2012/45

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DISCUSSION PAPER

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United we stand? Coordinating capacity investment and allocation in joint ventures

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November 2012

Abstract

Among the recent innovative strategies for coping with product variety and market risk some firms have partnered to leverage economies of scale and risk pooling by sharing manufacturing capacity. In this paper we study how to structure such a joint venture to achieve full efficiency at low transaction costs. Specifically, we study whether capacity should be owned jointly or separately. Overall, we find that the two ownership structures have complementary strengths and weaknesses in term of their incentives for coordinating capacity allocation and investment. On the one hand, capacity allocation is simple to coordinate under joint ownership, but may entail high transaction costs under separate ownership when the joint venture consists of many firms with different profit margins. On the other hand, capacity investments remain simple to coordinate under separate ownership, but are efficient under joint ownership only in the presence of large economies of scale or asymmetric demands or asymmetric profit margins, and would otherwise entail high transaction costs. Our analysis thus characterizes the trade-off between economic benefits and transaction costs in the choice of capacity ownership structure.

Keywords: joint ventures, non-cooperative game theory, newsvendor model, economies of scale, capacity ownership.

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1 Introduction

Product proliferation and the shortening of their lifecycle have made it more challenging for manufacturing firms to deliver the right product in the right quantity at the right time. To cope with this increased product variety and market risk, some firms have sought to leverage economies of scale and of risk pooling by investing in joint production capacity with other firms that use similar productive resources. For instance, Ford and Volkswagen jointly invested in the early 90s in a Latin-American joint venture, called Autolatina, with the goal of sharing the risk of operating in that volatile market and supporting a wide model range (Cavusgil et al. 2007). As another example, Syncrude, the world's largest producer of synthetic crude oil from oil sands, was created to offer its founding partners (including large and financially strong firms such as Imperial Oil) an opportunity to share capital demands and risks (Herd 2010). The trend towards forming alliances seems to have been growing, at least in the automotive industry (*The Economist* 2012).

Although joint operations may create many economic benefits, it also creates additional competition for manufacturing capacity, on top of the product competition in the market. For instance, Volkswagen, which accounted for the lion's share of Autolatina's output, blocked Ford's plans to introduce a small car through the joint venture, thereby pushing Ford to produce a small car on its own (Jackson and Turner 1994), and ultimately leading to the dissolution of the joint venture despite year-after-year profitability (Cavusgil et al. 2007). More recently, Ford, in its joint venture with Mazda and Changan, was accused of allocating too much capacity to its own cars and to prevent Mazda from meeting its fast-growing demand (Cheng 2010).

Although some may recommend to "be careful who you marry, and have a good prenuptial agreement" (Bradsher 1997), aligning everyone's interests for sharing capacity could require incommensurable effort. Aligning incentives for capacity allocation is indeed nontrivial. For instance, complex rules need to be adopted to align capacity allocation priorities in the Hunter Valley Coal Chain, a consortium in charge of the world's largest coal export operation (HVCC 2009). When coordinating decisions to achieve full efficiency turns out to be too complicated to be contractually specified, i.e., entails high transaction costs, economic benefits may need to be sacrificed, even though such benefits are often the primary reason for which joint ventures are created.

Is it possible to achieve the best of both worlds, i.e., to adopt simple and efficient coordinating mechanisms for sharing capacity? The existence of such mechanisms depends on the structure of capacity ownership. As Hansmann (1988) puts it, the costs of collective choice mechanisms are crucial in determining the efficiency of alternative assignments of ownership.

Should capacity be owned jointly or separately? Under joint ownership (JO), each unit of capacity is jointly owned by all partners, in proportion to their investments. (See Figure 1, left, for an illustration.) Accordingly, capacity allocation decisions must be approved by all partners. For instance at Syncrude, each owner has a specific undivided co-ownership interest in the assets of the joint venture (Syncrude 2012). By contrast under separate ownership (SO), each unit of capacity is owned by one firm only, and that firm is the only one entitled to choose how to use that unit of capacity. (See Figure 1, right, for an illustration.) For instance at Sevel-Nord, an alliance between Fiat and PSA, each partner owns half the production capacity and can decide to use it for its own products or for its partner's products in case of capacity shortage (Bidault and Schweinsberg 1996).

In this paper, we consider a joint venture (JV) between manufacturing firms that combine their resources to leverage economies of scale and risk pooling. Although there are various reasons for setting up a JV such as reinforcing competitive positions or developing joint organizational learning (Kogut 1988), we focus here on economies of scale and risk pooling. Moreover, we consider a situation in which firms keep the management of their product lines separate, either because profit margins are difficult to contract on or because firms may not want or be allowed to reveal them, let alone share them, for competitiveness or antitrust reasons. We study whether firms should own the JV's capacity jointly or separately, so as to align everyone's interests in investing and allocating capacity and achieve the full economic benefits of capacity sharing at low transaction costs.

Figure 1: Joint vs. Separate Ownership.

We model the JV operations as a two-stage game, similar to the newsvendor model (Van Mieghem 1999). In the first stage, firms jointly invest in production capacity without knowing demand. In the second stage, demand is realized and firms must decide how to allocate the joint production capacity. Because of demand uncertainty, capacity rationing will inevitably occur, thereby creating competition for manufacturing capacity. Coordinating capacity investment and allocation decisions at low transaction costs will be key to the long-term viability of the JV.

We find that SO arrangements may be associated with high transaction costs for coordinating capacity allocation when the JV consists of many firms with different profit margins. With only two firms, or when the firms have identical profit margins, as is the case at Sevel-Nord, capacity allocation can be orchestrated at low transaction costs. By contrast, capacity sharing under JO is always conceptually simple to administer even with a large number of firms.

However, JO may lead to capacity overinvestment, unless there exist sufficiently large economies of scale in production or capacity investment; or, if the demand for the lower profit margin products is sufficiently large; or, if the spread in profit margins between products is sufficiently important. If those conditions are met, as appears to be the case at Syncrude, full efficiency is achieved by having the JV operate on a non-profit basis. By contrast, SO arrangements lead to simple investment coordination, provided that production capacity allocation decisions are also coordinated.

Consequently, SO and JO arrangements appear to be complementary in strengths and in weaknesses: coordinating capacity allocation seems to be SO's main weakness and JO's main strength, and coordinating capacity investments seems to be SO's main strength and JO's main weakness.

The paper is organized as follows. We review the relevant literature in the next section and introduce the model in Section 3. Sections 4 and 5 characterize the equilibrium in the capacity investment and allocation games, respectively for the SO and the JO structures, and discuss when a coordinating mechanism achieves full efficiency at low transaction costs. We present our conclusions in Section 6. All proofs appear in the appendix.

2 Literature Review

JVs have generated a lot of academic research from the perspective of economics and strategic management theory because of their hybrid nature. In comparison, the management of their operations has received only limited attention. We first briefly review the economics and management literature and then review the operations management literature on outsourcing, mergers, inventory transshipments, and inventory centralization games.

Economics and Strategy of Joint Ventures. According to transaction cost economics, the emergence of JVs results from a trade-off between savings in production costs (e.g., due to scale of operations) and transaction costs (e.g., expenses for writing and enforcing contracts); see Kogut (1988). In this paper, we offer an operational perspective on transaction cost theory by showing that contractual complexity may grow exponentially with the number of JV partners. We study when it is feasible to achieve full economic efficiency with simple coordinating mechanisms, but note that trade-offs inevitably arise in other circumstances.

Hennart (1988) makes the distinction between equity JVs, when "two or more sponsors bring given assets to an independent legal entity and are paid for some or all of their contributions from the profits earned by the entity" and non-equity JVs, which consist of contractual arrangements such as supply agreements. Similarly, the International Financial Reporting Standards classify "joint arrangements" either as "joint ventures, [which] are arrangements in which the parties have rights to an investment," or as "joint operations, for which the parties have rights to the assets and obligations for the liabilities" (IFRS 2011). Consistent with those distinctions, we consider two forms of ownership, namely joint ownership and separate ownership, which mimic these two forms of joint arrangements.

Harrigan (1988) proposes a classification of JVs. According to her framework, we consider here a situation with high market uncertainty and either stagnating demand, leading firms to create a horizontal JV so as to consolidate capacity, or rapidly growing demand, leading firms to share supply capacity until critical mass is reached. Kogut (1988) indeed argues that one of the primary reasons for setting up a JV is to "take advantage of economies of scale and diversify risk."

Outsourcing. Considering two newsvendors separately owning their capacity, Van Mieghem (1999) studies when their capacity investment and production decisions can be coordinated, when one newsvendor has the option to outsource its production to the other. The key economic tension in this outsourcing model arises from demand uncertainty and the benefits of risk pooling (Van Mieghem 2003, Jordan and Graves 1995). Similarly here, demand uncertainty is at the core of competition for capacity, although risk pooling may not be the primary driver behind the creation of a JV. It is found that full coordination with outsourcing is in general not attained with fixed-price contracts and incomplete contracts, but can be achieved with state-dependent prices. In contrast to outsourcing arrangements in which capacity transfers are unidirectional, we consider here a situation in which capacity transfers can occur in either direction, thus offering even greater potential for risk pooling. Unlike Van Mieghem (1999), we find that coordination can be achieved with fixed-price contracts due to the bidirectional nature of transfers, but that their contractual complexity grows exponentially with the number of partners.

Mergers. A JV is a form of horizontal merger. Focusing on economies of scale in production, Cho (2011) and Cho and Wang (2012) study the impact of mergers on firms' profits and consumer prices, respectively without and with demand uncertainty. Zhu et al. (2012) empirically find that mergers improve profits mostly through increased market power but do not seem to result in increased operational efficiency. In contrast to this stream of research, which has primarily focused on the changes in industry competitiveness before and after a merger, we focus on the management of the operations of a particular alliance.

Inventory Transshipments. Considering independent newsvendors separately owning inventory, Anupindi et al. (2001) study how to coordinate *ex-post* inventory transshipments to reduce the local mismatches between supply and demand. In order to ensure participation of the firms in the transshipment process, they propose a "dual allocation rule" based on the dual solution of the corresponding transshipment optimization problem. Implementing the dual allocation rule remains challenging, however, as retailers may want to withhold their inventory (Granot and Sošić 2003) or may converge to a suboptimal equilibrium (Suakkaphong and Dror 2011). Instead of having recourse to an *ex-post* allocation rule, Rudi et al. (2001) and Hu et al. (2007) propose to coordinate transshipment decisions with *ex-ante* transshipment prices in a two-retailer distribution system. Dong and Rudi (2004), Zhang (2005), and Krishnan et al. (2011) consider more general distribution networks, with or without centralized chain store downstream. Huang and Sošić (2011) show that the dual allocation rule outperforms transshipment prices when retailers are asymmetric and that the opposite holds otherwise.

Despite their similarities, capacity and inventory sharing cannot be managed in the same way because production capacity is inherently more flexible than inventory. In particular, capacity is completely undifferentiated before it is used whereas inventory's usage costs may depend on who bought it and where it is stored. Consequently with capacity, it may not be optimal to satisfy the local demands first before satisfying the excess demand of other products, which is a necessary condition for the applicability of the dual allocation rule (Suakkaphong and Dror 2011). In fact, we show that operating capacity in the same way as inventory, i.e., sharing only the leftover capacity to satisfy the residual demand, can result in a substantial loss of efficiency.

Inventory Centralization. Similar to this paper, Hartman et al. (2000), Müller et al. (2002), Plambeck and Taylor (2005), and Kemahlıoğlu-Ziya and Bartholdi (2011) consider a situation in which the full benefits of risk pooling are reaped, with multilateral transfers of undifferentiated units of capacity. Yu et al. (2011) consider a similar setup with queuing systems. Adopting a cooperative game-theoretic framework, this literature studies how to allocate the benefits of pooling to make the coalition stable.

However, JVs are often subject to opportunistic behavior and internal conflicts despite being paved with the best intentions.[1] In fact, a JV's performance – and survival – is significantly affected by the tensions between cooperative and noncooperative behaviors (Park and Russo 1996, Kumar 2010). It is thus critical to understand the strategic tensions between the partners before setting up a JV. Reporting that more than half of JVs fail, Bamford et al. (2004) indeed argue that most JV failures could have been prevented had more time been spent upfront to better align the partners' strategic objectives. Luo (2002) confirms that contracting and cooperation have a complementary, and not substitute, impact on JV performance, since contracts provide an institutional framework for guiding the course of cooperation. Accordingly, we adopt in this paper a non-cooperative game-theoretic approach to study the strategic interactions arising in the creation and operation of the JV. Compared to cooperative game theory, our focus is on the enforceability of decisions at low transaction costs, and not on stability. Our approach can therefore be viewed as complementary to cooperative game theory, precisely delving into the contractual details that are typically assumed away in cooperative game-theoretic models (Plambeck and Taylor 2005, p. 140).

3 Model

We consider a JV between $n \geq 2$ firms that independently manage their product lines. For ease of exposition, we assume that each firm produces only one product, sold on distinct markets and using one single flexible piece of production equipment. For instance at Sevel-Nord, Fiat and PSA made almost identical vehicles, sold under the firms' respective brands through their respective distribution networks, with a large portion of their demand based in their respective home markets (Bidault and Schweinsberg 1996). Similarly at Autolatina, Volkswagen focused on the subcompact car segment whereas Ford focused on the midsize car segment and they kept separate distribution channels in Brazil (Cavusgil et al. 2007). This simple setting, which can be interpreted as a high-level, "black-box" description of a complex newsvendor network production facility (Van Mieghem and Rudi 2002), also captures most of the benefits of flexibility.

We model the JV's operations as a two-period game, akin to the newsvendor model, as it frequently occurs in the high-tech and the automotive industries (e.g., Jordan and Graves 1995, Van Mieghem 1999). In the first period, the JV partners decide on their investments in a common resource without knowing the demand for their products. In the second period, demand is realized and the firms choose how to allocate the joint capacity so as to fulfill their respective demands.

[1] For instance, EADS is renowned for being prone to endless conflicts between its two parents, the French and the German governments (Reiermann 2007). Even Renault-Nissan, which is often reckoned as one of the most successful alliances, has also been subject to internal jealousy (*The Economist* 2011).

In reality, this second period can consist of N production planning periods. Capacity investment decisions are indeed often strategic, e.g., are made once every decade, whereas production planning decisions are tactical, e.g., are made on a monthly basis. In that case, the probability of a particular demand realization can be interpreted as the expected frequency of that demand realization over the N planning periods. With multiple planning periods, it is even more important to structure the JV so as to coordinate capacity allocation decisions since each planning period could potentially give rise to a new conflict.

Let y_i be the effective capacity provided by Firm i in the capacity investment stage, and let $y = \sum_{i=1}^{n} y_i$ be the total effective capacity. As is common in many JVs, capacity investments may exhibit economies of scale. Accordingly, let $K(y)$ be the total cost to install capacity y, with $K(0) = 0$, $K'(y) > 0$, and $K''(y) \leq 0$.

After the investment stage, demand $\mathbf{D} = (D_1, \ldots, D_n)$ is realized. Demand is assumed to follow a multivariate continuous distribution $F_{1,\ldots,n}(\boldsymbol{\xi}) = \mathbb{P}[D_1 \leq \xi_1, \ldots, D_n \leq \xi_n]$, finite with probability 1, with expectation $\mathbb{E}[\mathbf{D}]$. Let us denote $F_i(\xi) = \mathbb{P}[D_1 \leq \infty, \ldots, D_{i-1} \leq \infty, D_i \leq \xi, D_{i+1} \leq \infty, \ldots, D_n \leq \infty]$ as the distribution of D_i, $\bar{F}_i(\xi) = 1 - F_i(\xi)$ as its complementary distribution, and $f_i(\xi) = F_i'(\xi)$ as its density; similarly, let $F(\xi) = \mathbb{P}[\sum_{i=1}^{n} D_i \leq \xi]$, $\bar{F}(\xi) = 1 - F(\xi)$, and $f(\xi) = F'(\xi)$ be the corresponding quantities for the total demand $D = \sum_{i=1}^{n} D_i$. With these notations, a shortage of capacity will occur if $D \geq y$. Demands are not restricted to be independent; in particular, the JV could benefit from risk pooling with negative correlation.

Upon observing their demands, the firms decide how to allocate capacity to maximize their profits. Let x_i be the quantity of Product i produced and $x = \sum_{i=1}^{n} x_i$ be the total production quantity. Similar to the literature on outsourcing (e.g., Van Mieghem 1999), inventory sharing (e.g., Anupindi et al. 2001) and inventory centralization games (e.g., Hartman et al. 2000), we assume exogenous prices and denote v_i as Product i's gross profit margin exempt of production costs. As is common in many JVs, production costs may exhibit economies of scale. Let $c(y)$ be the unit production cost if y units of capacity have been invested, with $c'(y) \leq 0$. Without loss of generality, we assume that products are ranked in decreasing order of profitability, i.e., $v_i > v_{i+1}$. (We shall also discuss the degenerate case in which $v_i = v_{i+1}$.) For simplicity, we assume that each unit of product uses exactly one unit of capacity. (This assumption is without loss of generality if $c'(y) = 0$.) To eliminate trivial cases, we assume that all products are profitable to make, i.e., $v_n \geq c(y)$, once capacity investments are sunk.

Throughout this paper, we assume perfect information. It turns out that, if the contractual mechanism coordinates production decisions, firms will be willing to truthfully reveal their demand, so our analysis is robust to that assumption. In addition, we consider a situation in which firms do not want or are not allowed to share their profit margins, either for competitiveness or antitrust reasons. In fact, profit margins are often hard to estimate, especially when they include non-manufacturing costs (e.g., distribution) or opportunity costs (e.g., growth in market share). Consequently, we only consider contracts that do not make payments dependent on profit margins.

Moreover, we assume that all actions of the firms, namely capacity investment and allocation decisions, are contractible, i.e., there is no double moral hazard, unlike Marinucci (2009) and Roels et al. (2010). Accordingly, there always exists, in principle, a contract that can attain full efficiency under decentralized decision-making. However, such contract may entail high transaction costs. We next characterize the system-optimal capacity investment and allocation decisions, which will serve as a natural benchmark to achieve full efficiency in the JV operations at low transaction costs.

3.1 First-Best Decisions

Let $\Pi(\mathbf{x}; \mathbf{D}, y)$ be the JV's profit, for a given production vector $\mathbf{x}$, demand realization $\mathbf{D}$, and capacity y. The first-best (FB) production plan, denoted $\mathbf{x}^*(\mathbf{D}, y)$, maximizes the JV's total profit, i.e., solves

$$\begin{array}{rll} \max_{\mathbf{x}} \Pi(\mathbf{x};\mathbf{D},y) = & \sum_{j=1}^{n}(v_j - c(y))x_j - K(y) & \\ \text{subject to} & \sum_{j=1}^{n} x_j \le y & \\ & 0 \le x_i \le D_i & \forall i. \end{array} \tag{1}$$

Because (1) is a continuous knapsack problem, $x_i^*(\mathbf{D}, y) = \min\{D_i, y - \sum_{j<i} x_j^*(\mathbf{D}, y)\}$ for all i. Let $S_i(y)$ denote the expected sales for Product i if the FB production plan is implemented, i.e., $S_i(y) = \mathbb{E}_{\mathbf{D}}[x_i^*(\mathbf{D}, y)]$.

Define $\Pi(y)$ as the expected profit at capacity y if the FB production plan is implemented, i.e., $\Pi(y) \equiv \mathbb{E}_{\mathbf{D}}[\Pi(\mathbf{x}^*(\mathbf{D}, y); \mathbf{D}, y)]$. In the following, we assume that $\Pi(y)$ is strictly pseudo-concave, i.e., has a unique maximum and no interior minimum. [2] Let y^* be the first-best (FB) capacity level, defined as the solution to the following optimality conditions:

$$\sum_{i=1}^{n}(v_i - c(y^*))S_i'(y^*) - c'(y^*)S(y^*) - K'(y^*) = 0. \tag{2}$$

3.2 Capacity Allocation and Investment Game

For the JV to be viable, capacity allocation and investment decisions must be approved by all JV partners. Otherwise, opportunistic behavior or conflicts could arise. Although there always exists, in principle, a contract that can lead to full efficiency, such contract may entail high transaction costs. In particular, that contract may require the JV partners to foresee an exponential number of contingencies or to define complicated rules to coordinate their decisions. In that case, efficiency may need to be sacrificed to preserve contractual simplicity. Is it possible to achieve the best of both worlds, i.e., high efficiency and contractual simplicity?

To answer that question, we need to consider the structure of capacity ownership. We consider two extreme forms of ownership structure, namely joint and separate ownership, illustrated in Figure 1. Under joint ownership (JO), each unit of capacity is jointly owned by all n partners, in proportion to their investments. By contrast under separate ownership (SO), each unit of capacity is owned by one firm only, and that firm is the only one entitled to choose how to use it.

3.2.1 Capacity Allocation Game.

In the production stage, the JV partners may need to trade capacity to make the best use of the joint capacity. Let q_{ij} denote the amount of capacity owned by Firm i and allocated to the production of Product j. In order to create incentives for firms to share capacity, capacity trades need to be rewarded. Accordingly, let $t_{ij}(q)$ be the price paid by Firm j to Firm i to use q units of its capacity. Although this transfer function can be quite general, we assume that it does not depend on the demand realizations; that is, price functions must be agreed on *ex-ante*, similar Van Mieghem's (1999) price-only contracts.[3]

Given a vector of capacity transfers $\mathbf{q}$, demand realization $\mathbf{D}$, and capacity investment $\mathbf{y}$, Firm i's profit, denoted as $\Pi_i(\mathbf{q}; \mathbf{D}, \mathbf{y})$, consists of the net revenue associated with Product i and the net profit from capacity trades, minus the firm's share of capacity investment costs. That is, $\Pi_i(\mathbf{q}; \mathbf{D}, \mathbf{y}) = (v_i - c(y))x_i(\mathbf{q}) - \sum_{j\neq i} t_{ji}(q_{ji}) + \sum_{j\neq i} t_{ij}(q_{ij}) - K(y)\frac{y_i}{y}$.

Under JO, Firm i owns only y_i/y share of each unit of capacity. Accordingly, if Firm i believes it is best to make x_j units of Product j, it can only contribute $(y_i/y)x_j$ units of capacity towards that goal, i.e., its capacity transfer to Firm j must be equal to $(y_i/y)x_j$.

Under SO, Firm i owns y_i units of capacity and is the only one entitled to choose how to allocate those y_i units. The available capacity for Product j is therefore the net result of capacity inflows

[2] This is in particular the case when $K''(y) = c'(y) = 0$. When $\Pi(y)$ has multiple stationary points, the conditions for coordinating capacity investments (Propositions 2 and 4) are only necessary.

[3] This restriction turns out to be without loss of generality, at least conceptually, since we will allow for state-dependent trade limits, which serve the same purpose as state-dependent prices in a dual fashion.

($\sum_{k\neq j} q_{kj}$) and outflows ($\sum_{k\neq j} q_{jk}$), in addition to the initial capacity investment (y_j). Accordingly, $x_j(\mathbf{q}) = \min\{D_j, y_j + \sum_{k\neq j} q_{kj} - \sum_{k\neq j} q_{jk}\}$. In addition, the JV partners may wish to set trade limits to better match supply and demand. For instance, trades may be forbidden if the trade recipient does not need or intend to sell back the traded capacity. On the other hand, trades should not be forced, consistent with the free spirit of the JV. Accordingly, we assume that trades must satisfy the following upper bound constraints: $0 \leq q_{ij} \leq g_{ij}(\mathbf{D}, \mathbf{y})$ for all i, j. Although these trade limits can be defined arbitrarily, we assume that they are such that the FB production decisions are always feasible. Ideally, these trade limits ($g_{ij}(\mathbf{D}, \mathbf{y})$) should be contractually simple to specify so as to keep transaction costs low.

Accordingly if Firm i were the only firm making decisions about capacity transfers and capacity allocations, it would choose capacity transfers so as to maximize its own profit:

$$\begin{array}{rlll}
\max_{\mathbf{q}} \Pi_i(\mathbf{q}; \mathbf{D}, \mathbf{y}) = & v_i x_i(\mathbf{q}) - \sum_{j\neq i} t_{ji}(q_{ji}) + \sum_{j\neq i} t_{ij}(q_{ij}) - c(y) x_i(\mathbf{q}) - K(y)\frac{y_i}{y} & & \\
\text{subject to} & \sum_{j=1}^{n} x_j(\mathbf{q}) \leq y & & \\
& 0 \leq x_j(\mathbf{q}) \leq D_j & \forall j & \\
& x_j(\mathbf{q}) = q_{ij}\frac{y}{y_i} & \forall j & [\text{JO}] \\
& x_j(\mathbf{q}) = \min\{D_j, y_j + \sum_{k\neq j} q_{kj} - \sum_{k\neq j} q_{jk}\} & \forall j & [\text{SO}] \\
& 0 \leq q_{ij} \leq g_{ij}(\mathbf{D}, \mathbf{y}) & \forall i, j & [\text{SO}],
\end{array} \tag{3}$$

in which the first two constraints require that production should not exceed total capacity and the production of Product j should not be greater than its demand, similar to (1). The third constraint applies to JO arrangements only, and the last two constraints apply to SO arrangements only. In the following, we will refer as $\Pi_i^{SO}(\mathbf{q}; \mathbf{D}, \mathbf{y})$ Firm i's profit under SO and as $\Pi_i^{JO}(\mathbf{q}; \mathbf{D}, \mathbf{y})$ Firm i's profit under JO.

In reality, capacity allocation decisions are made jointly. A firm will agree with the collective decision if that decision maximizes its profit. In addition, firms care about economic efficiency given that it is often the primary driver for creating a JV. This leads to the formulation of our first coordinating principle: Capacity trades are incentive-compatible if they lead to the FB production plan and they maximize each firm's individual profit.

Coordinating Condition 1 (IC_{Prod})**.** *There exists a* $\bar{\mathbf{q}}$ *such that* $\bar{\mathbf{q}} \in \arg\max \Pi_i(\mathbf{q}; \mathbf{D}, \mathbf{y})$ *for all* i *and* $\mathbf{x}(\bar{\mathbf{q}}) = \mathbf{x}^*$.

Fortunately, coordination leads to efficiency. If all firms agree on the capacity transfers, these capacity transfers will be such that the FB production plan is implemented, as shown next.

Lemma 1. *If* $\bar{\mathbf{q}} \in \arg\max \Pi_i(\mathbf{q}; \mathbf{D}, \mathbf{y})$ *for all* i*, then* $\mathbf{x}(\bar{\mathbf{q}}) = \mathbf{x}^*$.

Hence for (IC_{Prod}) to be satisfied, it is necessary and sufficient that all firms agree on a capacity transfer plan.

3.2.2 Capacity Investment Game.

We next consider the capacity investment game, which takes place under demand uncertainty. Let $\Pi_i(y_i; \mathbf{y}_{-i}) \equiv \mathbb{E}_{\mathbf{D}}[\max_{\mathbf{q}} \Pi_i(\mathbf{q}; \mathbf{D}, \mathbf{y})]$ denote Firm i's expected profit if it invests y_i and the other firms invest $\mathbf{y}_{-i} = (y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)$. Implicit in this expectation are all capacity transfers obtained from (3), for each possible demand realization. In particular, this expected profit depends on the structure of ownership.

If Firm i were autonomous in its choice of capacity investment, it would choose a capacity that maximizes its expected profit:

$$y_i = \arg\max_{y_i \geq 0} \Pi_i(y_i; \mathbf{y}_{-i}). \tag{4}$$

In reality, capacity investment decisions are made jointly. Under decentralized decision-making, a firm will agree with the collective investment decision if its own investment maximizes its indivual profit, given the other firms' investments. In addition, firms care about economic efficiency given that it is often the primary driver for creating a JV. This leads to the formulation of our second coordinating principle, which mirrors (IC_{Prod}): Capacity investment decisions are incentive-compatible if they lead to the FB investment in equilibrium.

Coordinating Condition 2 (IC_{Inv})**.** *There exists a* $\bar{\mathbf{y}}$ *such that* $\bar{y}_i = \arg\max_{y_i} \Pi_i (y_i; \mathbf{y}_{-i})$ *for all* i *and* $\sum_{i=1}^{n} \bar{y}_i = y^*$.

In the sequel, we will say that a contract is *coordinating* if both (IC_{Inv}) and (IC_{Prod}) are satisfied. We next investigate whether coordination is achievable at low transaction costs, for both cases of SO and JO.

4 Separate Ownership

We first consider separate ownership (SO) arrangements. We find that SO arrangements can yield full efficiency at low transactional costs when the JV consists of only two partners or when the firms have identical profit margins. Otherwise, they may be associated with high transaction costs, which grow exponentially with the number of firms. We first study the capacity allocation game and then the capacity investment game.

4.1 Coordinating Capacity Allocation

We first consider the production stage. After demand is realized, some firms may experience a capacity shortage while others are experiencing a capacity surplus. Naturally, firms who have an excess of capacity would like to sell it at the highest price whereas firms who experience a shortage would like to buy capacity at the lowest price. Hence, coordinating capacity trades to achieve full efficiency is nontrivial.

The next proposition shows that capacity trades can only be coordinated with extreme-point solutions $\mathbf{q}$ to (3). Hence for (IC_{Prod}) to be satisfied, one needs to properly define the capacity trade limits $g_{ij}(\mathbf{D}, \mathbf{y})$ for every possible demand realization $\mathbf{D}$. Proposition 1 however reveals that as many as 2^n scenarios may need to be considered to properly specify those trade limits and yield the FB production plan. Intuitively, each firm can be either in a buyer or in a seller position. Taken together, this yields 2^n scenarios.

Proposition 1. *Let* $\mathbf{q}^{SO} \in \arg\max \Pi_i (\mathbf{q}; \mathbf{D}, \mathbf{y})$ *for all* i *under SO. Then,* $q_{ij}^{SO} = 0$ *or* $q_{ij}^{SO} = g_{ij}(\mathbf{D}, \mathbf{y})$ *for all* i, j*, in which the function* $g_{ij}(\mathbf{D}, \mathbf{y})$ *is piecewise with as many as* 2^n *different pieces.*

For instance when $n = 2$, four demand scenarios need to be considered. Specifically, setting $g_{12}(\mathbf{D}, \mathbf{y}) = \min\{(D_2 - y_2)^+, (y_1 - D_1)^+\}$ and $g_{21}(\mathbf{D}, \mathbf{y}) = \min\{y_2, (D_1 - y_1)^+\}$ will satisfy (IC_{Prod}). Intuitively, each firm would like to position itself at the center of all trades, buying from all firms that have an excess of capacity and selling to all firms that have a shortage of capacity. Without those trade limits, capacity trades would then be impossible to coordinate. Those capacity trade limits thus play the same role, in a dual fashion, as the role played by state-dependent prices (Van Mieghem 1999).

The complexity of the specification of the capacity trade limits will thus limit the applicability of SO arrangements when either n is large or when the firms' profit margins are different. On the one hand when n is small, enumerating all demand contingencies remains manageable. For instance when $n = 2$, only four demand scenarios need to be foreseen. Moreover, it is easy to specify transfer prices $t_{ij}(q)$ so as to allocate capacity in priority to the most profitable products. For instance with linear transfer prices ($t_{ij}(q) = \lambda_{ij} q$), Condition ($IC_{Prod}$) is achieved when $v_1 - c(y) \geq \lambda_{21} \geq v_2 - c(y) \geq \lambda_{12} \geq 0$. On the other hand when firms have identical profit margins, any capacity allocation rule attains the FB production plan, as long as there is no waste. For instance, firms could satisfy their own demand first and then allocate the total residual capacity proportionally to the residual demands.

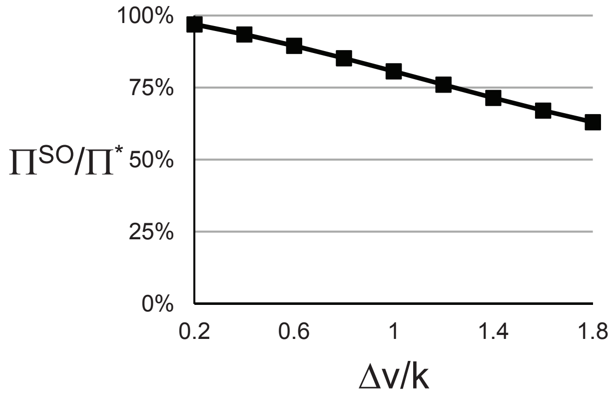

Figure 2: Efficiency of a simple SO capacity allocation mechanism that uses each firm's capacity to meet its own demand, and then optimally allocates the residual capacity among residual demands. The parameters are: $n = 3$, $D_i \sim U[0, 10]$, $K(y) = ky$, $c(y) = 0$, $v_{i-1} = v_i + \Delta v$, $k/[\sum_i v_i/n] = 50\%$.

But with a large number of firms exhibiting different profit margins, SO arrangements will probably be too complex to organize and administer if they are aimed at attaining full efficiency. The contract writing process will inevitably require coordinated efforts to structure capacity trades to attain full efficiency, violating the decentralized spirit of SO arrangements. Alternatively, capacity trades could be centrally managed, but the central nature of such an organization would also go against the decentralized nature of the SO philosophy. Under such centralized management, capacity trades could be structured so as to attain full efficiency at the price of being potentially complex or administered by an opaque mechanism.

Alternatively, efficiency could be sacrificed in favor of simple allocation rules, such as allocating residual capacity proportionally to products experiencing a capacity shortage, in decreasing order of profitability. This capacity allocation method is in fact similar to inventory transshipment games (e.g., Anupindi et al. 2001, Huang and Sošić 2011), in which inventory is first used to satisfy the local demands first, and leftovers are then pooled to optimally satisfy the residual demands. Figure 2 illustrates the loss of efficiency of such a rule with $n = 3$ and $v_{i-1} = v_i + \Delta v$ as a function of Δv. In that numerical example, we assume that all firms invest equally in the JV and that the total investment is equal to the FB investment, i.e., $y_i = y^*/n$, so that the loss of efficiency only comes from inefficient capacity allocations. Naturally, we observe that the greater the spread in profit margins, the greater the loss of efficiency associated with suboptimal capacity allocation. In fact, the total profit under SO is decreasing in $\Delta v/k$ whereas the FB profit (Π^*) is increasing in $\Delta v/k$. More importantly, the loss of efficiency can be very substantial. This illustrates the importance of treating capacity differently than inventory.

Irrespective of whether an optimal, but complex, or a simple, but inefficient, rule is adopted, such a rule must be specified, together with transfer payments, to induce proper capacity investments. In addition, the implementation of such rules must be tightly monitored to prevent the emergence of subcoalitions and parallel exchanges of capacity or to prevent firms from engaging themselves in shortage gaming by not truthfully revealing their demand (Granot and Sošić 2003).

Instead of sacrificing on efficiency, capacity allocation decisions could be resolved through a market mechanism, e.g., an auction, and therefore left unspecified in the contract. This incomplete contract approach, despite leading to *ex-post* efficient capacity allocations at low transaction costs, is not without challenges as it forces firms to foresee 2^n demand scenarios to guide their investment decisions, notwithstanding the unpredictability of its outcomes if the firms' bargaining power is unknown at the outset. Moreover, it leads to suboptimal investments in capacity, as we show next, consistent with transaction cost theory (Grossman and Hart 1986).

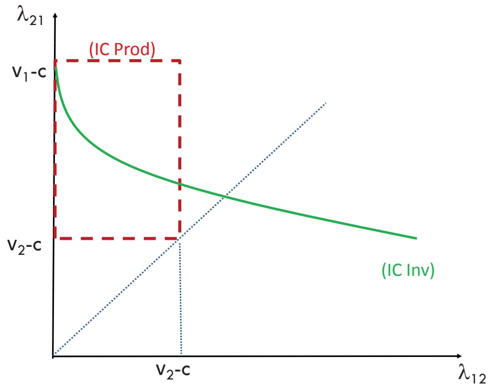

Figure 3: (IC_{Inv}) and (IC_{Prod}) under Separate Ownership.

4.2 Coordinating Capacity Investment

We next consider the capacity investment game. Because capacity allocation decisions must be foreseen when transfer prices are negotiated and capacity investment decisions are made, the complexity of capacity allocation decisions will propagate in the investment negotiation stage.

We focus on the case of $n = 2$ firms here since otherwise, transactions costs are high or efficiency may need to be sacrificed in the production stage. In addition with $n > 2$, there exist multiple solutions satisfying (IC_{Prod}), thus preventing any analysis of the capacity investment game without additional assumptions on the capacity trade flows. Moreover, we consider the case of linear transfer prices, i.e., $t_{ij}(q) = \lambda_{ij} q$, given their flexibility for coordinating (IC_{Prod}) and their simplicity, and we assume that the FB production plan is implemented. Specifically, (IC_{Prod}) is satisfied as long as $v_1 - c(y) \geq \lambda_{21} \geq v_2 - c(y) \geq \lambda_{12} \geq 0$.

Accordingly, the expected profit of Firm i in the investment stage can be expressed as follows:

$$\begin{aligned} \Pi_i^{SO}(y_i; y_{-i}) \quad &= \quad (v_i - c(y))S_i(y) - K(y)\frac{y_i}{y} \\ &\quad -\lambda_{ji}\mathbb{E}\left[x_i^*(y, D_1, D_2) - y_i\right]^+ + \lambda_{ij}\mathbb{E}\left[x_{-i}^*(y, D_1, D_2) - y_{-i}\right]^+. \end{aligned} \tag{5}$$

When $n = 2$, $v_1 \geq v_2$, $K''(y) = c'(y) = 0$, a Nash equilibrium can be shown to exist in the SO capacity investment game when $v_1 - c(y) \geq \lambda_{21} \geq v_2 - c(y) \geq \lambda_{12} \geq 0$ using Debreu-Glicksberg-Fan's theorem; see Lemma A-6 in the appendix. When (IC_{Prod}) holds strictly, that Nash equilibrium can be shown to be unique. (The proof is omitted for brevity.)

The next proposition provides a condition under which capacity investment decisions are coordinated. This result contrasts with outsourcing and subcontracting agreements, for which there exists no coordinating linear transfer-price contracts (Van Mieghem 1999). This difference in results stems from the fact that capacity transfers are bidirectional here, thus reducing both overage and underage costs of both firms, unlike outsourcing agreements, which involve only unidirectional transfers.

Proposition 2. *Suppose that $n = 2$. If there exists an interior Nash equilibrium $\mathbf{y}^{SO}$ in the SO capacity investment game, then (IC_{Inv}) is satisfied if and only if $\lambda_{21}\mathbb{P}[D_1 \geq y_1^{SO}] + \lambda_{12}\mathbb{P}[D_1 \leq y_1^{SO}, D_2 \geq y_2^{SO}] = K(y^*)/y^*$.*

Identifying the coordinating transfer prices thus involves the solution of a fixed-point problem, consisting of the equilibrium conditions characterizing the capacity investment game, obtained by differentiating (5), and the conditions stated in Proposition 2. Intuitively, transfer prices are functions of the respective investments, which are themselves functions of the transfer prices. Moreover, the computation of these transfer prices must consider all possible demand contingencies, depending on whether a firm experiences a shortage or a surplus of capacity. Hence the exponential complexity characterizing the coordination of production decisions carries over to the negotiation of their transfer prices.

Figure 3 illustrates the feasible space of SO linear contracts that satisfy (IC_{Inv}) as well as the space of contracts that satisfy (IC_{Prod}). It can be shown that the curve on which (IC_{Inv}) is satisfied is monotone when $S_1'(y^*) > 0$. In addition, it can be shown that profits evolve monotonically along the curve (i.e., $d\Pi_i^{SO}/d\lambda_i = -d\Pi_{-i}^{SO}/d\lambda_i \leq 0$), thus offering flexibility in terms of profit allocation.

Does there always exist an SO coordinating contract that satisfies both (IC_{Inv}) and (IC_{Prod})? The answer, when $n = 2$, is yes. For instance, setting $(\lambda_{21}, \lambda_{12}) = (v_1 - c, 0)$ coordinates capacity investment and allocation decisions when $K(y) = ky$ and $c(y) = c$. In that case, Firm 1 earns exactly its newsvendor profit. Incidentally, this coordinating contract illustrates the failure of incomplete contracts to coordinate capacity investment decisions, as was observed by Van Mieghem (1999) in the case of outsourcing. In a bargaining game, the total surplus is shared in fixed proportions depending on the firms' relative bargaining power; see Grossman and Hart (1986). Because the total surplus when Firm 2 sells a unit of capacity to Firm 1 is either equal to $v_1 - c$ if Firm 2 has excess capacity or $(v_1 - c) - (v_2 - c)$ if Firm 2 has a capacity shortage, the firms' transfer prices would then be state-dependent and therefore non-coordinating, even if Firm 2 had complete bargaining power. Hence, leaving the contract incomplete on capacity allocation will in general not induce efficient (i.e., FB) capacity investments, a classical result in transaction cost economics (Grossman and Hart 1986).

To summarize this section, we showed that SO arrangements could attain high efficiency at low transaction costs when either the JV consists of $n = 2$ firms or when the firms have identical profit margins. With $n > 2$ firms having different profit margins, the JV would have to either incur high transaction costs to reap the full economic benefits of the alliance, by either foreseeing as many as 2^n demand contingencies in the contract or by creating a central entity that would administer the *ex-post* capacity trades. Alternatively, efficiency would need to be sacrificed to keep transaction costs low.

5 Joint Ownership

We next consider joint ownership (JO) arrangements. In contrast to SO arrangements, we find that JO arrangements always lead to efficient capacity sharing at low transaction costs; specifically, they may involve only n different prices, satisfying $3n - 2$ linear inequalities. However, they may entail transaction costs at the investment level in order to prevent capacity overinvestment. The risk of overinvestment is however reduced in the presence of economies of scale, when the demand for the lower profit margin products is large, or when there is a large spread in profit margins.

5.1 Coordinating Capacity Allocation

We first consider the production stage. In contrast to SO, we show that capacity allocation decisions are easily coordinated under JO due to its centralized nature. In particular, they can easily be coordinated with linear transfer price functions, i.e., $t_{ij}(q) = \lambda_{ij} q$.

Proposition 3. *Let $t_{ij}(q) = \lambda_j q$ with $\lambda_j \geq 0$ under JO. Then, (IC_{Prod}) holds if and only if*
(i) $\lambda_i \geq \lambda_{i+1}$ for all i,
(ii) $\lambda_{i-1} \frac{y_i}{y} \geq v_i - \lambda_i \frac{(y - y_i)}{y} - c(y) \geq \lambda_{i+1} \frac{y_i}{y}$ for all i,
with $\lambda_0 = \infty$ and $\lambda_{n+1} = 0$.

Similar to SO arrangements, simple rules are applicable when firms have identical profit margins, such as allocating capacity proportionally to the demands, provided that they do not create waste. Full efficiency is always attained in that case and transaction costs can be kept at a minimum, although those rules may still need to be contractually specified.

5.2 Coordinating Capacity Investment

We next consider the investment stage. Given that linear contracts can easily coordinate production decisions (Proposition 3), we focus on linear transfer price contracts in the sequel and assume that the FB production plan is implemented. Accordingly, Firm i's profit, when making a capacity investment of y_i, can be expressed as follows:

$$\Pi_i^{JO}\left(y_i;\mathbf{y}_{-i}\right) \quad = \quad (v_i-\lambda_i-c(y))S_i(y)+\frac{y_i}{y}\left(\Lambda(y)-K(y)\right), \tag{6}$$

in which $\Lambda(y) \doteq \sum_{j=1}^{n}\lambda_j S_j(y)$. In this formulation, production costs are allocated proportionally to sales; alternatively, those costs could be allocated proportionally to capacity investments, similar to $K(y)$, without changing our results.

A Nash equilibrium in the capacity investment game can be shown to exist without economies of scale (i.e., $K''(y) = c'(y) = 0$) under Conditions (i)-(iii) stated in Lemma 3, using Debreu-Glicksberg-Fan's theorem; see Lemma A-7 in the appendix. Moreover, the equilibrium can be shown to be unique under mild conditions on the transfer prices. (The proof is omitted for brevity.)

The next proposition provides a condition under which capacity investment decisions are coordinated.

Proposition 4. *Suppose there exists an interior equilibrium in the capacity investment game. Then (IC_{Inv}) is satisfied if and only if $\Lambda(y) = K(y)$.*

Hence, Proposition 4 states that (IC_{Inv}) is satisfied if and only if the JV operates on a non-profit basis, as is done by Syncrude or many cooperatives. One of the Rochdale principles for cooperatives indeed requires that the capital contributed by cooperative members be common property of the cooperative and that any surplus be either reinvested in the business or redistributed to its members (ICA 2007). This clearing condition is in fact similar to the requirement that the profit margins of all, but one, partners must be set to zero in a vertical supply chain to mitigate double marginalization.

Under (IC_{Inv}), the relative shares of investment, if both firms invest in the JV (i.e., $y_i^{JO} > 0$) can be expressed as follows:

$$\frac{y_i^{JO}}{y^*} = \frac{(v_i-\lambda_i-c(y))S_i'(y^*)-c'(y^*)S_i(y^*)}{K'(y^*)-\Lambda'(y^*)}. \tag{7}$$

We make the following two observations. First, the relative shares of investment may not be proportional to capacity allocations. For instance at Autolatina, Volkswagen accounted for about 2/3 of the output despite owning only 51% of the JV (Jackson and Turner 1994).

Second, Equation (7) reveals that investments are driven by (i) service levels ($S_i'(y^*)$) and (ii) economies of scale in production ($c'(y^*)$). Economies of scale affect all firms, although firms with larger sales volumes ($S_i(y^*)$) will give them more weight. Service levels affect mostly the firms that have the lowest profit margins, since their probability of not fulfilling their demand will be the highest given that they have the lowest priority access to capacity. In particular without economies of scale in production ($c'(y) = 0$), the JV may degenerate into an outsourcing agreement, in which the most profitable firms outsource their production to the least profitable firms.

Moreover under (IC_{Inv}), Profit (6) simplifies to

$$\Pi_i^{JO}\left(y_i^{JO};\mathbf{y}_{-i}^{JO}\right) = (v_i-\lambda_i-c(y^*))S_i(y^*). \tag{8}$$

Hence, Firm i makes nonnegative profit if its transfer price (λ_i) is no greater than its profit margin ($v_i - c(y^*)$).

Taken together, Equations (7) and (8) reinforce the interpretation of JO arrangements as equity JVs. Specifically when $c'(y) = 0$, the relative investments (7) turn out to be proportional to the relative shares of profits. Hence, the more a firm invests in the JV, the greater share of total profits it collects. Moreover unlike profit-sharing agreements, JO arrangements do not make dividends dependent on profit margins, and they are therefore more robust to errors in profit margin evaluations. In addition, the profit shares are determined endogenously here, contingent on the respective investments, instead of being taken as exogenous as in traditional profit-sharing contracts.

Figure 4 illustrates the feasible space of linear JO contracts that satisfy (IC_{Inv}) when $n = 2$, as well as the feasible space of contracts that satisfy (IC_{Prod}) when the total investment is the FB investment, i.e., after substituting (7) into Conditions (i) and (ii) in Proposition 3, and when

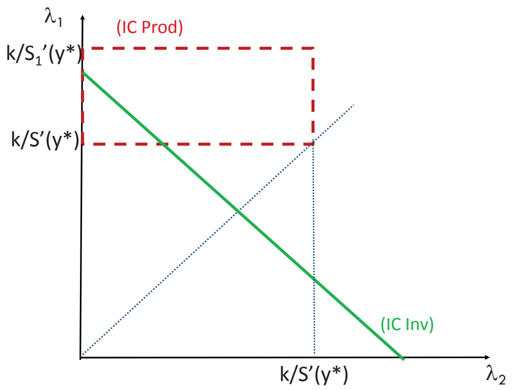

Figure 4: (IC_{Inv}) and (IC_{Prod}) under Joint Ownership.

$c'(y) = 0$ and $K'(y) = ky$. Unlike SO arrangements, the set of feasible JO contracts that satisfy (IC_{Inv}) does not depend on the equilibrium investments, i.e., does not result from a fixed-point mapping, and can therefore be represented as a straight line in the space of transfer prices. Hence in addition to be intuitive, the clearing condition (IC_{Inv}) is also simple to compute, an attractive feature for guiding contractual negotiations.

Lemma A-8 in Appendix shows that profits evolve monotonically along that line of feasible contracts (i.e., $d\Pi_i^{JO}/d\lambda_i \leq 0$). Similar to SO arrangements, JO arrangements lend themselves to various ways of allocating the total profit. Moreover, the equilibrium capacity investments evolve monotonically along that line of feasible contracts with linear costs of production (i.e., $dy_i^{JO}/d\lambda_i \leq 0$). Naturally, investing more makes you deserve a lower price and getting a lower price makes you invest more.

Does there always exist a fully coordinating JO contract that satisfies both (IC_{Inv}) and (IC_{Prod})? Unfortunately, the answer is not always, even when $n = 2$. Hence, JO arrangements, although (or because) they are associated with low transaction costs to coordinate capacity sharing, may not necessarily lead to the right capacity investment decisions in equilibrium. This is in contrast to SO arrangements which in general lead to the right capacity investments, but with high transaction costs to coordinate capacity allocation decisions.

To gain more insight into the conditions under which there exists a coordinating contract, we assume that $n = 2$ in the sequel. The next proposition provides a sufficient condition under which there exists a coordinating contract. The proposition assumes that both firms invest in the JV. Although similar insights can be derived for the case in which one firm does not invest in the JV, it should be noted that shared investment is the most interesting case of JV, as otherwise the JV would degenerate into an outsourcing agreement.

Proposition 5. *When $n = 2$, there exists a coordinating JO contract with $y_1^{JO}, y_2^{JO} > 0$ if*

$$1 + \frac{c'(y^*)S_2(y^*)K(y^*)}{(v_2 - c(y^*))K'(y^*)S_1(y^*)} \leq \frac{K(y^*)}{y^*K'(y^*)} \frac{y^*S'(y^*)}{S_1(y^*)}. \tag{9}$$

Although this condition is only sufficient, it turns out to be also necessary in the absence of economies of scale in production (i.e., when $c'(y) = 0$) or when the requirement that $\Pi_i^{JO}\left(y_i^{JO}; \mathbf{y}_{-i}^{JO}\right) \geq 0$ for all i is also added. From this, we already observe that economies of scale in production are beneficial, as they enlarge the set of coordinating contracts, a property that will be reinforced below.

We next explore the conditions under which Condition (9) is satisfied. We first study the effect of economies of scale in production or investment. Economies of scale in production ($c'(y) < 0$) make the left-hand side of the above inequality smaller, and therefore make Condition (9) easier to satisfy. Similarly, economies of scale in investment ($K''(y) < 0$) imply that $K(y) > K'(y)y$, and therefore also make Condition (9) easier to satisfy.

Corollary 1. *Coordination under JO is more likely to be achieved with economies of scale in production ($c'(y) < 0$) or investment ($K''(y) < 0$).*

To gain further insights, let us consider the (arguably contrived) case with no economies of scale. In that case, the line of contracts satisfying (IC_{Inv}) will always intersect the λ_1-axis below the upper boundary of the box defining the set of feasible contracts satisfying (IC_{Prod}), i.e., $\lambda_1 \leq k/S_1'(y^*)$; see Figure 4. As a result, the only cause of coordination failure is that the box defining the set of contracts satisfying (IC_{Prod}) may lie above the line defining the set of contracts satisfying (IC_{Inv}). Because $dy/d\lambda_i \geq 0$, $i = 1, 2$, for the equality $\Lambda(y) = K(y)$ to hold (by the implicit function theorem), shifting the line $\Lambda(y) = K(y)$ upwards results in greater investment y. In other words, the firms risk to overinvest in capacity if one wants to ensure that (IC_{Prod}) is satisfied.

When would firms not overinvest in capacity? When $c'(y) = 0$ and $K''(y) = 0$, Condition (9) simplifies to $y^*S'(y^*) \geq S_1(y^*)$. (With n firms, this condition is expressed as $y^* \sum_{j=1}^{i+1} S_j'(y^*) \geq \sum_{j=1}^{i} S_j(y^*)$ for all i when $K''(y) = c'(y) = 0$.) Graphically, the feasible set of contracts satisfying (IC_{Inv}) intersects the λ_1-axis above the lower boundary of the box defining the set of feasible contracts satisfying (IC_{Prod}) when $y^*S'(y^*) \geq S_1(y^*)$; see Figure 4.

Intuitively, Firm 2 is reluctant to allocate capacity to Product 1 unless it receives a high price for every unit of capacity used by Firm 1. This will happen if either Firm 1 is paying a high transfer price (λ_1) or if Firm 2 is capturing a large share (y_2/y) of that price. The risk with the first approach is that firms may end up overinvesting in capacity. In particular while the contract $(\lambda_1, \lambda_2) = (v_1 - c, v_2 - c)$ always satisfies (IC_{Prod}), it clearly leads to capacity overinvestment. Hence, the first approach will work only if the spread in profit margins is sufficiently large. The second approach, on the other hand, applies only if Firm 1 has limited incentives for investing in the JV, which, by (7), happens when $S_1'(y^*) \approx 0$. The next two corollaries formalize this intuition, in reverse order, by characterizing when the condition $y^*S'(y^*) \geq S_1(y^*)$ is satisfied.

Corollary 2. *Without economies of scale, coordination under JO is more likely to be achieved when, for a given demand D_1, D is stochastically large.*

In particular when D_1 and D_2 are independent, the condition is more likely to be met when D_2 is large (in the first stochastic order), which happens in particular when Product 2 has a large mean demand. When Product 2 has a large demand, the service level of Product 1 will be very high, i.e., $S_1'(y^*) \approx 0$, and therefore $y_2^{JO}/y^* \approx 1$ by (7). Hence, the JV will quickly degenerate into an outsourcing agreement in that case, i.e., Firm 1 outsources its production to Firm 2.

The next result shows that the requirement that $S_1(y^*) \leq y^*S'(y^*)$ is more likely to be satisfied with a large spread in profit margins. The result requires a specific condition, namely that there exists a $\bar{y}$ such that $S_1(y) - yS'(y) \leq 0$ if and only if $y \leq \bar{y}$. As shown in Lemmas A-9 and A-10 in the appendix, a sufficient condition is that either D_1 or D_2 is exponential and that demands are independent and have an increasing failure rate.

Corollary 3. *Suppose that D_1 and D_2 are such that there exists a $\bar{y}$ such that $S_1(y) - y\bar{F}(y) \leq 0$ if and only if $y \leq \bar{y}$. Without economies of scale, coordination under JO is more likely to be achieved when, for a given margin v_1, v_2 is small.*

Hence, the profit margins need to be sufficiently spread apart so that the transfer price paid by Firm 1 is sufficiently large to motivate Firm 2 to allocate capacity to Product 1, while being sufficiently smaller than $v_1 - c$ to prevent a situation of capacity overinvestment.

Incidentally, an industry observer, commenting on the failure of Autolatina, noted that the alliance came apart when the carmakers started using the plant to build similar models rather than sticking to complementary lines (Gao 2002). This observation is supported by Corollary 3, which advances that greater cooperation can be achieved with larger spreads in profit margins. Hence, dissimilar products not only reduce the intensity of competition in the marketplace, but also reduce competition for manufacturing capacity.

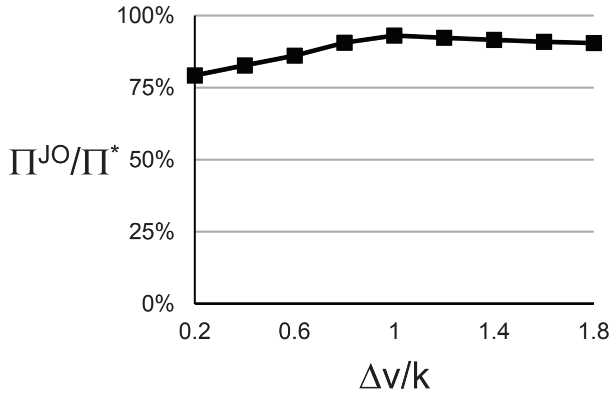

Figure 5: Efficiency of a simple JO arrangement that results in capacity overinvestment in the absence of economies of scale.

The parameters are: $n = 3$, $D_i \sim U[0, 10]$, $K(y) = ky$, $c(y) = 0$, $v_{i-1} = v_i + \Delta v$, $k/[\sum_i v_i/n] = 50\%$. The JO contract and equilibrium investments $(\boldsymbol{\lambda}, \mathbf{y})$ are found by jointly solving $\boldsymbol{\lambda} = \arg\min \sum_i \lambda_i$ subject to $\lambda_i \geq \lambda_{i+1}$ for all i and $\lambda_{i-1}\frac{y_i}{y} \geq v_i - \lambda_i \frac{(y-y_i)}{y} \geq \lambda_{i+1}\frac{y_i}{y}$ for all i, with $\lambda_0 = \infty$ and $\lambda_{n+1} = 0$; and $y_i = \arg\max(v_i - \lambda_i)S_i(y) + \frac{y_i}{y}\left(\sum_i \lambda_i S_i(y) - ky\right)$ for all i.

If Condition (9) does not hold, i.e., in the absence of economies of scale in production and investment, when the firms have relatively symmetric demands and symmetric profit margins, then the firms run into the risk of overinvesting in the JV. Figure 5 illustrates the loss of efficiency of JO arrangements associated with such capacity overinvestment. Among all contracts that satisfy (IC_{Prod}), the contract that minimizes the sum of transfer prices is selected. The intuition behind this selection rule comes from Figure 4, in which the contract that lies in the lower left corner of the (IC_{Prod}) feasible region minimizes the level of investment, and will thus minimize the magnitude of overinvestment when the feasible region of (IC_{Prod}) lies above the feasible region of (IC_{Inv}).

Consistent with Corollary 3, Figure 5 reveals that the loss of efficiency resulting from overinvestment generally decreases with the spread in profit margins. Hence, JO arrangements should succeed particularly where simple capacity allocation rules fail. Although simple capacity sharing rules (e.g., sharing proportionally to demand) are likely to perform well when profit margins are similar, they could lead to a large loss of efficiency when profit margins are very different. In fact, the loss of efficiency associated with JO arrangements overall exhibits the opposite behavior to the loss of efficiency associated with simple, but inefficient, SO arrangements displayed in Figure 2. This divergence in behaviors thus illustrates the complementary nature of the two capacity ownership structures.

In addition, Figure 5 reveals that the loss of efficiency associated with capacity overinvestment under JO, although significant, remains bounded, although the transfer prices have been selected in a rather ad-hoc fashion (i.e., to minimize $\sum_i \lambda_i$), thus giving room for potentially further improvement.

Hence in the absence of economies of scale or when the firms are too symmetric, the firms would need to rein in their desire to overinvest in capacity by forcing themselves to adopt a global, rather than individual, perspective on the joint investment. That is, coordinating investments under JO may entail high transaction costs. (On the positive side, it seems easier to restrict oneself to overinvest than to force oneself to not underinvest.) Hence, JO arrangements may be associated with high transaction costs at the capacity investment stage, to limit overinvestment, in contrast to SO arrangements which may be associated with high transaction costs at the capacity allocation stage.

6 Conclusion

This paper studies how to structure capacity ownership in manufacturing joint ventures to achieve full efficiency at low transaction costs. Specifically, we study whether capacity should be owned jointly or separately. We find that separate ownership and joint ownership have complementary strengths and weaknesses.

Specifically, separate ownership may entail high transaction costs for regulating capacity allocation, especially when a large number of firms participate in the joint venture. With only two firms or when the firms have identical profit margins, then capacity trades can be orchestrated in a simple manner and full efficiency can be attained at low contractual costs. If that is the case, capacity investments are easy to coordinate so as to achieve full efficiency. Otherwise, efficiency may need to be traded off for reducing transaction costs, either through the adoption of simple, but inefficient, capacity allocation rules or by leaving the contract incomplete, which would then result in inefficient investments.

By contrast, joint ownership leads to a simple and efficient administration of capacity allocation even with a large number of firms, but they may not lead to efficient capacity investments. Efficient capacity investments can be attained in a decentralized fashion with economies of scale or if the least profitable firms have a large demand, or if there is a wide spread in profit margins. If those conditions are met, the joint venture must be operated on a non-profit basis to reap all benefits of the venture. Otherwise, transaction costs must be incurred to rein in the firms' desire to overinvest in capacity.

To summarize, coordinating capacity allocation is the main strength of joint ownership and the main weakness of separate ownership, whereas coordinating investment decisions is the main strength of separate ownership and the main weakness of joint ownership. With limited economies of scale, separate ownership tends to perform best when firms have similar profit margins whereas joint ownership tends to perform best when they have very different profit margins. Overall, this paper highlights the importance of understanding how operational decisions are made before embarking in a joint venture.

Several questions remain. In particular, we have ignored competition in the market place. Although this is a mild assumption when firms operate in different markets (as was the case for PSA and Fiat at Sevel-Nord) or have differentiated products (as was initially the case for Ford and Volkswagen at Autolatina), this may not always be the case. With pricing power, we conjecture that demand rationing would happen less often, thereby reducing the intensity of competition at the manufacturing level. A potential caveat to this conjecture is that pricing power could reverse the priorities of the production allocation decisions if the firms have similar cost structures. By contrast without pricing power, competition in the marketplace may spill over in manufacturing, as a firm may want to crowd the market with its product to capture market share, as Volkswagen did with its Gol, blocking Ford's plans to sell a similar subcompact car (Bradsher 1997). The long-term viability of the joint venture is likely going to be even more fragile in that case.

A Proofs.

Proof. Proof of Lemma 1. Fix $\mathbf{D}$ and therefore $\mathbf{x}^*$. Under JO, there exists a unique matrix $\mathbf{q}(\mathbf{x}^*)$ such that $x_j^* = q_{ij}\frac{y}{y_i}$ for all j. Similarly under SO, we assumed that the trade limits $g_{ij}(\mathbf{D}, \mathbf{y})$ were such that the FB production plan is always feasible, i.e., there exists a matrix $\mathbf{q}(\mathbf{x}^*)$ satisfying $0 \leq q_{ij} \leq g_{ij}(\mathbf{D}, \mathbf{y})$ for all i, j such that $\mathbf{x}^*$ is implemented. Therefore, $\max_{\mathbf{x}} \Pi(\mathbf{x}; \mathbf{D}, y) = \Pi(\mathbf{x}^*; \mathbf{D}, y) = \sum_{i=1}^n \Pi_i\left(\mathbf{q}(\mathbf{x}^*); \mathbf{D}, \mathbf{y}\right) \leq \sum_{i=1}^n \max_{\mathbf{q}} \Pi_i\left(\mathbf{q}; \mathbf{D}, \mathbf{y}\right) = \sum_{i=1}^n \Pi_i\left(\bar{\mathbf{q}}; \mathbf{D}, \mathbf{y}\right) = \Pi(\mathbf{x}(\bar{\mathbf{q}}); \mathbf{D}, y)$. Because $\bar{\mathbf{q}}$ is feasible in (3) for all i, $\mathbf{x}(\bar{\mathbf{q}})$ is feasible in (1). Hence, $\mathbf{x}(\bar{\mathbf{q}})$ is optimal in (1). □

Lemma A-1. *Suppose that* $\mathbf{q}^{SO} \in \arg\max \Pi_i^{SO}\left(\mathbf{q}; \mathbf{D}, \mathbf{y}\right)$ *for all* i. *Then* $q_{ij}^{SO} = 0$ *or* $q_{ij}^{SO} = g_{ij}(\mathbf{D}, \mathbf{y})$ *for all* i, j.

Proof. Proof. Suppose that $0 < q_{ij}^{SO} < g_{ij}(\mathbf{D}, \mathbf{y})$. Then, the first-order optimality conditions yield $d\Pi_k^{SO}/dq_{ij} = 0$ for all k. We first show that $x_l = y_l + \sum_{k \neq l} q_{kl} - \sum_{k \neq i} q_{lk}$ for $l = i, j$. If $x_i < y_i + \sum_{k \neq i} q_{ki} - \sum_{k \neq i} q_{ik}$, $d\Pi_i^{SO}/dq_{ij} = t'_{ij}(q_{ij}) > 0$, a contradiction. Similarly if $x_j < y_j + \sum_{k \neq j} q_{kj} - \sum_{k \neq j} q_{jk}$, $d\Pi_j^{SO}/dq_{ij} = -t'_{ij}(q_{ij}) < 0$. Accordingly, $d\Pi_i^{SO}/dq_{ij} = -v_i + t'_{ij}(q_{ij}) + c(y) = 0$ and $d\Pi_j^{SO}/dq_{ij} = v_j - t'_{ij}(q_{ij}) - c(y) = 0$. Since $v_i \neq v_j$, this is a contradiction. □

Lemma A-2. *Any solution* $\mathbf{q}^{SO} \in \arg\max \Pi_i^{SO}(\mathbf{q}; \mathbf{D}, \mathbf{y})$ *must be such that* $q_{ij}^{SO} = (x_j^* - \sum_{k \neq i,j} q_{kj}^{SO} + \sum_{k \neq j} q_{jk}^{SO} - y_j)^+$ *for all* i, j.

Proof. Proof. Suppose, for contradiction, that $q_{ij}^{SO} > x_j^* - \sum_{k \neq i,j} q_{kj}^{SO} + \sum_{k \neq j} q_{jk}^{SO} - y_j$ and $q_{ij}^{SO} > 0$. By Lemma 1, $x_j(\mathbf{q}^{SO}) = x_j^*$. Hence, $q_{ij}^{SO} > x_j(\mathbf{q}^{SO}) - \sum_{k \neq i,j} q_{kj}^{SO} + \sum_{k \neq j} q_{jk}^{SO} - y_j$ and $q_{ij}^{SO} > 0$. Taking the derivative of Π_j^{SO} with respect to q_{ij} on that range of values yields $d\Pi_j^{SO}/dq_{ij} = -t'_{ij}(q_{ij}^{SO}) < 0$. Hence, Firm j will choose q_{ij} as small as possible, a contradiction. □

Lemma A-3. *Any solution* $\mathbf{q}^{SO} \in \arg\max \Pi_i^{SO}(\mathbf{q}; \mathbf{D}, \mathbf{y})$ *must be such that* $\sum_{k \neq j} q_{kj}^{SO} = (x_j^* + \sum_{k \neq j} q_{jk}^{SO} - y_j)^+$ *for all* j.

Proof. Proof. Suppose that $\mathbf{q}^{SO} \in \arg\max \Pi_i^{SO}(\mathbf{q}; \mathbf{D}, \mathbf{y})$. Pick any i, j. Assume first that $x_j^* - \sum_{k \neq i,j} q_{kj}^{SO} + \sum_{k \neq j} q_{jk}^{SO} - y_j \geq 0$. Then, $q_{ij}^{SO} = x_j^* - \sum_{k \neq i,j} q_{kj}^{SO} + \sum_{k \neq j} q_{jk}^{SO} - y_j$ by Lemma A-2, and therefore $\sum_{k \neq j} q_{kj}^{SO} = x_j^* + \sum_{k \neq j} q_{jk}^{SO} - y_j$. Assume next that $x_j^* - \sum_{k \neq i,j} q_{kj}^{SO} + \sum_{k \neq j} q_{jk}^{SO} - y_j < 0$. Then, $q_{ij}^{SO} = 0$ by Lemma A-2. Therefore, $x_j^* - \sum_{k \neq j,l} q_{kj}^{SO} + \sum_{k \neq j} q_{jk}^{SO} - y_j < q_{lj}^{SO}$ for all $l \neq i$. By Lemma A-2, we then have that $q_{lj}^{SO} = 0$ for all l. As a result, $\sum_{k \neq j} q_{kj}^{SO} = 0$ whenever $x_j^* + \sum_{k \neq j} q_{jk}^{SO} - y_j < 0$ and $\sum_{k \neq j} q_{kj}^{SO} = x_j^* + \sum_{k \neq j} q_{jk}^{SO} - y_j$ otherwise. □

Lemma A-4. *Suppose that* $\mathbf{q}^{SO} = \arg\max \Pi_i^{SO}(\mathbf{q}; \mathbf{D}, \mathbf{y})$ *for all* i. *Let* $\mathcal{N}(\mathbf{D}, \mathbf{y})$ *be the set of nodes* j *such that* $x_j^* + \sum_{k \neq j} g_{jk}(\mathbf{D}, \mathbf{y}) < y_j$. *Then, for all* $i \notin \mathcal{N}(\mathbf{D}, \mathbf{y})$, $\mathbf{q}^{SO}$ *solves:*

$$\begin{array}{rll} & \max_{\mathbf{q}} - \sum_{j \neq i} t_{ji}(q_{ji}) + \sum_{j \neq i} t_{ij}(q_{ij}) & \\ \textit{subject to} & \sum_{j \neq i} q_{ji} = \min\{D_i, (y - \sum_{j<i} D_j)^+\} + \sum_{j \neq i} q_{ij} - y_i & \\ & 0 \leq q_{ij} \leq g_{ij}(\mathbf{D}, \mathbf{y}) & \forall j, \end{array}$$

and such that $q_{ij}^{SO} = g_{ij}(\mathbf{D}, \mathbf{y})$ *for all* $i \in \mathcal{N}(\mathbf{D}, \mathbf{y})$ *and for all* $j \neq i$ *and* $q_{ji}^{SO} = 0$ *for all* $i \in \mathcal{N}(\mathbf{D}, \mathbf{y})$ *and for all* $j \neq i$.

Proof. Proof. By Lemmas 1 and A-3, $\mathbf{x}(\mathbf{q}^{SO}) = \mathbf{x}^*$ and $\sum_{k \neq j} q_{kj}^{SO} = (x_j^* + \sum_{k \neq j} q_{jk}^{SO} - y_j)^+$ for all j. Hence if $\mathbf{q}^{SO} = \arg\max \Pi_i^{SO}(\mathbf{q}; \mathbf{D}, \mathbf{y})$ for all i, then $\mathbf{q}^{SO}$ maximizes, for all i,

$$\begin{array}{rlll} \max \Pi_i^{SO}(\mathbf{q}; \mathbf{D}, \mathbf{y}) = & (v_i - c(y))x_i(\mathbf{q}) - \sum_{j \neq i} t_{ji}(q_{ji}) + \sum_{j \neq i} t_{ij}(q_{ij}) - K(y)\frac{y_i}{y} & & \\ \text{subject to} & \sum_{j=1}^{n} x_j(\mathbf{q}) \leq y & & \\ & 0 \leq x_j(\mathbf{q}) \leq D_j & \forall j & \\ & x_j(\mathbf{q}) = \min\{D_j, y_j + \sum_{k \neq j} q_{kj} - \sum_{k \neq j} q_{jk}\} & \forall j & \text{(A-1)} \\ & 0 \leq q_{ij} \leq g_{ij}(\mathbf{D}, \mathbf{y}) & \forall i, j & \\ & \mathbf{x}(\mathbf{q}) = \mathbf{x}^*, & & \\ & \sum_{k \neq j} q_{kj} = (x_j^* + \sum_{k \neq j} q_{jk} - y_j)^+ & \forall j. & \end{array}$$

Consider a particular solution $\mathbf{q}^{SO}$ and assume that $x_j^* + \sum_{k \neq j} q_{jk}^{SO} - y_j < 0$. By Lemma A-3, $\sum_{k \neq j} q_{kj}^{SO} = 0$, and therefore, $x_j(\mathbf{q}^{SO}) + \sum_{k \neq j} q_{jk}^{SO} - \sum_{k \neq j} q_{kj}^{SO} - y_j < 0$. Taking the derivative of Π_j with respect to q_{ji} on that range of values yields $d\Pi_j/dq_{ji} = t'_{ji}(q_{ij}^{SO}) > 0$. Hence, Firm j will choose q_{ji} as large as possible, namely, $g_{ji}(\mathbf{D}, \mathbf{y})$. Hence, the requirement that $\sum_{k \neq j} q_{kj} = (x_j^* + \sum_{k \neq j} q_{jk} - y_j)^+$ simplifies as follows: Either $x_j^* + \sum_{k \neq j} g_{jk}(\mathbf{D}, \mathbf{y}) < y_j$, i.e., $j \in \mathcal{N}(\mathbf{D}, \mathbf{y})$, in which case $q_{jk}^{SO} = g_{jk}(\mathbf{D}, \mathbf{y})$ for all $k \neq j$ and $q_{kj}^{SO} = 0$ for all $k \neq j$. Or $x_j^* + \sum_{k \neq j} g_{jk}(\mathbf{D}, \mathbf{y}) \geq y_j$, i.e., $j \notin \mathcal{N}(\mathbf{D}, \mathbf{y})$, in which case $\sum_{k \neq j} q_{kj}^{SO} = x_j^* + \sum_{k \neq j} q_{jk}^{SO} - y_j$. Replacing the last constraint in (A-1) with these two formulations and eliminating the redundant variables establishes the lemma. □

Lemma A-5. *For any* (i, j), *the function* $g_{ij}(\mathbf{D}, \mathbf{y})$ *is piecewise with as many as* 2^n *different pieces.*

Proof. Proof. By Lemma A-1, q_{ij}^{SO} is either equal to its upper bound or equal to zero. Let $\mathcal{A}(\mathbf{D},\mathbf{y})$ be an arc set such that $q_{ij}^{SO} = g_{ij}(\mathbf{D},\mathbf{y})$. From Lemma A-4, we must have $\sum_{j\neq i:(j,i)\in\mathcal{A}(\mathbf{D},\mathbf{y})} g_{ji}(\mathbf{D},\mathbf{y}) = \min\{D_i, (y-\sum_{j<i} D_j)^+\} + \sum_{j\neq i:(i,j)\in\mathcal{A}(\mathbf{D},\mathbf{y})} g_{ij}(\mathbf{D},\mathbf{y}) - y_i$ for all $i \notin \mathcal{N}(\mathbf{D},\mathbf{y})$. Since there are 2^n possible configurations for $\mathcal{N}(\mathbf{D},\mathbf{y})$, each function $g_{ij}(\mathbf{D},\mathbf{y})$ is thus piecewise with as many as 2^n different pieces. □

Proof. Proof of Proposition 1. The proof follows from Lemmas A-1 and A-5 in the appendix. □

Lemma A-6. *Suppose that $n = 2$, $K(y) = ky$, $c'(y) = 0$, $v_1 \geq \lambda_1 \geq v_2 \geq \lambda_2 \geq 0$, and $\lambda_1 \geq k$. Then, under the FB production plan, there exists a pure-strategy Nash equilibrium in the SO capacity investment game.*

Proof. Proof. Without loss of generality, let us rewrite $v_i \equiv v_i - c(y)$ since $c'(y) = 0$. Any equilibrium, if it exists, must satisfy the following first-order conditions:

$$\begin{aligned} \frac{d\Pi_1(y_1;y_2)}{dy_1} = v_1 S_1'(y) - k + \lambda_1 \mathbb{P}[y \geq D_1 \geq y_1] + \lambda_2 \mathbb{P}[y_1 \geq D_1, D_1+D_2 \geq y] &= 0, \text{ and} \\ \frac{d\Pi_2(y_2;y_1)}{dy_2} = v_2 S_2'(y) - k + \lambda_1 \mathbb{P}[y \leq D_1] + \lambda_2 \mathbb{P}[y_2 \leq D_2, D_1+D_2 \leq y] &= 0. \end{aligned} \tag{A-2}$$

From (A-2), we obtain that $\partial^2\Pi_1^{SO}(y_1;y_2)/\partial y_1^2$ is equal to

$$-(v_1-\lambda_1)f_1(y) - \lambda_1 \int_0^{y-y_1} f_{12}(y,\xi_2)d\xi_2 - (\lambda_1-\lambda_2)\int_{y-y_1}^{\infty} f_{12}(y_1,\xi_2)d\xi_2 - \lambda_2 \int_0^{y_1} f_{12}(\xi_1, y-\xi_1)d\xi_1,$$

which is nonpositive when $v_1 \geq \lambda_1 \geq \lambda_2 \geq 0$. Similarly, given that $f(y) = \int_0^y f_{12}(y-\xi_2,\xi_2)d\xi_2$, we find from (A-2) that $\partial^2\Pi_2^{SO}(y_2;y_1)/\partial y_2^2$ is equal to

$$-(\lambda_1 - v_2)f_1(y) - \lambda_2 \int_0^{y_2} f_{12}(y-\xi_2,\xi_2)d\xi_2 - (v_2-\lambda_2)f(y) - \lambda_2 \int_0^{y-y_2} f_{12}(\xi_1,y_2)d\xi_1,$$

which is nonpositive when $\lambda_1 \geq v_2 \geq \lambda_2 \geq 0$. Hence, $\Pi_1^{SO}(y_1;y_2)$ and $\Pi_2^{SO}(y_2;y_1)$ are continuous and concave when $v_1 \geq \lambda_1 \geq v_2 \geq \lambda_2 \geq 0$. Without loss of generality, the action spaces can be bounded. As a result, there exists a Nash equilibrium (Fudenberg and Tirole 1991, p. 34) and every equilibrium satisfies (A-2). □

Proof. Proof of Proposition 2. Any interior equilibrium must satisfy the following first-order conditions:

$$\begin{aligned} \frac{d\Pi_1^{SO}(y_1^{SO};y_2^{SO})}{dy_1} = 0 &= (v_1 - c(y))S_1'(y) - \tfrac{y_1}{y}K'(y) - \tfrac{y_2}{y^2}K(y) - c'(y)S_1(y) \\ &\quad + \lambda_{21}\mathbb{P}[y \geq D_1 \geq y_1] + \lambda_{12}\mathbb{P}[y_1 \geq D_1, D_1+D_2 \geq y], \quad \text{and} \\ \frac{d\Pi_2^{SO}(y_2^{SO};y_1^{SO})}{dy_2} = 0 &= (v_2 - c(y))S_2'(y) - \tfrac{y_2}{y}K'(y) - \tfrac{y_1}{y^2}K(y) - c'(y)S_2(y) \\ &\quad + \lambda_{21}\mathbb{P}[y \leq D_1] + \lambda_{12}\mathbb{P}[y_2 \leq D_2, D_1+D_2 \leq y]. \end{aligned} \tag{A-3}$$

Adding the equilibrium conditions (A-3) and using the first-order optimality condition (2) yields $\lambda_{21}\mathbb{P}[D_1 \geq y_1^{SO}] + \lambda_{12}\mathbb{P}[D_1 \leq y_1^{SO}, D_2 \geq y_2^{SO}] = K(y^*)/y^*$. Conversely, suppose that there exists an equilibrium satisfying $\lambda_{21}\mathbb{P}[D_1 \geq y_1^{SO}] + \lambda_{12}\mathbb{P}[D_1 \leq y_1^{SO}, D_2 \geq y_2^{SO}] = K(y^{SO})/y^{SO}$. Summing up the equilibrium conditions (A-3) yields $(v_1 - c(y^{SO}))S_1'(y^{SO}) + (v_2 - c(y^{SO}))S_2'(y^{SO}) = K'(y^{SO}) + c'(y^{SO})S(y^{SO})$. Because $\Pi(y)$ is strictly pseudo-concave, $y^{SO} = y^*$. □

Proof. Proof of Proposition 3. First, note that (i) implies that $\lambda_i \geq \lambda_j$ if and only if $i < j$; and, (i) and (ii) imply that $v_i - \lambda_i \frac{(y-y_i)}{y} - c(y) \geq \lambda_j \frac{y_i}{y}$ if and only if $i < j$.

Replacing $q_{ij} = x_j y_i / y$ for all i, j, $\mathbf{q}^{JO} = \arg\max \Pi_i^{JO}(\mathbf{q};\mathbf{D},\mathbf{y})$ if and only if $\mathbf{x}^{JO}$ solves, for all i:

$$\begin{aligned} \max \Pi_i^{JO}(\mathbf{x};\mathbf{D},\mathbf{y}) = \quad & (v_i - c(y))x_i - \lambda_i x_i \textstyle\sum_{j\neq i} \frac{y_j}{y} + \frac{y_i}{y}\sum_{j\neq i}\lambda_j x_j - K(y)\frac{y_i}{y} \\ \text{subject to} \quad & \textstyle\sum_{j=1}^n x_j \leq y \\ & 0 \leq x_j \leq D_j \qquad \forall j, \end{aligned}$$

with $q_{ij}^{JO} = x_j^{JO} y_i / y$ for all i, j.

Suppose that Conditions (i)-(ii) hold. Suppose that $x^{JO} < y$. For any $x_j < D_j$, we have $d\Pi_i^{JO}/dx_j = \lambda_j y_i/y \geq 0$ and $d\Pi_j^{JO}/dx_j = v_j - \lambda_j(1 - y_j/y) - c(y) \geq 0$ by (ii). Hence, all players agree that x_j must increase as much as possible until either $x_j = D_j$ or until $x = y$. Suppose now that $x = y$. For any $x_j^{JO} < D_j$, we have $d\Pi_i^{JO}/dx_j \geq d\Pi_i/dx_k$ if and only if $j < k$ by (i), for all $i \neq j, k$, and $d\Pi_j^{JO}/dx_j \geq d\Pi_j^{JO}/dx_k$ if and only if $j < k$ by (ii). Hence if $x_j < D_j$ and $x_k > 0$, all players agree that x_j must increase and x_k must decrease if $j < k$ until either $x_j = D_j$ or $x_k = 0$. As a result, $\mathbf{x}^{JO} = \mathbf{x}^*$. The converse is shown similarly. □

Lemma A-7. *Suppose that $n = 2$, $K(y) = ky$, $c'(y) = 0$, $\lambda_i \leq v_i$ for $i = 1, 2$, $\lambda_1 \geq \lambda_2 \geq 0$, and $y^* \leq \hat{y} \leq \bar{F}^{-1}(c/\lambda_1)$, in which $\hat{y}$ solves $\Lambda(\hat{y}) = k\hat{y}$. Then under the FB production plan, there exists a pure-strategy Nash equilibrium (y_1^{JO}, y_2^{JO}) in the JO capacity investment game.*

Proof. Proof. Without loss of generality, let us rewrite $v_i \equiv v_i - c(y)$ since $c'(y) = 0$. By the Weierstrass theorem, for any y_{-i}, $\Pi_i^{JO}(y_i; y_{-i})$ attains a maximum on $[-y_{-i}, \infty)$ because it is continuous and because $\lim_{y_i \to \infty} \Pi_i^{JO}(y_i; y_{-i}) = -\infty$. Hence, any equilibrium, if it exists, must solve

$$\frac{d\Pi_i^{JO}(y_i; y_{-i})}{dy_i} = (v_i - \lambda_i)S_i'(y) - k + \frac{y_i}{y}\Lambda'(y) + \frac{y_{-i}}{y^2}\Lambda(y) = 0, \quad i = 1, 2. \tag{A-4}$$

Summing up the necessary optimality conditions (A-4), we obtain that a Nash equilibrium (y_1^{JO}, y_2^{JO}), if it exists, must solve $v_1 S_1'(y) + v_2 S_2'(y) - k + \Lambda(y)/y - k = 0$ with $y = y_1^{JO} + y_2^{JO}$. For any $y < y^*$, $v_1 S_1'(y) + v_2 S_2'(y) - k > 0$ by (2) and $\Lambda(y)/y - k \geq 0$ because $y^* \leq \hat{y}$; hence $y_1^{JO} + y_2^{JO} \geq y^*$. On the other hand for any $y > \hat{y}$, $v_1 S_1'(y) + v_2 S_2'(y) - k < 0$ by (2), because $y^* \leq \hat{y}$, and $\Lambda(y)/y - k \leq 0$; hence, $y_1^{JO} + y_2^{JO} \leq \hat{y}$. Therefore, for any y_{-i}, $\Pi_i^{JO}(y_i; y_{-i})$ must attain its maximum on $[y^* - y_{-i}, \hat{y} - y_{-i}]$ for (y_i, y_{-i}) to be an equilibrium.

Moreover, if there exists an equilibrium with $y_i^{JO} < 0$, there also exists one with $y_i^{JO} \geq 0$ because $d\Pi_i^{JO}(0; y_{-i})/dy_i = (v_i - \lambda_i)S_i'(y) - k + \Lambda(y)/y \geq (v_i - \lambda_i)S_i'(y) \geq 0$, given that $\Lambda(y)/y - k \geq \Lambda(\hat{y})/\hat{y} - k = 0$ for any $y \leq \hat{y}$. Therefore, without loss of generality, one can restrict the strategy spaces to the compact sets $[(y^* - y_{-i})^+, (\hat{y} - y_{-i})^+]$.

When $y_2/y \leq (v_2 - \lambda_2)/(\lambda_1 - \lambda_2)$, the derivative of Firm 2's profit function is nonnegative:

$$\begin{aligned}
\frac{d\Pi_2^{JO}(y_2; y_1)}{dy_2} &= (v_2 - \lambda_2)S_2'(y) - k + \frac{\Lambda(y)}{y} + \frac{y_2}{y}\left(\Lambda'(y) - \frac{\Lambda(y)}{y}\right) \\
&\geq (v_2 - \lambda_2)S_2'(y) - k + \frac{\Lambda(y)}{y} + \frac{v_2 - \lambda_2}{\lambda_1 - \lambda_2}\left(\Lambda'(y) - \frac{\Lambda(y)}{y}\right) \\
&= \frac{v_2 - \lambda_2}{\lambda_1 - \lambda_2}\left(\lambda_1 S'(y) - \frac{\Lambda(y)}{y}\right) - k + \frac{\Lambda(y)}{y} \geq \left(1 - \frac{v_2 - \lambda_2}{\lambda_1 - \lambda_2}\right)\left(\frac{\Lambda(y)}{y} - k\right) \geq 0,
\end{aligned}$$

in which the first inequality used the fact that function $\Lambda(y)$ is concave increasing, the second inequality used the fact that $S'(y) \geq S'(\hat{y}) \geq k/\lambda_1$ for any $y \leq \hat{y}$, and the third inequality used the fact that $\Lambda(y)/y - k \geq \Lambda(\hat{y})/\hat{y} - k = 0$ for any $y \leq \hat{y}$.

On the other hand, when $y_2/y \geq (v_2 - \lambda_2)/(\lambda_1 - \lambda_2)$ and $y_1 \geq 0$, the second derivative of Firm 2's profit function is nonpositive:

$$\begin{aligned}
\frac{d^2\Pi_2^{JO}(y_2; y_1)}{dy_2^2} &= (v_2 - \lambda_2)S_2''(y) + \frac{y_2}{y}\Lambda''(y) + 2\frac{y_1}{y^2}\left(\Lambda'(y) - \frac{\Lambda(y)}{y}\right) \leq (v_2 - \lambda_2)S_2''(y) + \frac{y_2}{y}\Lambda''(y) \\
&= \left(v_2 - \lambda_2\frac{y_1}{y}\right)S''(y) + \left((\lambda_1 - \lambda_2)\frac{y_2}{y} - (v_2 - \lambda_2)\right)S_1''(y) \leq 0.
\end{aligned}$$

As a result, when $y_1 \geq 0$, $\Pi_2^{JO}(y_2; y_1)$ is quasiconcave, increasing when $y_2/y \leq (v_2 - \lambda_2)/(\lambda_1 - \lambda_2)$ and concave when $y_2/y \geq (v_2 - \lambda_2)/(\lambda_1 - \lambda_2)$. On the other hand, Firm 1's profit function $\Pi_1^{JO}(y_1; y_2)$ is concave for all $y_1 \geq 0$ because $(v_1 - \lambda_1)S_1(y)$, $(y_1/y)(\lambda_1 - \lambda_2)S_1(y)$, $(y_1/y)\lambda_2 S(y)$, and $-ky_1$ are all concave in y_1. Because both players' profit functions are continuous and quasiconcave on $[(y^* - y_{-i})^+, (\hat{y} - y_{-i})^+]$ and their respective strategy sets are compact, there exists a Nash equilibrium (Fudenberg and Tirole 1991, p. 34). □

Proof. Proof of Proposition 4. Suppose that there exists a Nash equilibrium such that $\sum_{i=1}^{n} y_i = y^*$. From (6), it must solve the equilibrium conditions: For $i = 1, \ldots, n$,

$$\frac{d\Pi_i^{JO}(y_i;\mathbf{y}_{-i})}{dy_i} = 0 \quad = \quad (v_i - c(y) - \lambda_i)S_i'(y) - c'(y)S_i(y) + \frac{y_i}{y}\left(\Lambda'(y) - K'(y)\right) + \frac{\sum_{j\neq i} y_j}{y^2}\left(\Lambda(y) - K(y)\right) \tag{A-5}$$

Summing up (A-5) over $i = 1, \ldots, n$, we obtain $0 = (v_1 - c(y^*))S_1'(y^*) + (v_2 - c(y^*))S_2'(y^*) - c'(y^*)S(y^*) - K'(y^*) + (n-1)\left(\Lambda(y^*) - K(y^*)\right)/y^*$. Because (2) holds, $\sum_{i=1}^{n} \lambda_i S_i(y^*) = K(y^*)$. Conversely, suppose that there exists a Nash equilibrium satisfying (A-5) such that $\sum_{i=1}^{n} \lambda_i S_i(y) = K(y)$. Summing up the equilibrium conditions (A-5), we obtain $(v_1 - c(y))S_1'(y) + (v_2 - c(y))S_2'(y) - c'(y)S(y) - K'(y) = 0$. Because $\Pi(y)$ is strictly pseudo-concave, there exists a unique y, namely y^*, such that this equality holds. □

Lemma A-8. *For any $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_n)$ such that $\Lambda(y^*) = K(y^*)$, $d\Pi_i^{JO}/d\lambda_i = -\sum_{j\neq i} d\Pi_j^{JO}/d\lambda_i \leq 0$, for all i and, when $c'(y) = 0$, $dy_i^{JO}/d\lambda_i = -\sum_{j\neq i} dy_j^{JO}/d\lambda_i \leq 0$ for all i.*

Proof. Proof. Applying the chain rule to (6), we get

$$\begin{aligned}\frac{d\Pi_i^{JO}(y_i;\mathbf{y}_{-i})}{d\lambda_i} &= \frac{\partial\Pi_i^{JO}(y_i;\mathbf{y}_{-i})}{\partial\lambda_i} + \frac{\partial\Pi_i^{JO}(y_i;\mathbf{y}_{-i})}{\partial y_i}\frac{dy_i}{d\lambda_i} + \sum_{j\neq i}\frac{\partial\Pi_i^{JO}(y_i;\mathbf{y}_{-i})}{\partial y_j}\frac{dy_j}{d\lambda_i} \\ &= \frac{\partial\Pi_i^{JO}(y_i;\mathbf{y}_{-i})}{\partial\lambda_i} = -S_i(y)\frac{\sum_{j\neq i} y_j}{y} = -\sum_{j\neq i}\frac{\partial\Pi_j^{JO}(y_j;\mathbf{y}_{-j})}{\partial\lambda_i},\end{aligned}$$

in which the first equality follows from the fact that $\partial\Pi_i^{JO}(y_i;\mathbf{y}_{-i})/\partial y_i = 0$ in equilibrium and that $\partial\Pi_i^{JO}(y_i;\mathbf{y}_{-i})/\partial y_j = 0$ for $j \neq i$ when $\Lambda(y^*) = K(y^*)$. Moreover, because $-S_i(y)(1-y_i)/y \leq 0$, $d\Pi_i^{JO}(y_i;\mathbf{y}_{-i})/d\lambda_i \leq 0$.

From (8) and (7), we obtain that $y_i^{JO}/y_j^{JO} = \left(\Pi_i^{JO}(y_i^{JO};\mathbf{y}_{-i}^{JO})/\Pi_j^{JO}(y_j^{JO};\mathbf{y}_{-j}^{JO})\right)\left(S_i'(y^*)S_j(y^*)/\left(S_i(y^*)S_j'(y^*)\right)\right)$ when $c'(y) = 0$. Hence, y_i^{JO}/y_j^{JO} is directly proportional to $\Pi_i^{JO}(y_i;\mathbf{y}_{-i})/\Pi_j^{JO}(y_j;\mathbf{y}_{-j})$. Therefore $dy_i^{JO}/d\lambda_i = -\sum_{j\neq i} dy_j^{JO}/d\lambda_i \leq 0$. □

Proof. Proof of Proposition 5. By Propositions 3 and 4, there exists a coordinating contract if and only if the following system of equations is feasible:

$$\begin{aligned}\lambda_1 S_1(y^*) + \lambda_2 S_2(y^*) &= K(y^*) \\ \lambda_1 \geq \lambda_2 &\geq 0 \\ v_1 - c(y^*) - \lambda_1 &\geq \frac{y_1^{JO}}{y^*}(\lambda_2 - \lambda_1) \\ v_2 - c(y^*) - \lambda_2 &\leq \frac{y_2^{JO}}{y^*}(\lambda_1 - \lambda_2) \\ v_2 - c(y^*) - \lambda_2\frac{y_1^{JO}}{y^*} &\geq 0.\end{aligned}$$

Consider the following polyhedron: $\mathcal{P} = \{(\lambda_1,\lambda_2) : \lambda_1 \geq \lambda_2 \geq 0, v_i - c(y^*) - \lambda_i \geq 0, i = 1,2\}$, which is nonempty since $v_2 \geq c(y^*)$. Because $\Lambda(y^*) \leq K(y^*)$ at $(\lambda_1,\lambda_2) = (0,0)$ and $\Lambda(y^*) \geq K(y^*)$ at $(\lambda_1,\lambda_2) = (v_1 - c(y^*), v_2 - c(y^*))$ since $\Pi(y^*) \geq 0$, the hyperplane $\{(\lambda_1,\lambda_2) : \Lambda(y^*) = K(y^*)\}$ separates $\mathcal{P}$. Therefore, the polyhedron $\mathcal{Q} = \{(\lambda_1,\lambda_2) \in \mathcal{P} : \lambda_1 S_1(y^*) + \lambda_2 S_2(y^*) = K(y^*)\}$ is non-empty.

For any point in $\mathcal{Q}$, the constraints $v_1 - c(y^*) - \lambda_1 \geq \frac{y_1^{JO}}{y^*}(\lambda_2 - \lambda_1)$ and $v_2 - c(y^*) - \lambda_2\frac{y_1^{JO}}{y^*} \geq 0$ are satisfied. Consider next the constraint $v_2 - c(y^*) - \lambda_2 \leq \frac{y_2^{JO}}{y^*}(\lambda_1 - \lambda_2)$. For any point in $\mathcal{Q}$, this constraint is satisfied if and only if, using (7),

$$\begin{aligned}& v_2 - c(y^*) - \lambda_2 \leq \frac{(v_2 - \lambda_2 - c(y^*))S_2'(y^*) - c'(y^*)S_2(y^*)}{K'(y^*) - \Lambda'(y^*)}(\lambda_1 - \lambda_2) \\ \Leftrightarrow\ & K'(y^*) - \Lambda'(y^*) \leq S_2'(y^*)(\lambda_1 - \lambda_2) - c'(y^*)S_2(y^*)\frac{\lambda_1 - \lambda_2}{v_2 - c(y^*) - \lambda_2} \\ \Leftrightarrow\ & \frac{K'(y^*)}{S'(y^*)} + \frac{c'(y^*)S_2(y^*)}{S'(y^*)}\frac{\lambda_1 - \lambda_2}{v_2 - c(y^*) - \lambda_2} \leq \lambda_1,\end{aligned}$$

given that $v_2 - \lambda_2 - c(y^*) \geq 0$ and that $K'(y^*) \geq \Lambda'(y^*)$ since $\lambda_i \leq v_i - c(y^*)$ and y^* solves (2). The hyperplane $\Lambda(y^*) = K(y^*)$ intercepts the λ_1-axis at $\lambda_1 = K(y^*)/S_1(y^*)$. Suppose that $v_1 - c(y^*) \geq K(y^*)/S_1(y^*)$, i.e., the point $(K(y^*)/S_1(y^*), 0) \in \mathcal{Q}$. Because $K(y^*)/S_1(y^*) \geq \frac{K'(y^*)}{S'(y^*)} + \frac{c'(y^*)S_2(y^*)}{S'(y^*)} \frac{K(y^*)}{S_1(y^*)(v_2 - c(y^*))}$ by assumption, the set $\{(\lambda_1, \lambda_2) \in \mathcal{Q} : v_2 - c(y^*) - \lambda_2 \leq \frac{y_2^{JO}}{y^*}(\lambda_1 - \lambda_2)\}$ is nonempty. Suppose now that $v_1 - c(y^*) \leq K(y^*)/S_1(y^*)$ and consider the point $(v_1 - c(y^*), (K(y^*) - (v_1 - c(y^*))S_1(y^*))/S_2(y^*)) \in \mathcal{Q}$. Because $v_1 - c(y^*) \geq \frac{K'(y^*)}{S'(y^*)}$ by (2) and because $v_1 \geq v_2$, this point also satisfies the constraint $\frac{K'(y^*)}{S'(y^*)} + \frac{c'(y^*)S_2(y^*)}{S'(y^*)} \frac{\lambda_1 - \lambda_2}{v_2 - c(y^*) - \lambda_2} \leq \lambda_1$. In that case also, the set $\{(\lambda_1, \lambda_2) \in \mathcal{Q} : v_2 - c(y^*) - \lambda_2 \leq \frac{y_2^{JO}}{y^*}(\lambda_1 - \lambda_2)\}$ is nonempty. □

Proof. Proof of Corollary 2. Denote $K(y) = ky$ and $c(y) = c$. Using (2), we obtain

$$S_1(y^*) - \bar{F}(y^*)y^* = S_1(y^*) - \frac{k - (v_1 - v_2)\bar{F}_1(y^*)}{v_2 - c} y^*.$$

The right-hand side of the expression is decreasing in y^* because its derivative with respect to y^* is equal to $((v_1 - c)\bar{F}(y_1^*) - k)/(v_2 - c) - (v_1 - v_2)f_1(y^*)/(v_2 - c) = -S_2'(y^*) - (v_1 - v_2)f_1(y^*)y^*/(v_2 - c) \leq 0$. Let $\tilde{y}^*$ be the solution to (2) when the total demand is $\tilde{D}$ and let $\tilde{S}(y)$ be the corresponding expected sales function. By (2), $\tilde{y}^* \geq y^*$. Because the right-hand side of the above equation only depends on D through y^*, keeping D_1 constant, we thus obtain that $S_1(y^*) - y^*[k - (v_1 - v_2)\bar{F}_1(y^*)]/(v_2 - c) \geq S_1(\tilde{y}^*) - \tilde{y}^*[k - (v_1 - v_2)\bar{F}_1(\tilde{y}^*)]/(v_2 - c)$. Hence if $S_1(y^*) \leq S'(y^*)y^*$, then $S_1(\tilde{y}^*) \leq \tilde{S}'(\tilde{y}^*)\tilde{y}^*$. □

Proof. Proof of Corollary 3. By (2), $dy^*/dv_2 > 0$. Consequently, there exists a threshold $\bar{v}_2$ such that $S_1(y) - y\bar{F}(y) \leq 0$ if and only if $v_2 < \bar{v}_2$. □

Lemma A-9. *Suppose $D_1 + D_2$ has an increasing failure rate (IFR). If D_1 is exponentially distributed, there exists a threshold $\bar{y} \in [0, \infty)$ such that $S_1(y) - y\bar{F}(y) \leq 0$ if and only if $y \leq \bar{y}$.*

Proof. Proof. In order to prove the existence of such a threshold, it is sufficient to show that the derivative of $S_1(y) - y\bar{F}(y)$, equal to $\bar{F}_1(y) - \bar{F}(y) + f(y)y$, is positive whenever $S_1(y) - y\bar{F}(y) = 0$ and $y > 0$. Given that D is IFR, $h(\xi) \equiv f(\xi)/\bar{F}(\xi)$ is increasing. Using Equation (2.1) in Barlow and Proschan (1965), we obtain $\bar{F}(y) = \exp\left[-\int_0^y h(\xi)d\xi\right] \geq \exp\left[-yh(y)\right]$, and therefore $yh(y) \geq -\ln\left(\bar{F}(y)\right)$. Accordingly, we obtain $\bar{F}_1(y) - \bar{F}(y) + f(y)y \geq \bar{F}_1(y) - \bar{F}(y) - \bar{F}(y)\ln\left(\bar{F}(y)\right)$, which is equal to $\bar{F}_1(y) - \frac{S_1(y)}{y} - \frac{S_1(y)}{y}\ln\left(\frac{S_1(y)}{y}\right)$ when $S_1(y) - y\bar{F}(y) = 0$. When D_1 follows a negative exponential distribution with parameter λ, $y = -\ln(\varphi)/\lambda$ and $S_1(y) = (1 - \varphi)/\lambda$, in which $\varphi = \bar{F}(y)$. Accordingly, we obtain that, when $S_1(y) = y\bar{F}(y)$, $\bar{F}_1(y) - \bar{F}(y) + f(y)y$ is bounded from below by $\varphi + \frac{1-\varphi}{\ln(\varphi)} + \frac{1-\varphi}{\ln(\varphi)}\ln\left(\frac{1-\varphi}{-\ln(\varphi)}\right)$. It can be then be verified that the lower bound is always positive for any $0 \leq \varphi < 1$, i.e., for any $y \in (0, \infty)$, completing the proof. □

Lemma A-10. *Suppose D_1 and D_2 are independent. If D_2 is exponentially distributed, there exists a threshold $\bar{y} \in [0, \infty)$ such that $S_1(y) - y\bar{F}(y) \leq 0$ if and only if $y \leq \bar{y}$.*

Proof. Proof. When D_1 and D_2 are independent, $\bar{F}(y) = \int_0^y \bar{F}_2(y - \xi)f_1(\xi)d\xi + \bar{F}_1(y)$ and $f(y) = \int_0^y f_2(y - \xi)f_1(\xi)d\xi$. Let $h_2(\xi) = f_2(\xi)/\bar{F}_2(\xi)$. Hence, given that D_2 is IFR, we have

$$\bar{F}_1(y) - \bar{F}(y) + f(y)y \quad = \quad \int_0^y \left(yf_2(y - \xi) - \bar{F}_2(y - \xi)\right) f_1(\xi)d\xi \geq (h_2(0)y - 1)\int_0^y \bar{F}_2(y - \xi)f_1(\xi)d\xi.$$

Let $\lambda = h_2(0)$. We must have that $\lambda y \geq 1$ when $S_1(y) - y\bar{F}(y) = 0$ because otherwise $\bar{F}_2(y - x) = \exp\left[-(y - x)\lambda\right] > \exp\left[-(y - x)/y\right] \geq x/y$ for all $x \leq y$, and therefore $\bar{F}(y) = \int_0^y \bar{F}_2(y - \xi)f_1(\xi)d\xi + \bar{F}_1(y) > \int_0^y (\xi/y)f_1(\xi)d\xi + \bar{F}_1(y) = S_1(y)/y$, a contradiction. As a result, when D_2 is exponentially distributed, we obtain:

$$\left(\bar{F}_1(y) - \bar{F}(y) + f(y)y\right)\Big|_{S_1(y) = y\bar{F}(y)} \geq \left((\lambda y - 1)\int_0^y \bar{F}_2(y - \xi)f_1(\xi)d\xi\right)\Big|_{S_1(y) = y\bar{F}(y)} \geq 0.$$

Hence, the function $S_1(y) - y\bar{F}(y) \leq 0$ crosses zero at most once and, if it does, it crosses it from below. □

References

Anupindi, R., Y. Bassok, E. Zemel. 2001. A general framework for the study of decentralized distribution systems. *Manufacturing Service Oper. Management* **3**(4) 349–368.

Bamford, J., D. Ernst, D. G. Fubini. 2004. Launching a world-class joint venture. *Harvard Bus. Rev.* **February** 1–10.

Barlow, R. E., F. Proschan. 1965. *Mathematical Theory of Reliability*. SIAM Classics in Applied Mathematics, Philadelphia, PA.

Bidault, F., M. Schweinsberg. 1996. *Fiat and Peugeot SevelNord Venture (A)*. IMD-3-0644.

Bradsher, K. 1997. After Latin venture fails, VW succeeds and Ford scrambles. *The New York Times* May 16.

Cavusgil, T., G. Knight, J. Riesenberger. 2007. *International Business: Strategy, Management, and the New Realities*. Prentice Hall.

Cheng, M. 2010. Shortage in production capacity resulting in the changan ford mazda spin-off. URL `http://www.upiu.com/business/2010/09/04/Shortage-in-production-capacity-resulting-in-the-Changan-Ford-Mazda-spin-off/UPIU-9361283624186/`. September 4. Retrieved September 2012.

Cho, S. H. 2011. Horizontal mergers in multi-tier decentralized supply chains. Carnegie-Mellon University Working Paper.

Cho, S. H., X. Wang. 2012. Newsvendor mergers. INFORMS 2012 presentation.

Dong, L., N. Rudi. 2004. Who benefits from transshipment? Exogenous vs. endogenous wholesale prices. *Management Sci.* **50**(5) 645–657.

Economist, (The). 2011. Renault's woes. *The Economist* URL `http://www.economist.com/node/18775491`. June 2.

Economist, (The). 2012. All for one, and one for all. *The Economist* October 6.

Fudenberg, D., J. Tirole. 1991. *Game Theory*. The MIT Press, Cambridge, MA.

Gao, P. 2002. A tune-up for China's auto industry. *McKinsey Quarterly* (1) 144–155.

Granot, D., G. Sošić. 2003. A three-stage model for a decentralized distribution system of retailers. *Oper. Res.* **51**(5) 771–784.

Grossman, S. J., O. D. Hart. 1986. The costs and benefits of ownership: A theory of vertical and lateral integration. *J. Polit. Econ.* **94**(4) 691–719.

Hansmann, H. 1988. Ownership of the firm. *Journal of Law, Economics & Organizations* **4**(2) 267–304.

Harrigan, K. R. 1988. Joint ventures and competitive strategy. *Strategic Management J.* **9**(2) 141–158.

Hartman, B. C., M. Dror, M. Shaked. 2000. Cores of inventory centralization games. *Games Econ. Behav.* **31**(1) 26–49.

Hennart, J.-F. 1988. A transaction costs theory of equity joint ventures. *Strategic Management J.* **9**(4) 361–374.

Herd, T. 2010. Joint ventures: Creating value against the odds. *Business Week* URL `http://www.businessweek.com/managing/content/nov2010/ca2010112_913040.htm#p1`. November 2. Retrieved September 2012.

Hu, X., I. Duenyas, R. Kapuscinski. 2007. Existence of coordinating transshipment prices in a two-location inventory model. *Management Sci.* **53**(8) 1289–1302.

Huang, X., G. Sošić. 2011. Transshipment of inventories: Dual allocations vs. transshipment prices. *Manufacturing Service Oper. Management* **12**(2) 299–318.

Hunter Valley Coal Chain. 2009. Attachment: Hunter valley coal chain alignment. URL http://www.accc.gov.au/content/item.phtml?itemId=886454&nodeId=fccddceca1e96b032470c19aec286c97&fn=Rio%20CA%20(attachment)%203%20Jul%2009.pdf. Retrieved September 2012.

ICA, (International Co-Operative Alliance). 2007. Statement of the cooperative identity. URL http://www.ica.coop/coop/principles.html. May 26. Retrieved November 2011.

IFRS. 2011. IFRS11 - Joint Arrangements. Tech. rep., IFRS. URL http://www.ifrs.org/Current+Projects/IASB+Projects/Joint+Ventures/IFRS+11+Joint+Arrangements/faqs.htm.

Jackson, K., R. Turner. 1994. Ford, VW begin divorce proceedings for Autolatina: Partners disagree over sharing models. *Automotive News* October 24.

Jordan, W. C., S. C. Graves. 1995. Principles on the benefits of manufacturing process flexibility. *Management Sci.* **41**(4) 577–594.

Kemahlıoğlu-Ziya, E., J. J. Bartholdi, III. 2011. Centralizing inventory in supply chains by using shapley value to allocate the profits. *Manufacturing Service Oper. Management* **13**(2) 146–162.

Kogut, B. 1988. Joint ventures: Theoretical and empirical perspectives. *Strategic Management J.* **9**(4) 319–332.

Krishnan, H., T. McCorrmick, J. Shao. 2011. Incentives for transshipment in a supply chain with decentralized retailers. *Manufacturing Service Oper. Management* **13**(3) 361–372.

Kumar, M. V. S. 2010. Are joint ventures positive sum games? The relative effects of cooperative and noncooperative behavior. *Strategic Management J.* **32**(1) 32–54.

Luo, Y. 2002. Contract, cooperation, and performance in international joint ventures. *Strategic Management J.* **23**(10) 903–919.

Marinucci, M. 2009. Optimal ownership in joint ventures with contributions of asymmetric partners. *Journal of Economics* **97**(2) 141–163.

Müller, A., M. Scarsini, M. Shaked. 2002. The newsvendor game has a nonempty core. *Games Econ. Behav.* **38**(1) 118–126.

Park, S. H., M. V. Russo. 1996. When competition eclipses cooperation: An event history analysis of joint venture failure. *Management Sci.* **42**(6) 857–890.

Plambeck, E. L., T. A. Taylor. 2005. Sell the plant? The impact of contract manufacturing on innovation, capacity, and profitability. *Management Sci.* **51**(1) 133–150.

Reiermann, C. 2007. New Franco-German battle over EADS? *Business Week* URL http://www.businessweek.com/globalbiz/content/jul2007/gb20070710_903960.htm. July 10. Retrieved November 2011.

Roels, G., U. S. Karmarkar, S. Carr. 2009. Contracting for collaborative services. *Management Sci.* **56**(5) 849–863.

Rudi, N., S. Kapur, D. F. Pyke. 2001. A two-location inventory model with transshipment and local decision making. *Management Sci.* **47**(12) 1668–1680.

Suakkaphong, N., M. Dror. 2011. Managing decentralized inventory and transshipment. *TOP* **19** 480–506.

Syncrude. 2012. The Syncrude project. URL http://www.syncrude.ca/users/folder.asp?FolderID=5626. Retrieved September 2012.

Van Mieghem, J. A. 1999. Coordinating investment, production and subcontracting. *Management Sci.* **45**(7) 954–971.

Van Mieghem, J. A. 2003. Capacity management, investment, and hedging: Review and recent developments. *Manufacturing Service Oper. Management* **5**(4) 269 302.

Van Mieghem, J. A., N. Rudi. 2002. Newsvendor networks: Inventory management and capacity investment with discretionary activities. *Manufacturing Service Oper. Management* **4**(4) 313–335.

Yu, Y., S. Benjaafar, Y. Gerchak. 2011. Capacity sharing and cost allocation among independent firms in the presence of congestion. University of Minnesota Working Paper.

Zhang, J. 2005. Transshipment and its impact on supply chain members' performance. *Management Sci.* **51**(10) 1534–1539.

Zhu, J., T. Boyaci, S. Ray. 2012. Horizontal mergers and supply chain performance. McGill University Working Paper.

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