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## Toward a theory of monopolistic competition

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### Abstract

We propose a general model of monopolistic competition, which encompasses existing models while being flexible enough to take into account new demand and competition features. The basic tool we use to study the market outcome is the elasticity of substitution at a symmetric consumption pattern, which depends on both the per capita consumption and the total mass of varieties. We impose intuitive conditions on this function to guarantee the existence and uniqueness of a free-entry equilibrium. Comparative statics with respect to population size, GDP per capita and productivity shock are characterized through necessary and sufficient conditions. Finally, we show how our approach can be generalized to the case of a multisector economy and extended to cope with heterogeneous firms and consumers.

**Keywords:** monopolistic competition, general equilibrium, additive preferences, homothetic preferences.

**JEL classification:** D43, L11, L13.

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# 1 Introduction

The theory of general equilibrium with imperfectly competitive markets is still in infancy. In his survey of the various attempts made in the 1970s and 1980s to integrate oligopolistic competition within the general equilibrium framework, Hart (1985) has convincingly argued that these contributions have failed to produce a consistent and workable model. Unintentionally, the absence of a general equilibrium model of oligopolistic competition has paved the way to the success of the CES model of monopolistic competition developed by Dixit and Stiglitz (1977). And indeed, the CES model has been used in so many economic fields that a large number of scholars believe that this is *the* model of monopolistic competition. For example, Head and Mayer (2014) observe that this model is “nearly ubiquitous” in the trade literature. However, owing to its extreme simplicity, the CES model dismisses several important effects that contradict basic findings in economic theory, as well as empirical evidence. To mention a few, unlike what the CES predicts, prices and firm sizes are affected by entry, market size and consumer income, while markups vary with costs.

In addition, tweaking the CES or using other specific models in the hope of obviating these difficulties does not permit to check the robustness of the results. For example, quadratic preferences are consistent with lower prices and higher average productivity in larger markets. By contrast, quadratic preferences imply that prices are independent of income, the reason being that they are nested in a quasi-linear utility. The effect of GDP per capita can be restored if preferences are indirectly additive, but then prices no longer depend on the number of competitors. If preferences are CES and the market structure is oligopolistic, markups depend again on the number of firms, but no longer on the GDP per capita. In sum, it seems fair to say that the state of the art looks like a scattered field of incomplete and insufficiently related contributions in search of a more general approach.

Our purpose is to build a *general* model of monopolistic competition that has the following two features. First, it encompasses all existing models of monopolistic competition, including the CES. Second, it displays enough flexibility to take into account demand and competition attributes in a way that will allow us to determine under which conditions many findings are valid. In this respect, we concur with Mrázová and Neary (2013) that “assumptions about the structure of preferences and demand matter enormously for comparative statics.” This is why we consider a setting in which preferences are unspecified and characterize preferences through necessary and sufficient conditions for each comparative static effect to hold. This allows us to identify which findings hold true against alternative preference specifications, and those which depend on particular classes of preferences. This should be useful to the applied economists in discriminating between the different specifications used in their settings. The flip side of the coin is the need to reduce the complexity of the problem. This is why, in the baseline model, we focus on competition among symmetric firms. We see this as a necessary step toward the development and analysis of a fully general theory of monopolistic competition.

By modeling monopolistic competition as a noncooperative game with a continuum of players, we are able to obviate at least two major problems.<sup>1</sup> First, we capture Chamberlin's central idea according to which the cross elasticities of demands are equal to zero, the reason being that each firm is negligible to the market. Second, whereas the redistribution of firms' profits is at the root of the non-existence of an equilibrium in general equilibrium with oligopolistic competition, we get rid of this feedback effect because individual firms are unable to manipulate profits. Thus, firms do not have to make full general equilibrium calculations before choosing their profit-maximizing strategy. Admittedly, the continuum assumption may be viewed as a *deus ex machina*. But we end up with a consistent and analytically tractable model in which firms are bound together through variables that give rise to income and substitution effects in consumers' demand. This has a far-fetched implication: even though firms do not compete strategically, *our model is able to mimic oligopolistic markets* and to generate within a general equilibrium framework findings akin to those obtained in partial equilibrium settings. As will be shown, the CES is the only case in which all effects vanish.

Our main findings may be summarized as follows. First, using an extension of the concept of differentiability to the case where the unknowns are functions rather than vectors, we determine a general demand system, which includes a wide range of special cases such as the CES, quadratic, CARA, additive, indirectly additive, and homothetic preferences. At any symmetric market outcome, the individual demand for a variety depends only upon its consumption when preferences are additive. By contrast, when preferences are homothetic, the demand for a variety depends upon its relative consumption level and the mass of available varieties. Therefore, when preferences are neither additive nor homothetic, *the demand for a variety must depend on its consumption level and the total mass of available varieties*.

Second, to prove the existence and uniqueness of a free-entry equilibrium and to study its properties, we need to impose some restrictions on the demand side of our model. Rather than making new assumptions on preferences and demands, we tackle the problem from the viewpoint of the theory of product differentiation. To be precise, the key concept of our model is the elasticity of substitution across varieties. We then exploit the symmetry of preferences over a continuum of goods to show that, under the most general specification of preferences, at any symmetric outcome the elasticity of substitution between any two varieties is a function of two variables only: the per variety consumption and the total mass of firms. Combining this with the absence of the business-stealing effect of oligopoly theory reveals that, at the market equilibrium, *firms' markup is equal to the inverse of the equilibrium value of the elasticity of substitution*. This result agrees with one of the main messages of industrial organization: the higher is the elasticity of substitution, the less differentiated are varieties, and thus the lower are firms' markup.

Therefore, it should be clear that the properties of the symmetric free-entry equilibrium depends

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<sup>1</sup>The idea of using a continuum of firms was proposed by Dixit and Stiglitz in their 1974 working paper, which has been published in Brakman and Heijdra (2004)

on how the elasticity of substitution function behaves when the per variety consumption and the mass of firms vary with the main parameters of the economy. This allows us to study the market outcome by means of simple analytical arguments. To be precise, by imposing plausible conditions to the elasticity of substitution function, we are able to disentangle the various determinants of firms' strategies. We will determine our preferred set of assumptions by building on what the theory of product differentiation tells us, as well as on empirical evidence.

Third, to insulate the impact of various types of preferences on the market outcome, we focus on symmetric firms and, therefore, on symmetric free-entry equilibria. We provide necessary and sufficient conditions on the elasticity of substitution for the existence and uniqueness of a free-entry equilibrium. Our setting is especially well suited to conduct detailed comparative static analyses in that we can determine the necessary and sufficient conditions for all the thought experiments undertaken in the literature. The most typical experiment is to study the impact of market size. What market size signifies is not always clear because it compounds two variables, i.e., the number of consumers and their willingness-to-pay for the product under consideration. The impact of population size and income level on prices, output and the number of firms need not be the same because these two parameters affect firms' demand in different ways. An increase in population or in income raises demand, thereby fostering entry and lower prices. But an income hike also raises consumers' willingness-to-pay, which tends to push prices upward. The final impact is thus a priori ambiguous.

We show that a larger market results in a lower market price and bigger firms if and only if the elasticity of substitution responds more to a change in the mass of varieties than to a change in the per variety consumption. This is so in the likely case where the entry of new firms does not render varieties much more differentiated. Regarding the mass of varieties, it increases with the number of consumers if varieties do not become too similar when their number rises. Thus, like most oligopoly models, *monopolistic competition exhibits the standard pro-competitive effects associated with market size and entry*. However, anti-competitive effects cannot be ruled out a priori. Furthermore, an increase in individual income generates similar, but not identical, effects if and only if varieties become closer substitutes when their range widens. The CES is the only utility for which price and output are independent of both income and market size.

Our setting also allows us to study the impact of a cost change on markups. When all firms face the same productivity hike, we show that *the nature of preferences determines the extent of the pass-through*. Specifically, a decrease in marginal cost leads to a lower market price, but a higher markup, if and only if the elasticity of substitution decreases with the per capita consumption. In this event, there is incomplete pass-through. However, the pass-through rate need not be smaller than one.

Last, we discuss three major extensions of our baseline model. In the first one, we focus on Melitz-like heterogeneous firms. In this case, when preferences are non-additive, the profit-maximizing strategy of a firm depends directly on the strategies chosen by all the other types' firms,

which vastly increases the complexity of the problem. Despite of this, we show that, regardless of the distribution of marginal costs, firms are sorted out by decreasing productivity order, while a bigger market sustains a larger number of active firms. We highlight the role of the elasticity of substitution by showing that it now depends on the number entrants and the cutoff costs for each type of firm. In the second, we consider a multisector economy. The main additional difficulty stems from the fact that the sector-specific expenditures depend on the upper-tier utility. Under a fairly mild assumption on the marginal utility, we prove the existence of an equilibrium and show that many of our results hold true for the monopolistically competitive sector. This highlights the idea that our model can be used as a building block to embed monopolistic competition in full-fledged general equilibrium models coping with various economic issues. Our last extension addresses the almost untouched issue of consumer heterogeneity in love-for-variety models of monopolistic competition. Consumers may be heterogeneous because of taste and/or income differences. Here, we will restrict ourselves to special, but meaningful, cases.

**Related literature.** Different alternatives have been proposed to avoid the main pitfalls of the CES model. Behrens and Murata (2007) propose the CARA utility that captures both price and size effects, while Zhelobodko et al. (2012) use general additive preferences to work with a variable elasticity of substitution, and thus variable markups. Bertolotti and Etro (2014) consider an additive indirect utility function to study the impact of per capita income on the market outcome, but price and firm size are independent of population size. Vives (1999) and Ottaviano et al. (2002) show how the quadratic utility model obviates some of the difficulties associated with the CES model, while delivering a full analytical solution. Bilbiie et al. (2012) use general symmetric homothetic preferences in a real business cycle model. Last, pursuing a different objective, Mrázová and Neary (2013) study a specific class of demand functions, that is, those which leaves the relationships between the elasticity and convexity of demands unchanged, and aim to predict the welfare effects of a wide range of shocks.

In the next section, we describe the demand and supply sides of our setting. The primitive of the model being the elasticity of substitution function, we discuss in Section 3 how this function may vary with the per variety consumption and the mass of varieties. In Section 4, we prove the existence and uniqueness of a free-entry equilibrium and characterize its various properties. The three extensions are discussed in Section 5, while Section 6 concludes.

## 2 The model and preliminary results

Consider an economy with a mass  $L$  of identical consumers, one sector and one production factor – labor, which is used as the numéraire. Each consumer is endowed with  $y$  efficiency units of labor, so that per capita labor income  $y$  is given and the same across consumers because the unit wage is

1.<sup>2</sup> On the supply side, there is a continuum of firms producing each a horizontally differentiated good under increasing returns. Each firm supplies a single variety and each variety is supplied by a single firm.

## 2.1 Consumers

Let  $\mathcal{N}$ , an arbitrarily large number, be the mass of “potential” varieties, e.g., the varieties for which a patent exists. Very much like in the Arrow-Debreu model where all commodities need not be produced and consumed, all potential varieties are not necessarily made available to consumers. We denote by  $N \leq \mathcal{N}$  the endogenous mass of available varieties.

Since we work with a continuum of varieties, we cannot use the standard tools of calculus anymore. Rather, we must work in a functional space whose elements are functions, and not vectors. A *potential consumption profile*  $\mathbf{x} \geq 0$  is a (Lebesgue-measurable) mapping from the space of potential varieties  $[0, \mathcal{N}]$  to  $\mathbb{R}_+$ . Since a market price profile  $\mathbf{p} \geq \mathbf{0}$  must belong to the dual of the space of consumption profiles (Bewley, 1972), we assume that both  $\mathbf{x}$  and  $\mathbf{p}$  belong to  $L_2([0, \mathcal{N}])$ . This implies that both  $\mathbf{x}$  and  $\mathbf{p}$  have a finite mean and variance. The space  $L_2$  may be viewed as the most natural infinite-dimensional extension of  $\mathbb{R}^n$ .

Individual preferences are described by a *utility functional*  $\mathcal{U}(\mathbf{x})$  defined over  $L_2([0, \mathcal{N}])$ . In what follows, we make two assumptions about  $\mathcal{U}$ , which seem close to the “minimal” set of requirements for our model to be nonspecific while displaying the desirable features of existing models of monopolistic competition. First, for any  $N$ , the functional  $\mathcal{U}$  is *symmetric* in the sense that any Lebesgue measure-preserving mapping from  $[0, N]$  into itself does not change the value of  $\mathcal{U}$ . Intuitively, this means that renumbering varieties has no impact on the utility level.

Second, the utility function exhibits *love for variety* if, for any  $N \leq \mathcal{N}$ , a consumer strictly prefers to consume the whole range of varieties  $[0, N]$  than any subinterval  $[0, k]$  of  $[0, N]$ , that is,

$$\mathcal{U}\left(\frac{X}{k}I_{[0,k]}\right) < \mathcal{U}\left(\frac{X}{N}I_{[0,N]}\right), \quad (1)$$

where  $X > 0$  is the consumer’s total consumption of the differentiated good and  $I_A$  is the indicator of  $A \subseteq [0, N]$ .

**Proposition 1.** *If  $\mathcal{U}(\mathbf{x})$  is continuous and strictly quasi-concave, then consumers exhibit love for variety.*

The proof is given in Appendix 1. The convexity of preferences is often interpreted as a “taste for diversification” (Mas-Collel et al., 1995, p.44). Our definition of “love for variety” is weaker than that of convex preferences because the former, unlike the latter, involves symmetric consumption only. This explains why the reverse of Proposition 1 does not hold. Since (1) holds under any

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<sup>2</sup>At a free-entry equilibrium, a consumer’s income is equal to  $y$  because profits are zero. Distinguishing between  $y$  and  $L$  allows us to discriminate between the effects generated by a change in the per capita income and the number of consumers.



monotone transformation of  $\mathcal{U}$ , our definition of love for variety is ordinal in nature. In particular, our definition does not appeal to any parametric measure such as the elasticity of substitution in CES-based models.

To determine the inverse demand for a variety, we assume that the utility functional  $\mathcal{U}$  is differentiable in  $\mathbf{x}$  in the following sense: there exists a unique function  $D(x_i, \mathbf{x})$  from  $\mathbb{R}_+ \times L_2$  to  $\mathbb{R}$  such that, for any given  $N$  and for all  $\mathbf{h} \in L_2$ , the equality

$$\mathcal{U}(\mathbf{x} + \mathbf{h}) = \mathcal{U}(\mathbf{x}) + \int_0^N D(x_i, \mathbf{x}) h_i di + o(\|\mathbf{h}\|_2) \quad (2)$$

holds,  $\|\cdot\|_2$  being the  $L_2$ -norm.<sup>3</sup> The function  $D(x_i, \mathbf{x})$  is the *marginal utility of variety  $i$* . In what follows, we focus on utility functionals that satisfy (2) for all  $\mathbf{x} \geq 0$  and such that the marginal utility  $D(x_i, \mathbf{x})$  is decreasing and differentiable with respect to the consumption  $x_i$  of variety  $i$ . That  $D(x_i, \mathbf{x})$  does not depend directly on  $i \in [0, N]$  follows from the symmetry of preferences. Evidently,  $D(x_i, \mathbf{x})$  strictly decreases with  $x_i$  if  $\mathcal{U}$  is strictly concave.

The reason for restricting ourselves to decreasing marginal utilities is that this property allows us to work with well-behaved demand functions. Indeed, maximizing the functional  $\mathcal{U}(\mathbf{x})$  subject to (i) the budget constraint

$$\int_0^N p_i x_i di = Y \quad (3)$$

where  $Y \geq y$  is the consumer's income and (ii) the availability constraint

$$x_i \geq 0 \text{ for all } i \in [0, N] \quad \text{and} \quad x_i = 0 \text{ for all } i \in ]N, \mathcal{N}]$$

yields the following inverse demand function for variety  $i$ :

$$p_i = \frac{D(x_i, \mathbf{x})}{\lambda} \quad \text{for all } i \in [0, N], \quad (4)$$

where  $\lambda$  is the Lagrange multiplier of the consumer's optimization problem. Expressing  $\lambda$  as a function of  $Y$  and  $\mathbf{x}$ , we obtain

$$\lambda(Y, \mathbf{x}) = \frac{\int_0^N x_i D(x_i, \mathbf{x}) di}{Y}, \quad (5)$$

which is the marginal utility of income at the consumption profile  $\mathbf{x}$  under income  $Y$ .

The marginal utility function  $D(x_i, \mathbf{x})$  also allows determining the Marshallian demand. Indeed, because the consumer's budget set is convex and weakly compact in  $L_2([0, \mathcal{N}])$ , while  $\mathcal{U}$  is differentiable (which means that (2) holds) and strictly quasi-concave, there exists a unique

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<sup>3</sup>For symmetric utility functionals, (2) is equivalent to assuming that  $\mathcal{U}$  is Frechet-differentiable in  $L_2$  (Dunford and Schwartz, 1988). The concept of Frechet-differentiability extends the standard concept of differentiability in a fairly natural way.

utility-maximizing consumption profile  $\mathbf{x}^*(\mathbf{p}, Y)$  (see Appendix 1). Plugging  $\mathbf{x}^*(\mathbf{p}, Y)$  into (4) – (5) and solving (4) for  $x_i$ , we obtain the Marshallian demand for variety  $i$ :

$$x_i = \mathcal{D}(p_i, \mathbf{p}, Y), \quad (6)$$

which is weakly decreasing in its own price.<sup>4</sup> In other words, when there is a continuum of varieties, *decreasing marginal utilities are a necessary and sufficient condition for the Law of demand to hold.*

To illustrate, consider the following examples used in the literature.

**1. Additive preferences.**<sup>5</sup> (i) Assume that preferences are additive over the set of available varieties (Spence, 1976; Dixit and Stiglitz, 1977):

$$\mathcal{U}(\mathbf{x}) \equiv \int_0^{\mathcal{N}} u(x_i) di, \quad (7)$$

where  $u$  is differentiable, strictly increasing, strictly concave and such that  $u(0) = 0$ . The CES and the CARA (Bertoletti, 2006; Behrens and Murata, 2007) are special cases of (7). The marginal utility of variety  $i$  depends only upon its own consumption:

$$D(x_i, \mathbf{x}) = u'(x_i).$$

Bertoletti and Etro (2014) have recently proposed a new approach to modeling monopolistic competition, in which preferences are expressed through the following indirect utility function:

$$\mathcal{V}(\mathbf{p}, Y) \equiv \int_0^{\mathcal{N}} v(p_i/Y) di,$$

where  $v$  is differentiable, strictly decreasing and strictly convex. It is readily verified that the marginal utility of variety  $i$  is now given by

$$D(x_i, \mathbf{x}) = \Lambda(\mathbf{x})(v')^{-1}(-\Lambda(\mathbf{x})x_i)$$

where  $\Lambda(\mathbf{x}) \equiv Y\lambda(\mathbf{x}, Y)$ .

**2. Homothetic preferences.** A tractable example of non-CES homothetic preferences is the translog proposed by Feenstra (2003). By appealing to the duality principle in consumption theory, these preferences are described by the following expenditure function:

$$\ln E(\mathbf{p}) = \ln \mathcal{U}_0 + \frac{1}{\mathcal{N}} \int_0^{\mathcal{N}} \ln p_i di - \frac{\beta}{2\mathcal{N}} \left[ \int_0^{\mathcal{N}} (\ln p_i)^2 di - \frac{1}{\mathcal{N}} \left( \int_0^{\mathcal{N}} \ln p_i di \right)^2 \right].$$

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<sup>4</sup>Since  $D$  is continuously decreasing in  $x_i$ , there exists at most one solution of (4) with respect to  $x_i$ . If there is a finite choke price ( $D(0, \mathbf{x}^*)/\lambda < \infty$ ), there may be no solution. To encompass this case, the Marshallian demand should be formally defined by  $\mathcal{D}(p_i, \mathbf{p}, y) \equiv \inf\{x_i \geq 0 \mid D(x_i, \mathbf{x}^*)/\lambda(y, \mathbf{x}^*) \leq p_i\}$ .

<sup>5</sup>The idea of additive utilities and additive indirect utilities goes back at least to Houthakker (1960).

In this case, there is no closed-form expression for the marginal utility  $D(x_i, \mathbf{x})$ . Nevertheless, it can be shown that  $D(x_i, \mathbf{x})$  may be written as follows:

$$D(x_i, \mathbf{x}) = \varphi(x_i, A(\mathbf{x})),$$

where  $\varphi(x, A)$  is implicitly defined by

$$\varphi x + \beta \ln \varphi = \frac{1 + \beta A}{\mathcal{N}},$$

while  $A(\mathbf{x})$  is a scalar aggregate determined by

$$A = \int_0^{\mathcal{N}} \ln \varphi(x_i, A) di.$$

The aggregate  $A(\mathbf{x})$  thus plays a role similar to that of  $\lambda$  (respectively,  $\Lambda$ ) under additive (respectively, indirectly additive) preferences.

**3. Non-additive preferences.** Consider the quadratic utility:

$$\mathcal{U}(\mathbf{x}) \equiv \alpha \int_0^{\mathcal{N}} x_i di - \frac{\beta}{2} \int_0^{\mathcal{N}} x_i^2 di - \frac{\gamma}{2} \int_0^{\mathcal{N}} \left( \int_0^{\mathcal{N}} x_i di \right) x_j dj, \quad (8)$$

where  $\alpha, \beta,$  and  $\gamma$  are positive constants. The marginal utility of variety  $i$  is given by

$$D(x_i, \mathbf{x}) = \alpha - \beta x_i - \gamma \int_0^{\mathcal{N}} x_j dj,$$

which is linear decreasing in  $x_i$ . Note that  $D$  decreases with the aggregate consumption across varieties:

$$X \equiv \int_0^{\mathcal{N}} x_j dj,$$

which captures the idea that the marginal utility of a variety decreases with the total consumption.

Another example of non-additive preferences, which also captures the idea of love for variety is given by the entropy-like utility:

$$\mathcal{U}(\mathbf{x}) \equiv U(X) + X \ln X - \int_0^{\mathcal{N}} x_i \ln x_i di,$$

where  $U$  is increasing and strictly concave. The marginal utility of variety  $i$  is

$$D(x_i, \mathbf{x}) = U'(X) - \ln \left( \frac{x_i}{X} \right),$$

which decreases with  $x_i$ .

Since the bulk of the applied literature focuses on additive or homothetic preferences, we find it important to provide a full characterization of the corresponding demands (the proof is given in

Appendix 3).

**Proposition 2.** *The marginal utility  $D(x_i, \mathbf{x})$  of variety  $i$  depends only upon (i) the consumption  $x_i$  if and only if preferences are additive and (ii) the consumption ratio  $\mathbf{x}/x_i$  if and only if preferences are homothetic.*

Proposition 2 can be illustrated by using the CES:

$$\mathcal{U}(\mathbf{x}) \equiv \left( \int_0^{\mathcal{N}} x_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}},$$

where  $\sigma > 1$  is the elasticity of substitution across varieties. The marginal utility  $D(x_i, \mathbf{x})$  is given by

$$D(x_i, \mathbf{x}) = A(\mathbf{x}) x_i^{1/\sigma} = A(\mathbf{x}/x_i),$$

where  $A(\mathbf{x})$  is the aggregate given by

$$A(\mathbf{x}) \equiv \left( \int_0^{\mathcal{N}} x_j^{\frac{\sigma-1}{\sigma}} dj \right)^{-1/\sigma}.$$

We show below that our setting can easily be accommodated to cope with two more general specifications of preferences.

**The number of varieties as a consumption externality.** In their 1974 working paper, Dixit and Stiglitz (1977) argued that the mass of varieties could enter the utility functional as a specific argument.<sup>6</sup> In this case, the number of available varieties has the nature of a consumption externality generated by the entry of firms.

A well-known example is given by the augmented-CES, which is defined as follows:

$$\mathcal{U}(\mathbf{x}, N) \equiv N^\nu \left( \int_0^{\mathcal{N}} x_i^{\frac{\sigma-1}{\sigma}} di \right)^{\sigma/(\sigma-1)}. \quad (9)$$

In Benassy (1996),  $\nu$  is a positive constant that captures the consumer benefit of a larger number of varieties. The idea is to separate the love-for-variety effect from the competition effect generated by the degree of product differentiation, which is inversely measured by  $\sigma$ . Blanchard and Giavazzi (2003) takes the opposite stance by assuming that  $\nu = -1/\sigma(N)$  where  $\sigma(N)$  increases with  $N$ . Under this specification, increasing the number of varieties does not raise consumer welfare but intensifies competition among firms.

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<sup>6</sup>Note that  $N$  can be written as a function of the consumption functional  $\mathbf{x}$  in the following way  $N = \mu\{x_i > 0, \forall i \leq \mathcal{N}\}$ . However, this raises new issues regarding differentiability of the utility functional.

Another example is the quadratic utility proposed by Shubik and Levitan (1971):

$$\mathcal{U}(\mathbf{x}, N) \equiv \alpha \int_0^{\mathcal{N}} x_i di - \frac{\beta}{2} \int_0^{\mathcal{N}} x_i^2 di - \frac{\gamma}{2N} \int_0^{\mathcal{N}} \left( \int_0^{\mathcal{N}} x_i di \right) x_j dj. \quad (10)$$

The difference between (8) and (10) is that the former may be rewritten as follows:

$$\alpha X - \frac{\beta}{2} \int_0^{\mathcal{N}} x_i^2 di - \frac{\gamma}{2} X^2,$$

which is independent of  $N$ , whereas the latter becomes

$$\alpha X - \frac{\beta}{2} \int_0^{\mathcal{N}} x_i^2 di - \frac{\gamma}{2N} X^2,$$

which ceteris paribus strictly increases with  $N$ .

Introducing  $N$  as an explicit argument in the utility functional  $\mathcal{U}(\mathbf{x}, N)$  may change the indifference surfaces. Nevertheless, the analysis developed below remains valid in such cases. Indeed, the marginal utility function  $D$  already includes  $N$  as an argument through the support of  $\mathbf{x}$ , which varies with  $N$ .

**An example of asymmetric preferences.** Assume that preferences are asymmetric in that the utility functional  $\mathcal{U}(\mathbf{x})$  is given by

$$\mathcal{U}(\mathbf{x}) = \tilde{\mathcal{U}}(\mathbf{a} \cdot \mathbf{x}), \quad (11)$$

where  $\tilde{\mathcal{U}}$  is a symmetric functional that satisfies (2),  $\mathbf{a} \in L_2([0, \mathcal{N}])$  a weight function, and  $\mathbf{a} \cdot \mathbf{x}$  the function  $(\mathbf{a} \cdot \mathbf{x})_i \equiv a_i x_i$  for all  $i \in [0, \mathcal{N}]$ . If  $x_i = x_j$ ,  $a_i > a_j$  means that all consumers prefer variety  $i$  to variety  $j$ , perhaps because the quality of  $i$  exceeds that of  $j$ .

The preferences (11) can be made symmetric by changing the units in which the quantities of varieties are measured. Indeed, for any  $i, j \in [0, \mathcal{N}]$  the consumer is indifferent between consuming  $a_i/a_j$  units of variety  $i$  and one unit of variety  $j$ . Therefore, by using the change of variables  $\tilde{x}_i \equiv a_i x_i$  and  $\tilde{p}_i \equiv p_i/a_i$ , we can reformulate the consumer's program as follows:

$$\max_{\tilde{\mathbf{x}}} \tilde{\mathcal{U}}(\tilde{\mathbf{x}}) \quad \text{s.t.} \quad \int_0^{\mathcal{N}} \tilde{p}_i \tilde{x}_i di \leq Y.$$

In this case, by rescaling of prices, quantities and costs by the weights  $a_i$ , the model involves symmetric preferences and heterogeneous firms, a setting we study in 5.2.

## 2.2 Firms

There are increasing returns at the firm level, but no scope economies that would induce a firm to produce several varieties. Each firm supplies a single variety and each variety is produced by a single firm. Consequently, a variety may be identified by its producer  $i \in [0, N]$ . Firms are *symmetric*: to produce  $q$  units of its variety, a firm needs  $F + cq$  efficiency units of labor, which means that firms share the same fixed cost  $F$  and the same marginal cost  $c$ . Being negligible to the market, each firm chooses its output (or price) while accurately treating some market aggregates as given. However, for the market to be in equilibrium, firms must accurately guess what these market aggregates will be.

In monopolistic competition, unlike oligopolistic competition, working with quantity-setting and price-setting firms yield the same market outcome (Vives, 1999). However, it turns out to be convenient to assume that firms are quantity-setters because the scalar  $\lambda$  encapsulates all the income effects. Thus, firm  $i \in [0, N]$  maximize its profits

$$\pi(q_i) = (p_i - c)q_i - F \tag{12}$$

with respect to its output  $q_i$  subject to the inverse market demand function  $p_i = LD/\lambda$ . Since consumers share the same preferences, the consumption of each variety is the same across consumers. Therefore, product market clearing implies  $q_i = Lx_i$ . Firm  $i$  accurately treats the market aggregates  $N$  and  $\lambda$ , which are endogenous, parametrically.

## 3 Market equilibrium

In this section, we characterize the market outcome when the number  $N$  of firms is exogenously given. This allows us to determine the equilibrium output, price and per variety consumption conditional upon  $N$ . In the next section, the zero-profit condition pins down the equilibrium number of firms.

When the number  $N$  of firms is given, a *market equilibrium* is given by the functions  $\bar{\mathbf{q}}(N)$ ,  $\bar{\mathbf{p}}(N)$  and  $\bar{\mathbf{x}}(N)$  defined on  $[0, N]$ , which satisfy the following four conditions: (i) no firm  $i$  can increase its profit by changing its output, (ii) each consumer maximizes her utility subject to her budget constraint, (iii) the product market clearing condition

$$\bar{q}_i = L\bar{x}_i \quad \text{for all } i \in [0, N]$$

and (iv) the labor market balance

$$c \int_0^N q_i di + NF = yL$$

hold.

### 3.1 Existence and uniqueness of a market equilibrium

The study of market equilibria, where the number of firms is exogenous, is to be viewed as an intermediate step toward monopolistic competition, where the number of firms is endogenized by free entry and exit. Since the focus is on symmetric free-entry equilibria, we find it reasonable to study symmetric market equilibria, which means that the functions  $\bar{\mathbf{q}}(N)$ ,  $\bar{\mathbf{p}}(N)$  and  $\bar{\mathbf{x}}(N)$  become scalars, i.e.,  $\bar{q}(N)$ ,  $\bar{p}(N)$  and  $\bar{x}(N)$ . For this, consumers must have the same income, which holds when profits are uniformly distributed across consumers. Therefore, the budget constraint (3) must be replaced by the following expression:

$$\int_0^N p_i x_i di = Y \equiv y + \frac{1}{L} \int_0^N \pi_i di. \quad (13)$$

Being negligible to the market, each firm accurately treats  $Y$  as a given.

Plugging  $D$  into (12), the program of firm  $i$  is given by

$$\max_{x_i} \pi(x_i, \mathbf{x}) \equiv \left[ \frac{D(x_i, \mathbf{x})}{\lambda} - c \right] L x_i - F.$$

Setting

$$D'_i \equiv \frac{\partial D(x_i, \mathbf{x})}{\partial x_i} \quad D''_i \equiv \frac{\partial D^2(x_i, \mathbf{x})}{\partial x_i^2},$$

the first-order condition for profit-maximization are given by

$$D(x_i, \mathbf{x}) + x_i D'_i = [1 - \bar{\eta}(x_i, \mathbf{x})] D(x_i, \mathbf{x}) = \lambda c, \quad (14)$$

where

$$\bar{\eta}(x_i, \mathbf{x}) \equiv -\frac{x_i D'_i}{D}$$

is the elasticity of the inverse demand for variety  $i$ . Note the difference with an oligopoly model: if the number of firms were discrete and finite, firms should account for their impact on  $\lambda$  (see 4.2.4 for an illustration).

In what follows, we determine the market equilibrium for a given  $N$ . Since  $\lambda$  is endogenous, we seek necessary and sufficient conditions for a unique (interior or corner) equilibrium to exist regardless of the value of  $\lambda c > 0$ . Indeed, finding conditions that depend upon  $c$  only turns out to be a very cumbersome task. The argument involves three steps.

**Step 1.** For (14) to have at least one positive solution  $\bar{x}(N)$  regardless of  $\lambda c > 0$ , it is sufficient to assume that, for any  $\mathbf{x}$ , the following Inada conditions hold:

$$\lim_{x_i \rightarrow 0} D = \infty \quad \lim_{x_i \rightarrow \infty} D = 0. \quad (15)$$

Indeed, since  $\bar{\eta}(0, \mathbf{x}) < 1$ , (15) implies that  $\lim_{x_i \rightarrow 0} (1 - \bar{\eta})D = \infty$ . Similarly, since  $0 < (1 - \bar{\eta})D < D$ , it ensues from (15) that  $\lim_{x_i \rightarrow \infty} (1 - \bar{\eta})D = 0$ . Because  $(1 - \bar{\eta})D$  is continuous, it follows from the intermediate value theorem that (14) has at least one positive solution. Note that (15) is sufficient, but not necessary. For example, if  $D$  displays a finite choke price exceeding the marginal cost, it is readily verified that (14) has at least one positive solution.

**Step 2.** The first-order condition (14) is sufficient if the profit function  $\pi$  is strictly quasi-concave in  $x_i$ . If the maximizer of  $\pi$  is positive and finite, the profit function is strictly quasi-concave in  $x_i$  for any positive value of  $\lambda c$  if and only if the second derivative of  $\pi$  is negative at any solution to the first-order condition. Since firm  $i$  treats  $\lambda$  parametrically, the second-order condition is given by

$$x_i D_i'' + 2D_i' < 0. \quad (16)$$

This condition means that firm  $i$ 's marginal revenue  $(x_i D_i' + D)L/\lambda$  is strictly decreasing in  $x_i$ . It is satisfied when  $D$  is concave, linear or not “too” convex in  $x_i$ . Furthermore, (16) is also a necessary and sufficient condition for the function  $\pi$  to be strictly quasi-concave for all  $\lambda c > 0$ , for otherwise there would exist a value  $\lambda c$  such that the marginal revenue curve intersects the horizontal line  $\lambda c$  more than once.

Observe also that (16) means that the revenue function is strictly concave. Since the marginal cost is independent of  $x_i$ , this in turn implies that  $\pi$  is strictly concave in  $x_i$ . In other words, when firms are quantity-setters, the profit function  $\pi$  is quasi-concave in  $x_i$  if and only if  $\pi$  is concave in  $x_i$  (see Appendix 4).

In sum, the profit function  $\pi$  is strictly quasi-concave in  $x_i$  for all values of  $\lambda c$  if and only if

**(A)** *firm  $i$ 's marginal revenue decreases in  $x_i$ .*

We show in Appendix 4 that **(A)** is equivalent to the well-known condition obtained by Caplin and Nalebuff (1991) for a firm's profits to be quasi-concave in its own price, i.e., the Marshallian demand  $\mathcal{D}$  is such that  $1/\mathcal{D}$  is convex in price. Since the Caplin-Nalebuff condition is the least stringent one for a firm's profit to be quasi-concave under price-setting firms, **(A)** is therefore the least demanding condition when firms compete in quantities.

Another sufficient condition commonly used in the literature is as follows (see, e.g., Krugman, 1979):

**(Abis)** *the elasticity of the inverse demand  $\bar{\eta}(x_i, x)$  increases in  $x_i$ .*

It is readily verified that **(Abis)** is equivalent to

$$-x_i \frac{D_i''}{D_i'} < 1 + \bar{\eta}.$$

whereas (16) is equivalent to



$$-x_i \frac{D_i''}{D_i'} < 2.$$

Since  $\bar{\eta} < 1$ , **(A<sub>bis</sub>)** implies **(A)**.

**Step 3.** Each firm facing the same demand and being negligible, the function  $\pi(x_i, \mathbf{x})$  is the same for all  $i$ . In addition, **(A)** implies that  $\pi(x_i, \mathbf{x})$  has a unique maximizer for any  $\mathbf{x}$ . As a result, the market equilibrium must be symmetric.

Using  $\pi_i \equiv (p_i - c)Lx_i - F$ , (13) boils down to labor market balance:

$$cL \int_0^N x_i di + FN = yL,$$

which yields the only candidate symmetric equilibrium for the per variety consumption:

$$\bar{x}(N) = \frac{y}{cN} - \frac{F}{cL}. \quad (17)$$

Therefore,  $\bar{x}(N)$  is unique and positive if and only if  $N \leq Ly/F$ . The product market clearing condition implies that the candidate equilibrium output is

$$\bar{q}(N) = \frac{yL}{cN} - \frac{F}{c}. \quad (18)$$

Plugging (18) into the profit maximization condition (22) shows that there is a unique candidate equilibrium price given by

$$\bar{p}(N) = c \frac{\sigma(\bar{x}(N), N)}{\sigma(\bar{x}(N), N) - 1}. \quad (19)$$

Clearly, if  $N > Ly/F$ , there exists no interior equilibrium. Accordingly, we have the following result: *If **(A)** holds and  $N \leq Ly/F$ , then there exists a unique market equilibrium. Furthermore, this equilibrium is symmetric.*

## 3.2 The elasticity of substitution

In this section, we define the elasticity of substitution, which will be central in our equilibrium analysis. To this end, we extend the definition proposed by Nadiri (1982, p.442) to the case of a continuum of goods.

Consider any two varieties  $i$  and  $j$  such that  $x_i = x_j = x$ . We show in Appendix 5 that the elasticity of substitution between  $i$  and  $j$ , conditional on  $\mathbf{x}$ , is given by

$$\bar{\sigma}(x, \mathbf{x}) = -\frac{D(x, \mathbf{x})}{x} \frac{1}{\frac{\partial D(x, \mathbf{x})}{\partial x}} = \frac{1}{\bar{\eta}(x, \mathbf{x})}. \quad (20)$$

Because the market outcome is symmetric, we may restrict the analysis to symmetric consump-

tion profiles:

$$\mathbf{x} = xI_{[0,N]}$$

and redefine  $\bar{\eta}(x_i, \mathbf{x})$  and  $\bar{\sigma}(x, \mathbf{x})$  as follows:

$$\eta(x, N) \equiv \bar{\eta}(x, xI_{[0,N]}) \quad \sigma(x, N) \equiv \bar{\sigma}(x, xI_{[0,N]}).$$

Furthermore, (20) implies that

$$\sigma(x, N) = 1/\eta(x, N). \quad (21)$$

Hence, along the diagonal, our original functional analysis problem boils down into a two-dimensional one.

Rewriting the equilibrium conditions (14) along the diagonal yields

$$\bar{m}(N) \equiv \frac{\bar{p}(N) - c}{\bar{p}(N)} = \eta(\bar{x}(N), N) = \frac{1}{\sigma(\bar{x}(N), N)}, \quad (22)$$

while

$$\bar{\pi}(N) \equiv (\bar{p}(N) - c)\bar{q}(N)$$

denotes the equilibrium operating profits made by a firm when there is a mass  $N$  of firms.

Importantly, (22) shows that, for any given  $N$ , *the equilibrium markup  $\bar{m}(N)$  varies inversely with the elasticity of substitution*. The intuition is easy to grasp. It is well known from industrial organization that product differentiation relaxes competition. When the elasticity of substitution is lower, varieties are worse substitutes, thereby endowing firms with more market power. It is, therefore, no surprise that firms have a higher markup when  $\sigma$  is lower. It also follows from (22) that the way  $\sigma$  varies with  $x$  and  $N$  shapes the market outcome. In particular, this demonstrates that assuming a constant elasticity of substitution amounts to adding very strong restraints on the way the market works.

Combining (17) and (19), we find that the operating profits are given by

$$\bar{\pi}(N) = \frac{cL\bar{x}(N)}{\sigma(\bar{x}(N), N) - 1}. \quad (23)$$

It is legitimate to ask how  $\bar{p}(N)$  and  $\bar{\pi}(N)$  vary with the mass of firms? There is no simple answer to this question. Indeed, the expression (23) suffices to show that the way the market outcome reacts to the entry of new firms depends on how the elasticity of substitution varies with  $x$  and  $N$ . This confirms why static comparative statics under oligopoly yields ambiguous results.

Thus, to gain insights about the behavior of  $\sigma$ , we give below the elasticity of substitution for the different types of preferences discussed in the previous section.

(i) When the utility is additive, we have:

$$\frac{1}{\sigma(x, N)} = r(x) \equiv -\frac{xu''(x)}{u'(x)}, \quad (24)$$

which means that  $\sigma$  depends only upon the per variety consumption when preferences are additive.

(ii) When the indirect utility is additive, it is shown in Appendix 6 that  $\sigma$  depends only upon the total consumption  $X = Nx$ . Since the budget constraint implies  $X = Y/p$ , (22) may be rewritten as follows:

$$\frac{1}{\sigma(x, N)} = \theta(X) \equiv -\frac{v'(1/X)}{v''(1/X)}X. \quad (25)$$

(iii) When preferences are homothetic, it ensues from Proposition 2 that

$$\frac{1}{\sigma(x, N)} = \varphi(N) \equiv \eta(1, N). \quad (26)$$

For example, under translog preferences, we have

$$\mathcal{D}(p_i, \mathbf{p}, Y(N)) = \frac{Y}{p_i} \left( \frac{1}{N} + \frac{\beta}{N} \int_0^N \ln p_j dj - \beta \ln p_i \right).$$

Therefore,  $\varphi(N) = 1/(1 + \beta N)$ .

Since both the direct and indirect CES utilities are additive, the elasticity of substitution is constant. Furthermore, since the CES is also homothetic, it must be that

$$r(x) = \theta(X) = \varphi(N) = \frac{1}{\sigma}.$$

It is, therefore, no surprise that the constant  $\sigma$  is the only demand side parameter that drives the market outcome under CES preferences.

As illustrated in Figure 1, the CES is the *sole* function that belongs to the above three classes of preferences. Furthermore, the expressions (24), (25) and (26) imply that the classes of additive, indirectly additive and homothetic preferences are disjoint, except for the CES that belongs to the three of them.

By contrast, the entropy utility does not belong to any of these three classes of preferences. Indeed, it is readily verified that

$$\sigma(x, N) = U'(Nx) + \ln N,$$

which decreases with  $x$ , whereas  $\sigma(x, N)$  is decreasing, U-shaped or increasing in  $N$ .

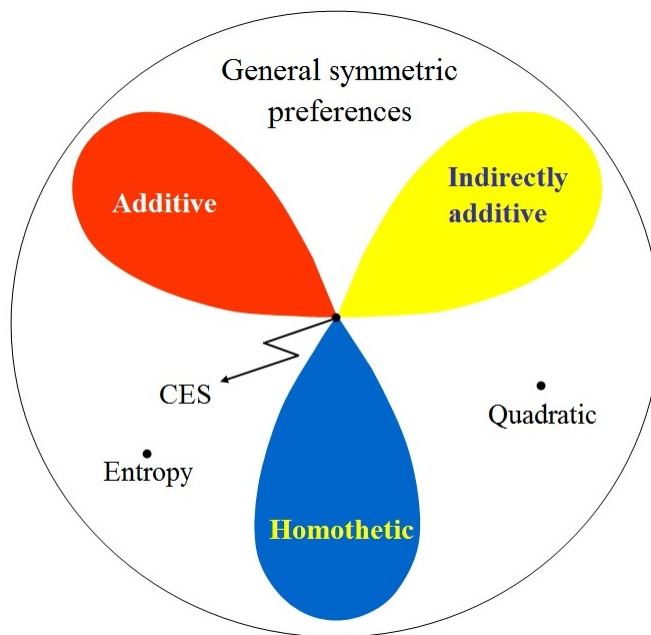


Fig. 1. The space of preferences

From now on, we consider the function  $\sigma(x, N)$  as the *primitive* of the model. There are two reasons for making this choice. First,  $\sigma(x, N)$  portrays what preferences are along the diagonal ( $x_i = x > 0$  for all  $i$ ). As a result, what matters for the equilibrium is how  $\sigma(x, N)$  varies with  $x$  and  $N$ . Second, the properties of the market outcome can be characterized by necessary and sufficient conditions stated in terms of the elasticity of  $\sigma$  with respect to  $x$  and  $N$ , which are denoted  $\mathcal{E}_x(\sigma)$  and  $\mathcal{E}_N(\sigma)$ . To be precise, the signs of these two expressions ( $\mathcal{E}_x(\sigma) \geq 0$  and  $\mathcal{E}_N(\sigma) \leq 0$ ) and their relationship ( $\mathcal{E}_x(\sigma) \geq \mathcal{E}_N(\sigma)$ ) will allow us to characterize *completely* the market equilibrium.

How  $\sigma$  varies with  $x$  is a priori not clear. Marshall (1920, Book 3, Chapter IV) has argued on intuitive grounds that the elasticity of the inverse demand  $\bar{\eta}(x_i, \mathbf{x})$  increases in sales.<sup>7</sup> In our setting, (20) shows that this assumption amounts to  $\partial \bar{\sigma}(x, \mathbf{x}) / \partial x < 0$ . However, this inequality does not tell us anything about the sign of  $\partial \sigma(x, N) / \partial x$  because  $x$  refers here to the consumption of *all* varieties. When preferences are additive, as in Krugman (1979), Marshall's argument can be applied because the marginal utility of a variety depends only upon its own consumption. But this ceases to be true when preferences are non-additive. Nevertheless, as will be seen in Section 4.3,  $\mathcal{E}_x(\sigma) < 0$  holds if and only if the pass-through is smaller than 100%. The literature on spatial pricing backs up this assumption, though it also recognizes the possibility of a pass-through exceeding 100% (Greenhut et al., 1987).

We now come to the relationship between  $\sigma$  and  $N$ . The literature in industrial organization suggests that varieties become closer substitutes when  $N$  increases, the reason being that adding new varieties crowds out the product space (Salop, 1979; Tirole, 1988). Therefore, assuming

<sup>7</sup>We thank Peter Neary for having pointed out this reference to us.

$\mathcal{E}_N(\sigma) > 0$  spontaneously comes to mind. As a consequence, the folk wisdom would be described by the following two conditions:

$$\mathcal{E}_x(\sigma) < 0 < \mathcal{E}_N(\sigma). \quad (27)$$

However, these inequalities turn out to be more restrictive than what they might seem at first glance. Indeed, they do not allow us to capture some interesting market effects and fail to encompass some standard models of monopolistic competition. For example, when preferences are quadratic, Bertolotti and Epifani (2014) have pointed out that the elasticity of substitution decreases with  $N$ :

$$\sigma(x, N) = \frac{\alpha - \beta x}{\beta x} - \frac{\gamma}{\beta} N.$$

This should not come as a surprise. Indeed, although spatial models of product differentiation and models of monopolistic competition are not orthogonal to each other, they differ in several respects. In particular, when consumers are endowed with a love for variety, they are inclined to spread their consumption over a wider range of varieties at the expense of their consumption of each variety. By contrast, in spatial models every consumer has a unique ideal variety. Therefore, providing a reconciliation of the two settings is not an easy task (Anderson et al., 1992). In what follows, we propose to study the impact of  $N$  on  $\sigma$  under the assumption that a consumer's total consumption  $Nx$  is arbitrarily fixed, as in spatial models of product differentiation, while allowing the per variety consumption  $x$  to vary with  $N$ , as in love-for-variety models.

In this case, it is readily verified that the following two relationships must hold simultaneously:

$$\begin{aligned} \frac{dx}{x} &= -\frac{dN}{N} \\ \frac{d\sigma}{\sigma} &= \frac{\partial\sigma}{\partial N} \frac{N}{\sigma} \frac{dN}{N} + \frac{\partial\sigma}{\partial x} \frac{x}{\sigma} \frac{dx}{x}. \end{aligned}$$

Plugging the first expression into the second, we obtain

$$\left. \frac{d\sigma}{dN} \right|_{Nx=const} = \frac{\sigma}{N} (\mathcal{E}_N(\sigma) - \mathcal{E}_x(\sigma)).$$

In this event, the elasticity of substitution increases with  $N$  if and only if

$$\mathcal{E}_x(\sigma) < \mathcal{E}_N(\sigma) \quad (28)$$

holds. This condition is less stringent than  $\partial\sigma/\partial N > 0$  because it allows the elasticity of substitution to decrease with  $N$ . In other words, entry may trigger more differentiation, perhaps because the incumbents react by adding new attributes to their products (Anderson et al., 1992). In addition, the evidence supporting the assumption  $\mathcal{E}_x(\sigma) < 0$  being mixed, we find it relevant to investigate the implications of the two cases,  $\mathcal{E}_x(\sigma) > 0$  and  $\mathcal{E}_x(\sigma) < 0$ . Note that  $\partial\sigma/\partial x = \partial\sigma/\partial N = 0$

in the CES case only.

## 4 Symmetric monopolistic competition

In the previous section, we have determined the equilibrium price, output and consumption conditional on the mass  $N$  of firms. Here, we pin down the equilibrium value of  $N$  by using the zero-profit condition. Therefore, a consumer's income is equal to her sole labor income:  $Y = y$ .

A *symmetric free-entry equilibrium* (SFE) is described by the vector  $(q^*, p^*, x^*, N^*)$ , where  $N^*$  solves the zero-profit condition

$$\bar{\pi}(N) = F, \quad (29)$$

while  $q^* = \bar{q}(N^*)$ ,  $p^* = \bar{p}(N^*)$  and  $x^* = \bar{x}(N^*)$ . The Walras Law implies that the budget constraint  $N^* p^* x^* = y$  is satisfied. Without loss of generality, we restrict ourselves to the domain of parameters for which  $N^* < \mathcal{N}$ .

Combining (22) and (29), we obtain a single equilibrium condition given by

$$\bar{m}(N) = \frac{NF}{Ly}, \quad (30)$$

which means that, at the SFE, the equilibrium markup is equal to the share of the labor supply spent on overhead costs. When preferences are non-homothetic, (17) and (19) show that  $L/F$  and  $y$  enter the function  $\bar{m}(N)$  as two distinct parameters. This implies that  $L$  and  $y$  have a different impact on the equilibrium markup, while a hike in  $L$  is equivalent to a drop in  $F$ .

### 4.1 Existence and uniqueness of a SFE

Differentiating (23) with respect to  $N$ , we obtain

$$\begin{aligned} \bar{\pi}'(N) &= \bar{x}'(N) \frac{d}{dx} \left[ \frac{cLx}{\sigma(x, yL/(cLx + F)) - 1} \right] \Big|_{x=\bar{x}(N)} \\ &= -\frac{y}{cN^2} \left( \sigma - 1 - x \frac{\partial \sigma}{\partial x} + \frac{cLx}{cLx + F} \frac{yL}{cLx + F} \frac{\partial \sigma}{\partial N} \right) \Big|_{x=\bar{x}(N)}. \end{aligned}$$

Using (17) and (29), the second term in the right-hand side of this expression is positive if and only if

$$\mathcal{E}_x(\sigma) < \frac{\sigma - 1}{\sigma} (1 + \mathcal{E}_N(\sigma)). \quad (31)$$

Therefore,  $\bar{\pi}'(N) < 0$  for all  $N$  if and only if (31) holds. This implies the following proposition.

**Proposition 3.** *Assume (A). There exists a unique free-entry equilibrium for all  $c > 0$  if and*

only if (31) holds. Furthermore, this equilibrium is symmetric.

Because the above proposition provides a necessary and sufficient condition for the existence of a SFE, we may safely conclude that the set of assumptions required to bring into play monopolistic competition must include (31). Therefore, throughout the remaining of the paper, we assume that (31) holds. This condition allows one to work with preferences that display a great of flexibility. Indeed,  $\sigma$  may decrease or increase with  $x$  and/or  $N$ . To be precise, varieties may become better or worse substitutes when the per variety consumption and/or the number of varieties rises, thus generating either price-decreasing or price-increasing competition. Evidently, (31) is satisfied when the folk wisdom conditions (27) hold.

Under additive preferences, (31) amounts to assuming that  $\mathcal{E}_x(\sigma) < (\sigma - 1)/\sigma$ , which means that  $\sigma$  cannot increase “too fast” with  $x$ . In this case, as shown by (30), there exists a unique SFE and the markup function  $m(N)$  increases with  $N$  provided that the slope of  $m$  is smaller than  $F/Ly$ . In other words, *a market mimicking anti-competitive effects need not preclude the existence and uniqueness of a SFE* (Zhelobodko et al., 2012). When preferences are homothetic, (31) holds if and only if  $\mathcal{E}_N(\sigma)$  exceeds  $-1$ , which means that varieties cannot become too differentiated when their number increases, which seems reasonable.

We consider (28) and (31) as our most preferred assumptions. The former, which states that the impact of a change in the number of varieties on  $\sigma$  dominates the impact of a change in the per variety consumption, points to the importance of the variety range for consumers, while the latter is a necessary and sufficient for the existence and uniqueness of a SFE. Taken together, (28) and (31) define a range of possibilities which is broader than the one defined by (27). We will refrain from following an encyclopedic approach in which all cases are systematically explored. However, since (28) need not hold for a SFE to exist, we will also explore what the properties of the equilibrium become when this condition is not met. In so doing, we are able to highlight the role played by (28) for some particular results to hold.

## 4.2 Comparative statics

In this subsection, we study the impact of a higher gross domestic product on the SFE. A higher total income may stem from a larger population  $L$ , a higher per capita income  $y$ , or both. Next, we will discuss the impact of firm’s productivity. To achieve our goal, it proves to be convenient to work with the markup as the endogenous variable. Setting  $m \equiv FN/(Ly)$ , we may rewrite the equilibrium condition (30) as a function of  $m$  only:

$$m\sigma \left( \frac{F}{cL} \frac{1-m}{m}, \frac{Ly}{F} m \right) = F. \quad (32)$$

Note that (32) involves the four structural parameters of the economy:  $L$ ,  $y$ ,  $c$  and  $F$ . Furthermore, it is readily verified that the left-hand side of (32) increases with  $m$  if and only if (31)

holds. Therefore, to study the impact of a specific parameter, we only have to find out how the corresponding curve is shifted.

Before proceeding, we want to stress that we provide below a complete description of the comparative static effects through a series of necessary and sufficient conditions. Doing this allows us to single out what seems to be the most plausible assumptions.

#### 4.2.1 The impact of population size

Let us first consider the impact on the market price  $p^*$ . Differentiating (32) with respect to  $L$ , we find that the right-hand side of (32) is shifted upwards under an increase in  $L$  if and only if (28) holds. As a consequence, the equilibrium markup  $m^*$ , whence the equilibrium price  $p^*$ , decreases with  $L$ . This is in accordance with Handbury and Weinstein (2013) who observe that the price level for food products falls with city size. In this case, (32) implies that the equilibrium value of  $\sigma$  increases, which amounts to saying that varieties get less differentiated in a larger market, very much like in spatial models of product differentiation.

Second, the zero-profit condition implies that  $L$  always shifts  $p^*$  and  $q^*$  in opposite directions. Therefore, firm sizes are larger in bigger markets, as suggested by the empirical evidence provided by Manning (2010).

How does  $N^*$  change with  $L$ ? Differentiating (23) with respect to  $L$ , we have

$$\left. \frac{\partial \bar{\pi}}{\partial L} \right|_{N=N^*} = \frac{cx}{\sigma(x, N) - 1} + \left. \frac{\partial \bar{x}(N)}{\partial L} \frac{\partial}{\partial x} \left( \frac{cLx}{\sigma(x, N) - 1} \right) \right|_{x=x^*, N=N^*}. \quad (33)$$

Substituting  $F$  for  $\bar{\pi}(N^*)$  and simplifying, we obtain

$$\left. \frac{\partial \bar{\pi}}{\partial L} \right|_{N=N^*} = \left[ \frac{cx\sigma}{(\sigma - 1)^3} (\sigma - 1 - \mathcal{E}_x(\sigma)) \right] \Big|_{x=x^*, N=N^*}.$$

Since the first term in the right-hand side of this expression is positive, (33) is positive if and only if the following condition holds:

$$\mathcal{E}_x(\sigma) < \sigma - 1. \quad (34)$$

In this case, a population growth triggers the entry of new firms. Furthermore, restating (30) as  $N/m(N) = Ly/F$ , it is readily verified that the increase in  $N^*$  is less proportional than the population hike if and only if  $m'(N) < 0$ , which is equivalent to (28).

Observe that (31) implies (34) when preferences are (indirectly) additive, while (34) holds true under homothetic preferences because  $\mathcal{E}_x(\sigma) = 0$ .

It remains to determine how the per variety consumption level  $x^*$  varies with an increase in population size. Combining (19) with the budget constraint  $x = y/(pN)$ , we obtain



$$\frac{Nx\sigma(x, N)}{\sigma(x, N) - 1} = \frac{y}{c}. \quad (35)$$

Note that  $L$  does not enter (35) as an independent parameter. Furthermore, it is straightforward to check that the left-hand side of (35) increases with  $x$  when (34) holds, and decreases otherwise. Combining this with the fact that (34) is also necessary and sufficient for an increase in  $L$  to trigger additional entry, the per variety consumption level  $x^*$  decreases with  $L$  if and only if the left-hand side of (35) increases with  $N$ , or, equivalently, if and only if

$$\mathcal{E}_N(\sigma) < \sigma - 1. \quad (36)$$

This condition holds if  $\sigma$  decreases with  $N$  or increases with  $N$ , but not “too fast,” which means that varieties do not get too differentiated with the entry of new firms. Note also that (28) and (36) imply (34). Evidently, (36) holds for (i) additive preferences, for in this case  $\mathcal{E}_N(\sigma) = 0$ , while  $\sigma > 1$ ; (ii) indirectly additive preferences, because, using (34) and  $\sigma(x, N) = 1/\theta(xN)$ , we obtain  $1 + \mathcal{E}_N(\sigma) = 1 + \mathcal{E}_x(\sigma) < \sigma$ ; and (iii) any preferences such that  $\sigma$  weakly decreases with  $N$ .

The following proposition comprises a summary.

**Proposition 4.** *If  $\mathcal{E}_x(\sigma)$  is smaller than  $\mathcal{E}_N(\sigma)$ , then a higher population size results in a lower markup and larger firms. Furthermore, if (36) holds, the mass of varieties increases less than proportionally with  $L$ , while the per variety consumption decreases with  $L$ .*

Note that the mass of varieties need not rise with the population size. Indeed,  $N^*$  falls when  $\mathcal{E}_N(\sigma)$  exceeds  $\sigma - 1$ . In this case, increasing the number of firms makes varieties very close substitutes, which strongly intensifies competition among firms. Under such circumstances, the benefits associated with diversity are low, thus implying that consumers value more and more the volumes they consume. This in turn leads a fraction of incumbents to get out of business.

When preferences are homothetic,  $\sigma$  depends upon  $N$  only. In this case, (35) boils down to

$$1 + \frac{N\varphi'(N)}{1 - \varphi(N)} > 0.$$

When  $\varphi'(N) < 0$ , this inequality need not hold. However, in the case of the translog where  $\varphi(N) = 1/(1 + \beta N)$ , (35) is satisfied, and thus  $x^*$  decreases with  $L$ .

What happens when  $\mathcal{E}_x(\sigma) > \mathcal{E}_N(\sigma)$ ? In this event, (31) implies that (36) holds. Therefore, the above necessary and sufficient conditions imply the following result: If  $\mathcal{E}_x(\sigma) < \mathcal{E}_N(\sigma)$ , then a higher population size results in a higher markup, smaller firms, a more than proportional rise in the mass of varieties, and a lower per variety consumption. As a consequence, a larger market may generate anti-competitive effects that take the concrete form of a higher market price and less efficient firms producing at a higher average cost. Such results are at odds with the main body of industrial organization, which explains why (28) is one of our most preferred conditions.

## 4.2.2 The impact of individual income

We now come to the impact of the per capita income on the SFE. One expects a positive shock on  $y$  to trigger the entry of new firms because more labor is available for production. However, consumers have a higher willingness-to-pay for the incumbent varieties and can afford to buy each of them in a larger volume. Therefore, the impact of  $y$  on the SFE is a priori ambiguous.

Differentiating (32) with respect to  $y$ , we see that the left-hand side of (32) is shifted downwards by an increase in  $y$  if and only if  $\mathcal{E}_N(\sigma) > 0$ . In this event, the equilibrium markup decreases with  $y$ .

To check the impact of  $y$  on  $N^*$ , we differentiate (23) with respect to  $y$  and get

$$\left. \frac{\partial \bar{\pi}(N)}{\partial y} \right|_{N=N^*} = \left[ \frac{\partial \bar{x}(N)}{\partial y} \frac{\partial}{\partial x} \left( \frac{cLx}{\sigma(x, N) - 1} \right) \right] \Big|_{x=x^*, N=N^*}.$$

After simplification, this yields

$$\left. \frac{\partial \bar{\pi}(N)}{\partial y} \right|_{N=N^*} = \frac{L}{N} \frac{\sigma - 1 - \sigma \mathcal{E}_x(\sigma)}{(\sigma - 1)^2} \Big|_{x=x^*, N=N^*}.$$

Hence,  $\partial \bar{\pi}(N^*)/\partial y > 0$  if and only if the following condition holds:

$$\mathcal{E}_x(\sigma) < \frac{\sigma - 1}{\sigma}. \quad (37)$$

Note that this condition is more stringent than (34). Thus, if  $\mathcal{E}_N(\sigma) > 0$ , then (37) implies (31).

As a consequence, we have:

**Proposition 5.** *If  $\mathcal{E}_N(\sigma) > 0$ , then a higher per capita income results in a lower markup and bigger firms. Furthermore, the mass of varieties increases with  $y$  if (37) holds, but decreases with  $y$  otherwise.*

Thus, when entry renders varieties less differentiated, the mass of varieties need not rise with income. Indeed, the increase in per variety consumption may be too high for all the incumbents to stay in business. The reason for this is that the decline in prices is sufficiently strong for fewer firms to operate at a larger scale. As a consequence, a richer economy need not exhibit a wider array of varieties.

Evidently, if  $\mathcal{E}_N(\sigma) < 0$ , the markup is higher and firms are smaller when the income  $y$  rises. Furthermore, (31) implies (37) so that  $N^*$  increases with  $y$ . Indeed, since varieties get more differentiated when entry arises, firms exploit consumers' higher willingness-to-pay to sell less at a higher price, which goes together with a larger mass of varieties.

Propositions 4 and 5 show that an increase in  $L$  is not a substitute for an increase in  $y$  and vice versa, except, as shown below, in the case of homothetic preferences. This should not come as a surprise because an increase in income affects the shape of individual demands when preferences are non-homothetic, whereas an increase in  $L$  shifts upward the market demand without changing

its shape.

Finally, observe that using (indirectly) additive utilities allows capturing the effects generated by shocks on population size (income), but disregard the impact of the other magnitude. If preferences are homothetic, it is well known that the effects of  $L$  and  $y$  on the market variables  $p^*$ ,  $q^*$  and  $N^*$  are exactly the same. Indeed,  $m$  does not involve  $y$  as a parameter because  $\sigma$  depends solely on  $N$ . Therefore, it ensues from (30) that the equilibrium price, firm size, and number of firms depend only upon the total income  $yL$ .

### 4.2.3 The impact of firm productivity

Firms' productivity is typically measured by their marginal costs. To uncover the impact on the market outcome of a productivity shock common to all firms, we conduct a comparative static analysis of the SFE with respect to  $c$  and show that *the nature of preferences determines the extent of the pass-through*. In particular, we establish that the pass-through rate is lower than 100% if and only if  $\sigma$  decreases with  $x$ , i.e.

$$\mathcal{E}_x(\sigma) < 0 \tag{38}$$

holds. Evidently, the pass-through rate exceeds 100% when  $0 < \mathcal{E}_x(\sigma)$ .

Figure 2 depicts (30). It is then straightforward to check that, when  $\sigma$  decreases with  $x$ , a drop in  $c$  moves the vertical line rightward, while the  $p^*$ -locus is shifted downward. As a consequence, the market price  $p^*$  decreases with  $c$ . But by how much does  $p^*$  decrease relative to  $c$ ?

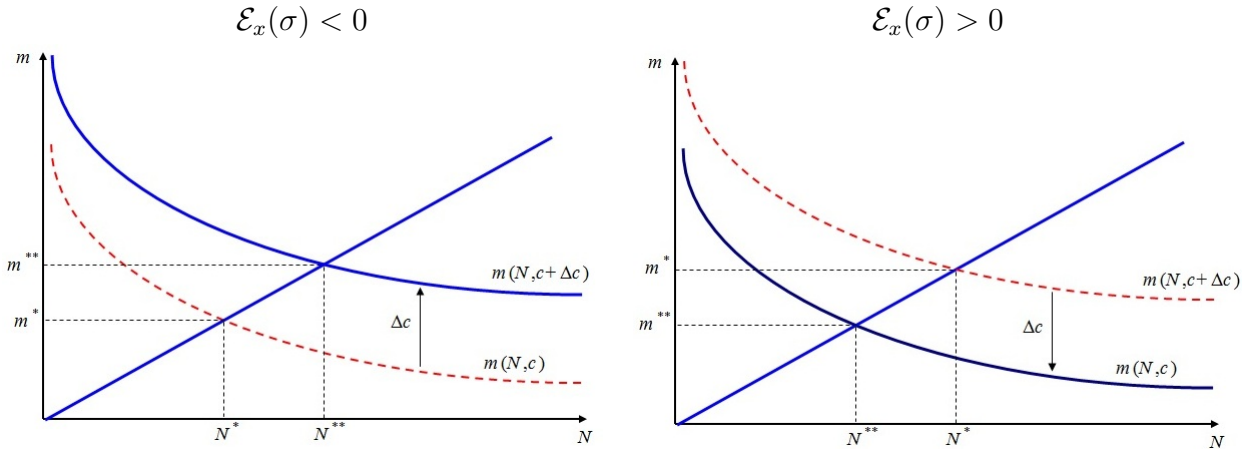


Fig. 2. Productivity and entry.

The left-hand side of (32) is shifted downwards under a decrease in  $c$  if and only if  $\mathcal{E}_x(\sigma) < 0$ . In this case, both the equilibrium markup  $m^*$  and the equilibrium mass of firms  $N^* = (yL/F) \cdot m^*$  increases with  $c$ . In other words, when  $\mathcal{E}_x(\sigma) < 0$ , *the pass-through rate is smaller than 1* because varieties becomes more differentiated, which relaxes competition. On the contrary, when  $\mathcal{E}_x(\sigma) > 0$ ,

the markup and the mass of firms decrease because varieties get less differentiated. In other words, competition becomes so tough that  $p^*$  decreases more than proportionally with  $c$ . In this event, *the pass-through rate exceeds 1*.

Under homothetic preferences, ( $\mathcal{E}_x(\sigma) = 0$ ),  $\bar{p}(N)$  is given by

$$\bar{p}(N) = \frac{c}{1 - \varphi(N)} \implies m(N) = \varphi(N).$$

As a consequence, (30) does not involve  $c$  as a parameter. This implies that a technological shock does affect the number of firms. In other words, the markup remains the same regardless of the productivity shocks, thereby implying that *under homothetic preferences the pass-through rate equal to 1*.

The impact of technological shocks on firms' size leads to ambiguous conclusions. For example, under quadratic preferences,  $q^*$  may increase and, then, decreases in response to a steadily drop in  $c$ .

The following proposition comprises a summary.

**Proposition 6.** *If the marginal cost of firms decreases, (i) the market price decreases and (ii) the markup and number of firms increase if and only if (38) holds.*

This proposition has an important implication. If the data suggest a pass-through rate smaller than 1, then it must be that  $\mathcal{E}_x(\sigma) < 0$ . In this case, (34) must hold while (31) is satisfied when  $\mathcal{E}_N(\sigma)$  exceeds  $-1$ , thereby a bigger or richer economy is more competitive and more diversified than a smaller or poorer one. Note that (31) does not restrict the domain of admissible values of  $\mathcal{E}_x(\sigma)$  for a pass-through rate to be smaller than 1, whereas (31) requires that  $\mathcal{E}_x(\sigma)$  cannot exceed  $(1 - 1/\sigma)(1 + \mathcal{E}_N(\sigma))$ . Recent empirical evidence shows that the pass-through generated by a commodity tax or by trade costs need not be smaller than 1 (Martin, 2012, and Weyl and Fabinger, 2013). Our theoretical argument thus concurs with the inconclusive empirical evidence: the pass-through rate may exceed 1, although it is more likely to be less than 1.

Let us make a pause and summarize what our main results are. We have found necessary and sufficient conditions for the existence and uniqueness of a SFE (Proposition 3), and provided a complete characterization of the effect of a market size or productivity shock (Propositions 4 to 6). Observing that (27) implies (31), (28) and (37), we may conclude that a unique SFE exists (Proposition 3), that a larger market or a higher income leads to lower markups, bigger firms and a larger number of varieties (Propositions 4 and 5), and that the pass-through rate is smaller than 1 (Proposition 6) *if* (27) holds. Recall, however, that less stringent conditions are available for each of the above-mentioned properties to be satisfied separately.

#### 4.2.4 Monopolistic versus oligopolistic competition

It should be clear that Propositions 4-6 have the same nature as results obtained in similar comparative analyses conducted in oligopoly theory (Vives, 1999). They may also replicate less standard

anti-competitive effects under some specific conditions (Chen and Riordan, 2008).

The markup (22) stems directly from preferences through the sole elasticity of substitution because we focus on monopolistic competition. However, in symmetric oligopoly models the markup emerges as the outcome of the interplay between preferences *and* strategic interactions. Nevertheless, at least to a certain extent, both settings can be reconciled.

To illustrate, consider the case of an integer number  $N$  of quantity-setting firms and of an arbitrary utility  $U(x_1, \dots, x_N)$ . The inverse demands are given by

$$p_i = \frac{U_i}{\lambda} \quad \lambda = \frac{1}{Y} \sum_{j=1}^N x_j U_j.$$

Assume that firms do not manipulate consumers' income through profit redistribution. Firm  $i$ 's profit-maximization condition is then given by

$$\frac{p_i - c}{p_i} = -\frac{x_i U_{ii}}{U_i} + \mathcal{E}_{x_i}(\lambda) = -\frac{x_i U_{ii}}{U_i} + \frac{x_i U_i + \sum_{j=1}^N x_i x_j U_{ij}}{\sum_{j=1}^N x_j U_j}. \quad (39)$$

Set

$$r_o(x, N) \equiv -\frac{x U_{ii}(x, \dots, x)}{U_i(x, \dots, x)} \quad r_c(x, N) \equiv \frac{x U_{ij}(x, \dots, x)}{U_i(x, \dots, x)} \quad \text{for } j \neq i.$$

The symmetry of preferences implies that  $r_o(x, N)$  and  $r_c(x, N)$  are independent of  $i$  and  $j$ .

Combining (39) with the symmetry condition  $x_i = x$ , we obtain the markup condition:

$$\frac{p - c}{p} = \frac{1}{N} + \left(1 - \frac{1}{N}\right) [r_o(x, N) + r_c(x, N)]. \quad (40)$$

The elasticity of substitution  $s_{ij}$  between varieties  $i$  and  $j$  is given by (see Appendix 5)

$$s_{ij} = -\frac{U_i U_j (x_i U_j + x_j U_i)}{x_i x_j [U_{ii} U_j^2 - 2U_{ij} U_i U_j + U_{jj} U_i^2]}. \quad (41)$$

When the consumption pattern is symmetric, (41) boils down to

$$s(x, N) = \frac{1}{r_o(x, N) + r_c(x, N)}. \quad (42)$$

Combining (40) with (42), we get

$$\frac{p - c}{p} = \frac{1}{N} + \left(1 - \frac{1}{N}\right) \frac{1}{s(x, N)}. \quad (43)$$

Unlike the profit-maximization condition (43), product and labor market balance, as well as the zero-profit condition, do not depend on strategic considerations. Since

$$\frac{p-c}{p} = \frac{1}{\sigma(x, N)} \quad (44)$$

under monopolistic competition, comparing (44) with (43) shows that *the set of Cournot symmetric free-entry equilibria is the same as the set of equilibria obtained under monopolistic competition if and only if  $\sigma(x, N)$  is given by*

$$\frac{1}{\sigma(x, N)} = \frac{1}{N} + \left(1 - \frac{1}{N}\right) \frac{1}{s(x, N)}.$$

As a consequence, by choosing appropriately the elasticity of substitution as a function of  $x$  and  $N$ , monopolistic competition is able to replicate not only the direction of comparative static effects generated in symmetric oligopoly models with free entry, but also their magnitude. Hence, we find it to say that *monopolistic competition under non-separable preferences mimics oligopolistic competition.*

### 4.3 When is the SFE socially optimal?

The social planner faces the following optimization problem:

$$\max \mathcal{U}(\mathbf{x}) \quad \text{s.t.} \quad Ly = cL \int_0^N x_i di + NF.$$

The first-order condition with respect to  $x_i$  implies that the problem may be treated using symmetry, so that the above problem may be reformulated as maximizing

$$\phi(x, N) \equiv \mathcal{U}(xI_{[0, N]})$$

subject to  $Ly = N(cLx + F)$ .

The ratio of the first-order conditions with respect to  $x$  and  $N$  leads to

$$\frac{\phi_x}{\phi_N} = \frac{NcL}{cLx + F}. \quad (45)$$

It is well known that the comparison of the social optimum and market outcome leads to ambiguous conclusions for the reasons provided by Spence (1976). We illustrate here this difficulty in the special case of homothetic preferences. Without loss of generality, we can write  $\phi(N, x)$  as follows:

$$\phi(N, x) = N\psi(N)x,$$

where  $\psi(N)$  is an increasing function of  $N$ . In this event, we get  $\phi_x x / \phi = 1$  and  $\phi_N N / \phi = 1 + N\psi' / \psi$ . Therefore, (45) becomes

$$\mathcal{E}_N(\psi) = \frac{F}{cLx},$$

while the market equilibrium condition (30) is given by

$$\frac{\varphi(N)}{1 - \varphi(N)} = \frac{F}{cLx}.$$

The social optimum and the market equilibrium are identical if and only if

$$\mathcal{E}_N(\psi) = \frac{\varphi(N)}{1 - \varphi(N)}. \quad (46)$$

It should be clear that this condition is unlikely to be satisfied unless strong restrictions are imposed on the utility. To be concrete, denote by  $A(N)$  the solution to

$$\mathcal{E}_N(A) + \mathcal{E}_N(\psi) = \frac{\varphi(N)}{1 - \varphi(N)},$$

which is unique up to a positive coefficient. It is then readily verified that (46) holds for all  $N$  if and only if  $\phi(x, N)$  is replaced with  $A(N)\phi(x, N)$ . Thus, contrary to the folk wisdom, the equilibrium and the optimum may be the same for utility functions that differ from the CES (Dhingra and Morrow, 2013). This finding has an unexpected implication.

**Proposition 7.** *If preferences are homothetic, there exists a consumption externality for which the equilibrium and the optimum coincide regardless of the values taken by the parameters of the economy.*

Hence, the choice of a particular consumption externality has subtle welfare implications, which are often disregarded in the literature. For example, if we multiply  $A(N)$  by  $N^\nu$ , where  $\nu$  is a constant, there is growing under-provision of varieties when the difference  $\nu - 1/(\sigma - 1) < 0$  falls, and growing over-provision when  $\nu - 1/(\sigma - 1) > 0$  rises. This has the following unsuspected implication: preferences given by  $A(N)\phi(x, N)$  are cardinal in nature. Indeed, taking a power transformation of  $N^\nu\phi(x, N)$  makes the discrepancy between the equilibrium and the optimum can be made arbitrarily large or small by choosing the appropriate value of the exponent  $\nu$ .

## 5 Extensions

In this section, we extend our baseline model to heterogeneous firms. We then consider the case of a multisector economy and conclude with a discussion of a setting in which consumers are heterogeneous.

### 5.1 Heterogeneous firms

It is natural to ask whether the approach developed in this paper can cope with Melitz-like heterogeneous firms? In what follows, we consider the one-period framework used by Melitz and

Ottaviano (2008), the mass of potential firms being given by  $\mathcal{N}$ . Prior to entry, risk-neutral firms face uncertainty about their marginal cost while entry requires a sunk cost  $F_e$ . Once this cost is paid, firms observe their marginal cost drawn randomly from the continuous probability distribution  $\Gamma(c)$  defined over  $\mathbb{R}_+$ . After observing its type  $c$ , each entrant decides to produce or not, given that an active firm must incur a fixed production cost  $F$ . Under such circumstances, the mass of entrants,  $N_e$ , typically exceeds the mass of operating firms,  $N$ . Even though varieties are differentiated from the consumer's point of view, firms sharing the same marginal cost  $c$  behave in the same way and earn the same profit at equilibrium. As a consequence, we may refer to any variety/firm by its  $c$ -type only.

The equilibrium conditions are as follows:

(i) the profit-maximization condition for  $c$ -type firms:

$$\max_{x_c} \Pi_c(x_c, \mathbf{x}) \equiv \left[ \frac{D(x_c, \mathbf{x})}{\lambda} - c \right] Lx_c - F; \quad (47)$$

(ii) the zero-profit condition for the cutoff firm:

$$(p_{\bar{c}} - \bar{c})q_{\bar{c}} = F,$$

where  $\bar{c}$  is the cutoff cost. As will be shown below, firms are sorted out by decreasing order of productivity, which implies that the mass of active firms is equal to  $N \equiv N_e \Gamma(\bar{c})$ ;

(iii) the product market clearing condition:

$$q_c = Lx_c$$

for all  $c \in [0, \bar{c}]$ ;

(iv) the labor market clearing condition:

$$N_e \left[ F_e + \int_0^{\bar{c}} (cq_c + F) d\Gamma(c) \right] = yL,$$

where  $N_e$  is the number of entrants;

(v) firms enter the market until their expected profits net of the entry cost  $F_e$  are zero:

$$\int_0^{\bar{c}} \Pi_c(x_c, \mathbf{x}) d\Gamma(c) = F_e.$$

When  $c_i > c_j$ , (47) implies

$$\left[ \frac{D(x_{c_i}, \mathbf{x})}{\lambda} - c_i \right] Lx_i < \left[ \frac{D(x_{c_j}, \mathbf{x})}{\lambda} - c_j \right] Lx_{c_j},$$

so that there is *perfect sorting across firms' types* at any equilibrium, while *firms with a higher*



productivity earn higher profits.

It also follows from (47) that  $p_c [1 - \bar{\eta}(x_c, \mathbf{x})] = c$ . Combining this with the inverse demands yields

$$D(x_c, \mathbf{x}) [1 - \bar{\eta}(x_c, \mathbf{x})] = \lambda c. \quad (48)$$

Dividing (48) for a type- $c_i$  firm by the same expression for a type- $c_j$  firm, we obtain

$$\frac{D(x_{c_i}, \mathbf{x}) [1 - \bar{\eta}(x_{c_i}, \mathbf{x})]}{D(x_{c_j}, \mathbf{x}) [1 - \bar{\eta}(x_{c_j}, \mathbf{x})]} = \frac{c_i}{c_j}. \quad (49)$$

The condition **(A)** of Section 3 means that, for any given  $\mathbf{x}$ , a firm's marginal revenue  $D(x, \mathbf{x}) [1 - \bar{\eta}(x, \mathbf{x})]$  decreases with  $x$ . Therefore, it ensues from (49) that  $x_i > x_j$  if and only if  $c_i < c_j$ . In other words, more efficient firms produce more than less efficient firms. Furthermore, since  $p_i = D(x, \mathbf{x})/\lambda$  and  $D$  decreases in  $x$  for any given  $\mathbf{x}$ , more efficient firms sell at lower prices than less efficient firms. As for the markups, (49) yields

$$\frac{p_i/c_i}{p_j/c_j} = \frac{1 - \bar{\eta}(x_{c_j}, \mathbf{x})}{1 - \bar{\eta}(x_{c_i}, \mathbf{x})}.$$

Consequently, more efficient firms enjoy higher markups – a regularity observed in the data (De Loecker and Warzynski, 2012) – if and only if  $\bar{\eta}(x, \mathbf{x})$  increases with  $x$ , i.e., **(A<sub>bis</sub>)** holds (see Section 3.1).

The following proposition is a summary.

**Proposition 8.** *Assume that firms are heterogeneous. If **(A)** holds, then more efficient firms produce larger outputs, charge lower prices and earn higher profits than less efficient firms. Furthermore, more efficient firms have higher markups if **(A<sub>bis</sub>)** holds, but lower markups otherwise.*

From now on, we assume that, for any  $\bar{c}$  and  $N_e$ , there exists a unique equilibrium  $\bar{\mathbf{x}}(\bar{c}, N_e)$  of the quantity game. Since the distribution  $G$  is given, the profit-maximization condition implies that  $\mathbf{x}^*$  is entirely determined by  $\bar{c}$  and  $N_e$ . In other words, regardless of the nature of preferences and the distribution of marginal costs, the heterogeneity of firms amounts to replacing the variable  $N$  by the two variables  $\bar{c}$  and  $N_e$ . As for  $x$ , it is replaced by  $c$  because the  $c$ -type firms sell at a price that depends on  $c$ , thus making  $c$ -specific the consumption of the corresponding varieties. As a consequence, the complexity of the problem goes from two to three dimensions. The equilibrium operating profits of the quantity game may thus be written as follows:

$$\Pi_c^*(\bar{c}, N_e) \equiv \max_{x_c} \left[ \frac{D(x_c, \mathbf{x}^*(\bar{c}, N_e))}{\lambda(\mathbf{x}^*(\bar{c}, N_e))} - c \right] Lx_c.$$

We have seen that  $\Pi_c^*(\bar{c}, N_e)$  decreases with  $c$ . In what follows, we impose the additional two conditions that rule out anti-competitive market outcomes.

**(B)** *The equilibrium profit of each firm's type decreases in  $\bar{c}$  and  $N_e$ .*

The intuition behind this assumption is easy to grasp: a larger number of entrants or a higher cutoff leads to lower profits, for, in both cases, the mass of active firms  $N$  rises. To illustrate, consider first the case of CES preferences where the equilibrium profits are given by

$$\Pi_c^*(\bar{c}, N_e) = \frac{yL}{\sigma N_e} \frac{c^{1-\sigma}}{\int_0^{\bar{c}} z^{1-\sigma} dG(z)}, \quad (50)$$

which implies **(B)**. More generally, using Zhelobodko et al. (2012), it is readily verified that any additive preference satisfies **(B)**. The same holds for indirectly additive preferences. To see this, observe that (50) boils down to

$$\Pi_c^*(\bar{c}, N_e) = \frac{yL}{N_e} \frac{v'(\bar{p}(c, y)/y) [\bar{p}(c, y) - c]}{\int_0^{\bar{c}} v'(\bar{p}(z, y)/y) \bar{p}(z, y) dG(z)}, \quad (51)$$

where  $\bar{p}(c, y)$  is the  $c$ -type firm profit-maximizing price solving the first-order condition  $p(1 - \theta(y/p)) = c$ . Clearly, **(B)** follows from (51).

In what follows, we assume that, for any  $\bar{c}$  and  $N_e$ , there exists a unique equilibrium  $\bar{x}(\bar{c}, N_e)$  of the quantity game and allow for the endogenous determination of  $\bar{c}$  and  $N_e$ .

**Free-entry equilibria.** A free-entry equilibrium with heterogeneous firms is defined by a pair  $(\bar{c}^*, N_e^*)$  that satisfies the zero-expected-profit condition for each firm

$$\int_0^{\bar{c}} [\Pi_c^*(\bar{c}, N_e) - F] d\Gamma(c) = F_e, \quad (52)$$

as well as the cutoff condition

$$\Pi_{\bar{c}}^*(\bar{c}, N_e) = F. \quad (53)$$

Dividing (52) by (53) yields the following new equilibrium condition:

$$\int_0^{\bar{c}} \left[ \frac{\Pi_c^*(\bar{c}, N_e)}{\Pi_{\bar{c}}^*(\bar{c}, N_e)} - 1 \right] d\Gamma(c) = \frac{F_e}{F}. \quad (54)$$

Let  $\bar{c} = g(N_e)$  be the locus of solutions to (53) and  $\bar{c} = h(N_e)$  the locus of solutions to (54).<sup>8</sup> A free-entry equilibrium is thus an intersection point of the two loci  $\bar{c} = g(N_e)$  and  $\bar{c} = h(N_e)$  in the  $(N_e, \bar{c})$ -plane. As implied by **(B)**,  $g(N_e)$  is downward-sloping in the  $(N_e, \bar{c})$ -plane. Furthermore, it is shifted upward when  $L$  rises. As for  $h(N_e)$ , it is independent of  $L$  but its slope is a priori undetermined. Yet, we will see that the sign of the slope of  $h(N_e)$  is critical for the impact of a population hike on the cutoff cost.

Three cases may arise. First, if the locus  $h(N_e)$  is upward-sloping, there exists a unique free-

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<sup>8</sup>We give below sufficient conditions for the left-hand side of (54) to be monotone in  $\bar{c}$  and  $N_e$ , two conditions that guarantee that the locus  $\bar{c} = h(N_e)$  is well defined.

entry equilibrium, and this equilibrium is stable. Furthermore, both  $N_e^*$  and  $\bar{c}^*$  increase with  $L$  (see Figure 3a). Let us now rotate the locus  $h(N_e)$  in a clockwise manner, so that  $h(N_e)$  gets less and less steep. The second case, which corresponds to the CES preferences, arises when  $h(N_e)$  is horizontal, which implies that  $N_e^*$  rises with  $L$  while  $\bar{c}^*$  remains constant.

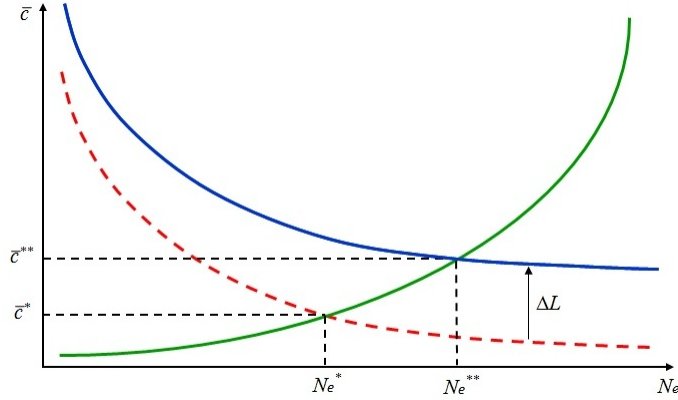


Fig. 3a. Cutoff and market size

Last, when  $h(N_e)$  is downward-sloping, two subcases must be distinguished. In the first one,  $h(N_e)$  is less steep than  $g(N_e)$ . As a consequence, there still exists a unique free-entry equilibrium. This equilibrium is stable and such that  $N_e^*$  increases with  $L$ , but  $\bar{c}^*$  now decreases with  $L$  (see Figure 3b). In the second subcase,  $h(N_e)$  is steeper than  $g(N_e)$ , which implies that the equilibrium is unstable because  $h(N_e)$  intersects  $g(N_e)$  from below. In what follows, we focus only upon stable equilibria.

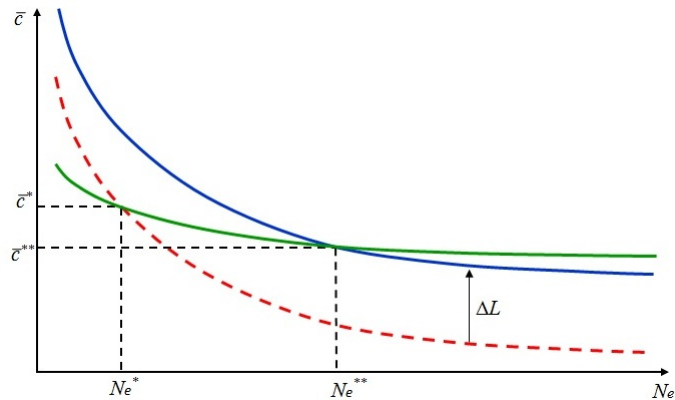


Fig. 3b. Cutoff and market size

In sum, we end up with the following properties of the free-entry equilibrium: (i) *the equilibrium mass of entrants always increases with  $L$*  but (ii) *the equilibrium cutoff cost may increase or decrease with  $L$* .

When firms are symmetric, we have seen that the sign of  $\mathcal{E}_N(\sigma(x, N))$  plays a critical role in comparative statics. The same holds here. The difference is that the mass of active firms is now determined by  $\bar{c}$  and  $N_e$  because  $N = \Gamma(\bar{c})N_e$ . As a consequence, how the mass of active firms responds to a population hike depends on the way the left-hand side of (54) varies with  $\bar{c}$  and  $N_e$ .

To this end, we will rewrite the left-hand side of (54) in terms of the elasticity of substitution. Observe, first, that the equilibrium profit of a type- $c$  firm gross of the fixed cost  $F$  is given by

$$\Pi_c^*(\bar{c}, N_e) = \frac{c}{\sigma_c(\bar{c}, N_e) - 1} q_c^*$$

where  $\sigma_c(\bar{c}, N_e)$  is the equilibrium value of the elasticity of substitution between any two  $c$ -type firms given by

$$\sigma_c(\bar{c}, N_e) \equiv \bar{\sigma}(x_c^*, \mathbf{x}^*(\bar{c}, N_e)).$$

Using the envelope theorem and the profit-maximization condition (47), we obtain:

$$\mathcal{E}_c(\Pi_c^*(\bar{c}, N_e)) = 1 - \sigma_c(\bar{c}, N_e). \quad (55)$$

Since

$$-\int_c^{\bar{c}} \frac{\mathcal{E}_z(\Pi_z^*(\bar{c}, N_e))}{z} dz = -\int_c^{\bar{c}} \frac{\partial \ln \Pi_z^*(\bar{c}, N_e)}{\partial z} dz = \ln \Pi_c^*(\bar{c}, N_e) - \ln \Pi_{\bar{c}}^*(\bar{c}, N_e),$$

it follows from (55) that

$$\frac{\ln \Pi_c^*(\bar{c}, N_e)}{\ln \Pi_{\bar{c}}^*(\bar{c}, N_e)} = \int_c^{\bar{c}} \frac{\sigma_z(\bar{c}, N_e) - 1}{z} dz.$$

Accordingly, (54) may be rewritten as follows:

$$\int_0^{\bar{c}} \left[ \exp \left( \int_c^{\bar{c}} \frac{\sigma_z(\bar{c}, N_e) - 1}{z} dz \right) - 1 \right] d\Gamma(c) = \frac{F_e}{F}. \quad (56)$$

Therefore, if  $\sigma_c(\bar{c}, N_e)$  increases with  $\bar{c}$ , the left-hand side of (54) increases with  $\bar{c}$ . Intuitively, when  $\bar{c}$  increases, the mass of firms rises as less efficient firms stay in business, which intensifies competition and lowers markups. In this case, the selection process is tougher. This is not the end of the story, however. Indeed, the competitiveness of the market also depends on how  $N_e$  affects the degree of differentiation across varieties.

The expression (56) shows that the left-hand side of (54) increases with  $N_e$  if and only if  $\sigma_c(\bar{c}, N_e)$  increases in  $N_e$ . This amounts to assuming that, for any given cutoff, the relative impact of entry on the low-productivity firms (i.e., the small firms) is larger than the impact on the high-productivity firms, the reason being that  $\mathcal{E}_c(\Pi_c^*(\bar{c}, N_e))$  decreases in  $N_e$  if and only if  $\sigma_c(\bar{c}, N_e)$  increases in  $N_e$ .

When  $\sigma_c(\bar{c}, N_e)$  increases both with  $\bar{c}$  and  $N_e$ , the locus  $h(N_e)$  is downward-sloping. Indeed,

when  $N_e$  rises, so does the left-hand side of (56). Hence, since  $\sigma_c(\bar{c}, N_e)$  also increases with  $\bar{c}$ , it must be that  $\bar{c}$  decreases for (56) to hold. As a consequence, we have:

**Claim 1.** *Assume (B). If  $\sigma_c(\bar{c}, N_e)$  increases with  $\bar{c}$  and  $N_e$ , then  $\bar{c}^*$  decreases with  $L$ .*

Given  $\bar{c}$ , the number of active firms  $N$  is proportional to the number of entrants  $N_e$ . Therefore, assuming that  $\sigma(\bar{c}, N_e)$  increases with  $N_e$  may be considered as the counterpart of one of our most preferred assumptions in the case of symmetric firms, that is,  $\sigma(x, N)$  increases with  $N$ . In this case, the pro-competitive effect generated by entry exacerbates the selection effect across firms. In response to a hike in  $L$ , the two effects combine to induce the exit of the least efficient active firms. This echoes Melitz and Ottaviano (2008) who show that a trade liberalization shock gives rise to a similar effect under quadratic preferences. Since  $N_e^*$  and  $\bar{c}^*$  move in opposite directions, it is hard to predict how  $L$  affects the equilibrium mass  $N^*$  of active firms. Indeed, how strong are the entry and selection effects depends on the elasticity of substitution and the distribution  $G$ .

Repeating the above argument shows that, when  $\sigma_c(\bar{c}, N_e)$  increases with  $\bar{c}$  and decreases with  $N_e$ , the locus  $h(N_e)$  is upward-sloping. In this event, we have:

**Claim 2.** *Assume (B). If  $\sigma_c(\bar{c}, N_e)$  increases with  $\bar{c}$  and decreases with  $N_e$ , then  $\bar{c}^*$  increases with  $L$ .*

When  $\sigma_c(\bar{c}, N_e)$  decreases with  $N_e$ , two opposing effects are at work. First, entry fosters product differentiation, and thus relaxes competition. This invites new firms to enter and allows less efficient firms to stay in business. On the other hand, the corresponding hike in  $\bar{c}$  tends to render varieties closer substitutes. What our result shows is that the former effect dominates the latter. Hence, the equilibrium mass of active firms  $N^* = \Gamma(\bar{c}^*)N_e^*$  unambiguously rises with  $L$ .

To conclude, the impact of population size on the number of entrants is unambiguous: a larger market invites more entry. By contrast, the cutoff cost behavior depends on how the elasticity of substitution varies with  $N_e$ . All in all this shows that the interaction between the entry and selection effects is non-trivial.<sup>9</sup>

Finally, by reformulating the game with price-setting firms, the same comparative statics analysis can be undertaken to study how the per capita income affects the market outcome.

**Pass-through.** We show here that the result on complete pass-through under homothetic preferences still holds when firms are heterogeneous. The first-order condition for a  $c$ -type firm is given by

$$\frac{p_c - c}{p_c} = \bar{\eta}(x_c, \mathbf{x}). \quad (57)$$

Consider a proportionate drop in marginal costs by a factor  $\mu > 1$ , so that the distribution of marginal costs is given by  $G(\mu c)$ . We first investigate the impact of  $\mu$  on firms' operating

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<sup>9</sup>Note that we do not say anything about the impact on the aggregate productivity. This one will depend upon the aggregation weights chosen, as well on the distribution of firms' types, which is here unspecified.

profits when the cutoff  $\bar{c}$  is unchanged. The cutoff firms now have a marginal cost equal to  $\bar{c}/\mu$ . Furthermore, under homothetic preferences,  $\bar{\eta}(x_c, \mathbf{x})$  does not depend on the income  $y$  and is positive homogeneous of degree 0. Therefore, (57) is invariant to the same proportionate reduction in  $c$ . As a consequence, the new price equilibrium profile over  $[0, \bar{c}]$  is obtained by dividing all prices by  $\mu$ .

We now show that the profits of the  $\bar{c}$ -type firms do not change in response to the drop in cost, so that the new cutoff is given by  $\bar{c}/\mu$ . Indeed, both marginal costs and prices are divided by  $\mu$ , while homothetic preferences imply that demands are shifted upwards by the same factor  $\mu$ . Therefore, the operating profit of the  $\bar{c}$ -type firms is unchanged because

$$\left(\frac{p_{\bar{c}}}{\mu} - \frac{\bar{c}}{\mu}\right) L\mu x_{\bar{c}} = (p_{\bar{c}} - \bar{c}) Lx_{\bar{c}} = F.$$

In sum, regardless of the cost distribution, *under homothetic preferences the equilibrium price distribution changes in proportion with the cost distribution*, thereby leaving unchanged the distribution of equilibrium markups, as in Proposition 6.

## 5.2 Multisector economy

Following Dixit and Stiglitz (1977), we consider a two-sector economy involving a differentiated good supplied under increasing returns and monopolistic competition, and a homogeneous good - or a Hicksian composite good - supplied under constant returns and perfect competition. Both goods are normal. Labor is the only production factor and is perfectly mobile between sectors. Consumers share the same preferences given by  $U(\mathcal{U}(\mathbf{x}), z)$  where the functional  $\mathcal{U}(\mathbf{x})$  satisfies the properties stated in Section 2, while  $z$  is the consumption of the homogeneous good. The upper-tier utility  $U$  is strictly quasi-concave, once continuously differentiable, strictly increasing in each argument, and such that the demand for the differentiated product is always positive.<sup>10</sup>

We choose the unit of the homogeneous good for its equilibrium price to be equal to 1. The budget constraint is now given by

$$\int_0^N p_i x_i di + z = E + z = Y,$$

where the expenditure  $E$  on the differentiated good is endogenous because competition across firms affects the relative price of this good.

Using the first-order condition for utility maximization yields

$$p_i = \frac{U'_1(\mathcal{U}(\mathbf{x}), z)}{U'_2(\mathcal{U}(\mathbf{x}), z)} D(x_i, \mathbf{x}).$$

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<sup>10</sup>Our results hold true if the choke price is finite but sufficiently high.

Let  $p$  be arbitrarily given. Along the diagonal  $x_i = x$ , this condition becomes

$$p = S(\phi(x, N), z)D(x, xI_{[0, N]}), \quad (58)$$

where  $S$  is the marginal rate of substitution between the differentiated and homogeneous goods:

$$S(\phi, z) \equiv \frac{U'_1(\phi(x, N), z)}{U'_2(\phi(x, N), z)}$$

and  $\phi(x, N) \equiv \mathcal{U}(xI_{[0, N]})$ .

The quasi-concavity of the upper-tier utility  $U$  implies that the marginal rate of substitution decreases with  $\phi(x, N)$  and increases with  $z$ . Therefore, for any given  $(p, x, N)$ , (58) has a unique solution  $\bar{z}(p, x, N)$ , which is the income-consumption curve. The two goods being normal, this curve is upward sloping in the plane  $(x, z)$ .

For any given  $x_i = x$ , the love for variety implies that the utility level increases with the number of varieties. However, it is reasonable to suppose that the marginal utility  $D$  of an additional variety decreases. To be precise, we assume that

**(C)** for all  $x > 0$ , the marginal utility  $D$  weakly decreases with the number of varieties.

Observe that **(C)** holds for additive and quadratic preferences. Since  $\phi(x, N)$  increases in  $N$ ,  $S$  decreases. As  $D$  weakly decreases in  $N$ , it must be that  $z$  increases for the condition (58) to be satisfied. In other words,  $\bar{z}(p, x, N)$  increases in  $N$ .

We now determine the relationship between  $x$  and  $m$  by using the zero-profit condition. Since by definition  $m \equiv (p - c)/p$ , for any given  $p$  the zero-profit and product market clearing conditions yield the per variety consumption as a function of  $m$  only:

$$x = \frac{F}{cL} \frac{1 - m}{m}. \quad (59)$$

Plugging (59) and  $p = c/(1 - m)$  into  $\bar{z}$ , we may rewrite  $\bar{z}(p, x, N)$  as a function of  $m$  and  $N$  only:

$$\hat{z}(m, N) \equiv \bar{z}\left(\frac{c}{1 - m}, \frac{F}{cL} \frac{1 - m}{m}, N\right).$$

Plugging (59) and  $p = c/(1 - m)$  into the budget constraint (3) and solving for  $N$  yields

$$N = \frac{Lm}{F} [Y - \hat{z}(m, N)]. \quad (60)$$

Since  $\bar{z}$  and  $\hat{z}$  vary with  $N$  identically,  $\hat{z}$  also increases in  $N$ . Therefore, (60) has a unique solution  $\hat{N}(m; Y)$  for any  $m \in [0, 1]$ .

Moreover, (60) implies that  $\partial \hat{N} / \partial Y > 0$ , while  $\partial \hat{N} / \partial L > 0$  because the income-consumption curve is upward-sloping. In other words, if the price of the differentiated product is exogenously given, an increase in population size or individual income leads to a wider range of varieties.

Since  $\hat{N}(m; Y)$  is the number of varieties in the two-sector economy, while  $Y = y$  because profits are zero at any SFE, the equilibrium condition (32) is to be replaced by the following expression:

$$m\sigma \left( \frac{F}{cL} \frac{1-m}{m}, \hat{N}(m) \right) = 1. \quad (61)$$

The left-hand side  $m\sigma$  of (61) equals zero for  $m = 0$  and exceeds 1 when  $m = 1$ . Hence, by the intermediate value theorem, the set of SFEs is non-empty. Moreover, it has an infimum and a supremum, which are both SFEs because the left-hand side of (61) is continuous. In what follows, we denote the corresponding markups by  $m_{\text{inf}}$  and  $m_{\text{sup}}$ ; if the SFE is unique,  $m_{\text{inf}} = m_{\text{sup}}$ . Therefore, the left-hand side of (61) must increase with  $m$  in some neighborhood of  $m_{\text{inf}}$ , for otherwise there would be an equilibrium to the left of  $m_{\text{inf}}$ , a contradiction. Similarly, the left-hand side of (61) increases with  $m$  in some neighborhood of  $m_{\text{sup}}$ .

Since  $\partial \hat{N} / \partial y > 0$ , (61) implies that an increase in  $y$  shifts the locus  $m\sigma$  upward if and only if  $\mathcal{E}_N(\sigma) > 0$ , so that the equilibrium markups  $m_{\text{inf}}$  and  $m_{\text{sup}}$  decrease in  $y$ . Consider now an increase in population size. Since  $\partial \hat{N} / \partial L > 0$ , (61) implies that an increase in  $L$  shifts the locus  $m\sigma$  upward if both  $\mathcal{E}_x(\sigma) < 0$  and  $\mathcal{E}_N(\sigma) > 0$  hold. In this event, the equilibrium markups  $m_{\text{inf}}$  and  $m_{\text{sup}}$  decrease in  $L$ .

Summarizing our results, we come to a proposition.

**Proposition 9.** *Assume (C). Then, the set of SFEs is non-empty. Furthermore, (i) an increase in individual income leads to a lower markup and bigger firms at the infimum and supremum SFEs if and only if  $\mathcal{E}_N(\sigma) > 0$  and (ii) an increase in population size yields a lower markup and bigger firms at the infimum and supremum SFEs if  $\mathcal{E}_x(\sigma) < 0$  and  $\mathcal{E}_N(\sigma) > 0$ .*

This extends to a two-sector economy what Propositions 4 and 5 state in the case of a one-sector economy where the SFE is unique. Proposition 8 also shows that the elasticity of substitution keeps its relevance for studying monopolistic competition in a multisector economy. In contrast, studying how  $N^*$  changes with  $L$  or  $y$  is more problematic because the equilibrium number of varieties depends on the elasticity of substitution between the differentiated and homogeneous goods.

### 5.3 Heterogeneous consumers

Accounting for consumer heterogeneity in models of monopolistic competition is not easy but doable. Assume that each consumer is characterized by a labor endowment  $y \geq 0$ , which need not be the same across consumers, and some taste parameter  $\theta$  belonging to the range  $\Theta$ . Let  $G$  be the joint probability distribution of labor endowment  $y$  and taste  $\theta$  over  $\mathbb{R}_+ \times \Theta$ . In what follows, we call  $G$  the income-taste distribution.

Let  $\mathcal{D}(p_i, \mathbf{p}; Y, \theta)$  be the Marshallian demand for variety  $i$  of a consumer with  $\theta$ -type preference where  $\theta$  is the taste parameter. When a consumer's income is equal to  $y$ , the aggregate demand



faced by firm  $i$  is given by

$$\Delta(p_i, \mathbf{p}) \equiv L \int_{\mathbb{R}_+ \times \Theta} \mathcal{D}(p_i, \mathbf{p}; y, \theta) dG(y, \theta). \quad (62)$$

Ever since the Sonnenschein-Mantel-Debreu theorem (Mas-Colell et al., 1995, ch.17), it is well known that the aggregate demand (62) need not inherit the properties of the individual demand functions. By contrast, as in Section 2, for each variety  $i$ , the aggregate demand  $\Delta(p_i, \mathbf{p})$  for variety  $i$  is decreasing in  $p_i$  regardless of the income-taste distribution  $G$ . A comparison with Hildenbrand (1983) and Grandmont (1987), who derived specific conditions for the Law of demand to hold when the number of goods is finite, shows how working with a continuum of goods, which need not be the varieties of a differentiated product, vastly simplifies the analysis.

The properties of  $\mathcal{D}$  crucially depend on the relationship between income and taste. Indeed, since firm  $i$ 's profit is given by  $\pi(p_i, \mathbf{p}) = (p_i - c)\Delta(p_i, \mathbf{p}) - F$ , the first-order condition for a symmetric equilibrium becomes

$$p \left[ 1 - \frac{1}{\varepsilon(p, N)} \right] = c,$$

where  $\varepsilon(p, N)$  is the elasticity of  $\Delta(p, \mathbf{p})$  evaluated at the symmetric outcome. If  $\varepsilon(p, N)$  is an increasing function of  $p$  and  $N$ , most of the results derived above hold true. Indeed, integrating consumers' budget constraints across  $\mathbb{R}_+ \times \Theta$  and applying the zero-profit condition yields the markup.

$$m(N) = \frac{NF}{L\bar{y}} \quad \text{where} \quad \bar{y} \equiv \int_{\mathbb{R}_+ \times \Theta} y dG(y, \theta). \quad (63)$$

Note that (63) differs from (30) only in one respect: the individual labor income  $y$  is replaced by the mean income  $\bar{y}$ , which is independent of  $L$ . Consequently, if  $\varepsilon(p, N)$  decreases both with  $p$  and  $N$ , a population hike or a productivity shock affects the SFE as in the baseline model (see Propositions 4 and 6). By contrast, the impact of an increase in  $\bar{y}$  is ambiguous because it depends on how  $\theta$  and  $y$  are related through the distribution  $G$ .

There is no reason to expect the aggregate demand to exhibit an increasing price elasticity even when the individual demands satisfy this property. To highlight the nature of this difficulty, we show in Appendix 7 that

$$\begin{aligned} \frac{\partial \varepsilon(p, N)}{\partial p} &= \int_{\mathbb{R}_+ \times \Theta} \frac{\partial \varepsilon(p, N; y, \theta)}{\partial p} s(p, N; y, \theta) dG(y, \theta) - \\ &\quad - \frac{1}{p} \int_{\mathbb{R}_+ \times \Theta} [\varepsilon(p, N; y, \theta) - \varepsilon(p, N)]^2 s(p, N; y, \theta) dG(y, \theta), \end{aligned} \quad (64)$$

where  $\varepsilon(p, N; y, \theta)$  is the elasticity of the individual demand  $\mathcal{D}(p_i, \mathbf{p}; y, \theta)$  evaluated at a symmetric outcome ( $p_i = p_j = p$ ), while  $s(p, N; y, \theta)$  stands for the share of demand of  $(y, \theta)$ -type consumers

in the aggregate demand, evaluated at the same symmetric outcome:

$$s(p, N; y, \theta) \equiv \frac{\mathcal{D}(p, \mathbf{p}; y, \theta)}{\Delta(p, \mathbf{p})} \Big|_{\mathbf{p}=pI_{[0, N]}}. \quad (65)$$

Because the second term of (64) is negative, the market demand may exhibit decreasing price elasticity even when individual demands display increasing price elasticities. Nevertheless, (64) has an important implication.

**Proposition 10.** *If individual demand elasticities are increasing and their variance is not too large, then the elasticity of the aggregate demand is increasing, and thus there exists a unique symmetric free-entry equilibrium.*

In this case, all the properties of Section 4 hold true because the elasticity of the aggregate demand satisfies **(Abis)**. Yet, when consumers are very dissimilar, like in the Sonnenschein-Mantel-Debreu theorem, the aggregate demand may exhibit undesirable properties.

The equation (64) shows that the effect of heterogeneity in tastes and income generally differ. In particular, consumers with different incomes and identical tastes have different willingness-to-pay for the same variety, which increases the second term in (64). By contrast, if consumers have the same income and only differ in their ideal variety, one may expect the second term in (64) to be close to zero when the market provides these varieties.

The main issue regarding consumer heterogeneity is to study how different types of consumer heterogeneity affect the variance of the distribution of individual elasticities. A first step in this direction has been taken by Di Comite et. al. (2014) who show how the main ingredients of Hotelling's approach to product differentiation - i.e. taste heterogeneity across consumers who have each a different ideal variety - can be embedded into the quadratic utility while preserving the properties of this model.

## 6 Concluding remarks

We have shown that monopolistic competition can be modeled in a much more general way than what is typically thought. Using the concept of elasticity of substitution, we have provided a complete characterization of the market outcome and of all the comparative statics implications in terms prices, firm size, and mass of firms/varieties. Somewhat ironically, the concept of elasticity of substitution, which has vastly contributed to the success of the CES model of monopolistic competition, thus keeps its relevance in the case of general preferences, both for symmetric and heterogeneous firms. The fundamental difference is that the elasticity of substitution ceases to be constant and now varies with the key-variables of the setting under study. We take leverage on this to make clear-cut predictions about the impact of market size and productivity shocks on the market outcome.

Furthermore, our framework is able to mimic a wide range of strategic effects usually captured by oligopoly models, and it does so without encountering several of the difficulties met in general equilibrium under oligopolistic competition. Finally, we have singled out our most preferred set of assumptions and given a disarmingly simple sufficient condition for all the desired comparative statics effects to hold true. But we have also shown that relaxing these assumptions does not jeopardize the tractability of the model. Future empirical studies should shed light on the plausibility of the assumptions discussed in this paper by checking their respective implications. It would be unreasonable, however, to expect a single set of conditions to be universally valid.

We would be the last to say that monopolistic competition is able to replicate the rich array of findings obtained in industrial organization. However, it is our contention that models such as those presented in this paper may help avoiding several of the limitations imposed by the partial equilibrium analyses of oligopoly theory. Although we acknowledge that monopolistic competition is the limit of oligopolistic equilibria, we want to stress that monopolistic competition may be used in different settings as a substitute for oligopoly models when these ones appear to be unworkable.

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# Appendix

**Appendix 1.** If the set  $B \in L^2[0, N]$  is convex and weakly compact, while the functional  $\mathcal{U} : B \rightarrow R$  is Frechet-differentiable and strictly quasi-concave, then  $\mathcal{U}$  has a unique maximizer  $x^* \in B$ .

**Proof.** Existence is implied by the following two results: (i) a quasi-concave Frechet-differentiable functional is weakly upper-semicontinuous (Barbu and Precupanu, 2012), and (ii) an upper-semicontinuous functional defined over a bounded weakly closed subset of a reflexive Banach space always has a maximizer (Céa, 1971). Indeed, a weakly compact set is bounded and weakly closed, while  $L^2[0, N]$  is a Hilbert space, thus a reflexive Banach space. Uniqueness follows from the strict quasi-concavity of  $\mathcal{U}$  and the convexity of  $B$ . Q.E.D.

**Appendix 2.** Proof of Proposition 1.

(i) We first show (1) for the case where  $N/k$  is a positive integer. Note that

$$1_{[0, N]} = \sum_{i=1}^{N/k} 1_{[(i-1)k, ik]} \quad (\text{A.1})$$

while symmetry implies

$$\mathcal{U} \left( \frac{X}{k} 1_{[(i-1)k, ik]} \right) = \mathcal{U} \left( \frac{X}{k} 1_{[0, k]} \right) \quad \text{for all } i \in \{2, \dots, N/k\}. \quad (\text{A.2})$$

Together with quasi-concavity, (A.1) – (A.2) imply

$$\mathcal{U} \left( \frac{X}{N} 1_{[0, N]} \right) = \mathcal{U} \left( \frac{k}{N} \sum_{i=1}^{N/k} \frac{X}{k} 1_{[(i-1)k, ik]} \right) > \min_i \mathcal{U} \left( \frac{X}{k} 1_{[(i-1)k, ik]} \right) = \mathcal{U} \left( \frac{X}{k} 1_{[0, k]} \right).$$

Thus, (1) holds when  $N/k$  is a positive integer.

(ii) We now extend this argument to the case where  $N/k$  is a rational number. Let  $r/s$ , where both  $r$  and  $s$  are positive integers and  $r \geq s$ , be the irredundant representation of  $N/k$ . It is then readily verified that

$$s1_{[0, N]} = \sum_{i=1}^r 1_{[N\{(i-1)k/N\}, N\{ik/N\}]} \quad (\text{A.3})$$

and

$$\mathcal{U} \left( \frac{X}{k} 1_{[N\{(i-1)k/N\}, N\{ik/N\}]} \right) = \mathcal{U} \left( \frac{X}{k} 1_{[0, k]} \right) \quad \text{for all } i \in \{2, \dots, r\} \quad (\text{A.4})$$

where the fractional part of the real number  $a$  is denoted by  $\{a\}$ .

Using (A.3) – (A.4) instead of (A.1) – (A.4) in the above argument, we obtain

$$\mathcal{U}\left(\frac{X}{N}1_{[0,N]}\right) = \mathcal{U}\left(\frac{1}{r}\sum_{i=1}^r\frac{X}{k}1_{[N\{(i-1)k/N\}, N\{ik/N\}]}\right) > \mathcal{U}\left(\frac{X}{k}1_{[0,k]}\right).$$

Thus, (1) holds when  $N/k$  is rational.

(iii) Finally, since  $\mathcal{U}$  is continuous while the rational numbers are dense in  $\mathbb{R}_+$ , (1) holds for any real number  $N/k > 1$ . Q.E.D.

**Appendix 3.** Proof of Proposition 2.

It is readily verified that the inverse demands generated by preferences (7) are given by  $D(x_i, \mathbf{x}) = u'(x_i)$ . The uniqueness of the marginal utility implies that preferences are additive. This proves part (i).

Assume now that  $\mathcal{U}$  is homothetic. Since a utility is defined up to a monotonic transformation, we may assume without loss of generality that  $\mathcal{U}$  is homogenous of degree 1. This, in turn, signifies that  $D(x_i, \mathbf{x})$  is homogenous of degree 0 with respect to  $(x_i, \mathbf{x})$ . Indeed, because  $t\mathcal{U}(\mathbf{x}/t) = \mathcal{U}(\mathbf{x})$  holds for all  $t > 0$ , (2) can be rewritten as follows:

$$\mathcal{U}(\mathbf{x} + \mathbf{h}) = \mathcal{U}(\mathbf{x}) + \int_0^N D\left(\frac{x_i}{t}, \frac{\mathbf{x}}{t}\right) h_i di + o(\|\mathbf{h}\|_2). \quad (\text{A.5})$$

The uniqueness of the marginal utility together with (A.5) implies

$$D\left(\frac{x_i}{t}, \frac{\mathbf{x}}{t}\right) = D(x_i, \mathbf{x}) \text{ for all } t > 0$$

which shows that  $D$  is homogenous of degree 0. As a result, there exists a functional  $\Phi$  belonging to  $L_2([0, \mathcal{N}])$  such that  $D(x_i, \mathbf{x}) = \Phi(\mathbf{x}/x_i)$ . Q.E.D.

**Appendix 4.** For simplicity, we assume here that profits are not redistributed to consumers, so that  $y$  is exogenous. When firms are quantity-setters or price-setters, firm  $i$ 's profits are given, respectively, by

$$\pi(x_i) = Lx_i \left[ \frac{D(x_i, \mathbf{x})}{\lambda} - c \right] - F,$$

$$\Pi(p_i) = (p_i - c)\mathcal{D}(p_i, \mathbf{p}, Y) - F.$$

We show that the following five conditions are equivalent:

- (i)  $\pi$  is strictly quasi-concave in  $x_i$  for any positive value of  $\lambda c$ ;
- (ii)  $\pi$  is strictly concave in  $x_i$  for any positive value of  $\lambda c$ ;
- (iii) the marginal revenue  $x_i D'_i + D$  is strictly decreasing in  $x_i$  for all  $x_i > 0$ ;
- (iv)  $\Pi$  is strictly quasi-concave in  $p_i$  for all positive values of  $c$ ;
- (v)  $1/\mathcal{D}$  is  $(-1)$ -convex in  $p_i$ .

We do this by proving the following claims: (i)  $\Rightarrow$  (iii)  $\Rightarrow$  (ii)  $\Rightarrow$  (i) and (iii)  $\Leftrightarrow$  (iv)  $\Leftrightarrow$  (v).

(i)  $\Rightarrow$  (iii). This is proven in subsection 3.1.

(iii)  $\Rightarrow$  (ii). Differentiating  $\pi(x_i)$  yields

$$\pi'_i = \frac{L}{\lambda} (x_i D'_i + D) - c,$$

so that  $\pi''_i < 0$  by (iii).

(ii)  $\Rightarrow$  (i). Straightforward.

(iii)  $\Leftrightarrow$  (iv). The first-order condition for maximizing  $\Pi$  may be written as follows:

$$p_i + \frac{\mathcal{D}}{\mathcal{D}'_i} = c. \quad (\text{A.6})$$

$\Pi$  is strictly quasi-concave in  $p_i$  for all  $c > 0$  if and only if (A.6) has at most one solution for each  $c$ . This, in turn, holds if and only if the left-hand side of (A.6) either strictly increases in  $p_i$ , or strictly decreases in  $p_i$ , for  $p_i > 0$ . Since

$$p_i + \frac{\mathcal{D}}{\mathcal{D}'_i} = p_i \left( 1 - \frac{1}{\varepsilon} \right)$$

and  $\varepsilon > 1$ , we have

$$p_i + \frac{\mathcal{D}}{\mathcal{D}'_i} > 0 \quad \lim_{p_i \rightarrow 0} \left( p_i + \frac{\mathcal{D}}{\mathcal{D}'_i} \right) = 0.$$

Therefore, the left-hand side of (A.6) must increase in the neighborhood of  $p_i = 0$ . This implies that the strict quasi-concavity of  $\Pi$  is equivalent to

$$\frac{d}{dp_i} \left( p_i + \frac{\mathcal{D}}{\mathcal{D}'_i} \right) > 0. \quad (\text{A.7})$$

Plugging  $p_i = D/\lambda$  into (A.7) and using  $\mathcal{D}(p_i) = D^{-1}(\lambda p_i) = x_i$ , it is readily verified that (A.7) becomes

$$\frac{d}{dx_i} (x_i D'_i + D) < 0,$$

i.e., the marginal revenue strictly decreases in  $x_i$ .

(iv)  $\Leftrightarrow$  (v)  $\Pi$  is strictly quasi-concave in  $p_i$  for all  $c > 0$  if and only if each solution to the first-order condition (A.6) satisfies the second-order condition

$$(p_i - c) \mathcal{D}''_i + 2 \mathcal{D}'_i < 0. \quad (\text{A.8})$$

Solving (A.6) for  $p_i - c$  and plugging the resulting expression into (A.8), we get

$$-\frac{\mathcal{D} \mathcal{D}''_i}{\mathcal{D}'_i} + 2 \mathcal{D}'_i < 0,$$

which is equivalent to  $(1/\mathcal{D})'' > 0$ , i.e.,  $1/\mathcal{D}$  is strictly convex. Q.E.D.



**Appendix 5.** We use an infinite-dimensional version of the definition proposed by Nadiri (1982). Setting  $D_i = D(x_i, \mathbf{x})$ , the elasticity of substitution between varieties  $i$  and  $j$  is given by

$$\bar{\sigma} = -\frac{D_i D_j (x_i D_j + x_j D_i)}{x_i x_j \left[ D_i' D_j^2 - \left( \frac{\partial D_i}{\partial x_j} + \frac{\partial D_j}{\partial x_i} \right) D_i D_j + D_j' D_i^2 \right]}.$$

Since  $\mathbf{x}$  is defined up to a zero measure set, it must be that

$$\frac{\partial D_i(x_i, \mathbf{x})}{\partial x_j} = \frac{\partial D_j(x_j, \mathbf{x})}{\partial x_i} = 0$$

for all  $j \neq i$ . Therefore, we obtain

$$\bar{\sigma} = -\frac{D_i D_j (x_i D_j + x_j D_i)}{x_i x_j (D_i' D_j^2 + D_j' D_i^2)}.$$

Setting  $x_i = x_j = x$  implies  $D_i = D_j$ , and thus we come to  $\bar{\sigma} = 1/\bar{\eta}(x, \mathbf{x})$ . Q.E.D.

**Appendix 6.** Let

$$\bar{\varepsilon}(p_i, \mathbf{p}, Y) \equiv -\frac{\partial \mathcal{D}(p_i, \mathbf{p}, Y)}{\partial p_i} \frac{p_i}{\mathcal{D}(p_i, \mathbf{p}, Y)}$$

be the elasticity of the Marshallian demand (6). At any symmetric outcome, we have

$$\varepsilon(p, N) \equiv \bar{\varepsilon}(p, pI_{[0, N]}).$$

Using the budget constraint  $p = y/Nx$  and (21) yields

$$\varepsilon(Y/Nx, N) = \eta(x, N) = \frac{1}{\sigma(x, N)}. \quad (\text{A.9})$$

When preferences are indirectly additive, using Roy's identity yields the demand function for variety  $i$  is given by

$$x_i = \frac{v'(p_i/Y)}{\int_0^N (p_k/Y) v'(p_k/Y) dk},$$

which implies that  $\varepsilon(Y/Nx, N) = 1 - \theta(Y/p)$  where  $\theta$  is given by (25). Combining this with (A.9), we get  $\sigma(x, N) = 1/\theta(Nx)$ . Q.E.D.

**Appendix 7.** At a symmetric outcome the aggregate demand elasticity is given by

$$\varepsilon(p, N) = \int_{\mathbb{R}_+ \times \Theta} \varepsilon(p, N; y, \theta) s(p, N; y, \theta) dG(y, \theta), \quad (\text{A.10})$$

where  $s(p, N; y, \theta)$  is the share of the  $(y, \theta)$ -type consumer's individual demand in the aggregate demand.

Differentiating (A.10) with respect to  $p$  yields

$$\frac{\partial \varepsilon(p, N)}{\partial p} = \int_{\mathbb{R}_+ \times \Theta} \left( \frac{\partial \varepsilon(p, N; y, \theta)}{\partial p} s + \varepsilon(p, N; y, \theta) \frac{\partial s}{\partial p} \right) dG(y, \theta). \quad (\text{A.11})$$

Using (65), we obtain

$$\mathcal{E}_p(s) = \varepsilon(p, N) - \varepsilon(p, N; y, \theta).$$

Hence,

$$\frac{\partial s}{\partial p} = \frac{s}{p} [\varepsilon(p, N) - \varepsilon(p, N; y, \theta)].$$

Note that

$$\int_{\mathbb{R}_+ \times \Theta} s(p, N; y, \theta) dG(y, \theta) = 1 \Rightarrow \int_{\mathbb{R}_+ \times \Theta} \frac{\partial s}{\partial p} dG(y, \theta) = 0. \quad (\text{A.12})$$

Therefore, plugging (A.12) into (A.11) and subtracting  $(\varepsilon(p, N)/p) \int_{\mathbb{R}_+ \times \Theta} (\partial s / \partial p) dG(y, \theta) = 0$  from both sides of (A.12), we obtain the desired expression (64). Q.E.D.

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