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Stabilization of the Modelling of a Radio-Frequency Quadrupole Based on Quasi-Helmholtz Projectors

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Abstract — **This paper presents a stabilization of a Radio-Frequency Quadrupole simulation based on the quasi-Helmholtz projectors. A boundary element method applied to this case undergoes a low-frequency breakdown i.e the associated system of equations becomes increasingly ill-conditioned for decreasing frequencies. This in practice implies that the convergence of iterative methods which are used to solve the linear system is poor. This paper reports that a quasi-Helmholtz projectors based stabilization is sufficient to handle the level of realism required by applications.**

1 INTRODUCTION

A Radio Frequency Quadrupole (RFQ) [1], [2] is one of the most important part of a LINear ACcelerator (LINAC). It is situated at the very beginning of the accelerator and its goals are focusing the beam coming out of the ion source, bunching it (making packets of particles), and giving a first acceleration to it. Because of its strategic function, an RFQ must be designed as carefully as possible. The electromagnetic solver and the motion solver should provide accurate results within a reasonable computation time. Presently, most of RFQ solvers are based on the hypothesis that the fields are quasi-static. For instance, Toutatis [3] is one of the most famous solvers used for RFQ simulations and it is based on the Poisson's equation.

The RFQ chosen for our simulations is the Myrrha's accelerator [9]. Fig. 1 shows the mesh of the Myrrhas RFQ. The accelerator is made of four rods which are supported by four stems. The beam is accelerated between these rods. Each given stem is connected to two rods of the same voltage while its two neighbours are connected to the two rods of opposite voltage. Actually, the rods and the stems constitute a resonant circuit that should resonate at a given frequency. The frequency is chosen in function of the rods' profile and the input beam energy in order to synchronize the beam and bunches with the RF fields.

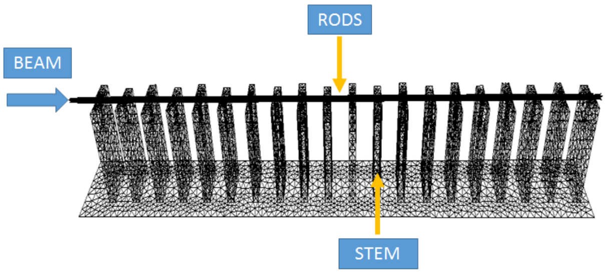

Figure 1: Mesh of Myrrha's RFQ.

The RFQ design is based on the well-known electrostatic quadrupole. In order to obtain an accelerating field and to bunch the beam, the rods must be curved in the direction of acceleration of the beam and an RF source must be used. The structure is only few wavelengths long. It means that the BEM system of equations is ill conditioned because of the low-frequency-breakdown. Iterative solvers such as GMRES [6] are often used for BEM solution because of the heavy computational cost of direct solvers. Iterative solvers such as GMRES tend to converge quickly when the system of equations is well-conditioned.

The goal of this work is to show that with a Boundary Element Method (BEM) solver, it is possible to perform effective field simulations for an RFQ when low frequency stabilization is used. The use of a full-wave solver may improve the accuracy of the fields when compared to Toutatis. Because the structure operates in a low-frequency regime the BEM impedance matrix suffers from a low-frequency breakdown [4], [5]. In order to decrease the number of iterations required by GMRES [6], [7] to solve the BEM system, several low-frequency preconditioners have been proposed (see [8] and references therein). In order to stabilize the simulations of the RFQ, this work will adopt the low-frequency preconditioner in [8].

2 SOLUTION METHODOLOGY

The preconditioner we used here was proposed in [8]. The main results of this paper are recalled here-

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under. The EFIE impedance matrix is given by

$$Z_k l = j\eta(k\Sigma(\overline{J}_k, \overline{J}_l) + \frac{1}{k}\Gamma(\overline{J}_k, \overline{J}_l))$$

with the two bilinear forms define as

$$\begin{aligned}
\Sigma : Vec(S)^2 &\rightarrow \mathbb{C} : \\
(\overline{J}_k, \overline{J}_l) &\rightarrow \int_S \int_S k < \overline{J}_k | \overline{J}_l > G d\overline{r}' d\overline{r} \\
\Gamma : Vec(S)^2 &\rightarrow \mathbb{C} : \\
(\overline{J}_k, \overline{J}_l) &\rightarrow \int_S \int_S [\nabla_s \cdot \overline{J}_k][\nabla_s \cdot \overline{J}_l] G d\overline{r}' d\overline{r}
\end{aligned}$$

where η is the free-space impedance, k is the wave number, $j \equiv \sqrt{-1}$, $\{\overline{J}_k\}_{k\in[1,...,N]}$ is a set of basis/testing functions, G is the Green's function, S is a compact connected piecewise smooth manifold, ∇_s is the Laplace-Beltrami operator and $Vec(S)$ is a vector fields on the tangent bundle of S. In this paper, S can refer to the discretized surface as well.

The basis/testing functions used in this paper are the well known RWGs [11]. Let be $E \equiv span < \overline{RWG}_1, ..., \overline{RWG}_N >$ the vector space of the current distributions spanned by the RWGs with N the total number of basis functions. Let be $T : \mathbb{C}^N \rightarrow E : (x_1, ..., x_N) \rightarrow \sum_{i=1}^N x_i \overline{RWG}_i$ the isomorphism between E and $\mathbb{C}^N$. $\mathbb{C}^N$ is called the coefficients space. Let $L(\mathbb{C}^N)$ be the vector space of linear applications in the coefficients space. Any bilinear form A discretized with the RWG is denoted by $[A] \in L(\mathbb{C}^N)$ and is a matrix.

The $[\Sigma]$ operator is of full rank when discretized with RWG while the $[\Gamma]$ operator has a non-null kernel that is spanned by the set of loop basis functions

$$ker([\Gamma]) \equiv \{x = (x_1, ..., x_N) | \nabla_s \cdot T(x) = 0\}$$

Since the Γ operator is inversely proportional to the frequency and the Σ operator is proportional to the frequency, the two operators tend to be unbalanced at low frequency. Since the Γ operator has a non-null kernel, one may build currents that, when evaluated through the EFIE impedance matrix, generate image vectors respectively proportional and inversely proportional to the frequency. Let S be a complementary space to $ker([\Gamma])$ such that $E = S \bigoplus ker([\Gamma])$. Let $[x] \in ker([\Gamma])$ be such that $\|x\|_2 = 1$, then one has

$$[Z][x] = j\eta k[\Sigma][x] \propto k$$

Now, let us take $[x] \in S$ such that $\|x\|_2 = 1$, one has

$$[Z][x] = j\eta(k[\Sigma][x] + \frac{1}{k}[\Gamma][x]$$

These two relations show that for a fixed geometry, $\exists K > 0, K \in \mathbb{R}$ such that $\forall k <$ one has $\|[Z][x]\|_2 << \|[Z][y]\|_2$ for $[x] \in ker(\Gamma)]$, $[y] \in S]$ and $\|[x]\|_2 = \|[y]\|_2 = 1$. This means, by definition, that one ends up with a wide spectrum of singular values. Let be $M = dim(ker([\Gamma]))$, there should be M singular values of much lower value than the $N - M$ other ones.

The idea of the low-frequency preconditioner consists of performing a rescaling of these two operators by rescaling the currents living in S and in ker($[\Gamma]$). It means, one needs to generate two bases for S and ker($[\Gamma]$) to perform the rescaling. Since it can be cumbersome to build the loop basis functions, an other approach is considered. A natural idea would be to invoke the Helmholtz decomposition theorem [12] and to try to build the space of non-solenoidal currents which is orthogonal to the solenoidal currents subspace (for the metric on the Riemann manifold). However, because one uses div-conforming basis functions, it can be shown that there is no way to build curl-free currents with RWG [13]. However, it is shown in [13] that one may build a subspace S that is orthogonal to the loop currents but the orthogonality has to be understood in the coefficients space with the canonic scalar product of $\mathbb{C}^N$. The space $S = span < S_1, ..., S_{N-M}$ is spanned by the well-known star basis functions. The coefficients of the star basis functions as linear combination of RWG can be gathered together into a matrix denoted $[S]$ in which each column corresponds to a star basis function. Similarly, the loop basis functions are gathered in the matrix $[L]$. The orthogonal property between the stars and the loops implies that $[S]^t[L] = 0$. Now, thanks to the orthogonality property, one may build two projectors that are used to rescale any current decomposed as a unique sum of two vectors, one defined in S and one defined in $ker([\Gamma])$. The two orthogonal projectors are given by

$$\begin{aligned}
[P_s] &= [S]([S]^T[S])^{-1}[S]^T \\
[P_l] &= [I] - [P_s]
\end{aligned}$$

Where $[P_l]$ is equal to $[P_l] = [L]([L]^t[L])^{-1}[L]^t$.

Now, one may define a preconditioner as follows

$$[P] = [P_s]\sqrt{k} + [P_l]\frac{1}{j\sqrt{k}}$$

And the BEM system of equations one has to solve becomes

$$[P][Z][P][x] = [P][E]$$

where $[E]$ is the excitation vector. Since the system of equations is solved with an iterative solver, the inversion inside the star projector $[P_s]$ does not need to be explicitly calculated. The inversion of this sparse matrix is performed with the help of an algebraic multi-grid solver [14], [15], [16] which performs the computation with a linear complexity.

3 NUMERICAL SIMULATIONS

In order to analyse numerically each step of the preconditioner, a section of the accelerator was investigated as shown Fig. 2. The accelerator is fed

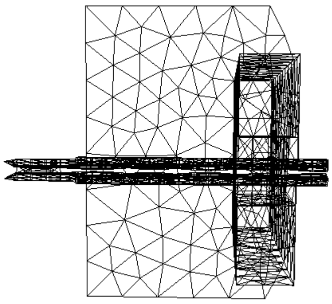

Figure 2: A section of the accelerator.

by an RF source as shown Fig. 3. The simulation was performed at 176Mhz. Fig 4 shows the spectrum of the impedance matrix and of the preconditioned impedance matrix. The condition number

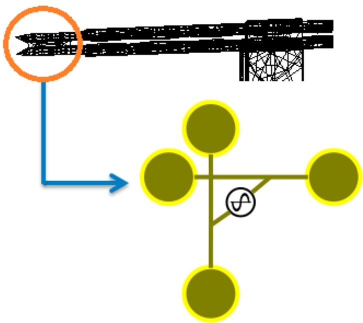

Figure 3: Feeding of the accelerator

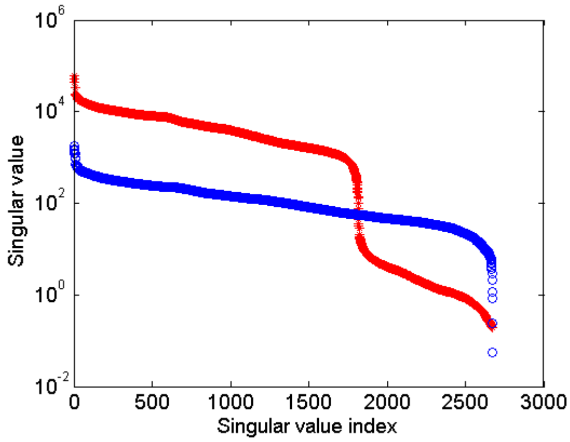

Figure 4: Spectrum of the impedance matrix in red and the spectrum of the preconditioned impedance matrix in blue.

of the preconditioned impedance matrix is 3.510^4, while the condition number of the impedance matrix is 3.510^5. Hence, the condition number is better by one order of magnitude. It might seem a relatively small improvement, but if one analyses a little bit closer the spectrum of the preconditioned impedance matrix, one may see that there are about 6 minimal singular values that are completely unbalanced (much lower in this case) than the rest of the spectrum. These 6 points induced only 6 additional iterations of GMRES. The rest of the spectrum is relatively flat, the ratio between the highest value and the smallest value is around 500. It means that one should expect a very good convergence of GMRES in comparison to the non-preconditioned BEM. As expected, for a residual error of 10^{-3}, GMRES needs 473 iterations for the non preconditioned BEM, while with the preconditioner GMRES only needs 148 iterations. This is a great improvement since the complexity of each additional iteration of GMRES grows superlinearly.

4 CONCLUSION

The preconditioner has proven to improve the GMRES convergence rate. The spectrum of the impedance matrix might be further improved with the help of a Calderon preconditioner. Indeed, the accelerator mesh is relatively irregular (the variance of triangle sizes is relatively big) as shown Fig 5. Since a Calderon preconditioner is immune from the mesh size variation, it may further improve the spectrum of the impedance matrix.

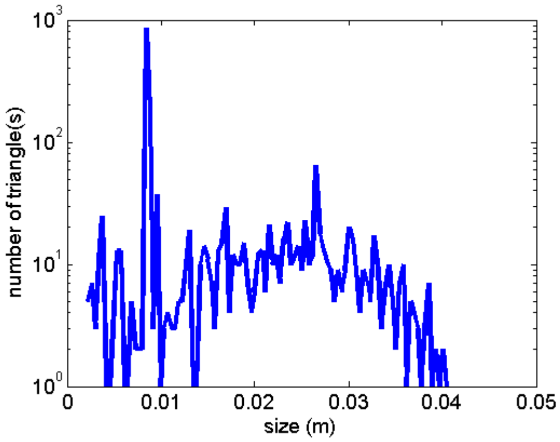

Figure 5: Triangle size distribution

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