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**A new formulation of the European day-ahead electricity
market problem and its algorithmic consequences**

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Abstract

A new formulation of the optimization problem implementing European market rules for non-convex day-ahead electricity markets is presented, that avoids the use of complementarity constraints to express market equilibrium conditions, and also avoids the introduction of auxiliary binary variables to linearise these constraints. Instead, we rely on strong duality theory for linear or convex quadratic optimization problems to recover equilibrium constraints imposed by most of European power exchanges facing indivisible orders. When only so-called stepwise preference curves are considered to describe continuous bids, the new formulation allows to take full advantage of state-of-the-art solvers, and in most cases, an optimal solution together with market clearing prices can be computed for large-scale instances without any further algorithmic work. The new formulation also suggests a very competitive Benders-like decomposition procedure, which helps to handle the case of interpolated preference curves that yield quadratic primal and dual objective functions, and consequently a dense quadratic constraint. This procedure essentially consists in strengthening classical Benders cuts locally. Computational experiments on real data kindly provided by main European power exchanges (Apx-Endex, Belpex and Epex spot) show that in the linear case, both approaches are very efficient, while for quadratic instances, only the decomposition procedure is tractable and shows very good results. Finally, when most orders are block orders, and instances are combinatorially very hard, the new MILP approach is substantially more efficient.

Keywords: Auctions/bidding, market coupling, equilibrium prices, mixed integer programming, large scale optimization.

JEL classifications: C61, D44

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1 Introduction

Day-ahead electricity markets and European rules

The liberalization of electricity markets in developed countries has led to market design issues addressed now for many years, that still provide with interesting research questions. In Europe, efforts are currently made toward greater integration of electricity markets (e.g. the *Price Coupling of Region* (PCR) project supported by Europex [6]).

Day-ahead electricity markets are designed as two-sided auctions in which participants submit orders to buy or sell electricity power during some hours of the following day, in some given areas. A market operator collecting these orders is in charge of defining an optimal matching, as well as supporting prices. Order matching depends in particular on cross-boarder flows that are physically admissible between areas that are part of the market, while computed prices should ideally support a Walrasian equilibrium. Participants agree on a set of rules driving the clearing process, such as rules for bid acceptance and price determination. The literature about spatio-temporal partial equilibrium, thought in a different setting, dates back at least to the fifties ([5, 12]).

The most complicating feature is the fact that some orders may be non-convex, in the sense that they yield, in the mathematical formulation of the market clearing problem, objects that don't have the convexity property (e.g. convex feasible sets, etc). For example, a participant can submit a block order (or binary order) for which a "fill-or-kill condition" must hold (the order can only be fully accepted or fully rejected). These block orders allow participants to reflect more accurately their production constraints and cost structures. This is mainly due to (a) non-convex production sets (e.g. minimum and maximum output levels at which a plant can operate) and (b) fixed (start-up) costs. With these convex and non-convex bids, the market operator who is in charge of selecting an optimal allocation (execution levels of orders), together with related supporting prices, is facing a mixed integer optimization problem that doesn't possess all the nice features of its continuous relaxation counterpart.

A primal program defining the optimal selection of bids ensures that the allocation is dispatchable, i.e. respects network security constraints. Computed prices should ideally support a Walrasian equilibrium (for price-taker participants, the market clears for these prices and no excess demand/supply remains, see e.g. [11]). In a well-behaved context where there are no non-convexities (e.g. no block orders), optimal dual variables (i.e. shadow prices) of the primal program provide with supporting equilibrium prices, as expressed by complementarity constraints relating primal and dual optimal variables, see for example [7, 15].

In a mixed integer context, classical strong duality fails, corresponding supporting prices cannot be determined, and it is known that in most cases, strict linear equilibrium prices (also called uniform prices) do not exist. With strict linear pricing, payments depend only and linearly on exchanged quantities. In particular, this prevents the use of transfer payments for executed bids that would otherwise incur a loss to the bidder. As a consequence, in the presence of indivisibilities, market design choices must be made to deal with this issue.

A proposed solution [9] is to accept orders even if they incur a loss to the bidder, or to reject

them, even if they would provide with a gain. Some side-payments are paid to participants in one of these two situations, to implement a Walrasian equilibrium. Several approaches have been proposed to compute market prices and corresponding compensations (so-called uplifts), see [15] for a review. Due to these uplifts, this is not a *strict* linear pricing scheme.

In Europe, the common trend is to implement a strict linear pricing scheme. The chosen counterpart is that some in-the-money *block orders* may be paradoxically rejected, and are not financially compensated. On the other hand, all out-of-the money order are rejected, while at-the-money orders may be rejected, or executed (potentially fractionally for continuous orders). This is for example the solution adopted in coupled markets such as CWE (Central Western Europe market, pooling Belgium, France, Germany, Luxembourg and the Netherlands), which will be extended soon to the North Western Europe market (NWE), including Nordic-Baltic countries and Great-Britain. The market clearing optimization problem of these markets is the main topic of this article, see [1] for a full list of requirements.

The classical way to formulate common European market requirements in a mathematical model is via the addition of dual and complementarity constraints to the primal program defining feasible dispatches. These complementarity constraints form a *subset* of those that would be a consequence of duality theory holding in a well-behaved convex situation (without block orders), see [1, 10, 15]. This is reviewed in section 2.

To handle these formulations, special purpose algorithms have been designed, see [15] for a review. The two best algorithms so far have been developed independently [1, 10, 15], COSMOS being used in practice in the CWE region since 2009. Both are decomposition-based branch-and-bound algorithms solving a main optimization problem and adding cuts to exclude incumbents for which no strict linear prices fulfilling auction requirements exist. The new algorithm *Euphemia* which will be used in the NWE region is based on COSMOS [1].

Contribution and structure of this article

In this article, we provide with a non-trivial reformulation of the European Market Model (EMM) that has several advantages.

Precisely, we show how EMM can be modelled as a mixed integer *linear* program without the introduction of auxiliary binary variables to linearise complementarity constraints, when only stepwise preference curves (see definitions below) are considered. When (linearly) interpolated preference curves are considered, EMM can be formulated as a mixed integer quadratically constrained program (MIQCP) with only one non-linear convex quadratic constraint (with integer variables).

In the linear case, the new formulation allows to take full advantage of the power of well-known state-of-the-art solvers such as Cplex or Gurobi. In both cases, the new formulation allows the use of a classical Benders decomposition. In particular, we derive in section 5 a Benders-like decomposition procedure with cuts that are stronger than those proposed in [10]. The new cuts are indeed obtained by strengthening classical Benders cuts derived from the new formulation locally (i.e. in branch-and-bound subtrees). This decomposition algorithm is needed when interpolated orders are considered, since today solvers are not able to deal with large-scale MIQCP problems

of this kind.

The course of the paper is the following: in section 2, we recall the basic setting, fix the notations and give the known MPCC formulation of EMM, as can be found in [15]. In section 3, we give the new formulation and give the proof of its equivalence. For the sake of simplicity, both sections 2 and 3 are presented with stepwise preference curves, dealing with linear models only. The result is adapted in section 4 when linearly interpolated preference curves are considered, using classical Dorn’s quadratic duality results. In section 5, we show how to derive a powerful decomposition procedure by the use of a Benders-like argument, in both the linear and quadratic cases. Finally, Section 6 is devoted to computational experiments. The conclusion also points out some interesting research questions to address further.

2 Day-ahead Electricity Markets and Linear Equilibrium Prices

2.1 Basic Market Coupling

This first section describes the basic setting of market coupling, as it can be found, for example, in [1, 7, 10, 15]. All of the results are well-known and recalled here with our notations. As these results are of main importance for all new results presented later on, proofs are provided in appendix.

2.1.1 Description and Notations

Indexing Sets: I is the set of hourly orders (i.e. continuous orders), J is the set of block orders, and K is the set of network elements (e.g. high voltage power lines or nodes, depending on the chosen network representation). The set of areas and periods are A and T , while N is a set indexing network constraints.

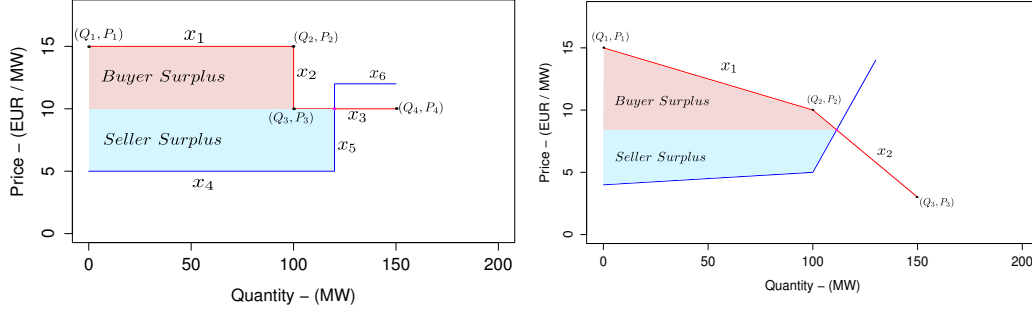
Decision variables: The variables $x_i \in [0, 1]$, $i \in I$ and $y_j \in \{0, 1\}$, $j \in J$ are decision variables which define the level of execution of a given order (x_i denotes a convex order, while y_j denotes a block order). The other variables n_k are used to describe feasible dispatches (the network model, see below).

Preference curves and Parameters of hourly orders

For hourly orders, participants submit points defining nodes of a preference curve stating what quantity is accepted to be exchanged in relation to the price. For each time slot and each area, aggregated supply and demand curves are computed, containing all the information needed for the clearing process.

A preference curve is specified by a finite set of points $\{(Q_s, P_s)\}_{s \in S}$: the curve is obtained via a linear interpolation in between these points.

Each two consecutive points (Q_s, P_s) and (Q_{s+1}, P_{s+1}) correspond to an order of quantity $(Q_{s+1} - Q_s)$.



Stepwise preference curves are such that $P_s = P_{s+1}$ if $Q_s \neq Q_{s+1}$.

For (linearly) interpolated orders, one can have $P_s \neq P_{s+1}$ and $Q_s \neq Q_{s+1}$. The right diagram corresponds to this situation. We deal with interpolated orders in section 4. The rest of this section, as well as section 3, deals with the situation depicted on the left diagram.

For sell orders $P_s \leq P_{s+1}$ (the curve is non-decreasing), while for buy orders $P_s \geq P_{s+1}$ (the curve is non-increasing).

An order $i \in I$ always comes from a preference curve corresponding to a given area and a given time slot. Nonetheless, we will denote its parameters by $P^i, Q_{i,t}^i$ for step orders, and $P_0^i, P_1^i, Q_{i,t}^i$ for interpolated orders, to ease the description of the model.

Instead of partitioning all orders into the sets of buy orders and sell orders, quantities for buy orders are counted positively, and negatively for sell orders. This is convenient to derive economic interpretations, to state network balance constraints, or the welfare maximizing objective.

Parameters of Block Orders

Let J denote the set of block orders. In today's markets, a block order $j \in J$ is related to a given area and specified by a price P^j and quantities Q_t^j for several periods $t \in T$. However, in our notations below, we will denote its parameters P^j and $Q_{l,t}^j$, allowing to consider quantities over multiple areas. This also eases the description of the current market model. The *binary* decision variable associated to the order is denoted y_j , determining if the order is entirely accepted or entirely rejected. Quantities Q_t^j are counted positively for buy orders, and negatively for sell orders, as for hourly orders, and for the same reasons.

Network Model

The network model is not central in this paper and the description provided here encompasses the most general formulation of a network with linear constraints. The set K contains network elements (inter-connectors or network nodes) and coefficients $e_{l,t}^k$ in (3) describe, for a given market (l, t) , how these elements are related to the net export position of this market. Then, constraints (4) describe the most general kind of linear constraints on these network elements. In particular, it can be specialized into a network flow model (usually called in this context "Available-to-Transfer

Capacity” network model), or a flow-based network model (as described e.g. in [7]). For ATC models, variables n_k correspond to flows through cross-boarder lines, while for flow-based models, they correspond to ‘critical network elements’ [1]. Shadow prices of constraints (4) are typically the prices computed when implicit auctions are used to determine congestion revenues of the TSO.

Objective function: The market coupling problem is modelled as a welfare maximisation program. This amounts to maximize the total seller and buyer surplus.

2.1.2 Primal Problem

$$\max_{x_i, y_j, n_k} \sum_i \left(\sum_{l,t} Q_{l,t}^i P^i \right) x_i + \sum_j \left(\sum_{l,t} Q_{l,t}^j P^j \right) y_j$$

subject to:

$$x_i \leq 1 \quad \forall i \in I \quad [s_i] \quad (1)$$

$$y_j \leq 1 \quad \forall j \in J \quad [s_j] \quad (2)$$

$$\sum_i Q_{l,t}^i x_i + \sum_j Q_{l,t}^j y_j = \sum_k e_{l,t}^k n_k, \quad \forall (l,t) \in A \times T \quad [p_{l,t}] \quad (3)$$

$$\sum_k a_{m,k} n_k \leq w_m \quad \forall m \in N \quad [u_m] \quad (4)$$

$$x_i, y_j \geq 0, \quad y_j \in \mathbb{Z}. \quad (5)$$

2.1.3 Duality and Equilibrium prices

Solving the market coupling problem implies to find prices supporting, ideally, a Walrasian equilibrium. In a convex situation where all orders are continuous orders, classical shadow prices ($p_{l,t}, u_m$ here) are equilibrium prices for the optimal bid allocation. This equilibrium property is a consequence of dual and complementarity constraints, as now recalled.

Definition 1. Let $p_{l,t}$ be a linear price system (unique price per period and per area). A step order is said to be:

(i) *in-the-money (ITM)* if $\sum_{l,t} Q_{l,t}^i (P^i - p_{l,t}) > 0$. For hourly orders, since an order has a precise location and time slot, the sum has only one term $Q_{l_0,t_0}^i (P^i - p_{l_0,t_0})$. So if $Q_{l_0,t_0}^i < 0$ (sell order), then $P^i < p_{l_0,t_0}$ and if $Q_{l_0,t_0}^i > 0$, then $P^i > p_{l_0,t_0}$. For block orders, this is true globally (the clearing price is such that a surplus is granted when all periods are considered).

(ii) *at-the-money* if $\sum_{l,t} Q_{l,t}^i (P^i - p_{l,t}) = 0$. For hourly orders, this means $Q_{l_0,t_0}^i (P^i - p_{l_0,t_0}) = 0$, and (assuming $Q_{l_0,t_0}^i \neq 0$), $P^i = p_{l_0,t_0}$: both bid and market prices are equal.

(iii) *out-of-the-money* if it is not ITM nor ATM (i.e. its execution would incur a loss):

$$\sum_{l,t} Q_{l,t}^i (P^i - p_{l,t}) < 0.$$

Definition 2 (Walrasian Equilibrium prices). *Let (x_i^*, y_j^*, n_k^*) be some dispatchable allocation (i.e. satisfying (1)-(5)), and $p_{l,t}^*$ a (uniform) price system. Then $(x_i^*, y_j^*, n_k^*, p_{l,t}^*)$ is a Walrasian equilibrium if:*

- Fully executed orders are ITM or ATM
- Fractionally executed orders are ATM
- Rejected orders are ATM or OTM

In such a situation, for a given order i : $\forall x_i \in [0, 1]$, $\sum_{l,t} Q_{l,t}^i (P_{l,t}^i - p_{l,t}^) x_i \leq \sum_{l,t} Q_{l,t}^i (P_{l,t}^i - p_{l,t}^*) x_i^*$.*

The inequality simply means that for these prices $p_{l,t}^*$, no other level of execution could be more profitable to the bidder.

Network equilibrium. Network resource prices u_m (shadow prices of constraints (4)) and electricity prices $p_{l,t}$ should also be coherent from an economic viewpoint. For two areas l and k , if no constraint on network resources involved in the transport of electricity from l to k or k to l is binding, then it should be considered they form but one market and have $p_{l,t} = p_{k,t}$. On another hand, if $p_{l,t} < p_{k,t}$, then the price difference should be related to network resource prices in some way. For ATC-based network representations (a network flow model), this price difference can only occur when the line from the market with lower price to the market with higher price is congested, in which case the price difference equals the congestion price. For flow-based representations, the price difference is equal to a weighted sum of resource prices [1].

Assumption. As it is not of main importance here, and to be more general, we will not discuss in details network equilibrium conditions that hold with the use of shadow prices (see e.g. [1, 7, 10] for relevant details about ATC or Flow-based network representations). *We will assume, as it is well-known, that dual and complementarity constraints (8) and (14) below hold if and only if these network equilibrium conditions are satisfied.*

We now review the fact that $p_{l,t}$ are Walrasian equilibrium prices, when one considers continuous orders only. We then show why strict linear pricing is in general impossible in the presence of indivisible orders.

Let consider the *continuous relaxation of the primal problem* above. Its dual problem is:

Dual Problem

$$\min_{s_i, s_j, p_{l,t}, u_m} \sum_i s_i + \sum_j s_j + \sum_m w_m u_m$$

subject to:

$$s_i + \sum_{l,t} Q_{l,t}^i p_{l,t} \geq \sum_{l,t} Q_{l,t}^i P^i \quad \forall i \in I \quad [x_i] \quad (6)$$

$$s_j + \sum_{l,t} Q_{l,t}^j p_{l,t} \geq \sum_{l,t} Q_{l,t}^j P^j \quad \forall j \in J \quad [y_j] \quad (7)$$

$$\sum_m a_{m,k} u_m - \sum_{l,t} e_{l,t}^k p_{l,t} = 0 \quad \forall k \in K \quad [n_k] \quad (8)$$

$$s_i, s_j, u_m \geq 0 \quad (9)$$

And related complementarity constraints are:

$$s_i(1 - x_i) = 0 \quad \forall i \in I \quad (10)$$

$$s_j(1 - y_j) = 0 \quad \forall j \in J \quad (11)$$

$$x_i(s_i + \sum_{l,t} Q_{l,t}^i p_{l,t} - \sum_{l,t} Q_{l,t}^i P^i) = 0 \quad \forall i \in I \quad (12)$$

$$y_j(s_j + \sum_{l,t} Q_{l,t}^j p_{l,t} - \sum_{l,t} Q_{l,t}^j P^j) = 0 \quad \forall j \in J \quad (13)$$

$$u_m(\sum_k a_{m,k} n_k - w_m) = 0 \quad \forall m \in N \quad (14)$$

Complementarity constraints typically express equilibrium conditions:

Lemma 1. *For an optimal solution (x_i, y_j, n_k) of the continuous relaxation of the primal program, take as linear prices $p_{l,t}$, the optimal dual variables of constraints (3). For these prices: (i) fully accepted orders are ITM or ATM, (ii) fractionally accepted orders are ATM and (iii) rejected orders are ATM or OTM.*

In particular, ITM orders are fully accepted and OTM orders are fully rejected.

Lemma 2. *Let (x_i, y_j, n_k) be a feasible point of the primal. If $p_{l,t}$ is a price system such that (i)–(iii) of lemma 1 hold, then one can define auxiliary variables s_i, s_j such that dual constraints (6)–(7), (9) and complementarity conditions (10)–(13) hold as well ('network equilibrium conditions' (8) and (14) are not considered).*

These two lemmas together with our previous assumption about network equilibrium conditions yield the following theorem and its direct corollary, recalling why strict linear pricing is in general impossible in the presence of block orders.

Theorem 1. *For a given bid selection (x_i^*, y_j^*, n_k^*) satisfying (1)–(5), strict linear equilibrium prices exist if and only if there are dual variables $s_i^*, s_j^*, p_{l,t}^*, u_m^*$ such that dual and complementarity constraints (6)–(14) are satisfied, in which case (x_i^*, y_j^*, n_k^*) is optimal for the welfare maximizing objective function of the primal program, $s_i^*, s_j^*, p_{l,t}^*, u_m^*$ is optimal for the dual, and both objective values are equal.*

Corollary 1. *Consider the primal problem including integer constraints. There exists a solution supported by strict linear equilibrium prices $p_{l,t}^*, u_m^*$ if and only if the continuous relaxation of the primal program admits an optimal solution (x_i^*, y_j^*, n_k^*) with $y_j^* \in \mathbb{Z}, \forall j \in J$.*

This result shows that strict linear prices exist if and only if there is no duality gap caused by integer constraints, which is not the case for most of real instances. Therefore, in general, strict linear pricing is simply proven infeasible.

2.2 The European Market Model: MPCC formulation

We describe here the model used everyday in Europe to compute clearing prices [1] and recall its MPCC formulation. Since strictly linear equilibrium pricing is proven infeasible, a solution is to compute equilibrium prices for hourly orders together with some restrictions concerning other non-convex orders. For example, one can allow block bids to be paradoxically executed (accepted while OTM) or paradoxically rejected (rejected while ITM), and set transfer payments to participants to reach an equilibrium situation.

In Europe, OTM orders are always rejected and paradoxically rejected ITM block orders are allowed but not compensated, while ATM orders may be rejected or accepted (potentially fractionally for continuous orders). This results in a suboptimal welfare (the pure welfare maximizing solution is rejected if no prices exist that satisfy these conditions).

The main requirements of EMM are: (i) strict linear prices, (ii) OTM orders must be rejected (block and hourly orders as well) and (iii) ITM *hourly* orders must be accepted.

The classical way to state this maximisation problem, that is to formulate European market rules, is to write primal, dual and all complementarity constraints excepted those of type (11) (see e.g. [15]). According to the interpretation given above, this corresponds to drop for block orders the requirement that they should be accepted if they are ITM. This yields a mathematical program with complementarity conditions (MPCC).

MPCC Formulation:

$$\max_{x,y,n,p,u} \sum_i \left(\sum_{l,t} Q_{l,t}^i P^i \right) x_i + \sum_j \left(\sum_{l,t} Q_{l,t}^j P^j \right) y_j$$

subject to constraints: (1) – (10) (i.e. primal - dual constraints) and (12) – (14) (complementarity constraints) but *not subject to* complementarity constraints of type (11). \square

This formulation involves non-linear constraints, and instances (which are very large in practice) would be hard or even impossible to solve with current MINLP solvers. For this reason, special purpose algorithms have been designed (see above).

We now show how to replace these complementarity constraints using duality theory.

3 A New Formulation

The advantage of this new formulation is threefold. First, in the MILP case, it allows to solve reasonably large-scale instances without any special purpose algorithm, by the use of state-of-the-art solvers such as Cplex or Gurobi. Second, one can derive from the new formulation a (Benders-like) decomposition algorithm that allows to deal with interpolated orders, in which case a MIQCP

must be solved. However, in the case of MILP instances, it turns out that the new formulation is more efficient (especially on instances with more block orders) than the decomposition procedure. Third, with a slight modification of the formulation, it could be allowed to consider the minimization problem of opportunity costs of paradoxically rejected orders or results about the deviations from perfect Walrasian equilibrium prices. How to use this in an interesting way is studied in more details in a paper in preparation.

We first state the formulation and then prove its equivalence in theorem 2 thereafter.

3.1 EMM with step orders as a MILP

The new formulation involves all primal and dual constraints as well as equality of objective functions condition (instead of a *subset of* complementarity constraints). To ensure the existence of a solution and reflect the choice of allowing some ITM block orders to be rejected, dual constraints of type (7) are modified, yielding constraints of type (21) below, where the M_j are large enough to deactivate the constraint when $y_j = 0$. Let remember that these constraints (7) together with complementarity constraints of type (11) would mean that ITM block orders must be accepted.

New MILP Formulation:

$$\max_{x,y,n,p,u} \sum_i \left(\sum_{l,t} Q_{l,t}^i P^i \right) x_i + \sum_j \left(\sum_{l,t} Q_{l,t}^j P^j \right) y_j$$

subject to:

$$\begin{aligned} \sum_i \left(\sum_{l,t} Q_{l,t}^i P^i \right) x_i + \sum_j \left(\sum_{l,t} Q_{l,t}^j P^j \right) y_j &\geq \\ \sum_i s_i + \sum_j s_j + \sum_m w_m u_m & \end{aligned} \quad (15)$$

$$x_i \leq 1 \quad \forall i \in I \quad (16)$$

$$y_j \leq 1 \quad \forall j \in J \quad (17)$$

$$\sum_i Q_{l,t}^i x_i + \sum_j Q_{l,t}^j y_j = \sum_k e_{l,t}^k n_k, \quad \forall (l,t) \in A \times T \quad (18)$$

$$\sum_k a_{m,k} n_k \leq w_m \quad \forall m \in N \quad (19)$$

$$s_i + \sum_{l,t} Q_{l,t}^i p_{l,t} \geq \sum_{l,t} Q_{l,t}^i P^i \quad \forall i \in I \quad (20)$$

$$s_j + \sum_{l,t} Q_{l,t}^j p_{l,t} \geq \sum_{l,t} Q_{l,t}^j P^j - M_j(1 - y_j) \quad \forall j \in J \quad (21)$$

$$\sum_m a_{m,k} u_m - \sum_{l,t} e_{l,t}^k p_{l,t} = 0 \quad \forall k \in K \quad (22)$$

$$x_i, y_j, s_i, s_j, u_m \geq 0, \quad y_j \in \mathbb{Z} \quad \forall j \in J \quad (23)$$

Theorem 2. *The MPCC and the New MILP formulations are equivalent. More precisely, the projection of both feasible sets on the space of decision variables $(x_i, y_j, n_k, p_{l,t}, u_m)$ are equal. Therefore, EMM with step orders can be modelled as a mixed integer linear program without the introduction of auxiliary binary variables.*

The proof of theorem 2 is presented in next section.

3.2 Block Order Selections and proof of theorem 2

We start by partitioning the set J of block orders into two subsets J_0 (rejected block orders) and J_1 (accepted block orders), corresponding to a block bid selection (block order variables y_j fixed to some values). Some of these selections admit linear prices fulfilling European market rules and some do not. The cost of fixing integer variables is reflected in the dual. This point of view helps to describe which of all possible bid selections are valid, according to European market rules. Indeed, these selections must be such that it is possible to get a feasible solution with $d_{j_1} = 0 \forall j_1 \in J_1$.

Let J_0, J_1 be a block bid selection. The corresponding primal, dual and complementarity constraints are:

Primal problem with a block bid selection

$$\max_{x_i, y_j, n_k} \sum_i \left(\sum_{l,t} Q_{l,t}^i P^i \right) x_i + \sum_j \left(\sum_{l,t} Q_{l,t}^j P^j \right) y_j$$

subject to:

$$x_i \leq 1 \quad \forall i \in I \quad [s_i] \quad (24)$$

$$y_j \leq 1 \quad \forall j \in J \quad [s_j] \quad (25)$$

$$y_{j_0} \leq 0 \quad \forall j_0 \in J_0 \quad [d_{j_0}] \quad (26)$$

$$-y_{j_1} \leq -1 \quad \forall j_1 \in J_1 \quad [d_{j_1}] \quad (27)$$

$$\sum_i Q_{l,t}^i x_i + \sum_j Q_{l,t}^j y_j = \sum_k e_{l,t}^k n_k, \quad \forall (l,t) \in A \times T \quad [p_{l,t}] \quad (28)$$

$$\sum_k a_{m,k} n_k \leq w_m \quad \forall m \in N \quad [u_m] \quad (29)$$

$$x_i, y_j \geq 0 \quad (30)$$

Dual problem with a block bid selection

$$\min \sum_i s_i + \sum_j s_j - \sum_{j_1} d_{j_1} + \sum_m w_m u_m$$

subject to:

$$s_i + \sum_{l,t} Q_{l,t}^i p_{l,t} \geq \sum_{l,t} Q_{l,t}^i P^i \quad \forall i \in I \quad [x_i] \quad (31)$$

$$s_{j_0} + d_{j_0} + \sum_{l,t} Q_{l,t}^{j_0} p_{l,t} \geq \sum_{l,t} Q_{l,t}^{j_0} P^{j_0} \quad \forall j_0 \in J_0 \quad [y_{j_0}] \quad (32)$$

$$s_{j_1} - d_{j_1} + \sum_{l,t} Q_{l,t}^{j_1} p_{l,t} \geq \sum_{l,t} Q_{l,t}^{j_1} P^{j_1} \quad \forall j_1 \in J_1 \quad [y_{j_1}] \quad (33)$$

$$\sum_m a_{m,k} u_m - \sum_{l,t} e_{l,t}^k p_{l,t} = 0 \quad \forall k \in K \quad [n_k] \quad (34)$$

$$s_i, s_j, d_{j_0}, d_{j_1}, u_m \geq 0 \quad (35)$$

Complementarity constraints

$$s_i(1 - x_i) = 0 \quad \forall i \in I \quad (36)$$

$$s_{j_0}(1 - y_{j_0}) = 0 \quad \forall j_0 \in J_0 \quad (37)$$

$$s_{j_1}(1 - y_{j_1}) = 0 \quad \forall j_1 \in J_1 \quad (38)$$

$$y_{j_0} d_{j_0} = 0 \quad \forall j_0 \in J_0 \quad (39)$$

$$(1 - y_{j_1}) d_{j_1} = 0 \quad \forall j_1 \in J_1 \quad (40)$$

$$u_m \left(\sum_k a_{m,k} n_k - w_m \right) = 0 \quad \forall m \in N \quad (41)$$

$$x_i \left(s_i + \sum_{l,t} Q_{l,t}^i p_{l,t} - \sum_{l,t} Q_{l,t}^i P^i \right) = 0 \quad \forall i \in I \quad (42)$$

$$y_{j_0} \left(s_{j_0} + d_{j_0} + \sum_{l,t} Q_{l,t}^{j_0} p_{l,t} - \sum_{l,t} Q_{l,t}^{j_0} P^{j_0} \right) = 0 \quad \forall j_0 \in J_0 \quad (43)$$

$$y_{j_1} \left(s_{j_1} - d_{j_1} + \sum_{l,t} Q_{l,t}^{j_1} p_{l,t} - \sum_{l,t} Q_{l,t}^{j_1} P^{j_1} \right) = 0 \quad \forall j_1 \in J_1 \quad (44)$$

Proof of Theorem 2.

Proof. Let $MPCC_{set}$ and $MILP_{set}$ be the feasible sets of the MPCC and MILP formulations respectively.

We show that $Proj_{(x,y,n,p,u)}(MPCC_{set}) = Proj_{(x,y,n,p,u)}(MILP_{set})$, where $Proj_{(x,y,n,p,u)}(*)$ denotes the projection on the space of decision variables (x, y, n, p, u) .

(i) Let $(x_i, y_j, n_k, p_{l,t}, u_m, s_i, s_j)$ be a feasible point of the MPCC formulation.

Let define $J_0 := \{j | y_j = 0\}$, $J_1 := \{j | y_j = 1\}$, $d_{j_1} := 0 \forall j_1 \in J_1$ and $d_{j_0} := M_{j_0} \forall j_0 \in J_0$. Since the parameters M_j and thus d_{j_0} have been chosen large enough, we can define new $s_{j_0}^{mod} = 0$ such that dual constraints of type (32) and complementarity constraints of type (37) above are satisfied. The new point $(x_i, y_j, n_k, p_{l,t}, u_m, s_i, s_j^{mod}, d_{j_0}, d_{j_1})$ satisfies constraints (24) - (44), that is all primal, dual and complementarity constraints corresponding to the primal and dual optimization problems where block order variables are fixed to some values. It is therefore optimal for the primal problem

with a block bid selection above. Moreover, by duality theory (implying equality of objective functions), constraint (15) is satisfied as well

($d_{j_1} = 0 \forall j_1 \in J_1$ in the dual objective function above). Due to constraints (32) – (33) and the given values of d_{j_0}, d_{j_1} , it is direct to check that the projection $(x_i, y_j, n_k, p_{l,t}, u_m, s_i, s_j^{mod})$ satisfies constraints (21). This shows that the projection $(x_i, y_j, n_k, p_{l,t}, u_m, s_i, s_j^{mod})$ satisfies (15) – (23), so is a feasible point of the new MILP formulation.

(ii) Now let $(x_i, y_j, n_k, p_{l,t}, u_m, s_i, s_j)$ be a feasible point of the MILP Formulation.

Let define $J_0, J_1, d_{j_0}, d_{j_1}$ as above at (i). The point $(x_i, y_j, n_k, p_{l,t}, u_m, s_i, s_j, d_{j_0}, d_{j_1})$ satisfies all primal and dual conditions, as well as equality of objective functions of the optimization problems with a block bid selection presented above. By duality theory (implying related complementarity constraints), it satisfies constraints (36) – (44). We can now define new $s_{j_0}^{mod} := s_{j_0} + d_{j_0}$ to satisfy constraints (7). Constraints (37) (same as (11)) may not be satisfied any more but the projection of the new point thus obtained, $(x_i, y_j, n_k, p_{l,t}, u_m, s_i, s_j^{mod})$, satisfies (1) – (10) and (12) – (15), and hence is a feasible point of the MPCC formulation. \square

4 Markets with interpolated preference curves

4.1 Primal Program, Dorn’s Dual and equilibrium prices

We now deal with interpolated (hourly) orders. Results are almost the same and a few minor changes are needed. The new formulation is also based on strong duality (for convex quadratic programs). We also recall the basic equilibrium conditions expressed by complementarity constraints in this different setting.

Consider the set of points $\{(P_s, Q_s)\}_{s \in S}$ defining a given preference curve. In this situation, the price P_s is the price at which the order of quantity $(Q_{s+1} - Q_s)$ starts to be accepted, and the price P_{s+1} is the price at which it is fully accepted. Intermediate marginal prices depend linearly on the level of execution of the order, as specified by the (interpolated) preference curve. We may have $P_s \neq P_{s+1}$ and $Q_s \neq Q_{s+1}$ for some s , but note that stepwise preference curves considered earlier are just particular cases of linearly interpolated curves. For sell orders $P_1^i \geq P_0^i$ (the curve is non-decreasing), while for buy orders $P_1^i \leq P_0^i$ (the curve is non-increasing).

Primal (Quadratic) Program

In the present case of linearly interpolated hourly orders, the objective function is:

$$\max_{x_i, y_j, n_k} \sum_i \left(\sum_{l,t} Q_{l,t}^i P_0^i x_i + \sum_{l,t} Q_{l,t}^i (P_1^i - P_0^i) \frac{x_i^2}{2} \right) + \sum_j \left(\sum_{l,t} Q_{l,t}^j P_{l,t}^j y_j \right)$$

subject to (primal constraints remain unchanged):

$$x_i \leq 1 \quad \forall i \in I \quad [s_i] \quad (45)$$

$$y_j \leq 1 \quad \forall j \in J \quad [s_j] \quad (46)$$

$$\sum_i Q_{l,t}^i x_i + \sum_j Q_{l,t}^j y_j = \sum_k e_{l,t}^k n_k, \quad \forall (l,t) \in A \times T \quad [p_{l,t}] \quad (47)$$

$$\sum_k a_{m,k} n_k \leq w_m \quad \forall m \in N \quad [u_m] \quad (48)$$

$$x_i, y_j \geq 0, \quad y_j \in \mathbb{Z}. \quad (49)$$

Dual (Quadratic) Program

The objective function is trivially concave (factors $Q_{l,t}^i(P_1^i - P_0^i)$ are non-positive).

Strong duality still holds for such convex quadratic programs (see e.g. [4, 8, 14]).

Compared to the dual presented in section 2.1.3., the objective function is:

$$\min_{s_i, s_j, p_{l,t}, u_m} \sum_i s_i + \sum_j s_j + \sum_m w_m u_m - \sum_i \left(\sum_{l,t} Q_{l,t}^i (P_1^i - P_0^i) \right) \frac{v_i^2}{2}$$

and feasibility inequalities of type (6) have an additional linear term:

$$s_i + \sum_{l,t} Q_{l,t}^i p_{l,t} \geq \sum_{l,t} Q_{l,t}^i P_0^i + \sum_{l,t} Q_{l,t}^i (P_1^i - P_0^i) v_i \quad \forall i \in I \quad [x_i] \quad (50)$$

$$s_j + \sum_{l,t} Q_{l,t}^j p_{l,t} \geq \sum_{l,t} Q_{l,t}^j P^j \quad \forall j \in J \quad [y_j] \quad (51)$$

$$\sum_m a_{m,k} u_m - \sum_{l,t} e_{l,t}^k p_{l,t} = 0 \quad \forall k \in K \quad [n_k] \quad (52)$$

$$s_i, s_j, u_m \geq 0 \quad (53)$$

Lemma 3. *If (x_i, y_j, n_k) is an optimal solution of the primal program, there exists a dual optimal solution $(s_i, s_j, p_{l,t}, u_m, v_i)$ such that $v_i = x_i \quad \forall i \in I$.*

Proof. It is a direct application of Dorn's quadratic duality theorem (see e.g. [4], [8] or [14]). \square

When stating primal, dual and complementarity constraints, or primal and dual constraints with equality of objective functions, we will thus be allowed to replace v_i with x_i , since such a solution of the dual program exists. This is indeed needed for the economic interpretations.

Complementarity Constraints

Compared to the previous case with stepwise preference curves and complementarity constraints (10) – (14), one has just to replace complementarity constraints of type (12) by:

$$x_i (s_i + \sum_{l,t} Q_{l,t}^i p_{l,t} - \sum_{l,t} Q_{l,t}^i P_0^i - \sum_{l,t} Q_{l,t}^i (P_1^i - P_0^i) x_i) = 0, \quad (54)$$

where lemma 3 has been used to replace v_i by x_i . With this taken into account, the Walrasian equilibrium interpretation of (the new) complementarity conditions still holds:

- if $x_i = 1$ (the order is fully accepted), equations of type (54) imply

$$s_i = \sum_{l,t} Q_{l,t}^i P_0^i + \sum_{l,t} Q_{l,t}^i (P_1^i - P_0^i) 1 - \sum_{l,t} Q_{l,t}^i p_{l,t} \geq 0, \text{ that is } \sum_{l,t} Q_{l,t}^i (P_1^i - p_{l,t}) \geq 0.$$

Recalling that there is in fact only one term in the sum and the sign convention for quantities, this means that $p_{l,t} \geq P_1$ for sell orders and $p_{l,t} \leq P_1$ for buy orders, as expected (the comparison of market price and P_1 corresponds to the situation where the order is fully executed).

- if $0 < x_i < 1$ (the order is partially accepted), equations of type (10) imply $s_i = 0$ and equations of type (54) then imply $\sum_{l,t} Q_{l,t}^i P_0^i + \sum_{l,t} Q_{l,t}^i (P_1^i - P_0^i) x_i = \sum_{l,t} Q_{l,t}^i p_{l,t}$. In this case, $p_{l,t}$ is equal to $P_0^i + (P_1^i - P_0^i) x_i$, i.e. the marginal market price equals the interpolated bid price *for this level of execution*.
- if $x_i = 0$, equations (10) imply $s_i = 0$ and inequalities (50) (with $v_i = x_i = 0$) imply

$$\sum_{l,t} Q_{l,t}^i P_0^i \leq \sum_{l,t} Q_{l,t}^i p_{l,t}.$$

This means that $p_{l,t} \leq P_0$ for sell orders and $p_{l,t} \geq P_0$ for buy orders, as expected.

MPCC formulation in the quadratic case

As in the previous case, a first MPCC formulation can be given. One has just to replace in the previous MPCC formulation the objective function, as well as dual and complementarity constraints (6) by (50) and (12) by (54).

4.2 EMM with interpolated orders: new formulation

We give here the new formulation analogue to the one presented in section 3. For the sake of clarity, we rewrite here all constraints in extenso, as they will be used in section 5.

MIQCP Formulation

$$\max \sum_i \left(\sum_{l,t} Q_{l,t}^i P_0^i x_i + \sum_{l,t} Q_{l,t}^i (P_1^i - P_0^i) \frac{x_i^2}{2} \right) + \sum_j \left(\sum_{l,t} Q_{l,t}^j P_{l,t}^j y_j \right)$$

subject to:

$$\begin{aligned} \sum_i \left(\sum_{l,t} Q_{l,t}^i P_0^i x_i + \sum_{l,t} Q_{l,t}^i (P_1^i - P_0^i) \frac{x_i^2}{2} \right) + \sum_j \left(\sum_{l,t} Q_{l,t}^j P_{l,t}^j y_j \right) \geq \\ \sum_i s_i + \sum_j s_j + \sum_m w_m u_m - \sum_{l,t} Q_{l,t}^i (P_1^i - P_0^i) \frac{x_i^2}{2} \end{aligned} \quad (55)$$

and: (16) – (19), (21) – (23), and (50) with $v_i = x_i \ \forall i \in I$ instead of (20). Explicitly:

$$x_i \leq 1 \quad \forall i \in I \quad (56)$$

$$y_j \leq 1 \quad \forall j \in J \quad (57)$$

$$\sum_i Q_{l,t}^i x_i + \sum_j Q_{l,t}^j y_j = \sum_k e_{l,t}^k n_k, \quad \forall (l,t) \in A \times T \quad (58)$$

$$\sum_k a_{m,k} n_k \leq w_m \quad \forall m \in N \quad (59)$$

$$s_i + \sum_{l,t} Q_{l,t}^i p_{l,t} \geq \sum_{l,t} Q_{l,t}^i P^i + \sum_{l,t} Q_{l,t}^i (P_1^i - P_0^i) x_i \quad \forall i \in I \quad (60)$$

$$s_j + \sum_{l,t} Q_{l,t}^j p_{l,t} \geq \sum_{l,t} Q_{l,t}^j P^j - M_j(1 - y_j) \quad \forall j \in J \quad (61)$$

$$\sum_m a_{m,k} u_m - \sum_{l,t} e_{l,t}^k p_{l,t} = 0 \quad \forall k \in K \quad (62)$$

$$x_i, y_j, s_i, s_j, u_m \geq 0 \quad y_j \in \mathbb{Z} \quad \forall j \in J \quad (63)$$

Theorem 3. *Both formulations are equivalent. More precisely, the projection on the space of main variables $(x_i, y_j, n_k, p_{l,t}, u_m)$ of both feasible sets are equal.*

Proof. The proof is very similar to the proof of theorem 2. It is just needed to adapt primal and dual problems with a block bid selection to the quadratic setting, and to replace in the proof constraint (15) by (55), constraints of type (6) or (31) by (50) and constraints of type (12) by (54), taking into account lemma 3 according to which we can consider optimal dual variables $v_i = x_i \quad \forall i \in I$. \square

5 A Decomposition Method

Here, we derive from our new formulation a Benders-like decomposition algorithm, where cuts are added when no prices exist for a given incumbent of the primal problem (dispatchable allocation). It is in this sense similar to the two best algorithms [1, 10] mentioned earlier (the algorithm briefly described in [1] is the proprietary algorithm in charge of solving CWE market instances on which further technical developments for European market integration will rely). The cuts we propose are stronger than the cuts proposed in [10]. Quadratic instances of the new formulation cannot be solved with today solvers, and such an algorithm is needed to solve efficiently real life instances. The derivation is in a first stage very close to [2], and in particular relies on the Farkas lemma and the finiteness of the number of vertices of the polytope defining the feasible set of a slave program.

To simplify notations, in all this section, only one market (one area and one period) is considered, but all of what follows can be carried out with several areas and periods. We are sometimes referring to corresponding previous constraints involving the network structure, but the adaptations needed are minor and direct.

Exposition is made first in the linear case. It is shown hereafter how to handle the quadratic case in a similar way.

5.1 The linear case

Consider the primal problem defined in section 2.1.2:

$$\max \quad obj := \sum_i Q^i P^i x_i + \sum_j Q^j P^j y_j,$$

subject to (1) – (5), with only one market (no network and only one period), i.e. with N empty and $\sum_k e_{i,t}^k n_k := 0$, to simplify notations.

Consider now a branch-and-bound procedure and let (x_i^*, y_j^*, n_k^*) be an incumbent of this primal program. According to constraints (15), (20) – (23) of the new MILP formulation, supporting prices exist if and only if there exist s_i, s_j, p_m (p_m denoting the market price) such that:

$$-s_i - Q^i p_m \leq -Q^i P_0^i \quad \forall i \in I \quad [u_i] \quad (64)$$

$$-s_j - Q^j p_m \leq -Q^j P^j + M_j(1 - y_j^*) \quad \forall j \in J \quad [u_j] \quad (65)$$

$$\sum_i s_i + \sum_j s_j \leq obj^* \quad [u_\sigma] \quad (66)$$

$$s_i, s_j \geq 0 \quad (67)$$

where obj^* denotes the corresponding optimal value of the objective function for this incumbent.

According to the Farkas lemma[13], a solution to a linear system $Ax \leq b, x \geq 0$ exists if and only if $\forall y \geq 0, yA \geq 0 \Rightarrow yb \geq 0$. The existence of a supporting price is so equivalent to:

$$\sum_i -Q^i P_0^i u_i + \sum_j -Q^j P^j u_j + \sum_j M_j(1 - y_j^*) u_j + obj^* u_\sigma \geq 0$$

$\forall (u_i, u_j, u_\sigma)$ such that:

$$-u_i + u_\sigma \geq 0 \quad (68)$$

$$-u_j + u_\sigma \geq 0 \quad (69)$$

$$-\sum_i Q^i u_i - \sum_j Q^j u_j = 0 \quad [p_m] \quad (70)$$

$$u_i, u_j, u_\sigma \geq 0, \quad (71)$$

The condition being trivially satisfied if $u_\sigma = 0$, we can assume $u_\sigma := 1$ (normalization).

Rearranging terms, prices exist if and only if :

$$\sum_i Q^i P^i u_i + \sum_j Q^j P^j u_j - \sum_j M_j (1 - y_j^*) u_j \leq \sum_i Q^i P^i x_i^* + \sum_j Q^j P^j y_j^*$$

$\forall (u_i, u_j) \in P$ with P defined by the constraints:

$$u_i \leq 1 \tag{72}$$

$$u_j \leq 1 \tag{73}$$

$$\sum_i Q^i u_i + \sum_j Q^j u_j = 0 \tag{74}$$

$$u_i, u_j \geq 0, \tag{75}$$

This yields:

Lemma 4. *For a given incumbent (x_i^*, y_j^*) , an equilibrium price exist if and only if:*

$$\max_{(u_i, u_j) \in P} \sum_i Q^i P^i u_i + \sum_j Q^j P^j u_j - \sum_j M_j (1 - y_j^*) u_j \leq obj^*. \tag{76}$$

Lemma 5. *Let (u_i^*, u_j^*) denotes an optimal solution to the optimization problem stated in Lemma 4. Then $y_j^* = 0 \Rightarrow u_j^* = 0$.*

Proof. Because the numbers M_j are very (arbitrarily) large fixed numbers, if $y_j^* = 0$, the objective could not be optimal for any vertex of P with $u_j \neq 0$. Accordingly, this could also be shown by noting that constraints of *the dual* of the left-hand side program are constraints (20) – (23) with $y_j = y_j^*$ fixed, and that u_j are the shadow prices of constraints (21). If $y_j^* = 0$, the corresponding constraint (21) is not binding because of the choice of the M_j ($s_j \geq 0$ is binding instead), and $u_j = 0$. \square

Note that the numbers M_j are used here only in proofs, and will be avoided in the final procedure described below.

The criterion of *Lemma 4* admits a nice interpretation. Indeed, let consider the continuous relaxation of the primal problem with the additional constraints that all blocks at 0 in the incumbent are held at 0 in this relaxation. From the two previous lemmas, it follows that the value of the objective function for this relaxation cannot be greater than the value for the current incumbent. Formally:

Theorem 4. *For an incumbent (x_i^*, y_j^*) , consider the polytope*

$P^{F^*} := P \cap \{(u_i, u_j) \mid u_j = 0 \text{ if } y_j^* = 0\}$. *Then an equilibrium price exist if and only if*

$$\max_{(u_i, u_j) \in P^{F^*}} \sum_i Q^i P^i u_i + \sum_j Q^j P^j u_j \leq obj^*,$$

where obj^* denotes the optimal value associated with the incumbent of the objective function, in which case equality holds as well. \square

Proof. It is a direct corollary of lemma 4 and lemma 5. Also, since (x_i^*, y_j^*) is feasible for the left-hand side, if the inequality holds, equality holds as well. \square

Now suppose there are no prices for the feasible dispatch, then:

$$\max_{(u_i, u_j) \in P} \sum_i Q^i P^i u_i + \sum_j Q^j P^j u_j - \sum_j M_j (1 - y_j^*) u_j > obj^*,$$

and using lemma (5), we even have:

$$\max_{(u_i, u_j) \in P} \sum_i Q^i P^i u_i + \sum_j Q^j P^j u_j > obj^*.$$

We then add the following valid inequality (see lemma (4)), with (u_i^*, u_j^*) optimal for the relaxed problem:

$$\sum_i Q^i P^i u_i^* + \sum_j Q^j P^j u_j^* - \sum_j M_j (1 - y_j) u_j^* \leq \sum_i Q^i P^i x_i + \sum_j Q^j P^j y_j. \quad (77)$$

At this stage, we can already note that there is a finite number of such inequalities to add, which is bounded by the number of vertices of the bounded polyhedron P .

These cuts are not strong as such because of the M_j (a small change in the variables allows to satisfy the new constraint when LP relaxations are considered), but it is possible to strengthen them:

Theorem 5. *For each new incumbent in the branch-and-bound for which no supporting price exists, the inequality $\sum_{j|y_j^*=1} (1 - y_j) \geq 1$ is valid in the subtree.*

Proof. Consider a new incumbent (x_i, y_j) in the subtree originating from the current incumbent (x_i^*, y_j^*) for which no equilibrium price exists, that is for which:

$$\sum_i Q^i P^i x_i^* + \sum_j Q^j P^j y_j^* < \sum_i Q^i P^i u_i^* + \sum_j Q^j P^j u_j^*.$$

If $\sum_{j|y_j^*=1} (1 - y_j) = 0$, using lemma 5, $\sum_j M_j (1 - y_j) u_j^* = 0$ and the valid inequality (77) for (x_i, y_j) reduces to:

$$\sum_i Q^i P^i u_i^* + \sum_j Q^j P^j u_j^* \leq \sum_i Q^i P^i x_i + \sum_j Q^j P^j y_j.$$

But for a new incumbent (x_i, y_j) in the subtree originating from (x^*, y^*) ,

$$\sum_i Q^i P^i x_i + \sum_j Q^j P^j y_j = obj \leq obj^* < \sum_i Q^i P^i u_i^* + \sum_j Q^j P^j u_j^*,$$

and no such solution can admit an (European) equilibrium price. \square

Note that this improves on the "no-good" cuts proposed in [10]: $\sum_{j|y_j^*=1} (1 - y_j) + \sum_{j|y_j^*=0} y_j \geq 1$ which essentially correspond to valid inequalities (77) we have locally strengthened.

5.2 The quadratic case

Again, for an incumbent (x_i^*, y_j^*) in a branch-and-bound solving the primal problem, we apply the Farkas lemma to constraints (55) and (60) – (63) of the new formulation to test the existence of linear prices. This yields the equivalent condition (again neglecting network details):

$$\begin{aligned} \forall (u_i, u_j) \in P, \quad & \sum_i Q^i P_0^i u_i + \sum_i Q^i (P_1^i - P_0^i) x_i^* u_i + \sum_j Q^j P^j u_j - \sum_j M_j (1 - y_j^*) u_j \\ & \leq \sum_i Q^i P_0^i x_i^* + \sum_i Q^i (P_1^i - P_0^i) (x_i^*)^2 + \sum_j Q^j P^j y_j^* \end{aligned} \quad (78)$$

where P is the polytope defined above in the linear case.

Note that we can only apply the Farkas Lemma to the new formulation after having chosen the dual solution $v_i = x_i \forall i \in I$: if we consider inequality (55) with unknown v_i instead of $v_i = x_i^* \forall i \in I$ fixed to the given values, the inequality is not linear any more in the unknown 'dual variables' and the Farkas lemma doesn't apply.

Mainly two things should be noted concerning this condition. First, it is a *linear* condition which relates two 'quadratic quantities' (with fixed values) which are close to the original quadratic objective function of the primal program.

But, and this is the second point, contrary to condition (76), both right and left-hand sides do not correspond to the objective function of the initial primal program (with additional terms involving M_j for the left-hand side). This last point was used in the preceding arguments to derive the decomposition procedure with our locally valid cuts.

Nonetheless, though it is not direct, it is possible to recover the analogue result:

Lemma 6. *For a given incumbent (x_i^*, y_j^*) , an equilibrium price exist if and only if:*

$$\max_{(u_i, u_j) \in P} \sum_i Q^i P^i u_i + \sum_i Q^i (P_1^i - P_0^i) \frac{u_i^2}{2} + \sum_j Q^j P^j u_j - \sum_j M_j (1 - y_j^*) u_j \leq obj^*, \quad (79)$$

where obj^* denotes the optimal value of the quadratic objective function associated with the current incumbent.

Proof. See appendix for details. □

Observe however that condition (78) asks to solve a linear program and is more efficient as a tester for the existence of equilibrium prices than condition (79).

We can now adapt to the quadratic case the decomposition algorithm with exactly the same cuts:

Theorem 6. *In the quadratic case also, for each new incumbent in the branch-and-bound for which no supporting price exists, cuts of the form $\sum_{j|y_j^*=1} (1 - y_j) \geq 1$ are valid in the subtree.*

Proof. The proof is exactly the same as in Theorem 5. Just replace condition (77) by its counterpart derived from (79) (i.e. with quadratic terms). \square

Note also that like in the previous linear case, a consequence of lemma 6 is:

Theorem 7. *For an incumbent (x_i^*, y_j^*) , consider the polytope $P^F := P \cap \{(u_i, u_j) \mid u_j = 0 \text{ if } y_j^* = 0\}$. Then an equilibrium price exists if and only if*

$$\max_{(u_i, u_j) \in P^F} \sum_i Q^i P_0^i u_i + \sum_i Q^i (P_1^i - P_0^i) \frac{u_i^2}{2} + \sum_j Q^j P^j u_j \leq obj^*, \quad (80)$$

where obj^* denotes the optimal value of the quadratic objective function associated with the incumbent, in which case equality holds as well.

6 Computational Results

In this section, we mainly address three questions related to the new formulation. First, how state-of-the-art solvers behave on real instances, when the whole model (15)-(23) is provided? Second, how the Benders-like algorithm behaves in comparison to the first approach? Third, how both approaches behave on very combinatorial linear instances? We present here answers to these three questions. Apx-Endex kindly provided us with real data from 2011. Statistics computed over the whole year 2011 (i.e. 365 instances) are presented. In appendix, we present in more details results for randomly chosen instances. The sample is very representative of the whole year.

Computational experiments have been carried out with AIMMS [3] (with Cplex 12.5), on a computer running Windows 7 64 bits, with a four cores CPU i5 @ 3.10 Ghz, and 4 GB of RAM. Even with such a modest platform, results turn out to be very positive. The decomposition procedure has been implemented using lazy constraint callbacks (available in Cplex since version 12.3, which also allows for *locally valid* lazy constraints).

Concerning practical requirements for an algorithm, main European power exchanges ask for a time limit of ten minutes, and we have adopted this stopping criterion for all tests below.

For both approaches (the new formulation and the decomposition procedure), we have computed the number of instances solved up to optimality, the (geometric) average time needed to find these optimal solutions, and the (geometric) average of the final absolute MIP gap when only a suboptimal solution is available in time. To provide with a more robust insight, we also provide with the number of visited nodes for the new MILP approach, and the number of cuts generated in the decomposition approach.

6.1 Historical instances with stepwise preference curves

Piecewise linear preference curves have been transformed into stepwise preference curves to get MILP instances. To do this, for each two consecutive points of the preference curve such that $Q_i \neq Q_{i+1}$ and $P_i \neq P_{i+1}$, a point (Q^*, P^*) has been inserted in between, with $Q^* = Q_i$ and $P^* = P_{i+1}$.

A particular attention has been devoted to numerical issues. One drawback of the new formulation is the so-called big-M constants involved in the constraints. As it is well-known, this may result in numerically ill-conditioned instances. It appeared that very tight tolerance parameters must be set to obtain correct solutions.

Instances contain orders for 4 areas (Belgium, France, Germany and the Netherlands), and span the whole day (24 hours, excepted twice per year, 23 and 25 hours respectively). There are approximatively 50 000 hourly orders (preference curve segments) and 600 block orders per instance.

	Solved instances	Running time (solved instances, sec)	Final abs. gap (unsolved instances)	Nodes (solved - unsolved) instances	Cuts (solved - unsolved) instances
New MILP formulation	84%	104.42	418.16	43 - 33584	/
Decomposition Procedure	72.78%	6.47	402.05	16 - 1430	8 - 3492

The new MILP formulation allows to solve most of the instances without any algorithmic work and to obtain very good suboptimal solutions when the instance cannot be solved up to optimality. The decomposition procedure is much faster on most instances but most of the time doesn't help to solve hard instances that the MILP approach cannot solve. The fact that the new MILP formulation approach takes in average more time for solved instances is mainly due to the time needed to solve the root node relaxation. However, for quadratic instances (with interpolated orders), the decomposition procedure is the only tractable approach.

Comparing runs with and without solver's cut generation procedures, it turned out that they were not useful and were indeed slowing down the process in both the decomposition procedure and the full model approaches. In fact, for the full model approach, this may be explained by the presence of big-M's and the fact that most of the cuts generated may be very weak in practice. Concerning the decomposition procedure, in most cases, many good solutions to the primal program are easily found and cuts are not of main interest, the main part of the procedure (from a running time point of view) consisting in rejecting incumbents when no European prices exist.

6.2 Historical instances with interpolated orders

When interpolated orders are considered, the resulting new MIQCP optimization problem cannot be solved with today's solvers (e.g. Cplex or Gurobi), and only the decomposition procedure can be relevantly assessed.

To check for the existence of prices for a given new incumbent, the linear condition (78) has been used, and the locally valid local cut of theorem 6 is added when no equilibrium prices exist.

	Solved instances	Running time (solved instances, sec)	Final abs. gap (unsolved instances)	Nodes (solved - unsolved) instances	Cuts (solved - unsolved) instances
Decomposition Procedure	70.41%	16.70	370.91	11 - 619	7 - 1382

As it can be seen, most of instances are solved up to optimality, and a very small gap remains when only a suboptimal solution is found within ten minutes.

6.3 Instances with (almost) only block orders

We have built 50 instances where orders are almost all block orders in the following way. Starting from historical instances, all block orders have been relocated to one area only and are spanning only one hour of the day. Two small continuous orders (one buy order and one sell order) have been added for the sole purpose to have an instance with at least one feasible solution (a matching of orders is possible). The difference between both approaches in this case is remarkable:

	Solved instances	Running time (solved instances, sec)	Final abs. gap (unsolved instances)	Nodes (solved - unsolved) instances	Cuts (solved - unsolved) instances
New MILP formulation	100%	4.17	/	40797 - /	-
Decomposition Procedure	78%	13.82	9303.16	64564 / 937172	1662 / 82497

In this case, the new MILP formulation approach is much more powerful. One possible explanation is the high number of block order selections for which no prices exist, which are enumerated by the decomposition. On another hand, with the full model approach, the solver may be able to branch more efficiently. The difference of performances between the two approaches was more impressive on a less powerful platform. This difference would therefore be more important for instances with more block orders.

7 Conclusions

We have proposed a new formulation for European day-ahead electricity markets that turns out to be (a) tractable and (b) very competitive as long as stepwise preference curves describing hourly orders are considered. More than 80 % of the historical instances of 2011 can be solved up to optimality, and for the other ones, the final gap is very small. We have also compared this approach with a decomposition procedure derived directly from the new formulation, which appeared to solve most instances faster but was not helpful on hard instances that the new formulation approach was not able to solve. Unfortunately, the simple use of the analogue new formulation is no longer successful when piecewise linear preference curves are considered. Today's state-of-the-art MIQCP solvers are not able to deal with large-scale programs with this structure. On the other hand, the Benders-like decomposition approach derived from the new formulation allows managing these cases in an efficient way. Finally, the new MILP formulation performs much better than the decomposition approach on small very combinatorial linear instances, and this could be exploited in auctions with more block orders. Another interesting point is that an approach similar to the new formulation allows considering other objective functions over the set of constraints defining European market rules. In particular, with a similar modelling technique, it would be possible to consider, for example, an objective function whose aim is to minimize the total opportunity costs

of paradoxically rejected block orders. In a article in preparation, we study how this modelling technique can be used from a market design analysis point of view.

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A Proofs of section 1

Proof of lemma 1.

Proof. Let consider a hourly order i :

(i) if $x_i = 1$ (the order is fully accepted), complementarity constraints of type (12) imply

$$s_i = \sum_{l,t} Q_{l,t}^i (P^i - p_{l,t}) \geq 0, \text{ i.e. the order is in-the-money or at-the-money.}$$

(ii) if $0 < x_i < 1$ (the order is partially accepted), complementarity constraints of type (10) imply

$$s_i = 0 \text{ and those of type (12) then imply } \sum_{l,t} Q_{l,t}^i P^i = \sum_{l,t} Q_{l,t}^i p_{l,t}, \text{ i.e. the order is ATM.}$$

(iii) if $x_i = 0$, complementarity constraints (10) imply $s_i = 0$ and then dual constraints (6) imply

$$\sum_{l,t} Q_{l,t}^i P^i \leq \sum_{l,t} Q_{l,t}^i p_{l,t}, \text{ i.e. the order is ATM or OTM.} \quad \square$$

Proof of lemma 2.

Proof. Assume $p_{l,t}$ are prices satisfying (i) – (iii) of Lemma 1. Let define $s_i, s_j \geq 0$ as follows:

$$(a) \quad s_i = \sum_{l,t} Q_{l,t}^i (P^i - p_{l,t}) \geq 0 \text{ if } x_i = 1 \text{ and likewise for } s_j,$$

$$(b) \quad s_i = \sum_{l,t} Q_{l,t}^i (P^i - p_{l,t}) = 0 \text{ if } 0 < x_i < 1 \text{ and likewise for } s_j,$$

$$(c) \quad s_i = 0 \geq \sum_{l,t} Q_{l,t}^i (P^i - p_{l,t}) \text{ if } x_i = 0 \text{ and likewise } s_j.$$

Conditions (i) – (iii) of lemma 1 ensure that in cases (a) and (b), $s_i = \sum_{l,t} Q_{l,t}^i (P^i - p_{l,t})$ is non negative (well defined), and that in case (c) $s_i = 0$ is greater or equal to $\sum_{l,t} Q_{l,t}^i (P^i - p_{l,t})$. It is then direct to check that constraints (6) – (7), (9) and (10) – (13) are satisfied. \square

Proof of theorem 1.

Proof. (i) Let (x_i^*, y_j^*, n_k^*) be a point satisfying (1) – (5) and suppose there exist $s_i^*, s_j^*, p_{l,t}^*, u_m^*$ satisfying (6) – (9) such that (10) – (14) are satisfied as well. Lemma 1 shows that $p_{l,t}$ are Walrasian equilibrium prices. As stated above, we assume that network dual and complementarity conditions (8) and (14) represent network equilibrium related to the chosen network model. In this case, strict linear prices exist.

(ii) To show the converse, assume $p_{l,t}$ satisfy (i) – (iii) of lemma 1 and that $p_{l,t}, u_m$ are network equilibrium prices. As stated before, we can then assume that $p_{l,t}, u_m$ satisfy (8) and (14) (network dual and complementarity conditions). Lemma 2 shows that it is also possible to assign values to s_i, s_j such that all constraints (6) – (14) are satisfied.

(iii) Assume (x_i, y_j, n_k) and (s_i, s_j, u_m) are primal and dual feasible respectively. By duality theory, they satisfy complementarity conditions if and only if equality of objective functions holds, in which case both are optimal for their respective problem. \square

Proof of Corollary 1.

Proof. Theorem 1 shows that for a given feasible solution of the primal (x_i^*, y_j^*, n_k^*) , strict linear equilibrium prices exist if and only if this solution is optimal for the continuous relaxation. \square

B Proof of lemma 6

Proof. (i) If (European) equilibrium prices exist, condition (78) holds, and necessarily:

$$\forall (u_i, u_j) \in P,$$

$$\begin{aligned} & \sum_i Q^i P_0^i u_i + \sum_j Q^j P^j u_j - \sum_j M_j (1 - y_j^*) u_j \\ & \leq \sum_i Q^i P_0^i x_i^* + \sum_i Q^i (P_1^i - P_0^i) [(x_i^*)^2 - x_i^* u_i] + \sum_j Q^j P^j y_j^* \\ & \leq \sum_i Q^i P_0^i x_i^* + \sum_i Q^i (P_1^i - P_0^i) \left[\frac{(x_i^*)^2}{2} - \frac{u_i^2}{2} \right] + \sum_j Q^j P^j y_j^*, \end{aligned}$$

where the first inequality is condition (78) rearranged, and where for the last inequality, we use the fact that if c_{ij} are coefficients of a negative semi-definite matrix, then:

$$\sum_{ij} c_{ij} x_i (x_j - u_j) \leq \frac{1}{2} \left(\sum_{ij} c_{ij} x_i x_j - \sum_{ij} c_{ij} u_i u_j \right).$$

Rearranging, we now get the necessary condition (79):

$$\max_{(u_i, u_j) \in P} \sum_i Q^i P_0^i u_i + \sum_i Q^i (P_1^i - P_0^i) \frac{(u_i)^2}{2} + \sum_j Q^j P^j u_j - \sum_j M_j (1 - y_j^*) u_j \leq obj^*,$$

where obj^* is the value of the quadratic objective function of the model for the current incumbent.

(ii) Let prove that this condition is also sufficient and let obj^* correspond to the optimal value associated to a new incumbent (x_i^*, y_j^*) . The left-hand side QP of inequation (79) is the continuous relaxation of the primal QP of section 4.1 with an additional term $-\sum_j M(1 - y_j^*)u_j$ in the objective function (taking into account the minor adaptations needed if one wants to consider a network representation).

The incumbent $(x_i^*, y_j^*) \in P$, so is feasible for this left-hand side QP and is therefore optimal for it (terms with the M_j cancel if $u_j = y_j^*$, so the expression is exactly the same on both sides).

By lemma 3, for this QP, there exist dual optimal variables $(s_i, s_j, p_{l,t}, v_i)$ such that $v_i = x_i^*$. Constraints of the dual are exactly constraints (60) – (63) (mutatis mutandis to take a network representation into account), and by strong duality for quadratic programs [4], we have:

$$\begin{aligned} & \sum_i s_i + \sum_j s_j - \sum_i Q^i (P_1^i - P_0^i) \frac{(x_i^*)^2}{2} \\ &= \sum_i Q^i P_0^i x_i^* + \sum_i Q^i (P_1^i - P_0^i) \frac{(x_i^*)^2}{2} + \sum_j Q^j P^j y_j^* - \sum_j M_j (1 - y_j^*) y_j^* \\ &\leq obj^* = \sum_i Q^i P_0^i x_i^* + \sum_i Q^i (P_1^i - P_0^i) \frac{(x_i^*)^2}{2} + \sum_j Q^j P^j y_j^*, \end{aligned}$$

so, rearranging, constraint (55) (equality of objective functions) is satisfied as well. This shows that for our incumbent (x_i^*, y_j^*) , we can define $(s_i, s_j, p_{l,t})$ such that all constraints (55) – (63) are satisfied. Therefore, an equilibrium price (or equilibrium prices when several areas or time slots are considered) satisfying EMM conditions exists for the solution (x_i^*, y_j^*) : one just needs to consider the dual optimal solution of the left-hand side QP for which $v_i = x_i^*$. \square

C Tables

C.1 Linear Instances

		New MILP formulation			Decomposition approach			
Instance	# Block orders	Run. Time	Final Gap	Nodes	Run. Time	Final Gap	Nodes	Cuts
1	766	600.37	495.89	27478	600.276	1463.35	842	3442
2	477	64.912			2.106			0
3	731	277.042		18331	600.401	106.75	2204	3594
4	566	64.819		12	2.527		17	2
5	683	57.097			2.168			0
6	513	47.283		84	6.537		46	27
7	658	79.577		473	99.7		458	706
8	604	51.028		3	2.371			0
9	571	36.348		2	1.685		2	0
10	655	136.891		5292	600.651	179.12	993	3625
11	686	54.335		77	6.708		85	29
12	692	69.156		2	2.73		2	0
13	640	93.773		9	3.369		21	3
14	618	85.692		7	3.635		6	5
15	550	92.368		57	6.567		75	16
16	591	59.857		9	3.885		7	7
17	685	117.781		91	3.37		15	4
18	699	600.339	252.83	27679	600.588	268.65	1408	3042
19	578	71.32		10	122.27		133	570
20	703	600.308	235.22	39225	600.604	517.47	1046	3173

C.2 Quadratic Instances (decomposition approach)

Instance	# Block orders	Run. Time	Final Gap	Nodes	Cuts
1	766	600.21	1160.50	565	1343
2	477	5.27			0
3	731	600.51	145.21	961	1382
4	566	8.47		13	2
5	683	8.30		1	0
6	513	10.95		32	7
7	658	13.01		59	16
8	604	5.82			0
9	571	4.31		1	0
10	655	601.10	256.69	357	1478
11	686	14.12		63	15
12	692	7.66			0
13	640	20.03		34	21
14	618	600.48	202.48	388	1291
15	550	180.29		247	366
16	591	42.67		40	58
17	685	44.43		82	66
18	699	600.56	204.42	753	1138
19	578	15.23		7	7
20	703	600.42	1740.25	457	1280

C.3 Instances with (almost) only block orders

		New MILP Formulation			Decomposition approach			
Instance	Block orders	Run. Time	Final Gap	Nodes	Run. Time	Final Gap	Nodes	Cuts
1	526	7.18		75561	600.14	28497.24	493464	132118
2	508	12.18		168467	540.00		1777391	121336
3	612	1.34		15348	2.32		8784	367
4	594	9.95		114400	15.41		81721	2756
5	671	4.74		53026	4.88		18312	847
6	766	8.80		90938	129.04		1156506	17312
7	714	1.82		17111	10.25		70038	1517
8	497	1.16		16210	459.08		1090631	106874
9	460	0.53		6216	0.56		4219	84
10	579	0.31		2474	1.01		2437	199
11	668	0.16		725	0.19		473	15
12	684	0.70		6733	2.45		29995	310
13	650	1.84		19433	7.58		71328	988
14	682	1.48		13224	2.43		10835	374
15	487	14.68		192265	600.01	6099.59	794340	142957
16	477	1.09		15481	302.75		699328	69114
17	597	0.16		792	0.47		5716	20
18	740	3.12		28904	28.44		105697	4312
19	794	5.91		57537	113.37		366836	14008
20	823	1.01		9677	600.03	209922.61	155204	63899

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- R. WEISMANTEL (2006), Lectures on mixed nonlinear programming.
- A. SHAPIRO (2010), Stochastic programming: modeling and theory.