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Algebraic Principles as a Tool for Energy Saving

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This paper discusses algebraic approaches of control design for a set of Single Input – Single Output (SISO) delayed systems that are further developed and discussed. The first principle utilises a special ring R_{QM}, - a set of RQ-meromorphic functions. The second one is based on a ring of proper and stable rational functions R_{PS} and can be considered as a special case. Controller parameters are derived through the general solution of linear Diophantine equations in the appropriate ring. A final controller can be tuned by the scalar real parameter m_0>0. The methodology is illustrated by a comparison with another approach, some analyses of a tuning parameter and example. The simulations are performed in the Matlab environment.

1. Introduction

This paper presents two possibilities of how to design controllers for simple delayed systems. Not only in the input-output relation can delay be modelled - but also, the dynamics can be affected by a time lag. There exist several approaches to control design methods; nowadays, three main groups dominate. The first group contains approaches based on the Smith predictor structure - or more precisely, its modifications, see Normey-Rico and Camacho (2008) for a review of the most frequently used dead-time compensators. These methods apply in mechanical, Araújo and Santos (2018) and chemical engineering as well, Hamdy and Ramadan (2017). They assume a controlled system model in feedback loops, in the sense of IMC (Internal Model Controllers); see, e.g. Levine (2019) for an example of the use of the IMC principle for a chemical process. The second group consists of predictive-based and finite-spectrum-assignment approaches, mainly using state-space descriptions, introduced already by Watanabe et al. (1983). The third family of approaches - described in this paper - is based on algebraic tools and methods. Their common theoretical basis was established in works by Kamen (1975) and Morse (1976) - and especially, in the celebrated book by Vidyasagar (1985). A review of these methods for various systems - including delayed ones, was later published in Kučera (1993).
The first design in the paper utilises a ring of stable quasipolynomial meromorphic functions (RQM) omitting any approximation - initially developed for delay systems in Zítek and Kučera (2003), and extended in Pekař and Prokop (2017). The second one is based on a ring of stable and proper rational functions (RPS), see Prokop and Corriou (1997). Many industrial processes can be modelled by stable systems with a delay time term. This contribution considers controller design for first-order (stable) delayed (FODS) and second-order (SODS) models. Control syntheses are performed for both systems. Some of the developed controllers are no longer in PI/PID structures.

2. Quasipolynomial meromorphic function approach

The definition of the R_{QM} ring can be found in Pekař and Prokop (2017). An element T(s) of the R_{QM} ring is the ratio of two quasipolynomials $y(s)/x(s)$, for which it holds that: $\sup_{Re\, s\geq 0} |T(s)| < \infty$, where s is the Laplace transform

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variable and Re means the real part. The term is formally stable, see Loiseau et al. (2002) for details. A quasipolynomial $x(s)$ of degree n with real coefficients x_{ij} and non-negative delays $\vartheta_{i,j}$ means:

$$x(s) = s^n + \sum_{i=0}^{n}\sum_{j=1}^{h_i} x_{ij}s^i e^{-\vartheta_{ij}s} \tag{1}$$

where h_i are given by the number of terms for a particular degree s^i. Quasipolynomial Eq(1) is exponentially stable if there is no finite s_0 such that $x(s_0) = 0$ and $\mathrm{Re}\,s_0 \geq 0$. For stability analyses of systems with delays based on the pole loci, see, e.g., Michiels and Niculescu (2014). The transfer function of a system with time delay is considered as a ratio of two quasipolynomial fractions in R_{QM}. As an example, a FODS can be expressed by:

$$G(s) = \frac{\dfrac{Ke^{-\tau s}}{s+m_0e^{-\theta s}}}{\dfrac{Ts+1}{s+m_0e^{-\theta s}}} = \frac{\dfrac{b(s)}{m(s)}}{\dfrac{a(s)}{m(s)}} = \frac{B(s)}{A(s)},\ m_0 > 0 \tag{2}$$

where K is the static gain, T expresses the time constant, τ means the input-output delay, the value of θ has the meaning a general delay, and m_0 is a selectable real parameter satisfying $m(s)$ to be stable; i.e., either $\theta = 0$ and $m_0 > 0$ or $\theta \neq 0$ and $0 < m_0\,e^{-\theta s} < \pi/2$. The former option yields R_{PS} (which is satisfactory for stable processes); and the latter agrees with R_{QM} (suggested for unstable systems).
The control loop is considered to be a simple feedback system with a controller $G_R(s) = Q(s)/P(s)$ and a controlled plant $G(s) = B(s)/A(s)$, depicted in Figure 1.

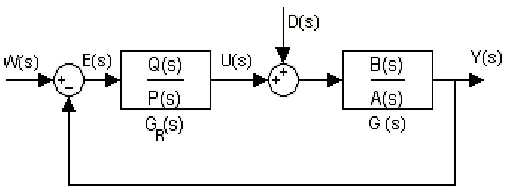

Figure 1: Feedback control loop

The aim of the control synthesis is to stabilise a feedback control system, obtain asymptotic tracking and attenuate load disturbance $d(t)$. Regarding the first requirement, it can be formulated in an elegant way in R_{QM} by the Diophantine equation:

$$A(s)P_0(s) + B(s)Q_0(s) = 1 \tag{3}$$

where, $\{P_0(s), Q_0(s)\}$ is a particular solution and all stabilising controllers can be expressed in a parametric form:

$$\frac{Q(s)}{P(s)} = \frac{Q_0(s)+A(s)Z(s)}{P_0(s)-B(s)Z(s)},\ P_0(s)-B(s)Z(s) \neq 0 \tag{4}$$

where, $Z(s) \in R_{QM}$ is arbitrary. The special choice of this element can ensure further control conditions. Details and proofs can be found e.g. in Pekař and Prokop (2017). Let the reference, $w(t)$, and load disturbance, $d(t)$, be expressed in the Laplace transform by $W(s)=H_W(s)/F_W(s)$, $D(s)=H_D(s)/F_D(s)$, $H_W(s), F_W(s), H_D(s), F_D(s) \in R_{QM}$ .The conditions for asymptotic tracking and disturbance attenuation result from the expression for the control error $E(s)$:

$$E(s) = \frac{A(s)P(s)}{A(s)P(s)+B(s)Q(s)}W(s) - \frac{B(s)P(s)}{A(s)P(s)+B(s)Q(s)}D(s) \tag{5}$$

It must hold that, $E(s) \in R_{QM}$; i.e., it is demanded that both $F_w(s)$ and $F_D(s)$ divide the products $A(s)P(s)$ and $B(s)P(s)$ simultaneously. Details about the divisibility in R_{QM} and R_{PS} can be found; e.g., in Zítek and Kučera (2003). The most frequent case is that both signals - $w(t)$ and $d(t)$, can be considered as step-wise functions. Let $A(s)$, $B(s)$ have no zero at $s = 0$ for simplicity´s sake. For the case of the R_{PS} ring, it is equivalent to reaching the absolute term of $P(s)$ equals zero. This condition is impossible to reach in R_{QM}, where it is required that $P(0) = 0$. The absolute term in $P(s)$ reads $\lambda\left(1-e^{-\tau s}\right)$, where λ stands for a selected real parameter. Usually, $\lambda = (m_0)^n$, where n means the order of the controlled system.

The controller design for a stable FODS resolves Eq(3) by the choice $Q_0 = 1$; yielding:

$$P_0(s) = \frac{s + m_0 - Ke^{-\tau s}}{Ts + e^{-\vartheta s}} \tag{6}$$

Now, parameterise the solution according to Eq(4) to obtain controllers that asymptotically reject the disturbance:

$$P(s) = \frac{s + m_0 - Ke^{-\tau s}}{Ts + e^{-\vartheta s}} - \frac{Ke^{-\tau s}}{s + m_0} Z(s) \tag{7}$$

The numerator of $P(s)$ has to include at least one zero root. It is appropriate to have $P(s)$ in a simple form, which is fulfilled, e.g., when:

$$Z(s) = \left(\frac{m_0}{K} - 1\right)\frac{s + m_0}{Ts + e^{-\vartheta s}} \tag{8}$$

providing

$$P(s) = \frac{s + m_0\left(1 - e^{-\tau s}\right)}{Ts + e^{-\vartheta s}}, Q = \frac{m_0}{K} \tag{9}$$

The final controller's structure is as follows:

$$G_R(s) = \frac{m_0\left(Ts + e^{-\vartheta s}\right)}{K\left(s + m_0\left(1 - e^{-\tau s}\right)\right)} \tag{10}$$

Note that the controller is of the anisochronic type because of a delay in the transfer function denominator. Naturally, it is possible to take $m(s)$ as a quasipolynomial - instead of the polynomial $m(s)$. This option would make a controller more complex. The importance of $m(s)$ reveals from the closed-loop transfer function:

$$G_{WY}(s) = \frac{Y(s)}{W(s)} = \frac{m_0 e^{-\tau s}}{s + m_0} \tag{11}$$

i.e. $m(s)$ appears as a characteristic (quasi)polynomial of the control loop. The obtained control structure can be easily compared with the well-known Smith predictor structure; see, e.g., Pekař and Gazdoš (2019).
The second-order delayed system (SODS) in this contribution is supposed in the form:

$$G(s) = \frac{Ke^{-\tau s}}{(Ts + 1)^2} \tag{12}$$

The control design for (SODS) given by Eq(3) takes the form:

$$(Ts + 1)^2 P_0(s) + Ke^{-\tau s} Q_0(s) = (s + m_0)^2 \tag{13}$$

By the choice of $Q_0(s) = 1$; the solution of Eq(13) is obtained as follows:

$$P_0(s) = \frac{(s + m_0)^2 - Ke^{-\tau s}}{(Ts + 1)^2} \tag{14}$$

and the general solution of Eq(4) is given by:

$$\frac{Q(s)}{P(s)} = \frac{1 + \dfrac{(Ts + 1)^2}{(s + m_0)^2} Z(s)}{\dfrac{(s + m_0)^2 - Ke^{-\tau s}}{(s + a_0)^2} - \dfrac{Ke^{-\tau s}}{(s + m_0)^2} Z(s)} \tag{15}$$

The choice for $Z(s)=\dfrac{\kappa(s+m_0)^2}{(Ts+1)^2}$ gives *P(s)* in a very simple form where *κ* is a real free parameter. By choosing $\kappa = (m_0^2/K) - 1$, Eq(15) gives the final form of the controller $G_R(s)$ as:

$$G_R(s)=\frac{m_0^2}{K}\frac{(Ts+1)^2}{s^2+2m_0s+m_0^2\left(1-e^{-\tau s}\right)} \tag{16}$$

One of the possible settings for m_0 can be found in Klán and Gorez (2003) for PID and PI controllers, as the "equalisation principle" or "balanced tuning". Since controller Eq(19) is in the quasipolynomial form, its denominator has an infinite number of poles. The construction of this controller is more complex than usual PI or PID controllers. However, modern PLC systems facilitate the use of advanced functions of so-called Anisochronic controllers Eq(10) and Eq(16).

3. Numerical examples

Regarding controllers Eq(10) and Eq(16), designed via a R_{QM} ring, the comparison of the m_0 setting can be done for the R_{PS} with balanced tuning. Consider the FODS approximating model:

$$G(s)=\frac{2}{3.45s+1}e^{-8.20s} \tag{17}$$

controlled by the controller given by Eq(10) first. It is necessary to approximate Eq(10) by the standard PI controller given by the transfer function:

$$G_{R,PI}(s)=K_C\left(1+\frac{1}{T_Is}\right) \tag{18}$$

where the controller gain K_C and its integral time constant T_I are to be set properly. Define the following functions:

$$F_1(G):=\lim_{s\to 0} sG(s) \qquad F_2(G):=\lim_{s\to\infty} G(s) \tag{19}$$

One can verify that -

$$\left.\begin{aligned}\frac{K_C}{T_I}&=F_1(G_{R,PI})\\ K_C&=F_2(G_{R,PI})\end{aligned}\right\}\Rightarrow T_I=\frac{F_2(G_{R,PI})}{F_1(G_{R,PI})} \tag{20}$$

By applying Eq(19) and Eq(20) to Eq(10), it is easy to ascertain that:

$$K_C=\frac{m_0T}{K} \qquad T_I=T \tag{21}$$

Rules for the PI controller Eq(18) according to the balanced tuning principle were derived by (Klán and Gorez, 2003)

$$K_C=\frac{1+(1-\Theta)^2}{2K_P} \qquad T_I=T_{ar}\frac{1+(1-\Theta)^2}{2} \tag{22}$$

where T_{ar} is the average residence time and the normalised delay reads $\Theta=\tau/T_{ar}$. It holds for a FODS that $T_{ar}=T+\tau$, see Pekař and Gazdoš (2019); that is $\Theta=L$.
By substituting K_C from Eq(22) into Eq(23), the tuning rule yields:

$$m_0=\frac{1+(1-\Theta)^2}{2T} \tag{23}$$

Unfortunately, the comparison of Eq(22) and Eq(23) for T_I gives no direct rule for m_0. This means that balanced tuning can be fully met only in a special case. For model Eq(17), one gets $m_0=0.16$ from Eq(23) - which is very close to the value obtained by Prokop et al. (2011) $m_0 = 0.15$. Control responses for this setting are displayed in Figure 2. Regarding the SODS approximating model:

$$G(s) = \frac{2}{(2.21s+1)^2} e^{-6.70s} \tag{24}$$

controlled by Eq(16). The functions given by Eq(19) cannot be used due to the derivative action. However, by neglecting the delay term in Eq(16) and by subsequent comparison with the standard (feasible) PID law:

$$G_{R,PID}(s) = K_C \left(1 + \frac{1}{T_I s} + \frac{T_D s}{\tau_F s + 1} \right) \tag{25}$$

where T_D is the derivative time constant and $\tau_F < T_D$ stands for a filtering constant.

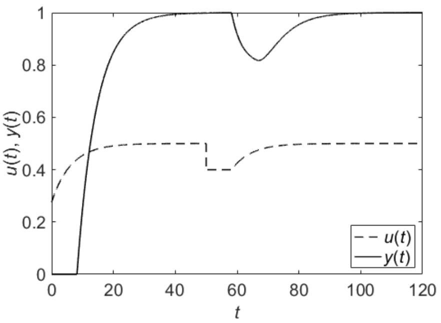

Figure 2: Control responses for Eq(17) and Eq(10) with $m_0 = 0.16$

The following estimation of the integral time constant can be derived $T_I = 2T - \frac{1}{2m_0}$, see Pekař and Gazdoš (2019) for further details. The point is balanced tuning attempt to reach the equality of the following two criteria:

$$ITAE := \int_0^\infty t|e(t)|\, dt \quad ITAD := T_I \int_0^\infty t|e(t)|\, dt \quad e(t) = \frac{de(t)}{dt} \tag{26}$$

The task is to find the value of m_0 such that the equality of the criteria in Eq(16) hold for T_I given by Eq(15). Numerical experiments for $d(t) = 0$, give $m_0 = 1.58$, for which the absolute difference between ITAE and ITAD is minimised - (ITAE = 32.1 vs. ITAD = 28.9). Note that if $d(t) = 0.1 \cdot \eta(t - 50)$ (where $\eta(\cdot)$ stands for the stepwise function), the optimal value reads $m_0 = 1.48$ (ITAE = 121.9 vs. ITAD = 98.9).
The comparison of control responses for Eq(24) and Eq(16) with $m_0 = 0.3$ and $m_0 = 1.58$ are given in Figure 3. Note that the initial control action is very high - ($u(0) = 6.1$) in the latter case.

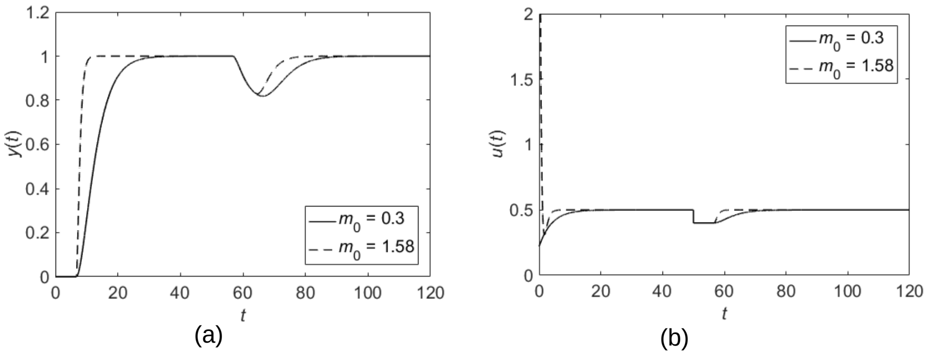

Figure 3: (a) Controlled output and (b) control action for Eq(24) and Eq(16) with $m_0 = 0.3$ *vs.* $m_0 = 1.58$

4. Conclusions

The contribution presents an algebraic Control Design for first and second-order systems with time delay. Control synthesis is performed through a solution of a Diophantine equation in the stable quasipolynomial meromorphic functions R_{QM} ring. This approach utilises quasipolynomials - and yields a class of Smith predictors as controllers (Smith, 1958). As a special case, a rational R_{PS} function approach using polynomials generates a class of generalised PID controllers. Both design methodologies represent the scalar tuning parameter $m_0 >$ 0; that can be adjusted by various strategies. The advantage of the proposed approach is its applicability to stable and unstable linear systems with retarded, neutral, lumped, and even distributed delays. No delay approximation, implies no information loss. On the contrary - nonlinear, time-varying, or delay-varying systems cannot be subject to this approach. A nontrivial task, which needs to be elaborated more in future research, is to set real parameter(s) value(s) such that the particular quasipolynomial is stable.
The proposed methodology has wide applications. The first one is in autotuning schemes as shown in Prokop et al. (2016). Autotuning schemes represent a combination with relay feedback estimation. Further utilization can be seen in processes in chemical and biochemical processes as shown in Vojtěšek et al. (2017). The class of delay systems can be frequently found in energy systems.

Acknowledgements

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