1-\nu_j^q\right)^l\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{g+l}}\right)^{\frac{1}{q}}
\end{array}\right\rangle.
\end{aligned}
$$

The Proof is completed.

From the structure of the proposed q-RPtNoFHM operator, it has been analyzed that it satisfies the following properties.

Theorem 2 (Idempotency). *If all $A_i=\langle(\alpha_i,\sigma_i),(u_i,\eta_i,v_i)\rangle$ $(i=1,2,\cdots,n)$ are equal with A, then*

$$q-RPtNoFHM\,(A_1,A_2,\cdots,A_n)=A.$$

Proof. Since $A_i=\langle(\alpha_i,\sigma_i),(u_i,\eta_i,v_i)\rangle=A$ for any i, we can get

$$
\begin{aligned}
q-RPtNoFHM\,(A_1,A_2,\cdots,A_n)&=\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}(A_i)^g\otimes(A_j)^l\right)^{\frac{1}{g+l}}\\
&=\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}(A)^g\otimes(A)^l\right)^{\frac{1}{g+l}}\\
&=\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}(A)^{g+l}\right)^{\frac{1}{g+l}}=A
\end{aligned}
$$

Therefore, $q-RPtNoFHM\,(A_1,A_2,\cdots,A_n)=A$

Theorem 3 (Boundedness). *Let $A_i=\langle(\alpha_i,\sigma_i),(u_i,\eta_i,v_i)\rangle$ $(i=1,2,\cdots,n)$ be a collection of q-RPtNoFN. If $A^-=\min_{\leq i\leq n}\{A_i\},A^+=\max_{\leq i\leq n}\{A_i\}$, then*

$$A^-\leq q-RPtNoFHM\,(A_1,A_2,\cdots,A_n)\leq A^+.$$

Proof. Since $A^+ = \max_{\leq i \leq n}\{A_i\}$, according to the Theorem 2, we can obtain

$$q-RPtNoFHM(A_1,A_2,\cdots,A_n) = \left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}(A_i)^g \otimes (A_j)^l\right)^{\frac{1}{g+l}}.$$
$$\leq \left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}(A^+)^g \otimes (A^+)^l\right)^{\frac{1}{g+l}}$$
$$\leq \left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}(A^+)^{g+l}\right)^{\frac{1}{g+l}} = A^+$$

Similarly, we can get $A^- \leq q-RPtNoFHM(A_1,A_2,\cdots,A_n)$.

Therefore $A^- \leq q-RPtNoFHM(A_1,A_2,\cdots,A_n) \leq A^+$.

Theorem 4 (Monotonicity). *Suppose $(A_1,A_2,\cdots,A_n)$ and $(B_1,B_2,\cdots,B_n)$ are two sets of q-RPtNoFN, $A_i = \left\langle(\alpha_{A_i},\sigma_{A_i}),\left(\mu_{A_i},\eta_{A_i},\nu_{A_i}\right)\right\rangle$, and $B_i = \left\langle(\alpha_{B_i},\sigma_{B_i}),\left(\mu_{B_i},\eta_{B_i},\nu_{B_i}\right)\right\rangle$, $(i=1,2,\cdots,n)$. For any i, if there is $\alpha_{A_i} \leq \alpha_{B_i}$ and $\mu_{A_i} \leq \mu_{B_i},\eta_{A_i} \geq \eta_{B_i},\nu_{A_i} \geq \nu_{B_i}$ then*

$$q-RPtNoFHM(A_1,A_2,\cdots,A_n) \leq q-RPtNoFHM(B_1,B_2,\cdots,B_n).$$

Proof. Since there is $\alpha_{A_i} \leq \alpha_{B_i}, \mu_{A_i} \leq \mu_{B_i}, \eta_{A_i} \geq \eta_{B_i}$, and $\nu_{A_i} \geq \nu_{B_i}$ for any i

Then, we can get

$$\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_{A_i}^g\alpha_{A_j}^l\right)^{\frac{1}{g+l}} \leq \left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_{B_i}^g\alpha_{B_j}^l\right)^{\frac{1}{g+l}},$$

$$\left(\left(1-\left(\prod_{i=1}^{n}\prod_{j=1}^{n}\left(1-\left(\mu_{A_i}^g\mu_{A_j}^l\right)^q\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q}}\right)^{\frac{1}{g+l}} \leq \left(\left(1-\left(\prod_{i=1}^{n}\prod_{j=1}^{n}\left(1-\left(\mu_{B_i}^g\mu_{B_j}^l\right)^q\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q}}\right)^{\frac{1}{g+l}},$$

$$\left(1-\left(1-\left(\prod_{i=1}^{n}\prod_{j=1}^{n}\left(1-\left(1-\eta_{A_i}^q\right)^g\left(1-\eta_{A_j}^q\right)^l\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{g+l}}\right)^{\frac{1}{q}} \geq \left(1-\left(1-\left(\prod_{i=1}^{n}\prod_{j=1}^{n}\left(1-\left(1-\eta_{B_i}^q\right)^g\left(1-\eta_{B_j}^q\right)^l\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{g+l}}\right)^{\frac{1}{q}},$$

and

$$\left(1-\left(1-\left(\prod_{i=1}^{n}\prod_{j=1}^{n}\left(1-\left(1-\nu_{A_i}^q\right)^g\left(1-\nu_{A_j}^q\right)^l\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{g+l}}\right)^{\frac{1}{q}} \geq \left(1-\left(1-\left(\prod_{i=1}^{n}\prod_{j=1}^{n}\left(1-\left(1-\nu_{B_i}^q\right)^g\left(1-\nu_{B_j}^q\right)^l\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{g+l}}\right)^{\frac{1}{q}}.$$

According to the score function in Definition 9, we can get

$$q-RPtNoFHM(A_1,A_2,\cdots,A_n) \leq q-RPtNoFHM(B_1,B_2,\cdots,B_n).$$

Definition 12. Let $A_i = \langle(\alpha_i,\sigma_i),(u_i,\eta_i,\nu_i)\rangle$ $(i=1,2,\cdots,n)$ be a collection of q-RPtNoFN, $W=(w_1,w_2,\cdots,w_n)$ be a weight vector of A_i, where $w_i \geq 0$, and $\sum_{i=1}^{n} w_i = 1$. The q-RPtNoFWHM operator is defined as

$$q-RPtNoFWHM(A_1,A_2,\cdots,A_n) = \left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}(A_iw_i)^p \otimes \left(A_jw_j\right)^q\right)^{\frac{1}{p+q}}. \tag{9}$$

Theorem 5. *Let $A_i = \langle(\alpha_i, \sigma_i), (u_i, \eta_i, \nu_i)\rangle$ $(i = 1, 2, \cdots, n)$ be a collection of q-RPtNoFN, then the result obtained by using the q-RPtNoFWHM operator is still a q-RPtNoFN, i.e.,*

$$q - RPtNoFWHM\,(A_1, A_2, \cdots, A_n) \tag{10}$$

$$\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}(w_i\alpha_i)^g\left(w_j\alpha_j\right)^l\right)^{\frac{1}{g+l}},$$

$$\sqrt{\frac{1}{g+l}}\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}(w_i\alpha_i)^g\left(w_j\alpha_j\right)^l\right)^{\frac{1}{g+l}-1}\cdot$$

$$\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}(w_i\alpha_i)^g\left(w_j\alpha_j\right)^l\sqrt{\frac{g\sigma_i^2}{\alpha_i^2}+\frac{l\sigma_j^2}{\alpha_j^2}}\right)$$

$$= \left\langle \left(\left(1-\left(\prod_{i=1}^{n}\prod_{i=1}^{n}\left(1-\left(1-\left(1-\mu_i^q\right)^{w_i}\right)^g\left(1-\left(1-\mu_j^q\right)^{w_j}\right)^l\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q}}\right)^{\frac{1}{g+l}}, \right\rangle$$

$$\left(1-\left(1-\left(\prod_{i=1}^{n}\prod_{i=1}^{n}\left(1-\left(1-\left(\eta_i^{w_i}\right)^q\right)^g\right)\left(1-\left(\eta_j^{w_j}\right)^q\right)^l\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{g+l}}\right)^{\frac{1}{q}},$$

$$\left(1-\left(1-\left(\prod_{i=1}^{n}\prod_{i=1}^{n}\left(1-\left(1-\left(\nu_i^{w_i}\right)^q\right)^g\right)\left(1-\left(\nu_j^{w_j}\right)^q\right)^l\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{g+l}}\right)^{\frac{1}{q}}$$

Proof. Based on the operation laws of q-RPtNoFNs, we can get

$$A_iw_i = \left\langle \alpha_iw_i, \sigma_iw_i, \left(1-\left(1-\mu_i^q\right)^{w_i}\right)^{\frac{1}{q}}, \eta_i^{w_i}, \nu_i^{w_i}\right\rangle,$$

$$(A_iw_i)^g = \left\langle \begin{array}{c} \left((\alpha_iw_i)^g, g^{\frac{1}{2}}(\alpha_iw_i)^{g-1}(\sigma_iw_i)\right), \left(\left(1-\left(1-\mu_i^q\right)^{w_i}\right)^{\frac{1}{q}}\right)^g, \\ \left(1-\left(1-\left(\eta_i^{w_i}\right)^q\right)^g\right)^{\frac{1}{q}}, \left(1-\left(1-\left(\nu_i^{w_i}\right)^q\right)^g\right)^{\frac{1}{q}} \end{array}\right\rangle,$$

$$A_jw_j = \left\langle \alpha_jw_j, \sigma_jw_j, \left(1-\left(1-\mu_j^q\right)^{w_j}\right)^{\frac{1}{q}}, \eta_j^{w_j}, \nu_j^{w_j}\right\rangle$$

$$\left(A_jw_j\right)^l = \left\langle \begin{array}{c} \left(\left(\alpha_jw_j\right)^l, l^{\frac{1}{2}}\left(\alpha_jw_j\right)^{l-1}\left(\sigma_jw_j\right)\right), \left(\left(1-\left(1-\mu_j^q\right)^{w_j}\right)^{\frac{1}{q}}\right)^l, \\ \left(1-\left(1-\left(\eta_j^{w_j}\right)^q\right)^l\right)^{\frac{1}{q}}, \left(1-\left(1-\left(\nu_j^{w_j}\right)^q\right)^l\right)^{\frac{1}{q}} \end{array}\right\rangle,$$

and

$$
\begin{aligned}
&(A_i w_i)^g \otimes \left(A_j w_j\right)^l \\
&\quad (\alpha_i w_i)^g \left(\alpha_j w_j\right)^l, (\alpha_i w_i)^g \left(\alpha_j w_j\right)^l \sqrt{\frac{g\,(\alpha_i w_i)^{2(g-1)}\,(\sigma_i w_i)^2}{(\alpha_i w_i)^{2g}} + \frac{l\left(\alpha_j w_j\right)^{2(l-1)}\left(\sigma_j w_j\right)^2}{\left(\alpha_j w_j\right)^{2l}}} \\
&= \left\langle \begin{bmatrix}
\left(\left(1-(1-\mu_i^q)^{w_i}\right)^{\frac{1}{q}}\right)^g \left(\left(1-(1-\mu_j^q)^{w_j}\right)^{\frac{1}{q}}\right)^l, \\
\left(\left(\left(1-\left(1-(\eta_i^{w_i})^q\right)^g\right)^{\frac{1}{q}}\right)^q + \left(\left(1-\left(1-\left(\eta_j^{w_j}\right)^q\right)^l\right)^{\frac{1}{q}}\right)^q - \left(\left(1-\left(1-(\eta_i^{w_i})^q\right)^g\right)^{\frac{1}{q}}\right)^q \left(\left(1-\left(1-\left(\eta_j^{w_j}\right)^q\right)^l\right)^{\frac{1}{q}}\right)^q\right)^{\frac{1}{q}}, \\
\left(\left(\left(1-\left(1-(\nu_i^{w_i})^q\right)^g\right)^{\frac{1}{q}}\right)^q + \left(\left(1-\left(1-\left(\nu_j^{w_j}\right)^q\right)^l\right)^{\frac{1}{q}}\right)^q - \left(\left(1-\left(1-(\nu_i^{w_i})^q\right)^g\right)^{\frac{1}{q}}\right)^q \left(\left(1-\left(1-\left(\nu_j^{w_j}\right)^q\right)^l\right)^{\frac{1}{q}}\right)^q\right)^{\frac{1}{q}}
\end{bmatrix} \right\rangle \\
&\quad (\alpha_i w_i)^g \left(\alpha_j w_j\right)^l, (\alpha_i w_i)^g \left(\alpha_j w_j\right)^l \sqrt{\frac{g\,(\sigma_i w_i)^2}{(\alpha_i w_i)^2} + \frac{l\left(\sigma_j w_j\right)^2}{\left(\alpha_j w_j\right)^2}} \\
&= \left\langle \begin{bmatrix}
\left(\left(1-(1-\mu_i^q)^{w_i}\right)^{\frac{1}{q}}\right)^g \left(\left(1-(1-\mu_j^q)^{w_j}\right)^{\frac{1}{q}}\right)^l, \\
\left(1-\left(1-(\eta_i^{w_i})^q\right)^g \left(1-\left(\eta_j^{w_j}\right)^q\right)^l\right)^{\frac{1}{q}}, \left(1-\left(1-(\nu_i^{w_i})^q\right)^g \left(1-\left(\nu_j^{w_j}\right)^q\right)^l\right)^{\frac{1}{q}}
\end{bmatrix} \right\rangle
\end{aligned}
$$

Then we use the mathematical induction to get

$$
\begin{aligned}
&\qquad \sum_{i=1}^{n} (\alpha_i w_i)^g \left(\alpha_j w_j\right)^l, \sum_{i=1}^{n} (\alpha_i w_i)^g \left(\alpha_j w_j\right)^l \sqrt{\frac{g\,(\sigma_i w_i)^2}{(\alpha_i w_i)^2} + \frac{l\left(\sigma_j w_j\right)^2}{\left(\alpha_j w_j\right)^2}} \\
&\sum_{i=1}^{n} (A_i w_i)^g \otimes \left(A_j w_j\right)^l = \Bigg\langle \left(1-\prod_{i=1}^{n}\left(1-\left(1-(1-\mu_i^q)^{w_i}\right)^g \left(1-(1-\mu_j^q)^{w_j}\right)^l\right)\right)^{\frac{1}{q}}, \\
&\qquad \prod_{i=1}^{n}\left(1-\left(1-(\eta_i^{w_i})^q\right)^g\right)^{\frac{1}{q}} \left(\left(1-\left(\eta_j^{w_j}\right)^q\right)^l\right)^{\frac{1}{q}}, \prod_{i=1}^{n}\left(1-\left(1-(\nu_i^{w_i})^q\right)^g\right)^{\frac{1}{q}} \left(\left(1-\left(\nu_j^{w_j}\right)^q\right)^l\right)^{\frac{1}{q}} \Bigg\rangle
\end{aligned}
$$

$$
\begin{aligned}
&\qquad \sum_{i=1}^{n}\sum_{j=1}^{n} (\alpha_i w_i)^g \left(\alpha_j w_j\right)^l, \sum_{i=1}^{n}\sum_{j=1}^{n} (\alpha_i w_i)^g \left(\alpha_j w_j\right)^l \sqrt{\frac{g\,(\sigma_i w_i)^2}{(\alpha_i w_i)^2} + \frac{l\left(\sigma_j w_j\right)^2}{\left(\alpha_j w_j\right)^2}} \\
&\sum_{i=1}^{n}\sum_{j=1}^{n} (A_i w_i)^g \otimes \left(A_j w_j\right)^l = \Bigg\langle \left(1-\prod_{i=1}^{n}\prod_{j=1}^{n}\left(1-\left(1-(1-\mu_i^q)^{w_i}\right)^g \left(1-(1-\mu_j^q)^{w_j}\right)^l\right)\right)^{\frac{1}{q}}, \\
&\qquad \prod_{i=1}^{n}\prod_{j=1}^{n}\left(1-\left(1-(\eta_i^{w_2})^q\right)^g\right)^{\frac{1}{q}} \left(1-\left(1-\left(\eta_j^{w_j}\right)^q\right)^l\right)^{\frac{1}{q}}, \prod_{i=1}^{n}\prod_{j=1}^{n}\left(1-\left(1-(\nu_i^{w_2})^q\right)^g\right)^{\frac{1}{q}} \left(1-\left(1-\left(\nu_j^{w_j}\right)^q\right)^l\right)^{\frac{1}{q}} \Bigg\rangle
\end{aligned}
$$

What's more, the following result can be derived

$$\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}(\alpha_i w_i)^g\left(\alpha_j w_j\right)^l,$$
$$\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}(\alpha_i w_i)^g\left(\alpha_j w_j\right)^l\sqrt{\frac{g(\sigma_i w_i)^2}{(\alpha_i w_i)^2}+\frac{l\left(\sigma_j w_j\right)^2}{\left(\alpha_j w_j\right)^2}};$$

$$\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}(A_i w_i)^g\otimes\left(A_j w_j\right)^l=\left\langle\begin{array}{c}\left(1-\left(\prod_{i=1}^{n}\prod_{j=1}^{n}\left(1-\left(1-(1-\mu_i^q)^{w_i}\right)^g\left(1-(1-\mu_j^q)^{w_j}\right)^l\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q}},\\ \left[\begin{array}{c}\left(\prod_{i=1}^{n}\prod_{j=1}^{n}\left(1-\left(1-(\eta_i^{w_i})^q\right)^g\right)^{\frac{1}{q}}\left(1-\left(1-\left(\eta_j^{w_j}\right)^q\right)^l\right)^{\frac{1}{q}}\right)^{\frac{2}{n(n+1)}},\\ \left(\prod_{i=1}^{n}\prod_{j=1}^{n}\left(1-\left(1-(\nu_i^{w_i})^q\right)^g\right)^{\frac{1}{q}}\left(1-\left(1-\left(\nu_j^{w_j}\right)^q\right)^l\right)^{\frac{1}{q}}\right)^{\frac{2}{n(n+1)}}\end{array}\right]\end{array}\right\rangle$$

$$\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}(\alpha_i w_i)^g\left(\alpha_j w_j\right)^l\right)^{\frac{1}{g+l}},\sqrt{\frac{1}{g+l}}\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}(\alpha_i w_i)^g\left(\alpha_j w_j\right)^l\right)^{\frac{1}{g+l}-1}$$
$$\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}(\alpha_i w_i)^g\left(\alpha_j w_j\right)^l\sqrt{\frac{g(\sigma_i w_i)^2}{(\alpha_i w_i)^2}+\frac{l\left(\sigma_j w_j\right)^2}{\left(\alpha_j w_j\right)^2}}\right)$$

$$\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}(A_i w_i)^g\otimes\left(A_j w_j\right)^l\right)^{\frac{1}{g+l}}=\left\langle\begin{array}{c}\left(\left(1-\left(\prod_{i=1}^{n}\prod_{j=1}^{n}\left(1-\left(1-(1-\mu_i^q)^{w_i}\right)^g\left(1-(1-\mu_j^q)^{w_j}\right)^l\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q}}\right)^{\frac{1}{g+l}},\\ \left[\begin{array}{c}\left(1-\left(1-\left(\prod_{i=1}^{n}\prod_{j=1}^{n}\left(1-\left(1-(\eta_i^{w_i})^q\right)^g\right)\left(1-\left(1-\left(\eta_j^{w_j}\right)^q\right)^l\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{g+l}}\right)^{1/q},\\ \left(1-\left(1-\left(\prod_{i=1}^{n}\prod_{j=1}^{n}\left(1-\left(1-(\nu_i^{w_i})^q\right)^g\right)\left(1-\left(1-\left(\nu_j^{w_j}\right)^q\right)^l\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{g+l}}\right)^{1/q}\end{array}\right]\end{array}\right\rangle.$$

Likewise, we can infer that the q-RPtNoFWHM operator has some properties, including monotonicity and boundedness.

5. A MAGDM FOR AGED HEALTHCARE PRODUCT PURCHASE BASED ON q-RPtNoF INFORMATION

In this section, we established the MAGDM method based on the proposed operator under the q-RPtNoF information and illustrate with a numerical example related to aged healthcare product purchase.

5.1. Proposed MAGDM Approach

In the q-RPtNoF environment, let $A = \{A_1, A_2, \cdots, A_n\}$ denote n alternatives, $C^k = \{C_1^k, C_2^k, \cdots, C_m^k\}$ denote the set of m attributes evaluated by the k-th expert, and the attribute weight is $w = \{w_1, w_2, \cdots, w_m\}$. The q-RPtNoF information evaluated by the k-th expert on attribute C_j of alternative A_i is $A_{ij}^k = \left\langle \left(\alpha_{ij}^k, \sigma_{ij}^k\right), \left(u_{ij}^k, \eta_{ij}^k, v_{ij}^k\right) \right\rangle$ $(i = 1, 2, \cdots, n;\ j = 1, 2, \cdots, m;\ k = 1, 2, \cdots, z)$, where, u_{ij}^k denotes the degree to which alternative A_{ij}^k belongs to NFN $\left(\alpha_{ij}^k, \sigma_{ij}^k\right)$ under attribute C_j^k, v_{ij}^k denotes the degree to which alternative A_{ij}^k does not belong to NFN $\left(\alpha_{ij}^k, \sigma_{ij}^k\right)$ under attribute C_j^k, and η_{ij}^k denotes the neutrality degree of alternative A_{ij}^k belonging to NFN $\left(\alpha_{ij}^k, \sigma_{ij}^k\right)$ under attribute C_j^k. The set of $n$alternative and the set of m attribute constitute t decision matrices $D^k = \left(A_{ij}^k\right)_{n\times m}$, and try to determine the ranking of alternatives.

Below gives the steps of the MAGDM process for elderly healthcare products purchase in the q-RPtNoF environment.

Step 1 Normalizing the decision matrix:

To avoid the impaction of different dimensions of attributes on decision results, we should normalize the decision-making matrix $D^k = \left(A_{ij}^k\right)_{n\times m}$ to $\overline{D}^k = \left(\overline{A}_{ij}^k\right)_{n\times m}$.

For benefit-oriented attributes [61]:

$$\overline{\alpha}_{ij}^k = \frac{\alpha_{ij}^k}{\max\limits_i \left(\alpha_{ij}^k\right)}, \overline{\sigma}_{ij}^k = \frac{\sigma_{ij}^k}{\max\limits_i \left(\sigma_{ij}^k\right)} \cdot \frac{\sigma_{ij}^k}{\alpha_{ij}^k}, \overline{u}_{ij}^k = u_{ij}^k, \overline{v}_{ij}^k = v_{ij}^k \tag{11}$$

For cost-oriented attributes [61]:

$$\overline{\alpha}_{ij}^k = \frac{\min\limits_i \left(\alpha_{ij}^k\right)}{\alpha_{ij}^k}, \overline{\sigma}_{ij}^k = \frac{\sigma_{ij}^k}{\max\limits_i \left(\sigma_{ij}^k\right)} \cdot \frac{\sigma_{ij}^k}{\alpha_{ij}^k}, \overline{u}_{ij}^k = u_{ij}^k, \overline{v}_{ij}^k = v_{ij}^k. \tag{12}$$

Step 2 Aggregating the evaluation information for different groups:

Using q-RPtNoFWHM operator, the sets of t group information $\overline{A}_{ij}^k = \left\langle \left(\overline{\alpha}_{ij}^k, \overline{\sigma}_{ij}^k\right), \left(\overline{u}_{ij}^k, \overline{\eta}_{ij}^k, \overline{v}_{ij}^k\right) \right\rangle$ of alternative A_i are aggregated into $\overline{A}_{ij} = \left\langle \left(\overline{\alpha}_{ij}, \overline{\sigma}_{ij}\right), \left(\overline{u}_{ij}, \overline{\eta}_{ij}, \overline{v}_{ij}\right) \right\rangle$.

Step 3 Aggregating information about different attributes:

Using the q-RPtNoFWHM operator, the sets of m attribute information $\overline{A}_{ij} = \left\langle \left(\overline{\alpha}_{ij}, \overline{\sigma}_{ij}\right), \left(\overline{u}_{ij}, \overline{\eta}_{ij}, \overline{v}_{ij}\right) \right\rangle$ of alternative A_i are aggregated into $\overline{A}_i = \left\langle \left(\overline{\alpha}_i, \overline{\sigma}_i\right), \left(\overline{u}_i, \overline{\eta}_i, \overline{v}_i\right) \right\rangle$.

Step 4 By using q-RPtNoFN score function and exact function, the score value $S(A_i)$ and accuracy value $H(A_i)$ of A_i are calculated.

Step 5 Alternatives are ranked based on q-RPtNoFNs sorting rules, and the best alternative is selected.

5.2. Numerical Example

This section illustrates the above stated MAGDM method with an example related to aged healthcare product purchase, which can be read as follows.

Hypertension is a major health problem for the elderly, so many elderly people may buy antihypertensive drugs from the healthcare product market to reduce their blood pressure. There are four products with antihypertensive effect on the market, forming a set of alternatives $A = \{A_1, A_2, A_3, A_4\}$, Four attributes of product are considered as decision criteria, which are product efficacy (C_1), merchant service level (C_2), word-of-mouth (C_3), and price level (C_4), thus an attribute set $C = \{C_1, C_2, C_3, C_4\}$ is formed. The attribute information is normally distributed, and the corresponding weight is $w = \{0.25, 0.2, 0.3, 0.25\}^T$. At the same time, before purchasing antihypertensive drugs, an elderly may pay attention to the opinions of different groups, including the general elderly consumers (K_1), the professional medical staff (K_2), the close relatives and friends (K_3), and the corresponding group weight is $w_k = \{0.35, 0.4, 0.25\}^T$. According to the group decision information, the decision information matrices are constructed as Tables 1–3.

According to Step 1, the decision information in Tables 1–3 is standardized by using Formulas (10) and (11) to obtain standardized data, as shown in Tables 4–6.

According to Step 3, group decision information in Tables 4–6 are aggregated based on q-RPtNoFWHM information aggregator $\left(g = l = 2, q = 3\right)$, as shown in Table 7:

Table 1 Decision information matrix based on group K_1.

	C_1	C_2	C_3	C_4
A_1	<(88, 7), 0.7, 0.6, 0.5>	<(9, 0.8), 0.6, 0.6, 0.8>	<(4, 0.35), 0.4, 0.7, 0.8>	<(48, 4), 0.8, 0.6, 0.5>
A_2	<(60, 5), 0.3, 0.6, 0.8>	<(7, 0.7), 0.8, 0.5, 0.4>	<(5, 0.4), 0.7, 0.4, 0.5>	<(41, 3), 0.5, 0.7, 0.6>
A_3	<(72, 6), 0.3, 0.4, 0.6>	<(6, 0.55), 0.64, 0.35, 0.72>	<(4.5, 0.3), 0.74, 0.45, 0.62>	<(38, 3.2), 0.48, 0.45, 0.74>
A_4	<(92, 8.5)0.48, 0.27, 0.73>	<(8.5, 0.73), 0.38, 0.18, 0.53>	<(3.8, 0.29), 0.53, 0.71, 0.34>	<(51, 4.5), 0.66, 0.27, 0.49>

Table 2 Decision information matrix based on group K_2.

	C_1	C_2	C_3	C_4
A_1	<(66, 5), 0.47, 0.51, 0.6>	<(7, 0.53), 0.54, 0.39, 0.72>	<(5, 0.39), 0.64, 0.44, 0.62>	<(44, 4.1), 0.42, 0.48, 0.6>
A_2	<(69, 6), 0.65, 0.55, 0.53>	<(8, 0.72), 0.55, 0.78, 0.4>	<(4.7, 0.4), 0.55, 0.66, 0.43>	<(49, 4.3), 0.3, 0.5, 0.22>
A_3	<(85, 7.5), 0.45, 0.47, 0.3>	<(7.2, 0.53), 0.45, 0.23, 0.6>	<(3.5, 0.31), 0.43, 0.74, 0.44>	<(53, 4.8), 0.52, 0.32, 0.4>
A_4	<(70, 5.8), 0.44, 0.2, 0.67)	<(8.1, 0.72), 0.51, 0.66, 0.7>	<(4.6, 0.39), 0.61, 0.45, 0.77>	<(50, 4.7), 0.7, 0.7, 0.6>

Table 3 Decision information matrix based on group K_3.

	C_1	C_2	C_3	C_4
A_1	<(76, 6.8), 0.44, 0.47, 0.53>	<(7.8, 0.64), 0.31, 0.54, 0.66>	<(5.2, 0.44), 0.56, 0.45, 0.42>	<(38, 3.2), 0.52, 0.45, 0.64>
A_2	<(83, 7.7), 0.51, 0.51, 0.33>	<(6.2, 0.53), 0.45, 0.23, 0.67>	<(3.5, 0.31), 0.43, 0.74, 0.44>	<(53, 4.8), 0.52, 0.32, 0.45>
A_3	<(88, 7.5), 0.45, 0.34, 0.36>	<(5.2, 0.33), 0.45, 0.64, 0.78>	<(4.3, 0.34), 0.73, 0.55, 0.71>	<(48, 4.2), 0.57, 0.39, 0.44>
A_4	<(85, 7.1), 0.43, 0.43, 0.73>	<(8.2, 0.71), 0.6, 0.3, 0.53>	<(3.8, 0.29), 0.44, 0.71, 0.34>	<(51, 4.5), 0.66, 0.27, 0.49>

Table 4 Standardized information matrix based on group K_1.

	C_1	C_2	C_3	C_4
A_1	<(0.957, 0.066), 0.7, 0.6, 0.5>	<(1, 0.089), 0.6, 0.6, 0.8>	<(0.8, 0.077), 0.4, 0.7, 0.8>	<(0.941, 0.074), 0.8, 0.6, 0.5>
A_2	<(0.652, 0.049), 0.3, 0.6, 0.8>	<(0.778, 0.088), 0.8, 0.5, 0.4>	<((1, 0.08)), 0.7, 0.4, 0.5>	<(0.804, 0.049), 0.5, 0.7, 0.6>
A_3	<(0.783, 0.059), 0.3, 0.4, 0.6>	<(0.667, 0.063), 0.64, 0.35, 0.72>	<(0.9, 0.05), 0.74, 0.45, 0.62>	<(0.745, 0.06), 0.48, 0.45, 0.74>
A_4	<(1, 0.092), 0.48, 0.27, 0.73>	<(0.944, 0.078), 0.38, 0.18, 0.53>	<(0.76, 0.055), 0.53, 0.71, 0.34>	<(1, 0.088), 0.66, 0.27, 0.49>

Table 5 Standardized information matrix based on group K_2.

	C_1	C_2	C_3	C_4
A_1	<(0.775, 0.051), 0.47, 0.51, 0.6>	<(0.864, 0.056), 0.54, 0.39, 0.72>	<(1, 0.076), 0.64, 0.44, 0.62>	<(0.83, 0.08), 0.42, 0.48, 0.64>
A_2	<(0.812, 0.07), 0.65, 0.55, 0.53>	<(0.988, 0.09), 0.55, 0.78, 0.4>	<(0.94, 0.085), 0.55, 0.66, 0.43>	<(0.925, 0.079), 0.3, 0.5, 0.22>
A_3	<(1, 0.088), 0.45, 0.47, 0.33>	<(0.889, 0.054), 0.45, 0.23, 0.67>	<(0.7, 0.069), 0.43, 0.74, 0.44>	<(1, 0.091), 0.52, 0.32, 0.45>
A_4	<(0.824, 0.064), 0.44, 0.2, 0.67)	<(1, 0.089), 0.51, 0.66, 0.7>	<(0.92, 0.083), 0.61, 0.45, 0.77>	<(0.943, 0.092), 0.7, 0.7, 0.6>

Table 6 Standardized information matrix based on group K_3.

	C_1	C_2	C_3	C_4
A_1	<(0.864, 0.079), 0.44, 0.47, 0.53>	<(0.951, 0.074), 0.31, 0.54, 0.66>	<(1, 0.085), 0.56, 0.45, 0.42>	<(0.717, 0.056), 0.52, 0.45, 0.64>
A_2	<(0.943, 0.093), 0.51, 0.51, 0.33>	<(0.756, 0.064), 0.45, 0.23, 0.67>	<(0.673, 0.062), 0.43, 0.74, 0.44>	<(1, 0.091), 0.52, 0.32, 0.45>
A_3	<(1, 0.083), 0.45, 0.34, 0.36>	<(0.634, 0.029), 0.45, 0.64, 0.78>	<(0.827, 0.061), 0.73, 0.55, 0.71>	<(0.906, 0.077), 0.57, 0.39, 0.44>
A_4	<(0.966, 0.077), 0.43, 0.43, 0.73>	<(1, 0.087), 0.6, 0.3, 0.53>	<(0.731, 0.05), 0.44, 0.71, 0.34>	<(0.962, 0.083), 0.66, 0.27, 0.49>

Table 7 Group decision information matrix.

	C_1	C_2	C_3	C_4
A_1	<(0.242, 0.051), 0.374, 0.62, 0.7>	<(0.262, 0.06), 0.361, 0.662, 0.787>	<(0.26, 0.082), 0.368, 0.669, 0.766)>	<(0.237, 0.048), 0.404, 0.693, 0.702>
A_2	<(0.221, 0.075), 0.348, 0.687, 0.714)>	<(0.24, 0.065), 0.435, 0.716, 0.63>	<(0.254, 0.047), 0.4, 0.662, 0.636)>	<(0.252, 0.075), 0.301, 0.685, 0.594>
A_3	<(0.258, 0.082), 0.271, 0.636, 0.607>	<(0.209, 0.053), 0.351, 0.522, 0.759>	<(0.225, 0.048), 0.45, 0.716, 0.659>	<(0.248, 0.077), 0.347, 0.583, 0.675>
A_4	<(0.258, 0.06), 0.3, 0.51, 0.707>	<(0.275, 0.074), 0.338, 0.598, 0.734>	<(0.229, 0.058), 0.366, 0.685, 0.681>	<(0.272, 0.067), 0.46, 0.619, 0.673>

According to Step 4, the attribute information in Table 7 is aggregated by using the q-RPtNoFWHM information aggregator, and the comprehensive q-RPtNoFN of each alternative is obtained as follows.

$$\overline{A}_1 = \langle (0.055, 0.014), 0.23, 0.771, 0.86 \rangle;$$

$$\overline{A}_2 = \langle (0.054, 0.015), (0.231, 0.781, 0.762) \rangle;$$

$$\overline{A}_3 = \langle (0.052, 0.015), (0.227, 0.752, 0.772) \rangle;$$

$$\overline{A}_4 = \langle (0.056, 0.014), (0.232, 0.744, 0.786) \rangle.$$

Then the score values of each alternative are calculated using the q-RPtNoFN score function, respectively:

$$S(A_1) = -0.0535;\ S(A_2) = -0.0485$$
$$S(A_3) = -0.0456;\ S(A_4) = -0.05$$

According to Step 5, based on the score value of each alternative, the ranking of four alternatives is $A_3 > A_2 > A_4 > A_1$. As such, the best alternative is A_3. Therefore, when the elderly buy antihypertensive products, A_3 is the best.

5.3. Sensitivity Analysis

In the q-RPtNoFWHM operator proposed in this paper, the group experts' weight w_k and parameters $g, l,\ q$ are involved. The values of different parameters have a certain influence on the decision results. In this section, the influences of the above parameters on the decision results are discussed.

The influence of group weight w_k on decision-making results is discussed. Different w_k values have different effects on the ranking of alternatives, as shown in Table 8.

According to Table 8, different group weights have a great influence on alternative ranking. When $w_1 = 0.9$, $w_2 = 0.05$, $w_3 = 0.05$, i.e., when the elderly pay more attention to the opinions of ordinary elderly consumers (K_1), when they buy antihypertensive drugs, the ranking of four drugs is $A_4 > A_3 > A_2 > A_1$, A_4 is the best choice. When $w_3 = 0.05$, $w_2 = 0.9$, $w_3 = 0.05$, i.e., when they value the opinions of the professional medical staff (K_1), the ranking of four drugs is $A_3 > A_2 > A_1 > A_4$, A_3 is the best choice;when they put more emphasis on the opinions of their friends and relatives (K_3), the ranking is $A_3 > A_2 > A_4 > A_1$, A_3 is the best choice.

The influence of the change of parameters g, l on the ranking of alternatives is discussed, and the influence of the change of parameters g, l on the ranking of alternatives and the score value of each alternative is analyzed, as shown in Table 9 and Figures 1–6.

According to Table 9, the change of parameters g, l has a great influence on the ranking of alternatives. When $g = l = 0.2$, $A_1 > A_3 > A_2 > A_4$, then A_1 is the best; when $g = l = 0.5$ or 1, $A_3 > A_2 > A_4 > A_1$,then A_3 is the best. When the values of g, l are different, $g = 2$, $l = 0.2$, $A_2 > A_3 > A_4 > A_1$, then A_2 is the best; when $g = 0.2$, $l = 2$, $A_3 > A_4 > A_2 > A_1$, then A_3 is the best; when $g = 1$, $l = 9$, then A_4 is the best.

Furthermore, according to Figures 1–6, when one of the values of g or l is fixed and $q = 3$, the change of g or l has a great influence on the ranking of the four alternatives. In addition, when $q = 3$, g and l change at the same time, the score value of each alternative changes with it, and the value changes from big to small. It is thus clear that g and l have a more sensitive change in the ranking and score value of the alternative. Therefore, the elderly can adjust values of g and l according to their preferences in the actual decision-making process when purchasing antihypertensive drugs, and obtain the corresponding decision-making results.

Furthermore, change of q also has a certain influence on the sorting. Therefore, the influence of the change of q on the ranking and score of the four alternatives is further discussed. The results are shown in Figure 7, when $g = l = 2$, $q \in (1, 3)$, it has no great influence on the overall sorting of the four alternatives, indicating that the operator proposed in this paper has good stability. In addition, it is worth noting that when q changes from large to small, the score values of the four alternatives also show a trend of change from small to large.

Table 8 The influence of group weight w_k on alternative ranking ($q = 3$).

The Value of w_k	The Score of A_i	The Ranking Result
$w_1 = 0.9, w_2 = 0.05, w_3 = 0.05$	$S(A_1) = -0.0302; S(A_2) = -0.0263; S(A_3) = -0.0256; S(A_4) = -0.0248$	$A_4 > A_3 > A_2 > A_1$
$w_1 = 0.8, w_2 = 0.05, w_3 = 0.2$	$S(A_1) = -0.0486; S(A_2) = -0.0416; S(A_3) = -0.0407; S(A_4) = -0.0404$	$A_4 > A_3 > A_2 > A_1$
$w_1 = 0.05, w_2 = 0.9, w_3 = 0.05$	$S(A_1) = -0.0277; S(A_2) = -0.0258; S(A_3) = -0.0238; S(A_4) = -0.0288$	$A_3 > A_2 > A_1 > A_4$
$w_1 = 0.333, w_2 = 0.333, w_3 = 0.333$	$S(A_1) = -0.0536; S(A_2) = -0.0483; S(A_3) = -0.0457; S(A_4) = -0.0498$	$A_3 > A_2 > A_4 > A_1$
$w_1 = 0.05, w_2 = 0.05, w_3 = 0.9$	$S(A_1) = -0.0283; S(A_2) = -0.0253; S(A_3) = -0.0239; S(A_4) = -0.0269$	$A_3 > A_2 > A_4 > A_1$

Table 9 Influence of change of parameters g, l on the alternatives ranking ($q = 3$).

The Value of g, l	The Score of A_i	The Ranking Result
$g = l = 0.2$	$S(A_1) = -0.003; S(A_2) = -0.0027;$ $S(A_3) = -0.0025; S(A_4) = -0.0028$	$A_1 > A_3 > A_2 > A_4$
$g = l = 0.5$	$S(A_1) = -0.02; S(A_2) = -0.0178;$ $S(A_3) = -0.0167; S(A_4) = -0.0188$	$A_3 > A_2 > A_4 > A_1$
$g = l = 1$	$S(A_1) = -0.0383; S(A_2) = -0.0344;$ $S(A_3) = -0.0323; S(A_4) = -0.036$	$A_3 > A_2 > A_4 > A_1$
$g = l = 5$	$S(A_1) = -0.067; S(A_2) = -0.0618;$ $S(A_3) = -0.0577; S(A_4) = -0.0609$	$A_3 > A_4 > A_2 > A_1$
$g = 2, l = 0.2$	$S(A_1) = -0.0155; S(A_2) = -0.0132;$ $S(A_3) = -0.0133; S(A_4) = -0.0152$	$A_2 > A_3 > A_4 > A_1$
$g = 0.2, l = 2$	$S(A_1) = -0.0142; S(A_2) = -0.0139;$ $S(A_3) = -0.0125; S(A_4) = -0.0133$	$A_3 > A_4 > A_2 > A_1$
$g = 1, l = 9$	$S(A_1) = -0.0203; S(A_2) = -0.0197;$ $S(A_3) = -0.0171; S(A_4) = -0.0162$	$A_4 > A_3 > A_2 > A_1$
$g = 9, l = \mathrm{q}$	$S(A_1) = -0.0175; S(A_2) = -0.0155;$ $S(A_3) = -0.0151; S(A_4) = -0.0186$	$A_3 > A_2 > A_4 > A_1$

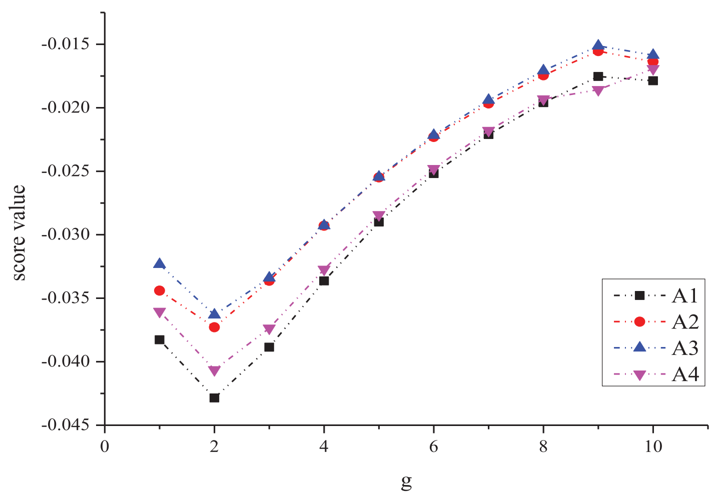

Figure 1 Ranking change of alternatives $A_i(i = 1,2,3,4)$ when $I = 2$, $q = 3$ and $g \in (1, 10)$.

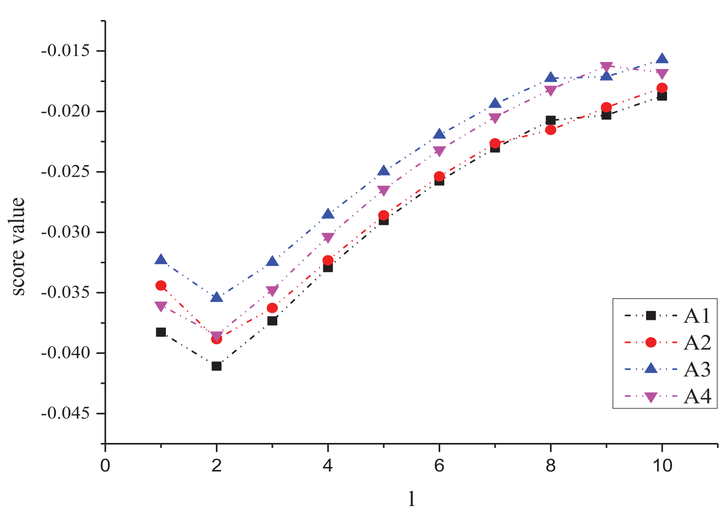

Figure 2 Ranking change of alternatives $A_i(i = 1,2,3,4)$ when $g = 2$, $q = 3$ and $l \in (1, 10)$.

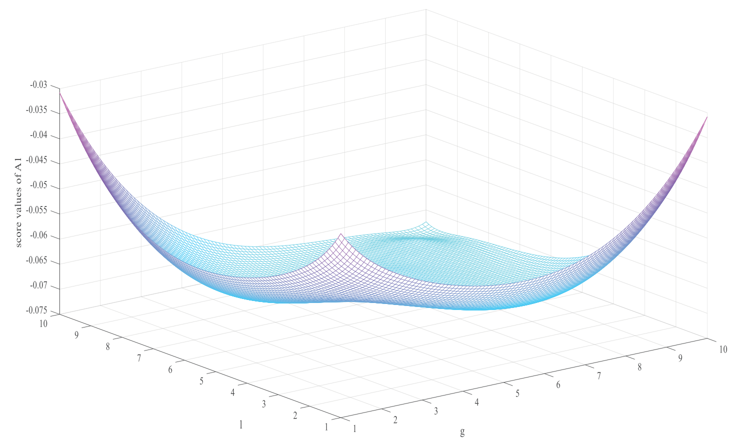

Figure 3 Scores of alternatives A_1 when $g, l \in (1, 10)$ and $q = 3$.

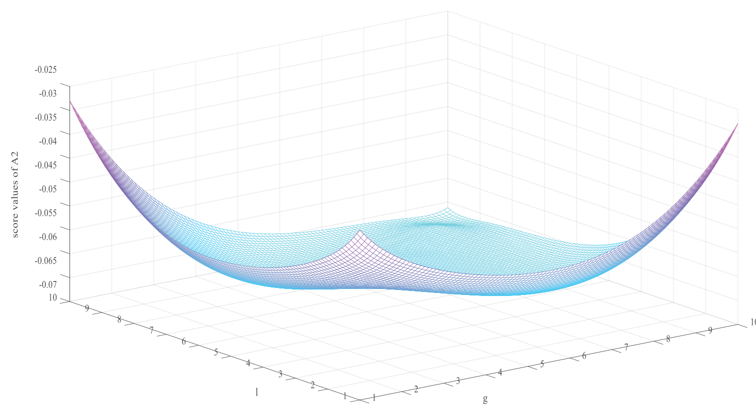

Figure 4 Scores of alternatives A_2 when $g, l \in (1, 10)$ and $q = 3$.

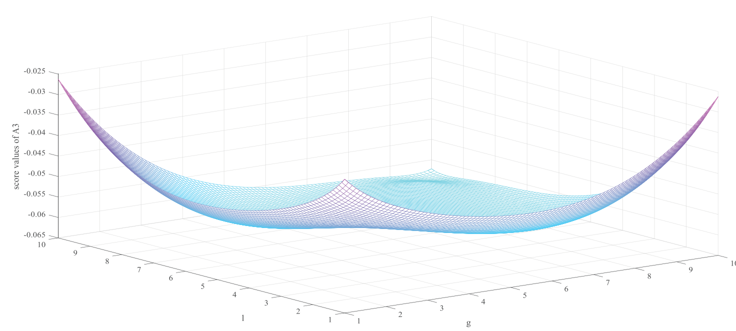

Figure 5 Scores of alternatives A_3 when $g, l \in (1, 10)$ and $q = 3$.

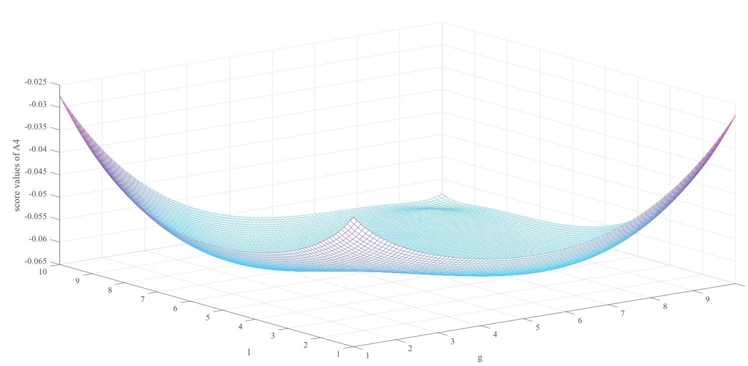

Figure 6 Scores of alternatives A_4 when $g, l \in (1, 10)$ and $q = 3$.

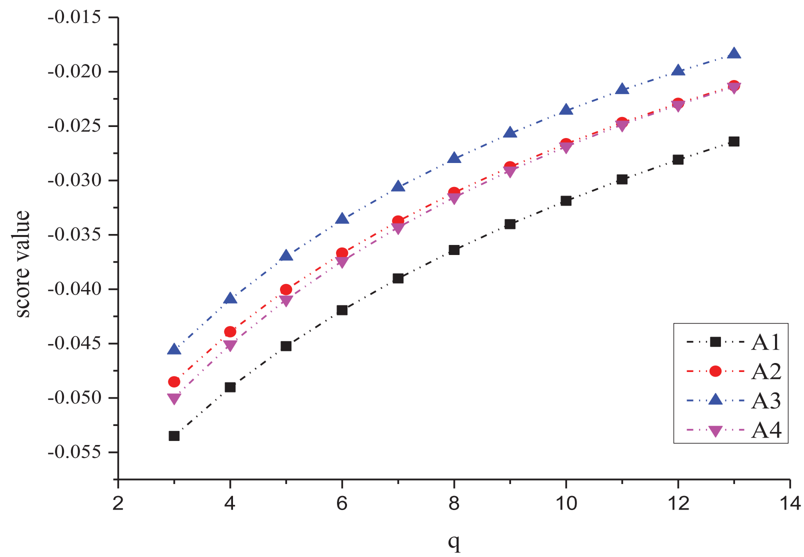

Figure 7 Ranking change of alternatives $A_i (i = 1, 2, 3, 4)$ when $l = 2$, $g = 2$ and $q \in (3, 13)$.

5.4. Comparative Analysis

Since the q-RPtNoFSs in this paper is proposed to describe the fuzzy information, the existing information aggregators cannot be directly used to aggregate the proposed q-RPtNoF information. For this reason, in order to make a reasonable comparison with the existing methods, the Bonferroni mean operator involved in the method proposed by Liu and Liu [62] is applied to the q-RPtNoF environment for information aggregation, and the parameters of Bonferroni mean operator are $p = q = 2$, then the best alternative is still A_3. Similarly, the Dombi Hamy Mean operator ($\lambda = 2$) in the method proposed by He *et al.* [56] is applied to the q-RPtNoF environment to get the ranking of alternatives, the best alternative is still A_3, which is the same as the best choice based on our method. It indicates the rationality of the operators proposed in this paper.

However, compared with the fuzzy sets proposed by Liu and Liu [62], Wang *et al.* [42], Wang *et al.* [31] and Yang and Zhu [18], q-RPtNoFSs take into account the multiple answers of decision-makers to attribute evaluation and can describe the fuzzy information more broadly. Compared with PtFS proposed by Cuong and Kreinovich [48] and Cuong [49], the q-RPtNoFSs takes into account the normal distribution of attribute information, further characterizes human social activities and natural phenomena, and is closer to human decision-making thinking. Compared with the q-Rung Picture Linguistic proposed by Li *et al.* [55] and q-RPtF Dombi Hamy Mean Operators proposed by He *et al.* [56] and T-Spherical Fuzzy Power Muirhead Mean Operators proposed by Liu *et al.* [57], the operators proposed in this paper take into account the opinions of heterogeneous groups and describe the correlation between different groups and attributes. In addition, in this paper, the preferences of different groups of opinions are considered, and the information aggregator is proposed to obtain different decision results according to different parameters, so the method in this paper has greater flexibility.

6. CONCLUSIONS

This paper discussed the case in which the decision-maker gives a variety of answer types in the actual decision-making process, and the sum of membership value for each answer is greater than 1, but the sum of q-power of them is less than 1 is considered. To this end, in this paper, the concepts of NFN and PtFS and q-ROFS were integrated, the concept of q-RPtNoFS is proposed, some basic theories of q-RPtNoFS were defined, q-RPtNoFHM operator and q-RPtNoFWHM operator were proposed and applied. The method proposed in this paper has the following advantages:

1. The method combines the concepts of the NFN and PtFs and q-ROFS, and puts forward the concept of q-RPtNoFS. The q-RPtNoFS not only interprets the information of normally distributed from human production activities and natural phenomena, but also describes multiple types of answer information for evaluating the same attribute. What's more, the q-RPtNoFS describes the characteristics that the sum of MBDs of different types of answers is greater than 1, but the sum of q-power of them is less than 1, which more broadly depicts the fuzzy information, and is closer to human decision-making thinking.
2. For the q-RPtFNoFWHM operator proposed in this paper, the decision-maker can adjust the values of parameters according to the subjective preference to obtain different alternatives. Therefore, the method proposed in this paper has strong flexibility.
3. Since the method in this paper considers the heterogeneous group opinion and the relationship between them, the decision-maker can get different decision results according to the group preference.

There are still many extensions to be made for this paper. In terms of basic theory, the concept of interval q-RPtNoFS and related theories can be further proposed, such as determining the similarity measurement method of q-RPtNoFS. In terms of information aggregation, it can be extended to the information aggregation model based on Muirhead Mean Operator or Einstein. In terms of application, it can be extended to the supply chain partner cooperation, logistics system or brain hemorrhage [63,64].

CONFLICTS OF INTEREST

The authors declared that they have no conflicts of interest to this work.

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