2014/36

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DISCUSSION PAPER

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Endogenizing long-term contracts in gas market models

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September 2014

Abstract

Up to now, the European natural gas trade was dominated by bilateral long-term upstream agreements between producers and midstreamers that fixed a minimum volume to be exchanged (Take Or Pay) and a price formula that was usually indexed on oil products prices. These arrangements were believed to allow: i) market risk sharing between the producer (who takes the price risk) and the midstreamer (who takes the volume risk) as well as ii) risk hedging since oil is considered as a trusted commodity by investors. The fall of the European demand combined with the increase of the oil price favored the emergence of a gas volume bubble that caused net losses for most of the European midstreamers who were bound by long-term agreements. As a result, some energy economists brought forward the idea of indexing contracts on gas spot prices. In this paper, we present an equilibrium model that endogenously captures the contracting behavior of both the producer and the midstreamer who strive to hedge their profit-related risk. The players choose between gas forward and oil-indexed contracts. Using the model we show that i) contracting can reduce the trade risk of both the producer and midstreamer, ii) oil-indexed contracts should be signed only when oil and gas spot prices are well correlated, otherwise, these contracts hold less interest for risk mitigation, iii) contracts are more needed when the upstream cost structure is CAPEX driven and iv) a too risk-averse behavior of the midstreamer might deprive upstream investments and the downstream consumer surplus.

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This text presents research results of the P7/36 PAI project COMEX, part of the Belgian Program on Interuniversity Poles of Attraction initiated by the Belgian State, Prime Minister Office, Science Policy Programming. The scientific responsibility is assumed by the authors.

1 Introduction

Take or Pay (TOP) and price indexation (PI) clauses are the two pillars of the long- term contracts that drove the development of the gas industry in various regional markets throughout the world. Long-term term contracts are concluded between producers and "mid-streamers" (we use this term throughout the paper as a generic name to refer to merchant, pipeline company or LSE). TOP clauses obligate the mid-streamer to take some contracted gas volume or to pay for it. They guarantee the developer of the resource a certain volume of sales for its production and hence provide some protection for its investment. Price clauses (with or without indexation) are also part of the traditional long-term contract. They either fix the price paid by the mid-streamer to the producer over some horizon or alternatively index it to the prices of a bundle of "competing fuels" (in fact oil products)[1]. Price clauses are meant to hedge both the producer and the mid-streamer: they grant the producer some volume and relative price stability and hence reduce the risk on its investment. The indexation also reassures the mid-streamer that the gas sold through TOP will remain competitive and hence marketable, at least as long as there exists something like a "competing fuel".

While long-term contracts can be found in other sectors, the early interest in gas contracts probably stems from the unusual disclosure obligation that characterized the industry at the time of its regulation in the USA. Access to these data was of considerable interest for economists as reflected in early work ([23] and [24]). These authors rationalized the existence of long-term gas contracts by the protection that these offered both against risk and exercise of market power (the hold up phenomenon). These arguments were later often invoked in general industrial economics literature (see [7], see also [9] for a later analysis still focusing on the gas industry).

The gas industry has significantly evolved since those early days but long-term contracts remain a subject of high interest, whether they still dominate the market or have almost completely disappeared from it. Long-term contracts are today largely absent from the US, the UK and Australian gas markets that are now driven by spot prices. "Long-term" essentially means two to three years today in these markets and contracts are financial of different types and maturities (see [28] for the US and [43] for the UK). But long-term contracts retain a role in US and Australian LNG exports as well as in some LNG imports by the UK. In contrast, Asia is still dominated by long-term contracts with some reflecting on when the transition to spot hub markets will take place (see [40] and previous work by these authors, see also [25]). Europe is in the middle of this transition with a significant fraction of its supply still linked to long-term contracts and an important development of spot price based hubs with in the mean time intense discussions on whether long-term contracts will remain in the future, at least in their present form.

This paper concentrates on risk sharing between producers and mid-streamers; this was one of the arguments put forward at the early days of the development of the gas contracts and it remains important today. Uncertainty has increased dramatically in today economy, including the energy sector. This contributed to a widespread application of the financial portfolio approach that originally relied on the diversification of positions in financial assets to manage risk. The works [18], [19] and [20] introduced this approach in gas economics at the time the US industry was in the final phase of its restructuring. These authors transposed the standard financial model to a physical portfolio of gas contracts with different maturities and exposures. Multistage stochastic programming was the underling methodology, reflecting that information on the different parameters influencing gas prices is progressively resolved with time and the portfolio adapted accordingly.

The application of portfolio theory to physical assets is now common in the literature,

[1] Gas was largely competing with oil products or other energy forms and the indexation clause could reflect that competition. This competition is contested today but those who advocate abandoning price clauses have so far little alternative to propose except for the sole reliance on the spot market.

particularly in the power sector. This paper extends that literature in two major steps. It first generalizes the portfolio model originally considered in [19] by casting it in an equilibrium model encompassing producers and mid-streamers. In other words, the problem is no longer to find the optimal portfolio of an agent exposed to risk but to find the equilibrium portfolios of agents that trade gas and share the risk involved in this trading. This offers the possibility to consider portfolios of both long and short-term contracts, combining traditional long term indexation and spot price exposure. As LNG development shows, producers that undertake large fixed cost investment still appreciate the protection of long-term gas contracts. The various crises that have taken place in the gas market since the eighties in the US, the UK and in Europe show that the short to medium term market is subject to shocks that cannot be managed through the sole traditional TOP and price indexation clauses. The flexibility of short-term financial contracts is thus also necessary. In more abstract terms, the energy market with its long-term commitment to physical assets and the long and short-term vagaries of the economy is today very incomplete in terms of risk hedging. It thus makes sense to call again on the portfolio approach to see the extent to which a mix of contracts can best manage it in a world where not all risks can be hedged.

Our second contribution is to model the behavior of agents by a multistage risk function. These functions have been extensively discussed in the financial and stochastic programming literature. They have been introduced for modeling equilibrium in power markets in [12] and later used in various other papers. We here extend a formulation of [11] and consider the so called "good-deal risk measure that presents the interest of being both "coherent" and "time consistent" in the sense of Artzner and coauthors (see [2] and [3]) . The use of the risk measure combines the risky payoffs into what we refer to as a risk adjusted payoff. Needless to say, standard indices used in portfolio theory, like expected payoff and standard deviation can also be computed ex post from the results of the risk function based model.

The remainder of the paper is organized as follows: Section 2 presents our model. The economic structure is described as well as the interaction between the producer and the mid-streamer. The risk measure is introduced and the optimization programs are stated and solved for both the producer and the mid-streamer. Section 3 gives the theoretical results of our model. Existence and uniqueness results are proved and discussed. Section 4 details the scope and perimeter of our study as well as the data used. Section 5 applies our model to a situation involving a producer with different production cost structure and a mid-streamer facing a demand risk that induces a significant gas price risk. Three cases are studied: the first one assumes that gas is only exchanged in the spot market; this case serves as a counterfactual for comparing the risk adjusted profits of the producer and the mid streamer evaluated in the other situations. The second case introduces a fixed price contract (where the price is endogenously determined). The third case adds an oil-indexed contract that can hedge part of the market risk. The results are presented in two versions depending on whether oil and gas demands are well correlated or not. We then examine the sensitivity of the optimal contract volumes with respect to the upstream cost structure (fraction of the total cost that is fixed or variable) in order to understand if the difference between the European conventional gas and the US shale gas production cost structures can induce different contracting behaviors. Our last result shows how the mid-streamer's risk aversion modifies the contract characteristics, upstream investments and the consumer surplus. The last section concludes the paper.

2 The model

2.1 The rationale for long-term contracts

Long-term contracts are still present in the European natural gas industry where they covered more than 70% of gas trade in 2011[6]. Historically, LTCs were signed between producers and mid-streamers for supplying the downstream, while at the same time supporting the development of the gas market and its infrastructure. Several spot markets (NBP in the

UK, the TTF in the Netherlands or the NCG in Germany) emerged after the massive 2007 liberalization of the European energy markets[2], to ensure the fluidity of gas exchanges. Simultaneously LTCs became less important for the development of the industry, as most of the European transport and storage infrastructure had been built and was already fully depreciated. Notwithstanding this constant increase of spot exchanges, which usually have a short duration (e.g. day ahead), long-term contracts between the mid-streamers and the producers, which usually last many years, remain regularly renewed [6]. It is thus important to recall some of the rationale underpinning long-term contracts.

As explained in the introduction, this paper focusses on risk aversion against uncertain payoffs as the main incentive motivating long term contracts between producers and mid-streamers. LTC also efficiently deal with the so called hold-up problem [22]: absent the contract the mid-streamer is in a strong position to extract high price concessions from a producer that has already built an infrastructure and needs to fully utilize it. We do not consider that problem in this paper. Uncertainty appears in almost all aspects of the gas chain, whether exploration and production, spot prices, demand (which depends on environmental conditions), oil price (which determine gas prices in LTCs) etc. Since contracts are signed before starting production (but after the discovery of the gas fields) and are used to hedge the market's uncertainty, this study mainly focuses on risk arising from spot gas and oil prices as well as downstream demand. The traditional interpretation of the long term gas contracts is that the price risk is supported by the producers that benefit from a guaranteed outlet for their infrastructure while the volume risk is supported by the mid-streamers that need to find the market to sell that volume. We briefly elaborate on these ideas.

Contracts and producers

LTCs guarantee producers a certain revenue to recoup their investment cost. Investments in gas production and transportation infrastructure are very costly[3]. Because LTCs constrain both the volume and the price of the gas exchange, they reduce the volatility of the producers' revenue that can then more easily trigger their investments [37]. [29] also argues that the existence of long-term contracts may reduce the perception of insecurity of supply by the consumers, which would increase their consumption (via investments in gas consumption technologies) and, therefore, the producers' payoff.

Contracts and mid-streamers

A gas mid-streamer is an intermediary between the producers and the consumers. It faces several market uncertainties in supply (or insecurity of supply), downstream demand as well as spot prices. Long-term contracts reduce the supply risk. Flexibility clauses[4] also guarantee the mid-streamer some leeway to meet with a fluctuating downstream demand, even if it may also need to resort to the spot market in case of a very high or low demand.

As explained before, experience on both sides of the Atlantic shows that high or low demand conditions happen and that contracts can themselves be at the origin of price and volume risks for the mid-streamer. This started in Western Europe in 2008 when TOP volumes became higher than downstream demand, forcing mid-streamers to take excess volumes at high indexed prices to dump them on the spot market at a loss. The phenomenon is still ongoing but it probably peaked in 2012, with the consequence that some producers then agreed to introduce spot indexation in their contracts in order to render the contracts more acceptable to mid-streamers.

[2]Starting from July 2007, energy markets have been opened to all consumers, including end-users.
[3]These costs can reach averages of 200 M$/Bcm/year and 120 M$/Bcm/year/1000km respectively.
[4]These are the terms of the contract that specify upper and lower bounds on the volume to be exchanged.

Contracts and consumers

Because the upstream part of the gas chain is concentrated (more than 70% of the European supply is provided by Russia, Norway and Algeria), the market can be subject to market power. Long-term contracts, once signed, constitute an efficient way to mitigate the impact of dominance. Indeed, producers can only exercise market power for the deliveries that exceed the TOP volume, which can be very small in some European countries (such as Italy). The argument does not apply to the behavior of the producers at the contracting stage. [1] even shows that under some market conditions, long-term contracts can increase competition in the upstream. This question has been at the origin of a considerable literature that we very briefly mention here. The principle idea of that work is that two Cournot players in the spot market have an incentive to engage in contracts in the forward market even though these contracts reduce their market power in the spot market. The authors suppose that the two stage (forward and spot) game is sub game perfect and that there is no possible arbitrage between the first and the second stage. Traders therefore anticipate the outcome of the spot game given their position in the forward market. The proof is based on the fact that the reaction functions in the forward market, taking into account the equilibrium in the spot market, are more elastic than the reaction functions of the spot market. A consequence of that property is that an infinite sequence of trading stages would result into more and more elastic reaction functions, leading at the end to a perfect competition equilibrium. The result has been extensively studied in the literature and it has been shown to be quite sensitive to the underlying assumptions. In particular [35] have shown that the result vanishes when extending the model to three stage games where traders can build capacities in the first stage, conclude forward contracts in the second stage and go to the spot market in the third stage. One could envisage applying the original Allaz Vila to a gas market with infinite production capacities. The [35] result would apply if one were to take capacities into account and specially in an investment problem as later treated in this paper. [30] demonstrated that long-term contracts in the upstream may benefit consumers by the reduction of uncertainty that they imply on consumption prices.

2.2 Model description and structure

Taking stock of the above, we construct a model of the gas market with long-term contracts featuring the following elements:

- A downstream consumer market and a spot market, both represented by stochastic demand functions.
- Contracting possibilities between mid-streamers and producers and clearing of the spot market in each period of the horizon (at least two periods or stages).
- Agents' risk aversion described by a risk-adjusted payoff derived from a risk function.
- Different possible contractual clauses, including an LTC with indexation of gas on oil price.

We simplify the model by considering a single producer and a single mid-streamer possibly linked by a long-term contract over three stages. The mid-streamer serves a downstream market and can buy from or sell to the spot market. The producer can also sell gas to the spot market. Competition is supposed perfect (agents are price takers). Consumers in the spot and downstream markets are modeled with linear inverse-demand functions. The LTC market takes place in three stages: the contracts constituting the portfolio are concluded in the first stage, the demand of the consumer market and the clearing of the spot market appear in the second and third stages. Demand curves in the the spot and downstream markets are random and correlated. Uncertainty is represented by a three states probability law, conditional on the previous level of the demand (see Figure 1). Randomness is modeled through the intercept of the inverse-demand functions (in the spot and downstream markets) as follows: starting from a node ω in the tree that corresponds to time t, the intercept in

$t+1$ can either have a high, medium or low evolution with respect to the initial value in node ω of period t. The possible future evolution of the markets' demand can therefore be summarized in the tree given in Figure 1. Node 0 is the tree's root. The downstream and spot markets' intercepts of the demand function are supposed to be perfectly correlated with the result that we know the state of one of them (high, low or medium), when we know the state of the other. Imperfect correlation can be modeled at the cost of additional formulation and computation.

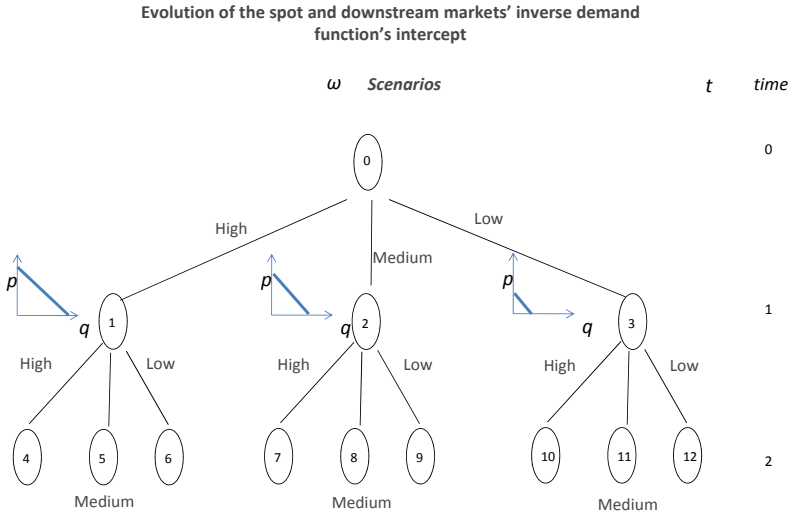

Figure 1:
The scenario tree

Each scenario node ω is weighted by a probability θ_ω estimated similarly by the producer and the mid-streamer (perfect information). Two nodes ω and $\omega\prime$ are denoted $\omega\prime \leq \omega$ if $\omega\prime$ is predecessor to ω in the tree. Scenario node ω's father is denoted by ω_f.
The players are rational, risk-averse and strive to optimize their risk-adjusted profit.

As a matter of notation and in order to avoid ambiguity we refer throughout the paper to the physical trade of a commodity as an "exchange" and reserve the word "trade" to the trading of financial products. We will also invariably call LTC any kind of direct exchange between the producer and the mid-streamer that does not go through the spot market, without any distinction on the duration of the contract. Two types of contracts will be considered: the first one, indexed by 1, binds the producer and mid-streamer by a fixed gas volume exchanged at a fixed price throughout the horizon of the contract. Both the volume and the price are endogenous and the volume will be considered as the TOP. The second kind of contract, indexed by 2, uses an exogenously given indexation formula to determine the contract price and the players need to determine the volume exchanged with that price indexation clause. We refer to oil as the underlying in the indexation clause in order to simplify the language; oil is here taken as a surrogate or a competing fuel, which was often a bundle of oil products, but could also include coal. The second case represents the historical contract with a fixed TOP and an competing fuel-indexed price whereas the first contract corresponds to the fixed price contracts that prevailed before the notion of the competitive fuel developed on the market. We shall see that these contracts have a financial

interpretation and that the first contract turns out to be a **forward** contract. In order to keep the presentation short, we did not consider contracts directly indexed on the spot price even though these could also mitigate the risk (depending on the clause such as for instance through caps or floors). This would be straightforward to do in our set up. The oil price is also assumed to be uncertain and will therefore be indexed by ω.

Figure 2 gives a schematic overview of the model.

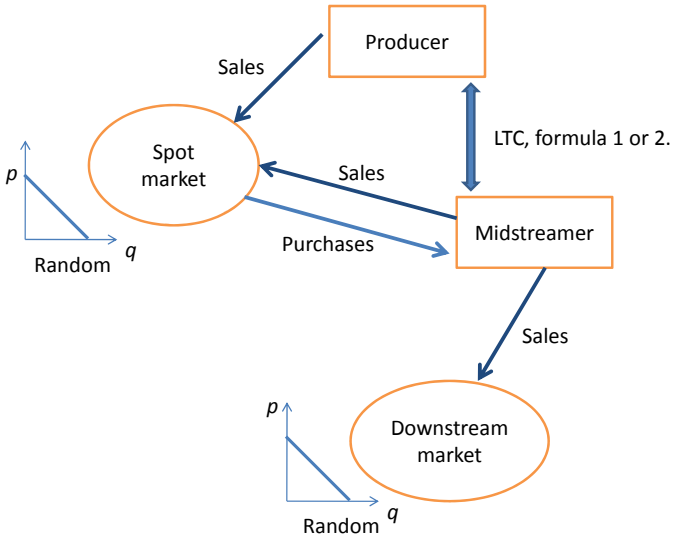

Figure 2:
Overview of the model

2.3 Notation

The following array gives the notation used in the study

Parameters

Ω	set of all scenario nodes ω
T	time
θ_ω	probability of node ω
a_ω, $-b_\omega$	intercept and slope of the inverse-demand curve in the downstream market, node ω
α_ω, $-\beta_\omega$	intercept and slope of the inverse-demand curve in the spot market, node ω
c	marginal production cost
π_ω	oil price at node ω, set exogenously.
λ^p	expected profit's weight in the producer's optimization program
λ^t	expected profit's weight in the mid-streamer's optimization program
$A^p,\ A^t$	risk measure parameters (producer and mid-streamer)

Variables

x_ω	gas volume sold or purchased (according to sign) by the producer in the spot market in node ω
up^1	gas volume sold by the producer to the mid-streamer using contract type 1
up^2	gas volume sold by the producer to the mid-streamer using contract type 2
ut^1	gas volume purchased by the mid-streamer from the producer using contract type 1
ut^2	gas volume purchased by the mid-streamer from the producer using contract type 2
h_ω	gas volume sold or purchased (according to its sign) by the mid-streamer in the spot market, at node ω
z_ω	gas volume sold by the mid-streamer to the downstream market, node ω
π^1	LTC price, contract type 1
π^2	dual variable associated with the contract type 2 clearing constraint
p_ω	spot market price, node ω
$p\prime_\omega$	downstream market price, node ω
νp_ω	proxy of the producer's risk-neutral probability, node ω
νt_ω	proxy of the mid-streamer's risk-neutral probability, node ω

2.4 Formulation

Players' decisions at period 0 (node 0) are the LTC volumes that will be used in the future by the mid-streamer to satisfy the local demand. More precisely, we will consider the LTC volumes $up(t)^1$ and $up(t)^2$ as first stage variables in the stochastic optimization programs of the two agents. We will use the tree formulation to represent both randomness and time using scenario nodes. This formulation dispenses with the writing of non-anticipativity constraints. The recourse actions are the sales/purchases in the spot market by the players.

We assume that the mid-steamer and the producer are both averse to the fluctuation of their profits and represent this aversion by a risk measure that we take as both coherent (see [2]) and time consistent (see [3]) since we are dealing with long-term incentives to contract. Following [8], we will use the good-deal risk measure that satisfies these two properties. Indeed, other standard coherent risk measures, such as the conditional value at risk CVaR [39] lack time consistency. From a computational point of view, the good-deal can be implemented in optimization models, via its dual formulation, as a Second-Order Cone Program [11].

2.4.1 The good-deal risk measure

Consider an incomplete market with an exchanged commodity (natural gas) and a random payoff denoted by Z_ω. Now we assume to simplify that we have two traded financial assets whose prices are $c1_\omega$ and $c2_\omega$ ($c1$ will be a risk-free asset and $c2$ an oil price derivative) and some contracts (constant or oil indexed price) that can be used to hedge part of the gas market's risk. The good-deal risk-measure calculates the maximum loss that can be suffered when considering a stochastic dynamic discount factor that dynamically prices all traded risky and non risky financial assets in such a way that no dynamic arbitrage is tolerated in

the prices of these assets and the corresponding stochastic discount factor's (price kernel) variance is bounded.

The risk function we are about to present is dynamic. To ease the explanation, we assume that randomness is captured by a scenario tree as the one that has been shown in Figure 1. We recall that each scenario node ω is weighted by a probability θ_ω. Two nodes ω and $\omega\prime$ are denoted $\omega\prime \leq \omega$ if $\omega\prime$ is predecessor to ω in the tree. Scenario node ω's father is denoted by ω_f.

The good-deal risk measure ρ is therefore defined as (with dual variables in parenthesis at the right of the constraints):

Definition 1. *The good-deal risk measure ρ:*

$$\begin{array}{rlll} \rho = & Max & -\sum_\omega \theta_\omega Z_\omega \left(\Pi_{\omega\prime\leq\omega}\zeta_{\omega\prime}\right) & \\ & \zeta_\omega & & \\ & s.t. & \forall\omega,\ \zeta_\omega \geq 0 & \\ & & \forall\omega,\ \displaystyle\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_{\omega\prime}}{\theta_\omega}\zeta_{\omega\prime}c1_{\omega\prime} = c1_\omega & (w1_\omega) \\ & & \forall\omega,\ \displaystyle\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_{\omega\prime}}{\theta_\omega}\zeta_{\omega\prime}c2_{\omega\prime} = c2_\omega & (w2_\omega) \\ & & \forall\omega,\ \displaystyle\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_{\omega\prime}}{\theta_\omega}\zeta^2_{\omega\prime} \leq A^2 & \end{array} \tag{1}$$

where

- The objective $-\sum_\omega \theta_\omega Z_\omega \left(\Pi_{\omega\prime\leq\omega}\zeta_{\omega\prime}\right)$ is the discounted expected loss.
- Constraints $\forall\omega, \zeta_\omega \geq 0$ forbid dynamic arbitrage.
- Constraints $\forall\omega,\ \displaystyle\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_{\omega\prime}}{\theta_\omega}\zeta_{\omega\prime}c1_{\omega\prime} = c1_\omega$ and $\forall\omega,\ \displaystyle\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_{\omega\prime}}{\theta_\omega}\zeta_{\omega\prime}c2_{\omega\prime} = c2_\omega$ bind the discount factor to the traded assets' prices, from period to period, so that the discounted expectation of these prices remains constant in time (martingale property).
- Constraints $\forall\omega,\ \displaystyle\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_{\omega\prime}}{\theta_\omega}\zeta^2_{\omega\prime} \leq A^2$ bound the variance of the stochastic discount factor.
- The parameter A is set exogenously and represents the risk-aversion level. The price vectors $c1$ and $c2$ are also set exogenously. They are the prices of the other than gas traded financial assets.

More details about the definition and utilization of the good-deal risk measure can be found in [33] and [8].

The dual formulation of the good deal

As stated in Definition 1, the good-deal risk measure is not directly usable in an optimization program. Indeed, its nested formulation may lead to computational difficulties. The dual formulation of Definition 1 solves this issue. This formulation is given below with the Lagrange variables being written in parenthesis (we here recall that ω_f designates the father of node ω).

Theorem 1. *The dual formulation of the good-deal risk measure is (see for instance [11] for the calculation. We also provide a demonstration in Appendix 2):*

$$
\begin{aligned}
\rho = \quad & Min \quad w1_0 c1_0 + w2_0 c2_0 + A\sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta_\omega^2} \\
& \eta_\omega \geq 0,\ w1_\omega,\ w2_\omega \\
& s.t. \quad \forall \omega, \quad \left(\eta_\omega + Z_\omega + c1_\omega(w1_{\omega_f} - w1_\omega) + c2_\omega(w2_{\omega_f} - w2_\omega) - A\sqrt{\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_{\omega\prime}}{\theta_\omega} \eta_{\omega\prime}^2} \right) \geq 0 \quad (\nu_\omega)
\end{aligned} \tag{2}
$$

We recall that 0 is the initial deterministic node of our scenario tree (see Figure 1); it is the ancestor of all the other nodes. Theorem 1 does not contain a nested formulation and hence leads to an implementation as a single conic program, which is therefore easier than the primal formulation. The mathematical properties of the formulation are further elaborated in Section 3
It is easy to demonstrate that the dual variables ν_ω can be interpreted as risk-neutral probabilities and that the following relation holds at the optimum:

$$
\rho = -\sum_\omega \nu_\omega Z_\omega \tag{3}
$$

2.4.2 The model's formulation: the forward contract. Case 0

In this section, we assume that neither oil nor oil products based contracts are considered. We also assume that agents do not hedge their position by trading oil or oil product derivatives. Only forward contracts are modeled. Therefore, in the players' good-deal definition, only a risk-free asset is traded and the oil derivative is ignored: $c1 = c2 = (1, 1, ...1)$. We refer to this situation as Case 0. The equilibrium model consists of agents' optimization problems (one problem for each agent) and market clearing constraints at the different stages of the market.

The producer's optimization problem:

The problem is stated as:

$$
\begin{aligned}
\text{Max} \quad & \lambda^p \textstyle\sum_\omega \theta_\omega p_\omega x_\omega \\
& +\lambda^p \textstyle\sum_\omega \theta_\omega \left(\pi^1 up^1\right) \\
& -\lambda^p \textstyle\sum_\omega \theta_\omega c(x_\omega + up^1) \\
& -(1-\lambda^p)\rho^p \\
\text{s.t.} \quad & \\
& 0 \leq up^1 \\
& \forall \omega,\ x_\omega \text{ free}
\end{aligned} \tag{4}
$$

where

- The term
  $\lambda^p \sum_\omega \theta_\omega p_\omega x_\omega$
  $+\lambda^p \sum_\omega \theta_\omega \left(\pi^1 up^1\right)$
  $-\lambda^p \sum_\omega \theta_\omega c(x_\omega + up^1)$

  is the expected producer's profit weighted by λ^p.

- The term: $(1-\lambda^p)\rho^p$

  is the producer's risk measure weighted by $1-\lambda^p$ where the risky payoff is stated as (the p index indicates that that this applies to the producer):

$$Z^p_\omega = p_\omega x_\omega + \pi^1 up^1 - c(x_\omega + up^1) \tag{5}$$

We recall that the prices of the two traded assets are $c1 = c2 = (1, 1, ...1)$.

The mid-streamer's optimization problem:

The problem (with dual variables in parenthesis at the right of the constraints) is:

$$\begin{array}{ll} \text{Max} & \lambda^t \sum_\omega \theta_\omega p_\omega h_\omega \\ & +\lambda^t \sum_\omega \theta_\omega p\prime_\omega z_\omega \\ & -\lambda^t \sum_\omega \theta_\omega \pi^1 ut^1 \\ & -(1-\lambda^t)\rho^t \\ \text{s.t.} & \forall \omega,\ z_\omega + h_\omega - ut^1 = 0 \quad (\mu_\omega) \\ & \forall \omega,\ 0 \leq z_\omega \\ & 0 \leq ut^1 \\ & \forall \omega,\ h_\omega \text{ free} \end{array} \tag{6}$$

where

- The term:
  $\lambda^t \sum_\omega \theta_\omega p_\omega h_\omega$
  $+\lambda^t \sum_\omega \theta_\omega p\prime_\omega z_\omega$
  $-\lambda^t \sum_\omega \theta_\omega \pi^1 ut^1$

  is the expected mid-streamer's profit weighted by λ^t.

- The term:

  $$(1-\lambda^t)\rho^t$$

  is the mid-streamer's risk measure weighted by $1-\lambda^t$ with the risky payoff being stated as (the t index indicates that that this applies to mid-streamers):

$$Z^t_\omega = p_\omega h_\omega + p\prime_\omega z_\omega - \pi^1 ut^1 \tag{7}$$

We recall that the traded assets prices are $c1 = c2 = (1, 1, ...1)$.

Conventionally $h_\omega \geq 0$ means that the mid-streamer sells gas in the spot market while $h_\omega \leq 0$ indicates that it buys gas in the spot market.

Equation $\forall \omega,\ z_\omega + h_\omega - ut^1 = 0$ is a sales=purchase condition for the mid-streamer, depending on whether this player buys or sells gas in the spot market.

The market clearing constraints.

The spot price p_ω is linked to the spot market demand by the inverse-demand function:

$$p_\omega = \alpha_\omega - \beta_\omega(x_\omega + h_\omega) \tag{8}$$

The downstream price $p\prime_\omega$ is linked to the downstream market consumption by the inverse-demand function:

$$p\prime_\omega = a_\omega - b_\omega z_\omega \tag{9}$$

The LTC sales equal purchases condition between the producer and mid-streamer (the corresponding dual variable is written between parenthesis):

$$up^1 - ut^1 = 0 \quad (\pi^1) \tag{10}$$

Since no market power is exerted in the LTC side of the market, the dual variable π^1 associated with the LTC supply/demand constraint represents the LTC clearing price.

Both the mid-streamer and the producer's objectives are concave functions of their decision variables. Indeed, for each player, the objective is composed of a difference between linear terms and the good-deal risk function which is convex (see Section 3).

2.4.3 Introducing oil indexation and derivatives. Case 1.

We now allow the players to hedge part of their risk on oil. This is referred to as Case 1. The way risk is hedged is as follows: there is an oil market and both players can sign an oil-indexed contract at a price linked to the oil price π_ω. At the same time they can also hedge their corresponding oil dependence by trading financial oil derivatives. This latter aspect is not necessary to the model: it is introduced to illustrate the flexibility offered by the risk functions for enriching the contract problem: former justification are introduced to justified the oil indexation clauses and more moderne hedging reasoning to mange the risk residual oil risk introduced by these clauses. Whether we include oil derivative or not in the good-deal risk function, the introduction of the indexation clause requires to specify that oil is exchanged at its price π_ω in the the good-deal risk function of both players and to add the corresponding oil-related contract payoff or cost. We also assume the existence of traded non-risky asset (whose price is scenario-independent), which implies that: $c1 = (1, 1, ..., 1)$ and $c2 = \pi$.

The producer's optimization problem

The producer's problem is then stated as:

$$\begin{array}{ll} \text{Max} & \lambda^p \sum_\omega \theta_\omega p_\omega x_\omega \\ & +\lambda^p \sum_\omega \theta_\omega \left(\pi^1 up^1\right) \\ & +\lambda^p \sum_\omega \theta_\omega \left((\pi^2 + \pi_\omega) up^2\right) \\ & -\lambda^p \sum_\omega \theta_\omega c(x_\omega + up^1) \\ & -(1-\lambda^p)\rho^p \\ \text{s.t.} & \\ & 0 \leq up^1,\ up^2 \\ & \forall \omega,\ x_\omega \text{ free} \end{array} \tag{11}$$

- The new term $(\pi^2 + \pi_\omega)up^2$ is the oil-indexed contract payoff. $\pi^2 + \pi_\omega$ can be viewed as the indexation formula where π_ω is the oil price and π^2 the marginal value of the oil-indexed contracts clearing constraint between the producer and mid-streamer. The producer's risky payoff is now:

$$Z^p_\omega = p_\omega x_\omega + \pi^1 up^1 + (\pi^2 + \pi_\omega)up^2 - c(x_\omega + up^1) \tag{12}$$

The mid-streamer's optimization problem:

The problem (with dual variables in parenthesis at the right of the constraints) is:

$$
\begin{array}{ll}
\text{Max} & \lambda^t \sum_\omega \theta_\omega p_\omega h_\omega \\
& +\lambda^t \sum_\omega \theta_\omega p\prime_\omega z_\omega \\
& -\lambda^t \sum_\omega \theta_\omega \pi^1 ut^1 \\
& -\lambda^t \sum_\omega \theta_\omega \left((\pi^2 + \pi_\omega) ut^2\right) \\
& -(1-\lambda^t)\rho^t \\
\text{s.t.} & \forall \omega,\ z_\omega + h_\omega - ut^1 = 0 \qquad (\mu_\omega) \\
& \forall \omega,\ 0 \leq z_\omega \\
& 0 \leq ut^1,\ ut^2 \\
& \forall \omega,\ h_\omega \text{ free}
\end{array}
\tag{13}
$$

- The new term $-(\pi^2 + \pi_\omega) ut^2$ is the oil-indexed contract cost. The mid-streamer's risky payoff is now:

$$
Z^t_\omega = p_\omega h_\omega + p\prime_\omega z_\omega - \pi^1 ut^1 - (\pi^2 + \pi_\omega) up^2 \tag{14}
$$

The market clearing constraints:

$$
up^1 - ut^1 = 0 \quad (\pi^1) \tag{15}
$$

$$
up^2 - ut^2 = 0 \quad (\pi^2) \tag{16}
$$

Both models (Case 0 and Case 1) are formulated as an equilibrium problem and solved in their complementarity form. The KKT conditions of the agents' problems together with the market clearing conditions that form the models are given in Appendix 1.

Before applying the model, we first present a set of theorems that guarantee the existence and/or uniqueness of the equilibrium.

3 Theoretical results

3.1 Main assumptions and notation

For the sake of simplicity and clarity of the demonstrations, we focus our attention throughout Section 3 on Case 0 where only the forward contract in signed. However, our results are still valid and can easily be generalized to Case 1. Since in that case, $c1 = c2$ (oil is ignored), to ease the presentation, we will remove from our notation the variables $w2^p$ and $w2^t$ which are redundant with $w1^p$ and $w1^t$ respectively and will call them w^p and w^t[5].
In the following equations, we use the convention that the scenario nodes are numbered from 0 to m, where 0 is the root of our scenario tree. We denote by Ω_l the subset of Ω constituted by the leaves of the tree (i.e. the nodes that do not have children nodes). We recall that ω_f denotes node ω's father in the tree. We will also denote by T the number of time periods: $T = \sum_\omega \theta_\omega$.

To simplify our results' presentation, we assume that there exists a risk free asset (whose price is scenario independent), which implies writing $c1 = (1, 1, ...1)$. This assumption is necessary to ensure the existence of the risk-neutral probabilities for the players.

We construct the scenario tree so that the parameters α_ω and a_ω are pairwise distinct in each time step and that starting from a scenario node ω, we will have in the coming period either an increase, a stagnation or a decrease of α_ω and a_ω. This means that we impose (with similar relations for a_ω):

[5] We can actually show that at the equilibrium, $w2^p = w1^p$ and $w2^t = w1^t$.

$$\alpha_4 > \alpha_1 = \alpha_5 > \alpha_6 \tag{17}$$
$$\alpha_7 > \alpha_2 = \alpha_8 > \alpha_9 \tag{18}$$
$$\alpha_{10} > \alpha_3 = \alpha_{11} > \alpha_{12} \tag{19}$$

Besides, we will assume that the slopes β_ω and b_ω are constant. Finally, the production cost function is assumed to be strictly convex and increasing with respect to the quantity produced.

One can then rewrite the objective functions of the players by including the formulation of the good-deal directly in the optimization programs. Substituting expression (2) directly in (4) and (6) one obtains:

The producer:

$$\begin{aligned} \text{Max} \quad & \lambda^p \sum_\omega \theta_\omega p_\omega x_\omega \\ & + \lambda^p \sum_\omega \theta_\omega \left(\pi^1 u p^1\right) \\ & - \lambda^p \sum_\omega \theta_\omega c(x_\omega + u p^1) \\ & - (1-\lambda^p)\left(w_0^p c1_0 + A^p \sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta p_\omega^2}\right) \end{aligned} \tag{20}$$

s.t.

$$\forall \omega,\ \eta p_\omega + Z_\omega^p + c1_\omega (w_{\omega_f}^p - w_\omega^p) - A^p \sqrt{\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_{\omega\prime}}{\theta_\omega} \eta p_{\omega\prime}^2} \geq 0 \qquad (\nu p_\omega) \tag{21}$$

$$\forall \omega,\ x_\omega \text{ free },\ 0 \leq u p^1,\ 0 \leq \eta p_\omega,\ \text{free } w_\omega^p \tag{22}$$

where the producer's profit Z_ω^p is:

$$Z_\omega^p = p_\omega x_\omega + \pi^1 u p^1 - c(x_\omega + u p^1) \tag{23}$$

The mid-streamer:

Max (24)

$$\lambda^t \sum_{\omega} \theta_\omega p_\omega h_\omega$$
$$+ \lambda^t \sum_{\omega} \theta_\omega p\prime_\omega z_\omega$$
$$- \lambda^t \sum_{\omega} \theta_\omega \pi^1 u t^1$$
$$- (1-\lambda^t) \left(w_0^t c1_0 + A^t \sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta t_\omega^2} \right)$$

s.t.

$$\forall \omega,\ z_\omega + h_\omega - u t^1 = 0 \tag{25}$$

$$\forall \omega,\ \eta t_\omega + Z_\omega^t + c1_\omega (w_{\omega_f}^t - w_\omega^t) - A^t \sqrt{\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_{\omega\prime}}{\theta_\omega} \eta t_{\omega\prime}^2} \geq 0 \qquad (\nu t_\omega) \tag{26}$$

$$\forall \omega,\ 0 \leq u t^1,\ z_\omega,\ \text{free } h_\omega,\ w_\omega^t \tag{27}$$

where the mid-streamer's profit Z_ω^t is:

$$Z_\omega^t = p_\omega h_\omega + p\prime_\omega z_\omega - \pi^1 u t^1 \tag{28}$$

Market clearing constraints:

The contract clearing constraint is

$$u p^1 = u t^1 \tag{29}$$

the spot price p_ω is:

$$p_\omega = \alpha_\omega - \beta_\omega (x_\omega + h_\omega) \tag{30}$$

and the downstream price $p\prime_\omega$ is:

$$p\prime_\omega = a_\omega - b_\omega z_\omega \tag{31}$$

All proofs can be found in Appendix 2.

3.2 Standard convexity (concavity) properties

We first prove the concavity of the producer and mid-streamer's objective functions as well as the convexity of their feasibility sets[6]. This is based on the following lemma:

Lemma 1. *$\forall (\delta_1, ..., \delta_n) \in \mathbb{R}^n$ such that $\forall i,\ \delta_i \geq 0$, the function $f : \mathbb{R}^n \longrightarrow \mathbb{R},\ (x_1, ..., x_n) \longrightarrow \sqrt{\sum_i \delta_i x_i^2}$ is convex.*

Theorem 2. *The players' objective functions are concave and their feasibility sets convex.*

[6]The convexity of the good-deal risk function is a well known property (see [8]). However, we present a demonstration of this property because some results developed herein will be used later on in our uniqueness result's demonstration

3.3 Preliminary assumptions and results

3.3.1 The complementarity formulation

It is important, before going further and for a proper interpretation of the contract model to clearly specify the mathematical nature of our problem. The general formulation given in expressions (20)-(31), is actually a Generalized Nash-Cournot problem because the objective function and the feasibility set of the producer depends on the mid-streamer's decision variables and conversely. Hence (since we have demonstrated the standard convexity properties), solving that model winds up to solving a QVI problem, for which there usually exist an infinity of solutions [36]. A particular solution of this model is obtained by imposing that the gas procurement contracts are actually traded in a market at a fixed price represented by the Lagrange multiplier of each contract clearing constraint (the assumptions underpinning the definition of the good-deal is that the finical contracts are traded). This implicitly states that the producer and the mid-streamer's Lagrange multipliers of their common constraints are the same, which makes us then rather look for the particular VI solution of the QVI problem (see for instance [17] and [14] for a better explanation of the link between Lagrange multipliers and VI solutions). This is a restrictive interpretation at least considering that these contracts were not effectively traded on markets even if the common wisdom was that the corresponding gas prices were "competitive" in the sense that they did not depart too much from being equal. The QVI formalism allows one to assess that conjecture by exploring solutions where implicit prices of contracts (their dual variables) are close but not equal. Different techniques exist for doing so and an illustration of their use in contracts (short term electricity contracts) can be found in de Maere and Smeers [34]. We leave that development for further work.

In order to write the KKT conditions of the game, we will need the objective functions to be differentiable with respect to the decision variables. In particular, given the use of the square root in the good-deal formulation, which is not differentiable in 0, we will add the following constraints to the players' feasibility sets:

The producer

$$\forall \omega,\ \eta p_\omega \geq \epsilon \tag{32}$$

The mid-streamer

$$\forall \omega,\ \eta t_\omega \geq \epsilon \tag{33}$$

Besides, to ensure the existence of the players' optimal decision variables, we will first bound all the variables as follows ($M \geq 0$):

The producer

$$\| \left(x, up^1, w^p, \eta p\right) \| \leq M \tag{34}$$

The mid-streamer

$$\| \left(h, z, ut^1, w^t, \eta t\right) \| \leq M \tag{35}$$

In practice, ϵ is set to a very small value and M to a very big one so that we do not perturb the model by adding constraints (32), (33), (34) and (35). However, we prove later that our problem is naturally bounded, which implies that for a value of M big enough, constraints (34) and (35) are not binding anymore. Constraints (34) and (35) make the feasibility sets of the players bounded and therefore compact, since they are also closed. Also, it is easy to check that the standard constraint qualification conditions hold for both the producer and the mid-streamer's feasibility sets. Therefore, we can conclude that the KKT conditions are necessary and sufficient to characterize the equilibrium that can be formulated as the solution of the Mixed Complementarity Problem (MCP) given in Appendix 1.

3.3.2 The spot and the downstream markets prices

In this section, we will prove that at the equilibrium (if it exists), the spot and downstream prices (as well as the marginal production cost) do not explicitly depend on the players' decision variables. Indeed (see the KKT conditions of Appendix 1), using equations (52a), (53a), (53h) and (54c), we will get $\forall \omega \in \Omega$:

$$c\prime(x_\omega + up^1) = \alpha_\omega - \beta\omega \left(x_\omega + up^1 - z_\omega\right) \tag{36}$$

$$\alpha_\omega - \beta\omega \left(x_\omega + up^1 - z_\omega\right) = a_\omega - b_\omega z_\omega \tag{37}$$

Which implies, knowing that the production cost function is strictly convex, that at the equilibrium, the net production $x_\omega + up^1$ and the downstream consumption z_ω depend only on the production cost and the spot and downstream inverse demand function parameters. Therefore, they are bounded. It also implies that the spot and downstream prices do not depend on the player's decision variables and are bounded. In the following, these prices can hence be considered as exogenous or parameters and will just be denoted by p_ω and $p\prime_\omega$.

3.3.3 Further assumptions and compactness results

Following [34] and [38], we will make the important assumption that the producer and the mid-streamer's risk measures are sufficiently similar. This can be formulated as follows: if we denote by

$$\mathcal{R}^p = \{\nu p_\omega \in \mathbb{R}^\Omega \text{ such that } \forall Z^p_\omega \in \mathbb{R}^m,\ \rho^p(Z^p) < +\infty\}$$

$$\mathcal{R}^t = \{\nu t_\omega \in \mathbb{R}^\Omega \text{ such that } \forall Z^t_\omega \in \mathbb{R}^m,\ \rho^t(Z^t) < +\infty\}$$

then the interior of the set $\mathcal{R} = \mathcal{R}^p \cap \mathcal{R}^t$ is not empty. We recall that the vectors νp (for the producer) and νt (for the mid-streamer) have been introduced in the dual formulation of the good-deal and are associated with constraints (21) and (21). They will be linked to the risk-neutral probabilities later on. This assumption is useful inasmuch as it ensures that the contracted volume $up^1 = ut^1$ remains bounded. Indeed, an equilibrium such that $up^1 = ut^1 \longrightarrow +\infty$ would lead to a producer and mid-streamer's risks $\longrightarrow -\infty$, which implies that the equilibrium cannot be a Nash equilibrium (since thanks to our assumption, we know that both players can have risk measures that give a finite risk). Therefore, given that the net production, the spot and downstream consumptions are bounded, we can already deduce that the variables $x_\omega, up^1, h_\omega, z_\omega, ut^1$ are bounded. We focus our interest now on the contract price.

Theorem 3. *At the equilibrium (if it exists), the contract price is bounded*

Hence, for a value of M big enough, constraints (34) and (35) are not binding anymore and the initial problem is not perturbed.

3.4 Risk-neutral probabilities and the formation of the contract price

If the players were risk-neutral, each scenario would have been weighted by θ_ω in their objectives. However, when considering the KKT conditions, one can notice that adding the good-deal risk measure to the players' objective leads to weighting each scenario node ω differently: by $\lambda^p\theta_\omega - \nu p_\omega$ for the producer and $\lambda^t\theta_\omega - \nu t_\omega$ for the mid-streamer. Actually, these new parameters represent the risk-neutral probabilities and will have a very important role in understanding the formation of the contract price. Before defining them theoretically, we first need to state two lemmas:

Lemma 2. *At the equlibrium,* $\forall \omega, \nu p_\omega < 0,\ \nu t_\omega < 0,\ \eta p_\omega > 0$ *and* $\eta t_\omega > 0$.

Lemma 3. $\omega \longrightarrow \lambda^p\theta_\omega - \nu p_\omega$ *and* $\omega \longrightarrow \lambda^t\theta_\omega - \nu t_\omega$ *are probability measures.*

Definition 2. *The producer's risk-neutral probabilities are* $\lambda^p\theta_\omega - \nu p_\omega$ *and the mid-streamer's risk-neutral probabilities are* $\lambda^t\theta_\omega - \nu t_\omega$.

The coming theorem is standard: it links the contract price to the spot prices so that at the equilibrium, no arbitrage is possible between the contract and the spot markets.

Theorem 4. *If a contract is signed, then the contract price is a risk-adjusted expectation of the spot price.*

Now, we will focus our interest on the solution set's properties.

3.5 Existence and uniqueness properties

Existence of the equilibrium can be proved in a similar way as in [21] and [34]. The proof introduces an additional player, the contract operator, whose role is to take care of the contract clearing constraint and demonstrates that at the equilibrium, his decision variables are bounded. To keep our paper's size reasonable enough, we decided not to report such a proof here and to leave it for a subsequent work, where we intend to focus on an existence result for a more general contract equilibrium formulation using a general formulation of a coherent and time-consistent risk function.
Hence, we rather focus here on a weak existence and uniqueness result.

3.5.1 The parametrized problem

The difficulty with proving uniqueness of the equilibrium is mainly related to convexity issues when the problem is seen as a whole. Indeed, if we want to apply the standard uniqueness results linked to the QVI/VI formulation, we need to guarantee the convexity of the whole feasible set with respect to all the decision variables. However, when one looks at constraint (21) for instance, one notices the presence of the profit Z^p in the constraint, that contains the product of two variables: $\pi^1 \times up^1$, which makes the feasibility set non-convex. Therefore, in the following, we will only demonstrate that if the contract price π^1 is set by the players exogenously, then the rest of the variables are unique.
In the rest of this section, we will assume that the contract price is set to a value π^1 and we will study the corresponding parametrized (by π^1) equilibrium problem. For the sake of clarity of the presentation, we will not index our problem by π^1 but one should bear in mind that the contract price is actually exogenous in this study. Besides, we will have to drop the contract clearing constraint (29).

3.5.2 The VI formulation

To demonstrate the uniqueness of the parametrized equilibrium, we will use the Variational Inequality (VI) type formulation of our Nash game. For that purpose, we will simplify our notation as follows (with the convention that vectors are written in columns):

The producer
We will denote by X^p the vector:

$$X^p =^t \left(x_{\omega,\omega\in\Omega}, up^1, w^p_{\omega,\omega\in\Omega}, \eta p_{\omega,\omega\in\Omega}\right) \in \mathbb{R}^{3m+1}$$

$\Pi^p(X^p, X^t)$ is the producer's objective function and its gradient F^p is:

$$F^p(X^p, X^t) = \begin{pmatrix} \nabla_{x_\omega}\Pi^p \\ \nabla_{up^1}\Pi^p \\ \nabla_{w^p_\omega}\Pi^p \\ \nabla_{\eta p_\omega}\Pi^p \end{pmatrix} \left(X^p, X^t\right) \tag{38}$$

where X^t is the mid-streamer's decision vector, as will be shown later. When the gradient is explicitly calculated, we get:

$$\nabla_{x_\omega}\Pi^p = \begin{pmatrix} \lambda^p\theta_\omega\left(\alpha_\omega - \beta_\omega\left(x_\omega + h_\omega\right)\right) - \lambda^p\theta_\omega c\prime(x_\omega + up^1) \\ \vdots \\ \omega \in \Omega \\ \vdots \end{pmatrix} \tag{39}$$

$$\nabla_{up^1}\Pi^p = \left(\lambda^p T\pi^1 - \lambda^p \textstyle\sum_\omega \theta_\omega c\prime(x_\omega + up^1)\right) \tag{40}$$

$$\nabla_{w^p_\omega}\Pi^p = \begin{pmatrix} -(1-\lambda^p)c1_0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{41}$$

and

$$\nabla_{\eta p_\omega}\Pi^p = \begin{pmatrix} -(1-\lambda^p)A^p\theta_\omega \dfrac{\eta p_\omega}{\sqrt{\displaystyle\sum_{\omega/\omega_f=0}\theta_\omega \eta p^2_\omega}} \\ \vdots \\ \omega/\omega_f = 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{42}$$

The mid-streamer We will denote by X^t the vector:

$$X^t =^t \left(h_{\omega,\omega\in\Omega}, z_{\omega,\omega\in\Omega}, ut^1, w^t_{\omega,\omega\in\Omega}, \eta t_{\omega,\omega\in\Omega}\right) \in \mathbb{R}^{4m+1}$$

$\Pi^t(X^p, X^t)$ is the producer's objective function and its gradient F^t is:

$$F^t(X^p, X^t) = \begin{pmatrix} \nabla_{h_\omega}\Pi^t \\ \nabla_{z_\omega}\Pi^t \\ \nabla_{ut^1}\Pi^t \\ \nabla_{w^t_\omega}\Pi^t \\ \nabla_{\eta t_\omega}\Pi^t \end{pmatrix}\left(X^p, X^t\right) \tag{43}$$

where

$$\nabla_{h_\omega}\Pi^t = \begin{pmatrix} \lambda^t\theta_\omega\left(\alpha_\omega - \beta_\omega\left(x_\omega + h_\omega\right)\right) \\ \vdots \\ \omega \in \Omega \\ \vdots \end{pmatrix} \tag{44}$$

$$\nabla_{z_\omega}\Pi^t = \begin{pmatrix} \lambda^t\theta_\omega\left(a_\omega - b_\omega z_\omega\right) \\ \vdots \\ \omega \in \Omega \\ \vdots \end{pmatrix} \tag{45}$$

$$\nabla_{ut^1}\Pi^t = \left(-\lambda^t T\pi^1\right) \tag{46}$$

$$\nabla_{w^t_\omega}\Pi^t = \begin{pmatrix} -(1-\lambda^t)c1_0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{47}$$

and

$$\nabla_{\eta t_\omega}\Pi^t = \begin{pmatrix} -(1-\lambda^t)A^t\theta_\omega \frac{\eta t_\omega}{\sqrt{\sum_{\omega/\omega_f=0}\theta_\omega \eta t_\omega^2}} \\ \vdots \\ \omega/\omega_f = 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{48}$$

In a condensed notation, we will have:

$$X = \begin{pmatrix} X^p \\ X^t \end{pmatrix} \in \mathbb{R}^{7m+2}$$

and

$$F = \begin{pmatrix} F^p \\ F^t \end{pmatrix} \in \mathbb{R}^{7m+2}$$

Note that the contract price has not been added to decision variable vector X and has not been considered in the calculation of the gradients.

As for the feasibility set, we will call K the set of variables X that verify both players' feasibility constraints (without the contract clearing condition):

$K = \{X \in \mathbb{R}^{7m+2}$ that verify $(21), (22), (51), (26), (27), (32), (33), (34), (35), (29), (30)$ and $(31)\}$

It is easy to check that K is compact and convex[7]. If we denote by $\underline{p} = \min\{p_\omega,\ \omega \in \Omega\}$ and $\overline{p} = \max\{p_\omega,\ \omega \in \Omega\}$, it is also easy to check that if $\underline{p} \leq \pi^1 \leq \overline{p}$, K is non-empty: $K \neq \emptyset$.

Theorem 5. *A vector X is a solution to our parametrized problem if and only if it is a solution of VI(F,K):*
$X \in K$ and $\forall X\prime \in K$, $-{}^tF(X)(X\prime - X) \geq 0$

3.5.3 Existence of the parametrized problem equilibrium

Now, it is time to state our parametrized equilibrium existence theorem:

Theorem 6. *$\forall \pi^1 \in [\underline{p}, \overline{p}]$, there exists at least one equilibrium to our parametrized problem.*

3.5.4 Uniqueness of the parametrized problem equilibrium

Uniqueness properties are often linked to function F's monotonicity or strict monotonicity. We will first demonstrate the following important lemmas:

Lemma 4. *If $\lambda_p = \lambda_t$, function -F is monotone.*

Remark 1. *Note that imposing $\lambda^p = \lambda^t$ is not very restrictive because it does not imply that the players have the same risk aversion level. Indeed, the latter is mainly driven by the coefficients A^p and A^t which may be different.*

Lemma 5. *If $\lambda_p = \lambda_t$, the parametrized equilibrium problem is integrable.*

We state now our parametrized equilibrium uniqueness theorem

Theorem 7. *If $\lambda_p = \lambda_t \ \in (0, 1)$, the equilibrium of the parametrized problem is unique.*

[7] This is due to the fact that the contract π^1, the spot p_ω and the downstream $p\prime_\omega$ prices are exogenous.

4 Calibration and data

We have applied our model to a stylized hypothetical transaction between a gas producer and a European gas mid-streamer serving the French gas demand, for which we could check the results on the basis of public sources. We use the PEG Nord (Point d'Echange du Gaz au Nord in France) price as a reference French market price. The NBP (National Balancing Point in the UK) plays the role of the spot market. The time scope of our study is [2012-2021] and each time-step corresponds to five years. The calibration of the inverse-demand functions is done in scenario node 0 (year 2012) as in [32]: we estimate the intercept and slope of an inverse-demand function using a reference (price, demand) point and an estimation of the elasticity γ[8]. We use elasticities $\gamma_{\text{NBP}} = 0.15$ for the spot market and $\gamma_{\text{PEG Nord}} = 0.05$ for the downstream market. We thus assume the downstream demand to be quasi-inelastic with respect to the price. The reference consumption and prices are taken from [5] and [26].

Results are presented in \$/Mbtu for prices and marginal costs and Bcm for volumes. In our reference case, the evolution of the inverse-demand function's parameters, the probability law and the risk function parameters are as follows:

ω	α(\$/MBtu)	β(\$/MBtu/Bcm)	a(\$/MBtu)	b(\$/MBtu/Bcm)	θ	A^p	A^t	λ^p	λ^t
1	75.0	0.58	89.8	0.34	0.33	1.07	1.07	0	0
2	86.2	0.58	81.6	0.34	0.33				
3	47.7	0.58	65.3	0.34	0.33				
4	82.5	0.58	94.3	0.34	0.11				
5	75.0	0.58	89.8	0.34	0.11				
6	67.5	0.58	85.3	0.34	0.11				
7	75.0	0.58	85.7	0.34	0.11				
8	68.2	0.58	81.6	0.34	0.11				
9	61.3	0.58	77.5	0.34	0.11				
10	52.5	0.58	68.6	0.34	0.11				
11	47.7	0.58	65.3	0.34	0.11				
12	42.9	0.58	62.0	0.34	0.11				

We remind that A^p and A^t are the risk-aversion parameters of the producer and the mid-streamer.
The production cost function is quadratic: it has been calibrated on a typical Asian production field (source [27]). This cost also includes the transportation cost to Europe.

5 Results

We focus on three situations: the first reference case assumes that gas is only exchanged in the spot market (no contract, $up^1 = up^2 = 0$). In the second case, we introduce the possibility to sign a constant price and volume contract (market-indexed); we determine this volume and study the inherent risk reduction, if any. The third case allows the actors to sign a constant price and volume contract, as well as an oil-indexed by constant volume contract. Two situations are then considered depending on the degree of correlation of oil price with the intercept of the gas demand price function.

5.1 The reference case: the spot trade only (no contract) (Case -1)

Figure 3 shows the producer and the mid-streamer's payoff in all possible market scenarios $\{1, 2...12\}$[9].

[8] Here we consider the elasticity of the demand over the price $\gamma = \frac{\mathrm{d}q}{\mathrm{d}p}\frac{p}{q}$.

[9] We recall that node 0 is the root node with known parameters. Since we are interested in randomness of the payoffs, we do not show scenario 0's payoff.

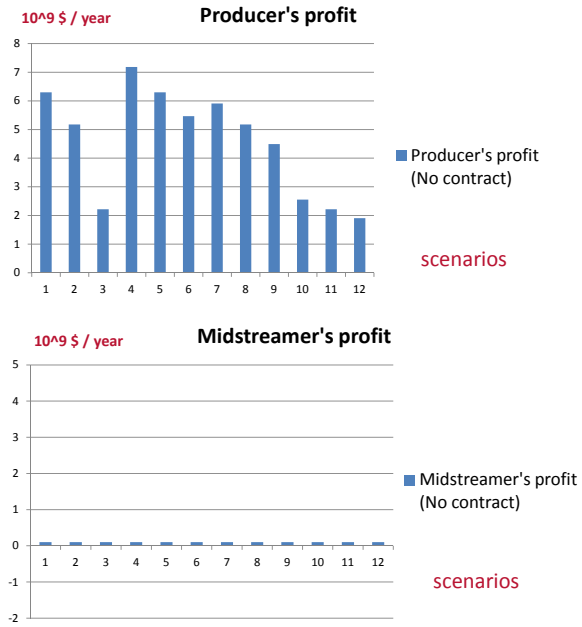

Figure 3:
The actors' payoff, Case -1. Only the spot market exchanges are allowed.

We notice that the producer's payoff fluctuates, which is due to the fact that the spot market price is random. Besides, the producer's payoff is maximal in scenario node 4 where the spot market inverse-demand function's intercept is highest. The same observation can be made for scenario nodes 10,11 and 12 and the minimum payoff and the lowest spot prices. The mid-streamer does not exert any market power, no contract is signed and the downstream market's price is sensitive to the volume. Hence, at the equilibrium, no arbitrage is possible between the spot and the downstream market prices and these are equal, which induces zero profit for the mid-streamer (the variable h_ω is free).
The producer's profit fluctuation leads to a non-zero risk: $\rho^p = -3.2\ 10^9$ \$. In other words, the producer's risk-adjusted profit expectation over all the time horizon is $-\rho^p = 3.2\ 10^9$ \$. The fact that the mid-streamer's profit is not random results in a zero profit and zero risk: $\rho^t = 0$ \$.

Risk (B\$)	Producer	Mid-streamer
	-3.2	0

5.2 The spot market and constant price and volume contract (Case 0)

The following reports result from a market situation where the producer and mid-streamer can conclude a contract. For that purpose, we use the reference case data set and allow for endogenous contracting.

Figure 4 shows the producer and the mid-streamer's payoff in all the possible market scenarios $\{1, 2...12\}$ for Case -1 (spot only) and Case 0 (spot + constant price and volume contract). We also report the producer's payoff due to LTC sales.

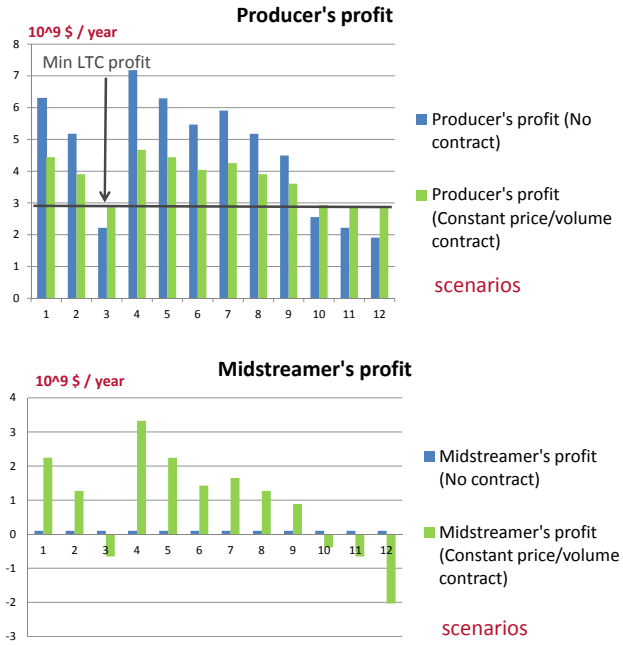

Figure 4:
The actors' payoff. Case -1 (100% spot) and Case 0 (spot market + constant price and volume LTC exchanges).

Both the producer and mid-streamer's payoffs fluctuate in Case 0. This is due to the fact that the fluctuation of the downstream price leads to an uncertain payoff because the LTC price (which is a cost for the mid-streamer) is assumed constant. Furthermore, comparing with Case -1, we notice that the contract reduces the fluctuations of the producer's payoff. In order to study the importance of the contract in the supply mix, we define the contract share parameter by:

$$CR = \langle \frac{up^1}{z_\omega} \rangle_\omega \tag{49}$$

where $\langle\rangle_\omega$ is a notation for the expected value. In other words, CR is the expected share of the contract in the total supply (from the mid-streamer's point of view). We find $CR = 85\%$ or an optimal contract volume of 35 Bcm, which is in line with what has been actually contracted in France in the previous years [6]. The contract price is 5.4 \$/Mbtu.

The following array summarizes the results of Case 0:

Risk (B\$)	Producer	Mid-streamer
	-3.4	-0.15

CR	85%

Comparing with the pure spot market case we observe that the producer and mid-streamer enter a contract with the result that they both increase their risk adjusted payoff. In order to do so the mid-streamer is willing to accept the risk of a large negative payoff in several scenario nodes (like in scenario node 12)
.

5.3 Spot market, constant price and volume contract and oil price-indexed contract (Case 1)

This case allows for oil price (π_ω) indexed contracts. The relevance of those contracts was undisputed before restructuring but it has been seriously questioned during the gas bubble and remains an open subject today. The important factor is the correlation between the oil price and the gas market demand. Since the downstream and spot markets inverse-demand functions are already perfectly correlated in our basic set up, we will mainly examine the correlation between the oil and gas spot prices, which is at the core of the debate on price indexation clauses. Let $Corr$[10] be the correlation parameter defined by:

$$Corr = 1 - \frac{1}{\text{Card}(\Omega)} \sum_\omega \frac{(\pi_\omega - \alpha_\omega)^2}{\langle\pi\rangle \ \langle\alpha\rangle} \tag{50}$$

where Card(Ω) is the number of elements of the scenario set Ω. A high correlation corresponds to a high value of $Corr$ (around 1) and vice-versa ($\leq 50\%$).

5.3.1 Oil is highly correlated with gas (Case 1.a)

We first assume $Corr \approx 1$ and allow for both oil indexed and constant price contracts. Figure 4 compares the producer and the mid-streamer's payoffs in all possible market scenarios $\{1, 2...12\}$ for Case -1 (spot market only) and Case 1.a. (spot + constant price and volume contract + oil-indexed contract, high correlation). We also report the producer's payoff due to LTC sales.

[10] We actually calculate the correlation between the oil price and the intercept of the spot market inverse-demand function, which is a proxy of the gas spot price.

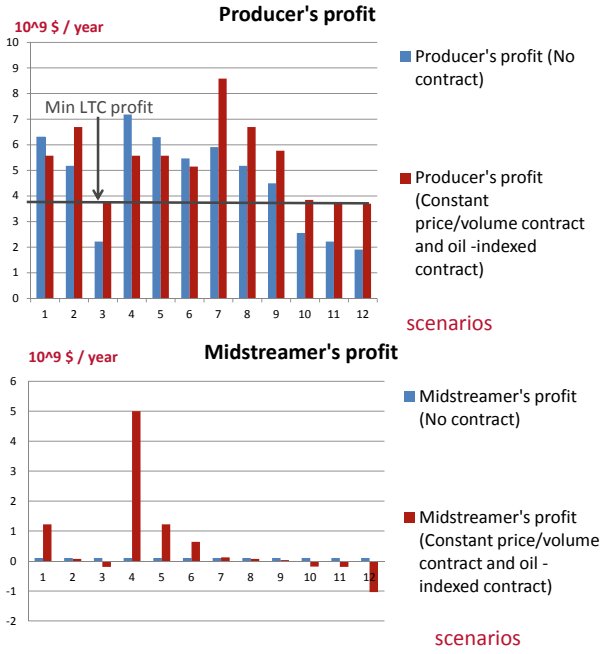

Figure 5:
The actors' payoff. Case -1 (100% spot) and Case 1.a. (spot market + constant price and volume + oil-indexed LTC exchanges). High correlation between oil and gas prices

Both the producer's and mid-streamer's payoffs fluctuate. This is due to the uncertainty of the oil price (which leads to a fluctuation of the oil-indexed contract) and the downstream price. Comparing to Case -1, the contracts reduce the fluctuations of the producer's payoff but increase the fluctuations of the mid-streamer's.
We find a $CR = 85\%$ ($up^1 + up^2 = 35$ Bcm) and calculate the respective shares of the oil-indexed and constant price contracts in Figure 6 (endogenously determined).

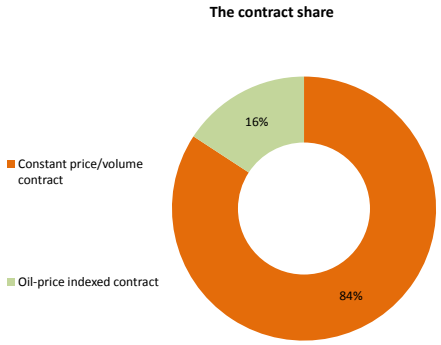

Figure 6:
The contract mix, Case 1.a. Spot market + constant price and volume + oil-indexed LTC exchanges. High correlation between oil and gas prices

The following array summarizes the results of Case 1.a:

Risk (B$)	Producer	Mid-streamer
	-4.2	-0.6

CR	85%
Oil indexation share in the contract	16%

We find that, in comparison with Case 0 with the sole forward contract, the introduction of the oil-indexed contract increases the risk adjusted payoffs of both the producer and mid-streamer. This indicates that the oil-priced contract absorbs a part of the risk in the market, due to its high correlation with the spot and downstream markets.

5.3.2 Oil is weakly correlated with gas (Case 1.b)

We now suppose $Corr = 0.5$ and redo the analysis with both the oil indexed and constant price contracts. Figure 7 shows the producer and the mid-streamer's payoff in all the possible market scenarios $\{1, 2...12\}$ for Case -1 (spot market only) and Case 1.b. (spot + constant price and volume contract + oil-indexed contract, low correlation). We also report the producer's payoff due to LTC sales.

Both the producer and mid-streamer's payoffs fluctuate in Case 1.b. This is due to the uncertainty of the oil price (which leads to a fluctuation of the oil-indexed contract) and

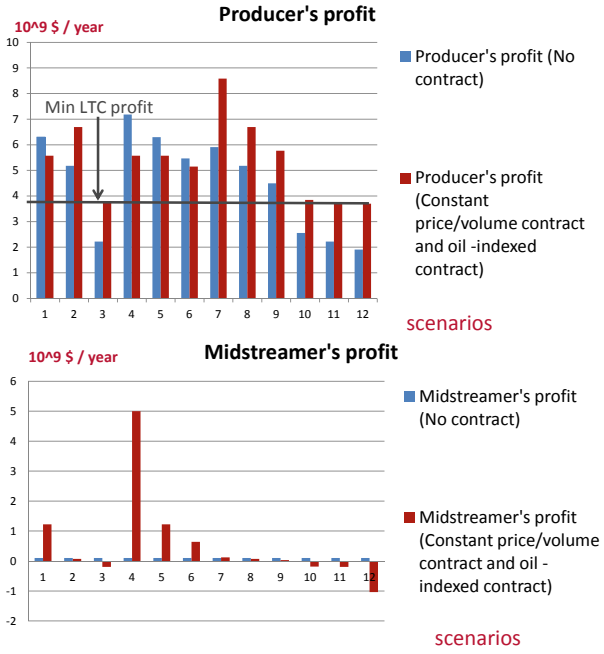

Figure 7:
The actors' payoff. Case -1 (100% spot) and Case 1.b. (spot market + constant price and volume + oil-indexed LTC exchanges). Low correlation between oil and gas prices

the downstream price. Comparing to Case -1, contracting reduces the fluctuations of the producer's payoff.
We again estimate $CR = 85\%$ (the total contracted volume is 35 Bcm) and calculate the share of the oil-indexed contract in the total contract mix. Figure 8 shows how the total contracted volume is allocated between the oil-indexed contract and the constant price and volume contract but that the share of the former is reduced: the forward contract is now much more effective, which is indeed what one should expect from the lower correlation between oil price and gas demand.

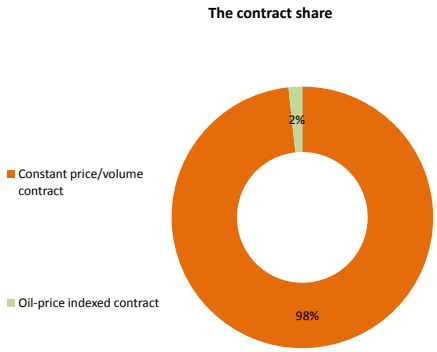

Figure 8:
The contract mix, Case 1.b. Spot market + constant price and volume + oil-indexed LTC exchanges. Low correlation between oil and gas prices

The following array summarizes the results of Case 1.b:

Risk (B$)	Producer	Mid-streamer
	-3.4	-0.15

CR	85%
Oil indexation share in the contract	2%

As expected from what has just been said, the comparison with Case 0 with the sole forward contract reveals that the oil-indexed contract does not improve the risk adjusted payoff of the players. This is reflected in the small 2% share of the oil indexed gas contract volume in the portfolio.

5.4 Endogenizing investment decisions

5.4.1 The link between investment and contracting decisions in gas markets

This last section explores the need for a producer to contract when confronted with capacity expansion decisions. A standard argument of the gas industry is that LTCs trigger upstream investments because they shield the producer from the threat of hold-up. [22]. We here discuss the other standard argument namely the capability of the contract to mitigate the revenue risk, inherent to the producer's will of covering its cost, accruing from downstream

uncertainties. We thus use our model in order to study the extent to which an investment decision might influence the producer's contracting behavior.

We consequently amend our model in order to introduce an investment decision inv. The production capacity is therefore bounded by inv in each time period and in each scenario. Like the contracting decisions, the investment inv can be considered as a first stage variable independent of the scenarios. We assume that the total upstream cost $cost$ is splitted between capital c_{in} and operations c_p costs as follows:

$$\begin{array}{lll} c_p = & \epsilon & cost \\ c_{in} = (1- & \epsilon) & cost \end{array}$$

where ϵ is a parameter such as $\epsilon \in [0, 1]$. $\epsilon = 0$ describes an industry where the production capital cost represents most of the upstream costs. This mimics the conventional gas production in Europe for instance, where operations costs might be negligible with respect to the initial investment cost. On the contrary, $\epsilon = 1$ occurs when the operational cost represents most of the production cost. This situation describes the current unconventional shale gas production in the US for instance, where the hydraulic fracking technique imposes a constant liquid injection to progressively free gas from the shale rocks in order to maintain a constant production rate.

The producer's optimization program is now modified as follows:

$$\begin{array}{ll} \text{Max} & \lambda^p \sum_\omega \theta_\omega p_\omega x_\omega \\ & +\lambda^p \sum_\omega \theta_\omega \left(\pi^1 up^1\right) \\ & -\lambda^p \sum_\omega \theta_\omega c_p(x_\omega + up^1) \\ & -\lambda^p \sum_\omega \theta_\omega c_{in}(inv) \\ & -(1-\lambda^p)\rho^p \\ \text{s.t.} & \forall \omega,\ 0 \leq x_\omega,\ up^1,\ inv \\ & \forall \omega,\ x_\omega + up^1 \leq inv \qquad (\upsilon_\omega) \end{array}$$

where

- The new term: $c_{in}(inv)$ is the investment cost.
- The constraint $\forall \omega,\ x_\omega + up^1 \leq inv$ is the production capacity constraint whose shadow cost is the dual variable υ_ω.

The following results are obtained from our standard data set (see Section 4).
The following figure gives the evolution of the contract volume up^1 over the parameter ϵ. To better understand the results, we concentrate on the sole forward contract formula up^1 in this study.

One observes that the contracted volume decreases with ϵ (-12% from $\epsilon = 0$ to $\epsilon = 1$). This indicates that the more the CAPEX dominates the upstream cost, the more there is an incentive to sign contracts in the gas trade. This result mimics the strategic behavior of European conventional gas producers who have shown a strong willingness to engage in long-term agreements before starting drilling. On the contrary, a producer which mainly faces OPEX costs is less likely to engage in long-term contracts. This is what has indeed been observed in the US shale gas revolution, where contract exchanges have shrinked in the previous decade or so: currently, they account for less than 30% of the gas trade, the rest being exchanged in the spot markets.

5.4.2 The impact of the mid-streamer's risk aversion on the consumer surplus in presence of investment decisions

As explained in the introduction, the recent gas bubble in Europe caused a net loss for most European mid-streamers, which triggered the discussion on spot-indexation ([41] and [42]). However, as seen in Section 5.2 and 5.3, despite the improvement of the risk adjusted

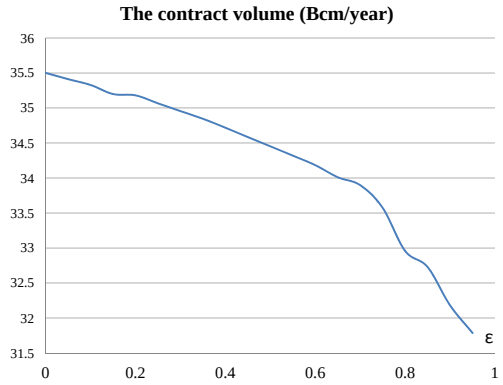

Figure 9:
The contract volume up^1 *over* ϵ

payoff of the mid-streamer, the contracts might also cause net losses for the mid-streamer (particularly in scenario nodes where the downstream demand is low such as 10, 11 or 12), which is the situation that mid streamers complained about during the recent bubble. This is mainly due to the fact that the forward contract price is a risk-adjusted expectation of the spot prices (theorem 4) but not an option that could buy out very unfavorable outcomes. A very risk averse mid-streamer with the sole opportunity to conclude forward type contracts would then reduce its contract volume in order to mitigate these losses. Such a reaction is actually currently being considered by some European mid-streamers. However, we have also shown before that in a capacity expansion decision context, a contract volume reduction would cause an upstream under-investment, that can lead to a strong increase of gas prices when the downstream demand is too high. As a consequence, one expects the consumer surplus to decrease if the mid-streamer becomes too risk-averse. The aim of this section is to show this effect.

In this study, we assume that the producer faces a non-zero capital cost: $\epsilon = 0.4$. Its risk aversion parameter is set to $A^p = 1.07$. Figure 10 shows the evolution of the contracted volume up^1 (formula 1), the invested capacity inv, the downstream gas price in node "2" $p\prime(2)$[11], as well as the average consumer surplus $\langle CS \rangle$, with respect to the mid-streamer's risk aversion parameter A^t. The consumer surplus CS at node ω is defined by:

$$\forall \omega \; CS_\omega = \int_0^{z_\omega} \left(p\prime_\omega(t) - p\prime_\omega(z_\omega) \right) dt$$

We recall that z_ω is the downstream consumption. In the case of a linear inverse-demand curve, the consumer surplus will be expressed by:

$$CS_\omega = \frac{1}{2} b_\omega z_\omega^2$$

and its average is[12]:

[11] We are interested in that price because node 2 is a medium scenario where the possible impact of under-investment is not extreme.

[12] Here, the average is calculated using the real probabilities θ_ω because the consumers are assumed to be risk-neutral in our model.

$$\langle CS \rangle = \frac{1}{2} \sum_{\omega} \theta_{\omega} b_{\omega} z_{\omega}^2$$

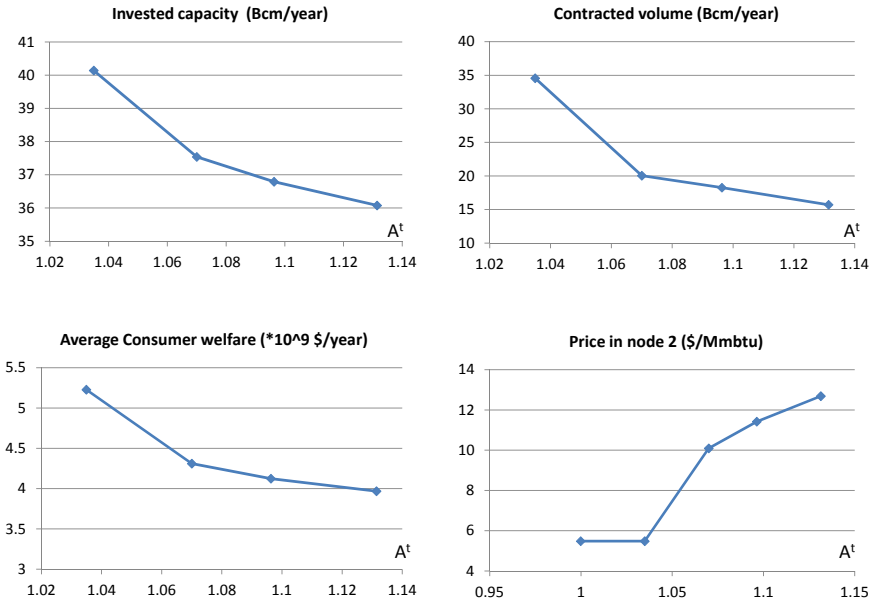

Figure 10:
The contract volume up[1]*, the investment inv, the downstream gas price in node "2"* $p\prime(2)$ *and the average consumer surplus* $\langle CS \rangle$ *over* A^t

The contract volume decreases with the mid-streamer's risk aversion. This is quite intuitive because the more risk-averse mid-streamer will only tolerate few losses. As a consequence, the producer invests less because contracts are not there to hedge its investment risk (see Section 5.4.1). Under-investment leads to high prices, as is also observed, and hence to a reduction of social welfare.

The conclusion of this section is straightforward. A very cautious mid-streamer limits its losses but deprives the consumer's welfare. Indeed, a very risk-averse mid-streamer prefers not taking the risk of high losses in the low demand scenarios. It will then favor the short-term spot purchase to satisfy the downstream demand. But the resulting lack of contract will also induce the producer to reduce its upstream investment, leading to a reduction of consumer surplus.

6 Conclusion

Oil price-indexed long-term contracts signed between producers and mid-streamers still dominate the gas supply activity in Europe. It is commonly argued that their objective is to share the gas market risk between the producer and the mid-streamer, in such a way that the producer takes the price risk and the mid-streamer the volume risk. Nevertheless, the growing importance of spot exchanges and the emergence of a gas bubble in Europe triggered the questioning of the relevance of these contracts in the gas trade.

The objective of this study is to model the optimal contracting behavior between the producer and the mid-streamer. The market risk is measured by to the good-deal risk function because it satisfies standard mathematical and useful economic properties. Both the producer and mid-streamer's incentive to contract are captured thanks to an equilibrium model, that has been solved in its complementarity form. In order to take into consideration the possible evolution of the LTC context in Europe (the spot indexation and renegotiation possibilities), two contract formulas are represented: the constant price and volume contract endogenously calculated (forward contract) and the oil price-indexed contract.

- Without any oil price indexation, our results suggest that signing contracts can benefit both the producers and mid-streamer because it reduces the market risk for both players and the fluctuation of the producer's payoff.
- Introducing the oil-indexation possibility may reduce even more the risk since part of that risk is taken care of by the oil price's fluctuation. However this risk reduction is sensitive to the correlation between the oil and gas spot prices. When the correlation is high, the optimal oil-indexed share of the contract is big and the risk reduction is important. On the contrary, if the correlation is low, no oil-indexed contract should be signed because such a contract does not reduce any risk in the gas market.
- A CAPEX driven production cost structure is more favorable to the signing of long-term contracts than an OPEX driven structure. Therefore, this result highlights the fact that the production of conventional gas, such as in Europe, is more likely to generate long-term contracts in the market than the production of unconventional gas, such as in the US.
- The more risk-averse the mid-streamer becomes, the fewer contracts are signed and fewer upstream investments are carried out. Thus, the consumer surplus might suffer from a lack of contracts in the market.

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7 Appendix 1

In this appendix, we write our equilibrium model in its complementarity form. For that purpose, we will calculate the KKT conditions of both the producer and mid-streamer that define the Nash equilibrium.

The producer's risk function is:

$$\begin{aligned}\rho^p = \quad & \text{Min} \quad w_0^p c1_0 + A^p \sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta p_\omega^2} \\ & \eta p_\omega \geq 0,\ w_\omega^p \\ & \text{s.t.} \quad \forall \omega, \quad \left(\eta p_\omega + Z_\omega^p + c1_\omega (w_{\omega_f}^p - w_\omega^p) - A^p \sqrt{\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_{\omega\prime}}{\theta_\omega} \eta p_{\omega\prime}^2} \right) \geq 0 \quad (\nu p_\omega)\end{aligned}$$

where the producer's payoff is:

$$Z_\omega^p = p_\omega x_\omega + \pi^1 u p^1 - c(x_\omega + u p^1)$$

The mid-streamer's risk function is:

$$\begin{aligned}\rho^t = \quad & \text{Min} \quad w_0^t c1_0 + A^t \sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta t_\omega^2} \\ & \eta t_\omega \geq 0,\ w_\omega^t \\ & \text{s.t.} \quad \forall \omega, \quad \left(\eta t_\omega + Z_\omega^t + c1_\omega (w_{\omega_f}^t - w_\omega^t) - A^t \sqrt{\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_{\omega\prime}}{\theta_\omega} \eta t_{\omega\prime}^2} \right) \geq 0 \quad (\nu t_\omega)\end{aligned}$$

where the mid-streamer's payoff is:

$$Z_\omega^t = p_\omega h_\omega + p\prime_\omega z_\omega - \pi^1 u t^1$$

In the following equations, we use the convention that the scenario nodes are numbered from 0 to m, where 0 is the summit of our scenario tree. We will denote by Ω_l the subset of Ω constituted by the leaves of the tree (i.e. the nodes that do not have children nodes). We recall that ω_f denotes node ω's father in the tree.

We first rewrite the objectives of the producer and the mid-streamer as in expressions (20) and (24).
The producer's maximization program:

$$\begin{aligned}&\text{Max} \\ &\lambda^p \sum_\omega \theta_\omega p_\omega x_\omega \\ &+ \lambda^p \sum_\omega \theta_\omega \left(\pi^1 u p^1\right) \\ &- \lambda^p \sum_\omega \theta_\omega c(x_\omega + u p^1) \\ &- (1-\lambda^p) \left(w_0^p c1_0 + A^p \sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta p_\omega^2} \right) \\ &\text{s.t.} \\ &\forall \omega,\ \eta p_\omega + Z_\omega^p + c1_\omega (w_{\omega_f}^p - w_\omega^p) - A^p \sqrt{\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_{\omega\prime}}{\theta_\omega} \eta p_{\omega\prime}^2} \geq 0 \qquad (\nu p_\omega) \\ &\forall \omega,\ x_\omega \text{ free },\ 0 \leq u p^1,\ 0 \leq \eta p_\omega,\ \text{free } w_\omega^p\end{aligned}$$

The mid-streamer's maximization program:

Max

$$\lambda^t \sum_{\omega} \theta_\omega p_\omega h_\omega$$
$$+ \lambda^t \sum_{\omega} \theta_\omega p\prime_\omega z_\omega$$
$$- \lambda^t \sum_{\omega} \theta_\omega \pi^1 u t^1$$
$$- (1-\lambda^t) \left(w_0^t c1_0 + A^t \sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta t_\omega^2} \right)$$

s.t.

$$\forall \omega,\ z_\omega + h_\omega - u t^1 = 0 \tag{51}$$

$$\forall \omega,\ \eta t_\omega + Z_\omega^t + c1_\omega (w_{\omega_f}^t - w_\omega^t) - A^t \sqrt{\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_{\omega\prime}}{\theta_\omega} \eta t_{\omega\prime}^2} \geq 0 \qquad (\nu t_\omega)$$

$$\forall \omega,\ 0 \leq u t^1,\ z_\omega,\ \text{free } h_\omega,\ w_\omega^t$$

The producer's KKT conditions:

$$\forall\omega \qquad \text{free } x_\omega \qquad \perp \quad \left(p_\omega - c\prime\left(x_\omega + up^1\right)\right)\left(\lambda^p\theta_\omega - \nu p_\omega\right) \qquad = 0 \quad (52a)$$

$$0 \le up^1 \qquad \perp \quad \pi^1 \sum_\omega \left(\lambda^p\theta_\omega - \nu p_\omega\right) - \sum_\omega c\prime\left(x_\omega + up^1\right)\left(\lambda^p\theta_\omega - \nu p_\omega\right) \qquad \le 0 \quad (52b)$$

$$\text{free } w_0^p \qquad \perp \quad -\left(1-\lambda^p\right)c1_0 - \sum_{\omega/\omega_f=0} \nu p_\omega c1_0 \qquad = 0 \quad (52c)$$

$$\forall\omega \notin \Omega_l \qquad \text{free } w_\omega^p \qquad \perp \quad \nu p_\omega c1_\omega - \sum_{\omega\prime/\omega\prime_f=\omega} \nu p_{\omega\prime} c1_{\omega\prime} \qquad = 0 \quad (52d)$$

$$\forall\omega/\omega_f = 0 \qquad 0 \le \eta p_\omega \qquad \perp \quad -\nu p_\omega - \left(1-\lambda^p\right)A^p\theta_\omega \frac{\eta p_\omega}{\sqrt{\sum_{\omega\prime/\omega\prime_f=0} \theta_{\omega\prime}\eta p_{\omega\prime}^2}} \qquad \le 0 \quad (52e)$$

$$\forall\omega/\omega_f > 0 \qquad 0 \le \eta p_\omega \qquad \perp \quad -\nu p_\omega + A^p \nu p_{\omega_f} \frac{\frac{\theta_\omega}{\theta_{\omega_f}}\eta p_\omega}{\sqrt{\sum_{\omega\prime/\omega\prime_f=\omega_f} \frac{\theta_{\omega\prime}}{\theta_{\omega_f}}\eta p_{\omega\prime}^2}} \qquad \le 0 \quad (52f)$$

$$\forall\omega \qquad 0 \ge \nu p_\omega \qquad \perp \quad \eta p_\omega + \left(p_\omega x_\omega + \pi^1 up^1 - c(x_\omega + up^1)\right) + c1_\omega\left(w_{\omega_f}^p - w_\omega^p\right) - A^p\sqrt{\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_{\omega\prime}}{\theta_\omega}\eta p_{\omega\prime}^2} \qquad \ge 0 \quad (52g)$$

The mid-streamer's KKT conditions:

$$\forall\omega \qquad \text{free } h_\omega \qquad \perp \quad p_\omega . \left(\lambda^t \theta_\omega - \nu t_\omega\right) + \mu_\omega \qquad = 0 \tag{53a}$$

$$\forall\omega \qquad 0 \leq z_\omega \qquad \perp \quad p\prime_\omega . \left(\lambda^t \theta_\omega - \nu t_\omega\right) + \mu_\omega \qquad \leq 0 \tag{53b}$$

$$0 \leq ut^1 \qquad \perp \quad -\pi^1 \sum_\omega \left(\lambda^t \theta_\omega - \nu t_\omega\right) - \sum_\omega \mu_\omega \qquad \leq 0 \tag{53c}$$

$$\text{free } w_0^t \qquad \perp \quad -\left(1-\lambda^t\right) c1_0 - \sum_{\omega/\omega_f=0} \nu t_\omega c1_0 \qquad = 0 \tag{53d}$$

$$\forall\omega \notin \Omega_l \qquad \text{free } w_\omega^t \qquad \perp \quad \nu t_\omega c1\omega - \sum_{\omega\prime/\omega\prime_f=\omega} \nu t_{\omega\prime} c1_{\omega\prime} \qquad = 0 \tag{53e}$$

$$\forall\omega \ / \ \omega_f = 0 \qquad 0 \leq \eta t_\omega \qquad \perp \quad -\nu t_\omega - \left(1-\lambda^t\right) A^t \theta_\omega \frac{\eta t_\omega}{\sqrt{\sum_{\omega\prime/\omega\prime_f=0} \theta_{\omega\prime} \eta t_{\omega\prime}^2}} \qquad \leq 0 \tag{53f}$$

$$\forall\omega \ / \ \omega_f > 0 \qquad 0 \leq \eta t_\omega \qquad \perp \quad -\nu t_\omega + A^t \nu t_{\omega_f} \frac{\frac{\theta_\omega}{\theta_{\omega_f}} \eta t_\omega}{\sqrt{\sum_{\omega\prime/\omega\prime_f=\omega_f} \frac{\theta_{\omega\prime}}{\theta_{\omega_f}} \eta t_{\omega\prime}^2}} \qquad \leq 0 \tag{53g}$$

$$\forall\omega \qquad \text{free } \mu_\omega \qquad \perp \quad z_\omega + h_\omega - ut^1 \qquad = 0 \tag{53h}$$

$$\forall\omega \qquad 0 \geq \nu t_\omega \qquad \perp \quad \eta t_\omega + \qquad \geq 0 \tag{53i}$$

$$\left(p_\omega h_\omega + p\prime_\omega z_\omega - \pi^1 ut^1\right)$$

$$+ c1_\omega \left(w_{\omega_f}^t - w_\omega^t\right) - A^t \sqrt{\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_{\omega\prime}}{\theta_\omega} \eta t_{\omega\prime}^2}$$

The spot, downstream and contract markets clearing conditions are:

$$\forall\omega \qquad p_\omega - (\alpha_\omega - \beta_\omega (x_\omega + h_\omega)) \qquad = 0 \tag{54a}$$

$$\forall\omega \qquad p\prime_\omega - (a_\omega - b_\omega z_\omega) \qquad = 0 \tag{54b}$$

$$\text{free } \pi^1 \qquad \perp \quad up^1 - ut^1 \qquad = 0 \tag{54c}$$

8 Appendix 2

This appendix gives the proofs our out lemmas and theorems:

8.1 Proof of theorem 1

Proof. The primal formulation of the good-deal is as follows:

$$\begin{array}{rll} \rho = & \underset{\zeta_\omega}{\text{Max}} & -\sum_\omega \theta_\omega Z_\omega \left(\Pi_{\omega\prime\leq\omega}\zeta_{\omega\prime}\right) \\ & \text{s.t.} & \forall\omega,\ \zeta_\omega \geq 0 \\ & & \forall\omega,\ \displaystyle\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_{\omega\prime}}{\theta_\omega}\zeta_{\omega\prime}c1_{\omega\prime} = c1_\omega \\ & & \forall\omega,\ \displaystyle\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_{\omega\prime}}{\theta_\omega}\zeta_{\omega\prime}c2_{\omega\prime} = c2_\omega \\ & & \forall\omega,\ \displaystyle\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_{\omega\prime}}{\theta_\omega}\zeta^2_{\omega\prime} \leq A^2 \end{array} \tag{55}$$

First we get rid of the nested formulation by changing variables and introducing ξ_ω such that:

$$\forall\omega\in\Omega,\ \xi_\omega = \theta_\omega\Pi_{\omega\prime\leq\omega}\zeta_{\omega\prime}$$

We obtain:

$$\begin{array}{rll} \rho = & \underset{\xi_\omega}{\text{Max}} & -\sum_\omega Z_\omega\xi_\omega \\ & \text{s.t.} & \forall\omega,\ \xi_\omega \geq 0 \\ & & \forall\omega,\ \displaystyle\sum_{\omega\prime/\omega\prime_f=\omega} \xi_{\omega\prime}c1_{\omega\prime} = \xi_\omega c1_\omega \\ & & \forall\omega,\ \displaystyle\sum_{\omega\prime/\omega\prime_f=\omega} \xi_{\omega\prime}c2_{\omega\prime} = \xi_\omega c2_\omega \\ & & \forall\omega,\ \displaystyle\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_\omega}{\theta_{\omega\prime}}\xi^2_{\omega\prime} - A^2\xi^2_\omega \leq 0 \end{array} \tag{56}$$

Which is equivalent to (duals are written between parenthesis):

$$\begin{array}{lll} \rho = & \text{Max} & -\sum_\omega Z_\omega \xi_\omega \\ & \xi_\omega & \\ & \text{s.t.} & \forall \omega,\ \xi_\omega \geq 0 \\ & & \forall \omega,\ \displaystyle\sum_{\omega\prime/\omega\prime_f=\omega} \xi_{\omega\prime} c1_{\omega\prime} = \xi_\omega c1_\omega \qquad (w1_\omega) \\ & & \forall \omega,\ \displaystyle\sum_{\omega\prime/\omega\prime_f=\omega} \xi_{\omega\prime} c2_{\omega\prime} = \xi_\omega c2_\omega \qquad (w2_\omega) \\ & & \forall \omega,\ \sqrt{\displaystyle\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_\omega}{\theta_{\omega\prime}} \xi^2_{\omega\prime}} - A\xi_\omega \leq 0 \qquad (\phi_\omega) \end{array} \tag{57}$$

It is easy to demonstrate that the objective function is concave, that the feasibility set is convex and that standard constraints qualifications hold. The dual and primal problems are therefore equivalent.
The dual problem is written below:

$$\begin{array}{lll} \rho = & \text{Inf} & \text{Sup} \qquad \mathcal{L}(\xi, w1, w2, \phi) \\ & w1_\omega \text{ free} & \xi_\omega \geq 0 \\ & w2_\omega \text{ free} & \\ & \phi_\omega \geq 0 & \end{array} \tag{58}$$

where the Lagrangian function is defined as follows:

$$\mathcal{L}: \quad \mathbb{R}^m \times \mathbb{R}^m \times \mathbb{R}^m \times \mathbb{R}^m \quad \longrightarrow \quad \mathbb{R}$$

$$\mathcal{L}(\xi, w1, w2, \phi) = -\sum_\omega Z_\omega \xi_\omega + \sum_{i=1,2,\ \omega} wi_\omega \left(\xi_\omega ci_\omega - \sum_{\omega\prime/\omega\prime_f=\omega} \xi_{\omega\prime} ci_{\omega\prime} \right) - \sum_\omega \phi_\omega \left(\sqrt{\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_\omega}{\theta_{\omega\prime}} \xi^2_{\omega\prime}} - A\xi_\omega \right) \tag{59}$$

Let us solve the right part of the optimization program (58):

$$\begin{array}{ll} \text{Sup} & \mathcal{L}(\xi, w1, w2, \phi) \\ \xi_\omega \geq 0 & \end{array} \tag{60}$$

The solution of problem (60) satisfies (assuming that $\forall \omega \in \Omega,\ \phi_\omega > 0$):

$$\forall \omega \in \Omega,\ \xi*_\omega = \frac{\left(-Z_\omega + \sum_i ci_\omega (wi_\omega - wi_{\omega_f}) + A\phi_\omega\right)^+}{\phi_{\omega_f}} \frac{\theta_\omega}{\theta_{\omega_f}} \sqrt{\sum_{\omega\prime/\omega\prime_f=\omega_f} \frac{\theta_{\omega_f}}{\theta_{\omega\prime}} \xi*^2_{\omega\prime}} \tag{61}$$

where x^+ designates the positive part of x: $x^+ = \max(x, 0)$. By putting expression (61) to the square and summing over all the $\omega\prime$ such that $\omega\prime$ and ω have the same ancestor, we obtain:

$$\forall \omega \in \Omega,\ \phi^2_\omega = \sum_{\omega\prime/\omega\prime_f=\omega} \left(\left(-Z_{\omega\prime} + \sum_i ci_{\omega\prime} (wi_{\omega\prime} - wi_\omega) + A\phi_{\omega\prime} \right)^+ \right)^2 \frac{\theta_{\omega\prime}}{\theta_\omega} \tag{62}$$

We can calculate the value of $\mathcal{L}$ at the optimum when $\xi = \xi*$

$$\mathcal{L}(\xi*, w1, w2, \phi) =$$
$$-\sum_{\omega} Z_\omega \xi *_\omega + \sum_{i=1,2,\ \omega} wi_\omega \left(\xi *_\omega ci_\omega - \sum_{\omega\prime/\omega\prime_f=\omega} \xi *_{\omega\prime} ci_{\omega\prime} \right) - \sum_{\omega} \phi_\omega \left(\sqrt{\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_\omega}{\theta_{\omega\prime}} \xi *^2_{\omega\prime}} - A\xi *_\omega \right) =$$
$$\sum_i wi_0 ci_0 + \phi_0 A + \sum_{\omega\neq 0} \left(-Z_\omega + \sum_{i=1,2} (wi_\omega ci_\omega - wi_{\omega_f} ci_\omega) + A\phi_\omega \right) \xi *_\omega - \sum_{\omega} \phi_\omega \sqrt{\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_\omega}{\theta_{\omega\prime}} \xi *^2_{\omega\prime}}$$

and using (61) with the following property: $\forall x \in \mathbb{R},\ x.x^+ = (x^+)^2$, we get:

$$\mathcal{L}(\xi*, w1, w2, \phi) = \textstyle\sum_i wi_0 ci_0 + \phi_0 A +$$
$$\sum_{\omega\neq 0} \frac{\left(\left(-Z_\omega + \sum_{i=1,2} (wi_\omega ci_\omega - wi_{\omega_f} ci_\omega) + A\phi_\omega \right)^+ \right)^2}{\phi_\omega} \frac{\theta_\omega}{\theta_{\omega_f}} \sqrt{\sum_{\omega\prime/\omega\prime_f=\omega_f} \frac{\theta_{\omega_f}}{\theta_{\omega\prime}} \xi *^2_{\omega\prime}} - \sum_{\omega} \phi_\omega \sqrt{\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_\omega}{\theta_{\omega\prime}} \xi *^2_{\omega\prime}}$$

and

$$\mathcal{L}(\xi*, w1, w2, \phi) = \textstyle\sum_i wi_0 ci_0 + \phi_0 A +$$
$$\sum_{\omega\neq 0} \frac{\left(\left(-Z_\omega + \sum_{i=1,2} wi_\omega ci_\omega - wi_{\omega_f} ci_\omega + A\phi_\omega \right)^+ \right)^2}{\phi_\omega} \frac{\theta_\omega}{\theta_{\omega_f}} \sqrt{\sum_{\omega\prime/\omega\prime_f=\omega_f} \frac{\theta_{\omega_f}}{\theta_{\omega\prime}} \xi *^2_{\omega\prime}} - \sum_{\omega} \phi_\omega \sqrt{\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_\omega}{\theta_{\omega\prime}} \xi *^2_{\omega\prime}}$$

Now using expression (62) we can deduce that:

$$\mathcal{L}(\xi*, w1, w2, \phi) = \textstyle\sum_i wi_0 ci_0 + \phi_0 A +$$
$$\sum_{\omega} \frac{\phi^2_\omega}{\phi_\omega} \sqrt{\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_\omega}{\theta_{\omega\prime}} \xi *^2_{\omega\prime}} - \sum_{\omega} \phi_\omega \sqrt{\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_\omega}{\theta_{\omega\prime}} \xi *^2_{\omega\prime}}$$

And finally:

$$\mathcal{L}(\xi*, w1, w2, \phi) = \sum_i wi_0 ci_0 + \phi_0 A \tag{63}$$

Now, getting back to expression (58) and adding equation (62), we have:

$$\begin{aligned} \rho = \quad & \text{Inf } \textstyle\sum_i wi_0 ci_0 + \phi_0 A \\ & w1_\omega \text{ free} \\ & w2_\omega \text{ free} \\ & \phi_\omega \geq 0 \\ & \phi^2_\omega = \sum_{\omega\prime/\omega\prime_f=\omega} \left(\left(-Z_{\omega\prime} + \sum_i ci_{\omega\prime} (wi_{\omega\prime} - wi_\omega) + A\phi_{\omega\prime} \right)^+ \right)^2 \frac{\theta_{\omega\prime}}{\theta_\omega} \end{aligned} \tag{64}$$

Now, we introduce a new variable $\eta_\omega \geq 0,\ \omega \in \Omega$ such that:

$$\forall \omega \in \Omega\ \eta_\omega = \left(-Z_\omega + \sum_i ci_\omega (wi_\omega - wi_{\omega_f}) + A\phi_\omega \right)^+ \tag{65}$$

And we can rewrite the optimization program (64) as follows:

$$\begin{aligned}\rho = \quad & \text{Inf } \textstyle\sum_i wi_0 ci_0 + A\sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta_\omega^2} \\ & w1_\omega \text{ free} \\ & w2_\omega \text{ free} \\ & \eta_\omega \geq 0 \\ & \eta_\omega + Z_\omega + c1_\omega(w1_{\omega_f} - w1_\omega) + c2_\omega(w2_{\omega_f} - w2_\omega) - A\sqrt{\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_{\omega\prime}}{\theta_\omega}\eta_{\omega\prime}^2} \geq 0\end{aligned} \tag{66}$$

□

8.2 Proof of lemma 1

Proof. Let us consider $x = (x_1, ..., x_n)$ and $y = (y_1, ..., y_n) \in \mathbb{R}^n$ and $s \in [0, 1]$. We have:

$$\begin{aligned} f(sx + (1-s)y) &= \sqrt{\sum_i \delta_i \left(sx_i + (1-s)y_i\right)^2} \\ &= \sqrt{\sum_i \delta_i \left(s^2x_i^2 + (1-s)^2y_i^2 + 2s(1-s)x_iy_i\right)} \end{aligned}$$

Using the Cauchy Schwarz inequality, we have:

$$\sum_i \delta_i x_i y_i \leq \sqrt{\sum_i \delta_i x_i^2}\sqrt{\sum_i \delta_i y_i^2}$$

Hence

$$\begin{aligned} & f(sx + (1-s)y) \\ & \leq \sqrt{s^2\sum_i \delta_i x_i^2 + (1-s)^2\sum_i \delta_i y_i^2 + 2s(1-s)\sqrt{\sum_i \delta_i x_i^2}\sqrt{\sum_i \delta_i y_i^2}} \\ & = \sqrt{\left(s\sqrt{\sum_i \delta_i x_i^2} + (1-s)\sqrt{\sum_i \delta_i y_i^2}\right)^2} \\ & = sf(x) + (1-s)f(y) \end{aligned}$$

□

8.3 Proof of theorem 2

Proof. This is a straightforward consequence of Lemma 1. □

8.4 Proof of theorem 3

Proof. Using equations (see Apendix 1) (52b), (53a) and (53c), we can deduce that:

$$\frac{1}{T}\sum_\omega \left(\lambda^t\theta_\omega - \nu t_\omega\right) p_\omega \leq \pi^1 \leq \frac{1}{T}\sum_\omega \left(\lambda^p\theta_\omega - \nu p_\omega\right) c\prime(x_\omega + up^1) \tag{67}$$

We will demonstrate later on (see Proof of lemma 3) that $\lambda^t\theta_\omega - \nu t_\omega$ and $\lambda^p\theta_\omega - \nu p_\omega$ are probability measures, which implies that they are bounded. Thus, π^1 is bounded since the spot price p_ω and the marginal production cost $c\prime(x_\omega + up^1)$ are also bounded.

□

8.5 Proof of lemma 2

Proof. $\forall \omega, \eta p_\omega > 0$ and $\eta t_\omega > 0$ is straightforward from (32) and (33).
We already have $\forall \omega / \omega_f = 0, \nu p_\omega < 0$. Indeed, if $\exists \omega / \omega_f = 0, \nu p_\omega = 0$, then using the slackness condition of equation (52e), we would have $\eta p_\omega = 0$, which is impossible. Let us now assume that $\exists \omega / \omega_f \neq 0$ and $\nu p_\omega = 0$. If $\nu p_{\omega_f} < 0$, then using the slackness condition of equation (52f), we will have $\eta p_\omega = 0$, which is impossible. If $\nu p_{\omega_f} = 0$ we make the previous reasoning by replacing ω by ω_f.
$\forall \omega, \nu t_\omega < 0$ can be demonstrated in a similar way. □

8.6 Proof of lemma 3

Proof. We know that $\forall \omega,\ \nu p(t) < 0$, which means that $\forall \omega,\ \lambda^p \theta_\omega - \nu p_\omega \geq 0$.
We have

$$\sum_{\omega / \omega_f = 0} \lambda^p \theta_\omega - \nu p_\omega = \lambda^p - \sum_{\omega / \omega_f = 0} \nu p_\omega \tag{68}$$

Using equation (52c), we can deduce that $\sum_{\omega / \omega_f = 0} \lambda^p \theta_\omega - \nu p_\omega = 1$.
Now, using (52d), we can easily calculate:

$$\forall \omega, \sum_{\omega\prime / \omega\prime_f = \omega} \lambda^p \theta_{\omega\prime} - \nu p_{\omega\prime} = \lambda^p \theta_\omega - \nu p_\omega \tag{69}$$

The demonstration is similar for $\omega \longrightarrow \lambda^t \theta_\omega - \nu t_\omega$. □

8.7 Proof of theorem 4

Proof. The proof is straightforward, given equations (52b), (53a) and (53c):

$$up^1 = ut^1 > 0 \Rightarrow \pi^1 = \frac{1}{T} \sum_\omega \left(\lambda^p \theta_\omega - \nu p_\omega \right) p_\omega = \frac{1}{T} \sum_\omega \left(\lambda^t \theta_\omega - \nu t_\omega \right) p_\omega \tag{70}$$

□

8.8 Proof of theorem 5

Proof. The proof is straightforward, because of the concavity of the players' objective functions and the convexity of their feasibility sets. See for instance [17] and [13] for a clear presentation of the equivalence between complementarity problems and the VI formulation. □

8.9 Proof of theorem 6

Proof. $\forall \pi^1 \in [\underline{p}, \overline{p}]$, K is non-empty, convex and compact. F is continuous over K. The existence of a solution to the VI is a consequence of [36]'s theorem 1 (which applies one of Brouwer's fixed point theorem). □

8.10 Proof of lemma 4

Proof. Let λ be such that $\lambda = \lambda^p = \lambda^t$.
Let X and $X\prime$ be two vectors in K and let us calculate $\left(-^t F(X) +^t F(X\prime) \right) (X - X\prime)$.

$$
\begin{aligned}
&\left(-^t F(X) +^t F(X\prime)\right)(X - X\prime) = \\
&\lambda \textstyle\sum_\omega \theta_\omega \beta_\omega \left(x_\omega + h_\omega - x\prime_\omega - h\prime_\omega\right)\left(x_\omega - x\prime_\omega\right) + \\
&\lambda \textstyle\sum_\omega \theta_\omega \left(c(x_\omega + up^1) - c(x\prime_\omega + up\prime^1)\right)\left(x_\omega - x\prime_\omega\right) + \\
&\lambda \textstyle\sum_\omega \theta_\omega \left(c(x_\omega + up^1) - c(x\prime_\omega + up\prime^1)\right)\left(up^1 - up\prime^1\right) + \\
&(1-\lambda) A^p \textstyle\sum_{\omega/\omega_f=0} \theta_\omega \left(\frac{\eta p_\omega}{\sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta p_\omega^2}} - \frac{\eta p\prime_\omega}{\sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta p\prime_\omega^2}} \right)\left(\eta p_\omega - \eta p\prime_\omega\right) + \\
&\lambda \textstyle\sum_\omega \theta_\omega \beta_\omega \left(x_\omega + h_\omega - x\prime_\omega - h\prime_\omega\right)\left(h_\omega - h\prime_\omega\right) + \\
&\lambda \textstyle\sum_\omega \theta_\omega b_\omega \left(z_\omega - z\prime_\omega\right)^2 + \\
&(1-\lambda) A^t \textstyle\sum_{\omega/\omega_f=0} \theta_\omega \left(\frac{\eta t_\omega}{\sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta t_\omega^2}} - \frac{\eta t\prime_\omega}{\sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta t\prime_\omega^2}} \right)\left(\eta t_\omega - \eta t\prime_\omega\right) +
\end{aligned}
\tag{71}
$$

and after some algebra:

$$
\begin{aligned}
&\left(-^t F(X) +^t F(X\prime)\right)(X - X\prime) = \\
&\lambda \textstyle\sum_\omega \theta_\omega \beta_\omega \left(x_\omega + h_\omega - x\prime_\omega - h\prime_\omega\right)^2 + \\
&\lambda \textstyle\sum_\omega \theta_\omega \left(c(x_\omega + up^1) - c(x\prime_\omega + up\prime^1)\right)\left(x_\omega + up^1 - x\prime_\omega - up\prime^1\right) + \\
&(1-\lambda) A^p \frac{\left(\sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta p_\omega^2} + \sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta p\prime_\omega^2}\right)}{\sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta p_\omega^2}\sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta p\prime_\omega^2}} \left(\sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta p_\omega^2}\sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta p\prime_\omega^2} - \sum_{\omega/\omega_f=0} \theta_\omega \eta p_\omega \eta p\prime_\omega \right) + \\
&\lambda \textstyle\sum_\omega \theta_\omega b_\omega \left(z_\omega - z\prime_\omega\right)^2 + \\
&(1-\lambda) A^t \frac{\left(\sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta t_\omega^2} + \sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta t\prime_\omega^2}\right)}{\sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta t_\omega^2}\sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta t\prime_\omega^2}} \left(\sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta t_\omega^2}\sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta t\prime_\omega^2} - \sum_{\omega/\omega_f=0} \theta_\omega \eta t_\omega \eta t\prime_\omega \right)
\end{aligned}
\tag{72}
$$

Using again the Cauchy Schwarz inequality, we have:

$$
\sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta p_\omega^2}\sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta p\prime_\omega^2} - \sum_{\omega/\omega_f=0} \theta_\omega \eta p_\omega \eta p\prime_\omega \geq 0 \tag{73}
$$

and

$$
\sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta t_\omega^2}\sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta t\prime_\omega^2} - \sum_{\omega/\omega_f=0} \theta_\omega \eta t_\omega \eta t\prime_\omega \geq 0 \tag{74}
$$

Therefore:

$$
\forall X \text{ and } X\prime \in K,\ \left(-^t F(X) +^t F(X\prime)\right)(X - X\prime) \geq 0 \tag{75}
$$

□

8.11 Proof of lemma 5

Proof. In this proof, we will use the equations issued from the KKT conditions given in Appendix 1 and their numbers.

If $\lambda_p = \lambda_t$, it is easy to derive that the Jacobian matrix of the KKT conditions is symmetric. Therefore, the problem is integrable and is equivalent to maximizing over the convex set K the following concave and continuous function:

$$\begin{array}{rccl} W: & K & \longrightarrow & \mathbb{R} \\ & X & \longrightarrow & \lambda \sum_\omega \theta_\omega \left(\alpha_\omega (x_\omega + h_\omega) - \frac{\beta_\omega}{2}(x_\omega + h_\omega)^2 + a_\omega z_\omega - \frac{b_\omega}{2} z_\omega^2 \right) \\ & & & -\lambda \sum_\omega \theta_\omega c(x_\omega + up^1) \\ & & & +\lambda \sum_\omega \theta_\omega \pi^1 (up^1 - ut^1) \\ & & & -(1-\lambda) \left(w_0^p c1_0 + A^p \sqrt{\sum_{\omega/\omega_f = 0} \theta_\omega \eta p_\omega^2} + w_0^t c1_0 + A^t \sqrt{\sum_{\omega/\omega_f = 0} \theta_\omega \eta t_\omega^2} \right) \end{array}$$

This result has a simple interpretation: when the players give the same weight to the profit in their objectives, the equilibrium is reached when the risk adjusted social welfare is maximized. Indeed, the expression of W can be splitted into five components:

$$W = W_{spot} + W_{down} + W_{producer} + W_{mid-streamer} - Risk \tag{76}$$

where

$$W_{spot} = \sum_\omega \theta_\omega \left(\alpha_\omega (x_\omega + h_\omega) - \frac{\beta_\omega}{2}(x_\omega + h_\omega)^2 - p_\omega (x_\omega + h_\omega) \right)$$

is the expected spot market consumer surplus. The term

$$W_{down} = \sum_\omega \theta_\omega \left(a_\omega z_\omega - \frac{b_\omega}{2} z_\omega^2 - p\prime_\omega z_\omega \right)$$

is the downstream market expected consumer surplus. The term

$$W_{producer} = \sum_\omega \theta_\omega \left(\pi^1 up^1 + p_\omega x_\omega - c(x_\omega + up) \right)$$

is the producer's net profit. The term

$$W_{mid-streamer} = \sum_\omega \theta_\omega \left(-\pi^1 ut^1 + p_\omega h_\omega + p\prime_\omega z_\omega \right)$$

is the mid-streamer's net profit and the term

$$Risk = w_0^p c1_0 + A^p \sqrt{\sum_{\omega/\omega_f = 0} \theta_\omega \eta p_\omega^2} + w_0^t c1_0 + A^t \sqrt{\sum_{\omega/\omega_f = 0} \theta_\omega \eta t_\omega^2}$$

is the sum of the players' risks (under constraints (21) and (26)). □

8.12 Proof of theorem 7

Proof. In this proof, we will also use the equations issued from the KKT conditions given in Appendix 1 and their numbers.
If X and $X\prime \in K$ are two solutions of $VI(F, K)$, then using lemma 4 we have:

$$\left(-{}^t F(X) + {}^t F(X\prime) \right) (X - X\prime) = 0 \tag{77}$$

Using expression (72), the fact that $0 < \lambda < 1$ and the fact that function c is strictly increasing, we can deduce that:

$$\forall \omega,\ x_\omega + h_\omega = x\prime_\omega + h\prime_\omega \tag{78}$$

$$\forall \omega,\ x_\omega + up^1 = x\prime_\omega + up\prime^1 \tag{79}$$

$$\forall \omega,\ z_\omega = z\prime_\omega \tag{80}$$

$$\exists \gamma p_0 \in \mathbb{R},\ \text{such that } \forall \omega/\omega_f = 0,\ \eta p_\omega = \gamma p_0\ \eta p\prime_\omega \tag{81}$$

$$\exists \gamma t_0 \in \mathbb{R},\ \text{such that } \forall \omega/\omega_f = 0,\ \eta t_\omega = \gamma t_0\ \eta t\prime_\omega \tag{82}$$

Note that equations (81) and (82) are deduced from the equality case of the general Cauchy Schwarz inequality.
It is easy to demonstrate that proposition (81) is equivalent to:

$$\forall \omega/\omega_f = 0,\ \frac{\eta p_\omega}{\sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta p_\omega^2}} = \frac{\eta p\prime_\omega}{\sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta p\prime_\omega^2}} \tag{83}$$

and proposition (82) is equivalent to:

$$\forall \omega/\omega_f = 0,\ \frac{\eta t_\omega}{\sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta t_\omega^2}} = \frac{\eta t\prime_\omega}{\sqrt{\sum_{\omega/\omega_f=0} \theta_\omega \eta t\prime_\omega^2}} \tag{84}$$

Therefore, so far, we have demonstrated that if X and $X\prime$ are two equilibria, then $\exists \kappa, \gamma p_0$ and $\gamma t_0 \in \mathbb{R}$ such that:

$$\forall \omega,\ x\prime_\omega = x_\omega - \kappa \tag{85}$$
$$\forall \omega,\ h\prime_\omega = h_\omega + \kappa \tag{86}$$
$$\forall \omega,\ up\prime^1 = up^1 + \kappa \tag{87}$$
$$\forall \omega,\ ut\prime^1 = ut^1 + \kappa \tag{88}$$
$$\forall \omega,\ z_\omega = z\prime_\omega \tag{89}$$
$$\forall \omega/\omega_f = 0,\ \eta p_\omega = \gamma p_0\ \eta p\prime_\omega \tag{90}$$
$$\forall \omega/\omega_f = 0,\ \eta t_\omega = \gamma t_0\ \eta t\prime_\omega \tag{91}$$

We will demonstrate that $\kappa = 0$.
We already know that X and $X\prime$ maximize the continuous concave function W, which implies that any convex combination of X and $X\prime$ also maximizes W.

We will calculate $W((1-\xi)X + \xi X\prime)$ and we will consider it as a function of $\xi \in [0,1]$:

$$\forall \xi \in [0,1],\ (1-\xi)X + \xi X\prime \text{ maximizes } W$$

$$\begin{aligned}
&W((1-\xi)X + \xi X\prime) = \\
&\lambda \textstyle\sum_\omega \theta_\omega \left(\alpha_\omega(x_\omega + h_\omega) - \frac{\beta_\omega}{2}((x_\omega - \kappa\xi)^2 + (h_\omega + \kappa\xi)^2) - \beta_\omega(x_\omega - \kappa\xi)(h_\omega + \kappa\xi) + a_\omega z_\omega - \frac{b_\omega}{2} z_\omega^2\right) \\
&-\lambda \textstyle\sum_\omega \theta_\omega c(x_\omega + up^1) \\
&-(1-\lambda)\left((w_0^p + \xi(w^p\prime_0 - w_0^p))c1_0 + A^p \sqrt{\sum_{\omega/\omega_f=0} \theta_\omega(\eta p_\omega + \xi(\eta p\prime_\omega - \eta p_\omega))^2}\right) \\
&-(1-\lambda)\left((w_0^t + \xi(w^t\prime_0 - w_0^t))c1_0 + A^t \sqrt{\sum_{\omega/\omega_f=0} \theta_\omega(\eta t_\omega + \xi(\eta t\prime_\omega - \eta p_\omega))^2}\right)
\end{aligned}$$

And after some algebraic developments, we have:

$$\begin{aligned}
&W((1-\xi)X + \xi X\prime) = \\
&Constant(\text{with respect to } \xi) \\
&-(1-\lambda)\left(\xi(w^p\prime_0 - w_0^p)c1_0 + A^p \sqrt{\sum_{\omega/\omega_f=0} \theta_\omega(\eta p_\omega + \xi(\eta p\prime_\omega - \eta p_\omega))^2}\right) \\
&-(1-\lambda)\left(\xi(w^t\prime_0 - w_0^t)c1_0 + A^t \sqrt{\sum_{\omega/\omega_f=0} \theta_\omega(\eta t_\omega + \xi(\eta t\prime_\omega - \eta t_\omega))^2}\right)
\end{aligned}$$

which is supposed to be constant with respect to ξ. This is possible if and only if:

$$\forall \omega / \omega_f = 0,\ \eta p_\omega = \eta p\prime_\omega \tag{92}$$

and

$$\forall \omega / \omega_f = 0,\ \eta t_\omega = \eta t\prime_\omega \tag{93}$$

and

$$w^p\prime_0 + w^t\prime_0 = w_0^p + w_0^t \tag{94}$$

Besides, we must also have:

$$\forall \xi \in [0,1],\ (1-\xi)X + \xi X\prime \in K \tag{95}$$

and in particular (using constraints (21) and (26) that are binding thanks to lemma 2):

$\forall \xi \in [0,1],\ \forall \omega,$

$$\begin{array}{l}(1-\xi)\eta p_\omega + \xi \eta p\prime_\omega + (1-\xi) Z_\omega^p + \xi Z^p\prime_\omega \\ +c1_\omega((1-\xi)w_{\omega_f}^p + \xi w^p\prime_{\omega_f} - (1-\xi)w_\omega^p - \xi w^p\prime_\omega) \\ -A^p \sqrt{\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_{\omega\prime}}{\theta_\omega} \left((1-\xi)\eta p_{\omega\prime} + \xi \eta p\prime_{\omega\prime}\right)^2} = 0\end{array}$$

$\forall \xi \in [0,1],\ \forall \omega,$

$$\begin{array}{l}(1-\xi)\eta t_\omega + \xi \eta t\prime_\omega + (1-\xi) Z_\omega^t + \xi Z^t\prime_\omega \\ +c1_\omega((1-\xi)w_{\omega_f}^t + \xi w^t\prime_{\omega_f} - (1-\xi)w_\omega^t - \xi w^t\prime_\omega) \\ -A^t \sqrt{\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_{\omega\prime}}{\theta_\omega} \left((1-\xi)\eta t_{\omega\prime} + \xi \eta t\prime_{\omega\prime}\right)^2} = 0\end{array}$$

given that:

$\forall \omega,$

$$\begin{array}{l}\eta p(t)_\omega + Z_\omega^{p(t)} + c1_\omega(w_{\omega_f}^{p(t)} - w_\omega^{p(t)}) \\ -A^{p(t)} \sqrt{\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_{\omega\prime}}{\theta_\omega} \eta p(t)_{\omega\prime}^2} = 0\end{array}$$

$$\begin{array}{l}\eta\prime p(t)_\omega + Z\prime_\omega^{p(t)} + c1_\omega(w\prime_{\omega_f}^{p(t)} - w\prime_\omega^{p(t)}) \\ -A^{p(t)} \sqrt{\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_{\omega\prime}}{\theta_\omega} \eta\prime p(t)_{\omega\prime}^2} = 0\end{array}$$

we have:

$\forall \xi \in [0,1],\ \forall \omega,$

$$\sqrt{\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_{\omega\prime}}{\theta_\omega} \left((1-\xi)\eta p_{\omega\prime} + \xi \eta p\prime_{\omega\prime}\right)^2} = (1-\xi)\sqrt{\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_{\omega\prime}}{\theta_\omega} \eta p_{\omega\prime}^2} + \xi \sqrt{\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_{\omega\prime}}{\theta_\omega} \eta p\prime_{\omega\prime}^2}$$

$\forall \xi \in [0,1],\ \forall \omega,$

$$\sqrt{\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_{\omega\prime}}{\theta_\omega} \left((1-\xi)\eta t_{\omega\prime} + \xi \eta t\prime_{\omega\prime}\right)^2} = (1-\xi)\sqrt{\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_{\omega\prime}}{\theta_\omega} \eta t_{\omega\prime}^2} + \xi \sqrt{\sum_{\omega\prime/\omega\prime_f=\omega} \frac{\theta_{\omega\prime}}{\theta_\omega} \eta t\prime_{\omega\prime}^2}$$

which is possible if and only if:

$$\forall \omega,\ \exists \gamma p_\omega \in \mathbb{R},\ \text{such that } \forall \omega\prime/\omega\prime_f = \omega,\ \eta p_{\omega\prime} = \gamma p_\omega\ \eta p\prime_{\omega\prime} \tag{96}$$

$$\forall \omega,\ \exists \gamma t_\omega \in \mathbb{R},\ \text{such that } \forall \omega\prime/\omega\prime_f = \omega,\ \eta t_{\omega\prime} = \gamma t_\omega\ \eta t\prime_{\omega\prime} \tag{97}$$

Given equations (52e), (52f), (53f) and (53g), we can directly deduce that the risk-neutral probabilities are the same for the two equilibria:

$$\nu p_= \nu p\prime \tag{98}$$

$$\nu t_= \nu t\prime \tag{99}$$

To finish the demonstration, we need to focus on some particular nodes of the scenario tree. To simplify the presentation, we will use the notation of the tree presented in Figure 1. However, our demonstration is more general and can be applied to any scenario tree.
Using constraints (21) and (26) for scenario nodes 4, 5 and 6, we have:
$\forall \omega \in \{4, 5, 6\}$
$\eta p_\omega = \gamma p_1 \eta p\prime_\omega$
and
$\eta p_\omega = -Z^p_\omega + c1_\omega w^p_1$
$\eta p\prime_\omega = -Z^p\prime_\omega + c1_\omega w^p\prime_1$

which means that (given that $c1_4 = c1_5 = c1_6$):

$$Z^p_4 - \gamma p_1 Z^p\prime_4 = Z^p_5 - \gamma p_1 Z^p\prime_5 = Z^p_6 - \gamma p_1 Z^p\prime_6 \tag{100}$$

Similarly, we can write the same equation for the mid-streamer:

$$Z^t_4 - \gamma t_1 Z^t\prime_4 = Z^t_5 - \gamma t_1 Z^t\prime_5 = Z^t_6 - \gamma t_1 Z^t\prime_6 \tag{101}$$

After some calculations, we will have (by combining equations (85)-(91), (100) and (101))[13]:

$$(1 - \gamma p_1) Z^p_4 - \kappa p_4 = (1 - \gamma p_1) Z^p_5 - \kappa p_5 = (1 - \gamma p_1) Z^p_6 - \kappa p_6 \tag{102}$$

$$(1 - \gamma t_1) Z^t_4 + \kappa p_4 = (1 - \gamma t_1) Z^t_5 + \kappa p_5 = (1 - \gamma t_1) Z^t_6 + \kappa p_6 \tag{103}$$

and[14]

$$\kappa = \frac{(1 - \gamma p_1)(Z^p_4 - Z^p_5)}{p_4 - p_5} = \frac{(1 - \gamma p_1)(Z^p_5 - Z^p_6)}{p_5 - p_6} \tag{104}$$

$$\kappa = \frac{(1 - \gamma t_1)(Z^t_5 - Z^t_4)}{p_4 - p_5} = \frac{(1 - \gamma t_1)(Z^t_6 - Z^t_5)}{p_5 - p_6} \tag{105}$$

Using equations (104) and (105), we have:

$$(1 - \gamma p_1)(Z^p_4 - Z^p_5) = (1 - \gamma t_1)(Z^t_5 - Z^t_4) \tag{106}$$

It is easy to demonstrate that if scenario node 4 is the one which has the highest intercept of the spot and downstream markets inverse demand function, then the producer and the mid-streamer's profits are the highest in this node, which implies that: $Z^{p(t)}_4 - Z^{p(t)}_5 \geq 0$[15]. Using equation (106), this means that $(1 - \gamma p_1)(1 - \gamma t_1) \leq 0$. Without any loss of generality we will assume that $(1 - \gamma p_1) \leq 0$ (otherwise, we will apply the coming reasoning to the mid-streamer).

[13] We recall that p_ω is the spot price at node ω.
[14] We recall that $p_4 > p_5 > p_6$.
[15] This is in particular implied by the fact that the contract price, which is a risk-adjusted expectation of the spot price (see equation (70)), is smaller than p_4.

Using equation (104), we have:

$$(1-\gamma p_1) \leq 0 \Longrightarrow \kappa \leq 0$$

Now, let us write constraint (21) for scenario node 1 and for vectors X and $X\prime$:

$$\eta p_1 + Z_1^p + c1_1(w_0^p - w_1^p) - A^p \sqrt{\sum_{\omega/\omega_f=1} \theta_\omega \eta p_\omega^2} = 0 \tag{107}$$

$$\eta p\prime_1 + Z^p\prime_1 + c1_1(w^p\prime_0 - w^p\prime_1) - A^p \sqrt{\sum_{\omega/\omega_f=1} \theta_\omega \eta p\prime_\omega^2} = 0 \tag{108}$$

which gives by subtracting (given that $c1_1 = 1$ and $\gamma p_1 = \gamma p\prime_1$):

$$Z_1^p - Z^p\prime_1 = (w_1^p - w^p\prime_1) - (w_0^p - w^p\prime_0) - A^p(1-\gamma p_1)\sqrt{\sum_{\omega/\omega_f=1} \theta_\omega \eta p\prime_\omega^2} \tag{109}$$

If we write constraint (21) for scenario node 4 and for X and $X\prime$, we would have:

$$w_1^p - w^p\prime_1 = \eta p_4 - \eta p\prime_4 + Z_4^p - Z^p\prime_4 \tag{110}$$

and combining equations (109) and (110), we will have:

$$\big((p_1 - \pi^1) - (p_4 - \pi^1)\big)\,\kappa = -(1-\gamma p_1)\eta p\prime_4 - (w_0^p - w^p\prime_0) - A^p(1-\gamma p_1)\sqrt{\sum_{\omega/\omega_f=1} \theta_\omega \eta p\prime_\omega^2} \tag{111}$$

Similarly, for nodes 5 and 6 we will have:

$$\big((p_1 - \pi^1) - (p_5 - \pi^1)\big)\,\kappa = -(1-\gamma p_1)\eta p\prime_5 - (w_0^p - w^p\prime_0) - A^p(1-\gamma p_1)\sqrt{\sum_{\omega/\omega_f=1} \theta_\omega \eta p\prime_\omega^2} \tag{112}$$

$$\big((p_1 - \pi^1) - (p_6 - \pi^1)\big)\,\kappa = -(1-\gamma p_1)\eta p\prime_6 - (w_0^p - w^p\prime_0) - A^p(1-\gamma p_1)\sqrt{\sum_{\omega/\omega_f=1} \theta_\omega \eta p\prime_\omega^2} \tag{113}$$

and by subtracting node 4 from node 5's relations:

$$(p_5 - p_4)\kappa = -(1-\gamma p_1)(\eta p\prime_4 - \eta p\prime_5) \tag{114}$$

Since $\eta p\prime_4 - \eta p\prime_5 = Z\prime_4^p - Z\prime_5^p$, we have $\eta p\prime_4 - \eta p\prime_5 \leq 0$. We already have $p_5 - p_4 < 0$ and $\kappa \leq 0$. Since $1 - \gamma p_1 \leq 0$, we can deduce that relation (114) is only possible if $kappa = 0$ and $1 - \gamma p_1 = 0$.
$\kappa = 0$ implies directly (from (85)-(91)) that:

$$\forall \omega,\ x\prime_\omega = x_\omega \tag{115}$$

$$\forall \omega,\ h\prime_\omega = h_\omega \tag{116}$$

$$\forall \omega,\ up\prime^1 = up^1 \tag{117}$$

$$\forall \omega,\ ut\prime^1 = ut^1 \tag{118}$$

$$\forall \omega,\ z\prime_\omega = z_\omega \tag{119}$$

$$\forall \omega,\ \nu p\prime_\omega = \nu p_\omega \tag{120}$$

$$\forall \omega,\ \nu t\prime_\omega = \nu t_\omega \tag{121}$$

which implies that the market variables are unique and therefore, the players' profits in all the scenarios are also unique. As a consequence, the good-deal risk variables are also unique ([8]):

$$\forall \omega,\ \eta p\prime_\omega = \eta p_\omega \tag{122}$$
$$\forall \omega,\ \eta t\prime_\omega = \eta t_\omega \tag{123}$$
$$\forall \omega,\ w^p\prime_\omega = w^p_\omega \tag{124}$$
$$\forall \omega,\ w^t\prime_\omega = w^t_\omega \tag{125}$$

□

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