Nonsingular Decaying Vacuum Cosmology and Entropy Production within the Generalized Second Law of Thermodynamics

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Abstract: Within the framework of the running vacuum cosmology models, we solve Friedmann's equations and calculate the entropy of the apparent horizon, checking that the Generalized Second Law of thermodynamics is satisfied which implies to compute the contribution from both the usual entropy inside it as well as the contribution from its surface. We also show that the model solves the horizon problem and obtain constraints on the model's parameters through our thermodynamical analysis.

I. INTRODUCTION

The ΛCDM model, although standard in cosmology for years, leaves many things unexplained, such as the nature of dark energy and dark matter. Moreover, it's incompatible with the standard model of particles (both models famously give wildly different results for the vacuum energy) and has other inconveniences such as the horizon problem. As such, many extensions of the ΛCDM model have been proposed. One subset of these are the running vacuum models. In this paper we set out to study said models and their compatibility with the Generalized Second Law of thermodynamics.

Running vacuum models are models where we consider that the cosmological constant (and therefore, the vacuum energy density ρ_Λ) "runs" with the Hubble parameter H, as if it were a one-loop beta function from Quantum Field theory. This idea is perfectly compatible with the Cosmological Principle which only asks for homogeneity and isotropy of space, not time. Not only are such models are a suitable extension of the ΛCDM models (in the sense that we can easily recover the ΛCDM models from them), but they also describe an initial inflationary period and a "graceful exit" from such period into the radiation dominated epoch, while also being compatible with current observations (see [8]). Moreover, they solve the horizon problem, they provide an explanation for the current elevated value of entropy inside the observable universe and they comply with the second law of thermodynamics (thanks to a positive constant term in our expression for $\rho_\Lambda(H)$), as we'll see in this paper.

Our goal in this work will simply be to check all these claims regarding our running vacuum model and also obtain some constrains on their parameters through the thermodynamical analysis. First we will solve Friedman's equations, finding $H(a)$ both for the early and the current universe. We will find expressions for the total entropy of the universe (including the entropy associated to the horizon) given by this model and we'll give a brief discussion of the Generalized Second Law of thermodynamics, checking whether it holds for our model.

In what follows we use natural units $c = \hbar = k_B = 1$, where k_B is Boltzmann's constant. A dot over a variable (for example, $\dot{H}$) denotes differentiation with respect to the cosmic time t while H' refers to differentiation with respect to the scale factor a. The scale factor a is taken to be dimensionless and it is normalised to 1 for the current universe, $a(t_0) = 1$.

II. THE RUNNING VACUUM MODEL

Let us begin with a studying the running vacuum model, both for the early and the current universe. Our assumption is that the vacuum density ρ_Λ runs with H, which we write in the following manner:

$$\frac{d\rho_\Lambda(H)}{d\ln H^2} = \frac{1}{(4\pi)^2}\sum_i \left[a_i M_i^2 H^2 + b_i H^4\right] \tag{II.1}$$

The sum is over all the particles of mass M_i that contribute to the vacuum energy density and a_i, b_i are dimensionless coefficients.

Justifying this assumption using modern physics is not trivial (one can turn toward [2] and [5] for this) but allowing the cosmological constant to evolve with the universe can be considered more natural than having it be a fixed value during its whole history. Note that we not allow terms of H with odd powers such as H and H^3; this is preferred in order to be compatible with the global covariance of QFT (see [5]).

Let us solve equation II.1. Using the chain rule $\frac{d\rho_\Lambda}{dH} = \frac{d\rho_\Lambda}{d\ln H^2}\frac{2}{H}$, integrating with respect to H and defining $\kappa^2 \equiv 8\pi G, c_0 \equiv \frac{k_0}{3M_{Pl}^2}, \nu \equiv \sum_i \frac{a_i M_i^2}{48\pi^2 M_{Pl}^2}$ and $\alpha \equiv \sum_i \frac{b_i}{96\pi^2}\frac{H_I^2}{M_{Pl}^2}$ where k_0 is an integration constant, G is Newton's gravitational constant and $M_{Pl}^2 = \frac{1}{8\pi G}$ is the reduced plank mass in natural units we obtain:

$$\rho_\Lambda(H) = \frac{3}{\kappa^2}\left(c_0 + \nu H^2 + \alpha\frac{H^4}{H_I^2}\right) \tag{II.2}$$

We also introduced the constant H_I which we define to be the Hubble ratio at the scale of inflation in order to make ν and α into dimensionless coefficients. Looking at the running vacuum model from the perspective of any sensible GUT yields $|\nu|, |\alpha| \ll 1$ (see [4]). Moreover, observations seem to indicate $\nu \sim 10^{-3}$ (see [8]).

Let us solve Friedmann's equations for the current matter-dominated universe, where we consider the radiation energy density ρ_r to be 0. Also note that for an expanding universe consistent with observations $H(a)$ decreases with a, so for our current universe the dominating terms in $\rho_\Lambda(H)$ will be the ones with low powers of H. We will therefore set $\alpha = 0$. Setting $\nu = \alpha = 0$ would just give us back the ΛCDM model so we will refrain from doing so. If we renormalize ρ_Λ by setting $\rho_\Lambda(H_0) = \rho_{\Lambda 0}$ we obtain a value for c_0 which yields $\rho_\Lambda(H) = \rho_{\Lambda 0} + \frac{3\nu}{\kappa^2}(H^2 - H_0^2)$. Now we also want these equations to satisfy the restriction

$$\rho_m'(a) + \frac{3}{a}(1+w)\rho_m(a) = -\rho_\Lambda' \tag{II.3}$$

where ρ_m is the energy matter energy density, and $w = \frac{p}{\rho}$ is the equation of state. The matter energy density ρ_m includes a contribution for both CDM and baryons. As usual, we set $w_m = 0$ for matter, $w_r = \frac{1}{3}$ for radiation and $w_\Lambda = -1$ for vacuum, both running or static. Equation II.3 is obtained from taking the covariant derivative of the Einstein equations, which must be zero thanks to the Bianchi identity which says $\nabla^\mu G_{\mu\nu} = 0$ where $G_{\mu\nu}$ is the Einstein tensor.

Normally taking this derivative of the Einstein equations under the assumptions of the ΛCDM model is what yields the fact that Λ is constant, but here we consider $\Lambda(H)$, which instead yields two different possible interpretations: either the gravitational constant G is not constant or there is creation/ annihilation of matter through interaction with the vacuum energy density. In this paper, we ignore the former and focus on the latter, so G will be constant for us and we'll have matter production through the decaying of vacuum, as we'll see later. The case of a non-constant G is explored in [5].

We write Friedmann equations as follows:

$$3H^2 = \kappa^2(\rho_m + \rho_r + \rho_\Lambda) \tag{II.4}$$

$$3H^2 + 2\dot{H} = -\kappa^2 \sum_{i=m,r,\Lambda} p_i = \kappa^2(\rho_\Lambda - \frac{1}{3}\rho_r) \tag{II.5}$$

We'll use them in order to find an explicit expression for $\rho_\Lambda(a)$. For the current universe we can write equation II.4 as $H^2 = \kappa^2(\rho_m + \rho_\Lambda)$. Differentiating this with respect to a yields $\frac{1}{\kappa^2}\frac{dH^2}{da} = \rho_m' + \rho_\Lambda'$. Now differentiating our expression for ρ_Λ with respect to a yields $\rho_\Lambda'(a) = \frac{3\nu}{\kappa^2}\frac{d(H^2(a))}{da}$ and from these two previous expressions we obtain $\rho_\Lambda'(a) = \frac{\nu}{1-\nu}\rho_m'$.

Now inserting this in equation II.3 we obtain $0 = \frac{3(1-\nu)}{a}\rho_m + \rho_m'$ which is a differential equation only involving ρ_m. Its solution is

$$\rho_m(a) = \rho_{m0} a^{-3(1-\nu)} \tag{II.6}$$

Thanks to this last expression we can explicitly calculate $\rho_m'(a)$ and insert it in equation II.3 to obtain $\rho_\Lambda'(a) = 3\rho_{m0}\nu a^{-4+3\nu}$. Integrating with respect to to a and imposing that for our current time $\rho_\Lambda(1) = \rho_{\Lambda 0}$ we obtain

$$\rho_\Lambda(a) = \rho_{\Lambda 0} + \frac{\nu\rho_{m0}}{1-\nu}\left(a^{-3(1-\nu)} - 1\right) \tag{II.7}$$

Now if we define the density parameter $\Omega_{m0} \equiv \frac{\rho_{m0}}{\rho_c}$ where $\rho_{c0} = \frac{3H_0^2}{\kappa^2}$ is the critical energy density of the current universe (which is equal to the current total energy density $\rho_{m0} + \rho_{\Lambda 0}$ since we consider that our universe has no spatial curvature) and inserting equations II.6 and II.7 into Eq. II.4 we obtain

$$H^2(a) = H_0^2\left[\frac{\Omega_{m0}}{1-\nu}\left(a^{-3(1-\nu)} - 1\right) + 1\right] \tag{II.8}$$

Remarkably, setting $\nu = 0$ gives us back the expression for $H(a)$ in the ΛCDM model.

Now let us look at the early universe, for which H is large. We will therefore now consider the effects of the term $\alpha\frac{H^4}{H_I^2}$ in our expression for $\rho_\Lambda(H)$. However, we will neglect the term c_0 in favour of terms $\nu H^2, \alpha\frac{H^4}{H_I^2}$. We could also set $\nu = 0$ and most of the results and conclusions we would obtain would be very similar, but we'll keep it in order to perform a more general analysis. Also, for the early universe all our matter is relativistic, so we set $\rho_m = 0$.

In summary, we write equations II.4 and II.5 (Friedmann's equations) as

$$3H^2 = \kappa^2(\rho_r + \rho_\Lambda(H)) \tag{II.9}$$

$$3H^2 + 2\dot{H} = \kappa^2(\rho_\Lambda(H) - \frac{1}{3}\rho_r) \tag{II.10}$$

Using that for the early universe we consider $\rho_\Lambda(H) = \frac{3}{\kappa^2}\left(\nu H^2 + \alpha\frac{H^4}{H_I^2}\right)$ we can solve these equations to obtain $H(a)$ for the early universe. The easiest way to do this is to put our expression for ρ_Λ in equations II.9 and II.10, and then take a linear combination of equations II.9 and II.10 to eliminate the term ρ_r, obtaining $\dot{H} = 2\left((\nu-1)H^2 + \alpha\frac{H^4}{H_I^2}\right)$.

Now we would like to solve this equation to obtain $H(a)$ so we first need to transform $\dot{H}$ into H'. Doing this is simple, writing $H = \frac{\dot{a}}{a}$ and using the chain law $\frac{d}{dt} = \frac{da}{dt}\frac{d}{da}$ gives us $\dot{H} = H'\dot{a} = H'Ha$.

Using this in our differential equation gives us $H' = \frac{2}{a}\left((\nu-1)H + \alpha\frac{H^3}{H_I^2}\right)$, which is a differential equation involving only $H(a)$. Its solution is

$$H(a) = \sqrt{\frac{1-\nu}{\alpha}}\frac{H_I}{\sqrt{1 + Da^{4(1-\nu)}}} \tag{II.11}$$

Where D is an arbitrary integration constant, although we need it to be positive if we want H to increase with a. Note that we also need $\alpha > 0$ for this to make sense, so this will now be a further assumption of our model.

With this explicit expression and Friedmann's equations for the early universe it is easy to derive explicit expressions for the energy densities $\rho_r(a)$ and $\rho_\Lambda(a)$:

$$\rho_r(a) = \frac{3H_I^2(1-\nu)^2 D a^{4(1-\nu)}}{\kappa^2\alpha(1+Da^{4(1-\nu)})^2} \tag{II.12}$$

$$\rho_\Lambda(a) = \frac{3H_I^2(1-\nu)(1+\nu D a^{4(1-\nu)})}{\kappa^2\alpha(1+Da^{4(1-\nu)})^2} \tag{II.13}$$

We shall simplify these expressions. First let us note that there exists a_{eq} such that $\rho_r(a_{eq}) = \rho_\Lambda(a_{eq})$. We call this the equilibrium time and it signifies the time where the early universe stops being dominated by dark energy (and therefore, the rapid expansion period of inflation stops) and begins to be dominated by relativistic matter.

Imposing $\rho_r(a_{eq}) = \rho_\Lambda(a_{eq})$ in their expressions yields $D = \frac{1}{1-2\nu} a_{eq}^{-4(1-\nu)}$. Defining $a_* \equiv (1-2\nu)^{1/(4-4\nu)} a_{eq}$ simplifies this to $D = a_*^{-4(1-\nu)}$.

We also define $\tilde{H}_I \equiv \sqrt{\frac{1-\nu}{\alpha}} H_I$ and $\hat{a} \equiv \frac{a}{a_*}$ so that $Da^{4(1-\nu)} = a_*^{-4(1-\nu)} a^{4(1-\nu)} = \hat{a}^{4(1-\nu)}$. This allows us to write the equation II.11 as

$$H(\hat{a})^2 = \frac{\tilde{H_I}^2}{1+\hat{a}^{4(1-\nu)}} \tag{II.14}$$

By defining $\tilde{\rho}_I \equiv \frac{3}{\kappa^2}\tilde{H}_I^2$ we can also simplify equations II.12 and II.13, writing them as

$$\rho_r(\hat{a}) = \tilde{\rho}_I \frac{(1-\nu)\hat{a}^{4(1-\nu)}}{[1+\hat{a}^{4(1-\nu)}]^2} \tag{II.15}$$

$$\rho_\Lambda(\hat{a}) = \tilde{\rho}_I \frac{1+\nu\hat{a}^{4(1-\nu)}}{\left[1+\hat{a}^{4(1-\nu)}\right]^2} \tag{II.16}$$

Let us have a closer look at these equations. For small a, $H = \tilde{H}_I$ is approximately constant and very large, and there is no radiation energy density but there is a big contribution from vacuum energy density, which leads to an accelerated expansion of the early universe; inflation. As a increases so does ρ_r while ρ_Λ decreases, signifying the decay of vacuum energy into relativistic particles. Then after an inflection point (at around a_{eq}) ρ_r decreases again with the expansion of the universe; particle creation has mostly stopped and we have entered the radiation epoch. We call this smooth transition the "graceful exit" from the inflation period. We'll see later that most entropy is created during inflation, becoming almost constant afterwards.

III. ENTROPY GENERATION

Let us look at the entropy generation associated to the running vacuum model. There are two different ways to do this. We could take as our thermodynamical system a set volume and let it evolve with the expansion of the universe (we call such volume the comoving volume) and calculate the entropy inside it. The other alternative is considering the entropy within the apparent horizon which is a type of cosmic horizon that we'll define later. The latter seems to make more physical sense, but it is not yet clear which one gives the correct thermodynamical description of the universe, as it is not clear whether the universe can be treated as a macroscopical thermodynamical system.

The entropy of the comoving volume can be easily calculated but we won't do that here, one can find the calculations in §5 and §6 of [1]. Instead we will focus on the apparent horizon and the volume it encloses, and we will calculate its entropy both for the early and late universes.

Let us talk about cosmological horizons. The particle cosmological horizon is the maximal distance from which light has had time to reach the observer since the beginning of the universe. The event horizon is the maximal distance from which light will ever reach the observer, until the end of the universe. More suitable to our thermodynamical considerations and different from the previous two is the apparent horizon l_h, whose definition can be found in [10]. For us, the apparent horizon for a universe with no spatial curvature will be $l_h = \frac{1}{H}$.

The Generalized Second Law of thermodynamics says we need to take into consideration not only the entropy caused by the particles inside the horizon but also the entropy of the horizon itself which follows the equation $S_\mathcal{A} = \frac{A}{4l_P^2}$ where A is the area of the horizon we are considering and $l_P = \sqrt{G}$ is Planck's length. This formula, initially intended to express the entropy of a black hole, was derived by Bekenstein and Hawking in 1974 (see [9]).

We are considering a spherical volume of radius l_h, the horizon. So the area will be $A = 4\pi l_h^2$. Using the Planck mass $M_P = \frac{1}{G}$ we write $S_\mathcal{A} = \frac{A}{4G} = \pi l_h^2 M_P^2$.

Now if we use $l_h = \frac{1}{H}$ we obtain a simple expression to calculate $S_\mathcal{A}$ depending solely on H: $S_\mathcal{A}(a) = \pi\frac{M_P^2}{H(a)^2}$. Using equations II.14 and II.8 we obtain

$$S_\mathcal{A}(\hat{a}) = \frac{\pi M_P^2[1+\hat{a}^{4(1-\nu)}]}{\tilde{H}_I^2} \tag{III.1}$$

$$S_\mathcal{A}(a) = \frac{\pi M_P^2}{H_0^2\left[1+\frac{\Omega_{m0}}{1-\nu}(a^{-3(1-\nu)}-1)\right]} \tag{III.2}$$

for the horizon entropy of the early and current universe, respectively.

For the current universe the entropy of the volume inside the horizon will just be the entropy associated to the particles, as there is no radiation. Since we consider these particles to be dust, their only entropy is the one associated to its existence. Let σ be the entropy per particle. We consider it to be constant, which just means that we consider that particles are created in thermodynamical equilibrium with surrounding particles. The entropy of the volume will be σ times the number of particles N. We write $N = nV$ where V is the volume we are considering (a sphere of radius l_h, so $V = \frac{4}{3}\pi l_h^3$) and n is the particle density. Writing again $l_h = \frac{1}{H}$ and setting $\sigma = k_B = 1$ (we set σ to be of the order of a unit of entropy k_B) we obtain $S_{\mathcal{V}}(a) = n(a)\frac{4\pi}{3H(a)^3}$, where n depends on a since we are considering the creation of particles through the decaying vacuum energy.

Let us now find an expression for $n(a)$. The energy of a dust particle is just its rest mass $mc^2 = m$ in natural units. The energy density is then the energy of a particle times the density of particles, $nm = \rho_m = \rho_{m0}a^{-3(1-\nu)}$. Defining $n_0 = \frac{\rho_0}{m_0}$ we obtain $n(a) = n_0 a^{-3(1-\nu)}$. Note that we could also have chosen to work with a constant number of particles but considering $m(a)$, but we will not do this on this paper. Using this expression we obtain

$$S_{\mathcal{V}}(a) = \frac{4\pi n_0 a^{-3(1-\nu)}(1-\nu)^{\frac{3}{2}}}{3H_0^3\left(1-\nu+\Omega_{m0}(a^{-3(1-\nu)}-1)\right)^{\frac{3}{2}}} \qquad \text{(III.3)}$$

for the current universe, where we used equation II.8. For the early universe, the entropy inside the horizon is the one corresponding to radiation, since we only have relativistic particles in our enclosed volume.

For this and since we have thermal equilibrium we'll use the well known entropy formula $S = V\frac{(p+\rho)}{T}$ (see the first appendix of [1]). Since for radiation $w_r = \frac{1}{3}$ this becomes $S_{\mathcal{V}} = \frac{4}{3}V\frac{\rho_r}{T_r}$, where T_r is the radiation temperature, shared by all particles since we suppose thermodynamical equilibrium in the volume inside the horizon, which is causally connected. Now we'll use the also well known relationship $\rho_r = \frac{\pi^2}{30}g_* T_r^4$ where g_* counts the massless degrees of freedom (see [3]). We use this last expression to eliminate T_r from our expression for $S_{\mathcal{V}}$, which becomes $S_{\mathcal{V}}(a) = \frac{4}{3}\left(\frac{\pi^2 g_*}{30}\right)^{1/4}\frac{4\pi}{3}\frac{\rho_r(a)^{3/4}}{H(a)^3}$.

If we include our expressions for $H(a)$ and $\rho_r(a)$ for the early universe (equations II.14 and II.15) and define a characteristic temperature for the inflation period through the relation between temperature and radiation energy density $\tilde{\rho}_I^{3/4} \equiv \left(\frac{\pi^2 g_*}{30}\right)^{3/4}\tilde{T}_I^3$ we obtain

$$S_{\mathcal{V}}(\hat{a}) = \frac{8\pi^3}{135}g_*\left(\frac{\tilde{T}_I}{\tilde{H}_I}\right)^3(1-\nu)^{\frac{3}{4}}\hat{a}^{3(1-\nu)} \qquad \text{(III.4)}$$

The Generalized Second Law of thermodynamics says that the total entropy will simply be $S_{total} = S_{\mathcal{A}} + S_{\mathcal{V}}$. We have explicit expressions for both terms depending on or a for the early universe (equations III.1 and III.4) and for the current universe (equations III.2 and III.3).

A quick look at these expressions shows that in both the early and the late universe the dominating term is $S_{\mathcal{A}}$ while $S_{\mathcal{V}}$ plays no major role in entropy. We note that for the early universe the entropy rapidly increases at the beginning ($\sim \hat{a}^4$), but then arrives at a plateau of stability in the current universe where it asymptotically approaches a constant value while slowly rising (see figure 1). This behaviour actually solves the entropy horizon problem as most entropy is generated during inflation

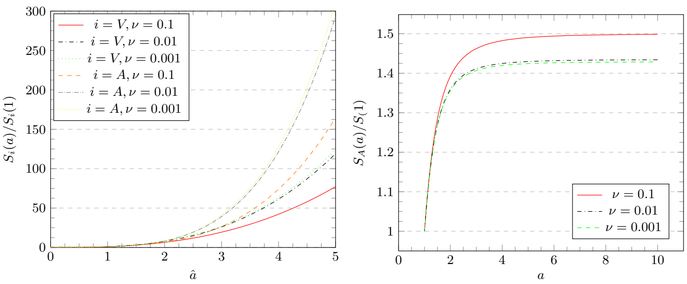

FIG. 1: Evolution of the horizon entropy and the entropy inside it with respect to $\hat{a}$ in the early universe (left) and evolution of the horizon entropy with respect to a in the current universe (right), both at different values of ν, normalized with respect to its value at $\hat{a} = 1$ and $a = 1$ respectively. In the early universe, note the fast increase and dominance of the horizon entropy, especially for small ν. In the current universe, note that the total entropy (the contribution of the entropy inside the horizon is negligible) eventually stabilizes.

where we have no particle horizon (since we have accelerated expansion) and therefore the universe is causally connected. Moreover, this yields an explanation of the high value of entropy observed in our universe, and the theoretical value we obtain is compatible with observations (see [1]).

Let us now check whether the second law of thermodynamics is satisfied. Normally one would write this as $\dot{S} \geq 0$ but that is not the full description of the second law: it is not enough that entropy increases; we also must ask that entropy eventually stops increasing in order to reach thermodynamical equilibrium in a system. Therefore, one also needs that eventually $\ddot{S} < 0$. For a perfect fluid of four-velocity u^α and entropy is $s^\alpha = n\sigma u^\alpha$ this law is expressed as $\nabla_\alpha s^\alpha \geq 0$.

Now we have that $\nabla_\alpha s^\alpha = u^\alpha \partial_\alpha s + s\nabla_\alpha u^\alpha = \dot{s} + 3Hs = \frac{1}{a^3}\frac{d(sa^3)}{dt} = \frac{1}{a^3}\frac{dS}{dt}$ since we are in the frame of our fluid where $u^\alpha = (1, \vec{0})$ and since $\nabla_\alpha u^\alpha = 3H$. Since a and $\dot{a}$ are always positive, $\nabla_\alpha s^\alpha \geq 0$ is equivalent to $S' \geq 0$. The second condition then reads $S'' < 0$. Doing this for the comoving volume one checks that both conditions are satisfied if $\nu > 0$ so thermodynamically a positive ν is preferred, which agrees with observations (see [8]).

Since $S_{total} = S_{\mathcal{V}} + S_{\mathcal{A}}$ and we have expressions for both terms in both the early and the current universe, we can easily differentiate them. We won't write these unhelpful long expressions for S'_{total} and S''_{total} here (one can find them in §7 of [1]) but one can check that for the current universe $S'_{total} > 0$ and for a big enough a, $S''_{total} < 0$ (see figure 2). In fact we only have this second condition as the universe transitions into its final de Sitter phase dominated by dark energy, so we can only obtain $S''_{total} < 0$ if we have a positive cosmological constant, $c_0 > 0$.

For the early universe we differentiate with respect to $\hat{a}$ (since it is more convenient and it is equivalent since $\frac{d\hat{a}}{da} > 0$) and we obtain $S'_{total} > 0$ as expected, but also $S''_{total} >$

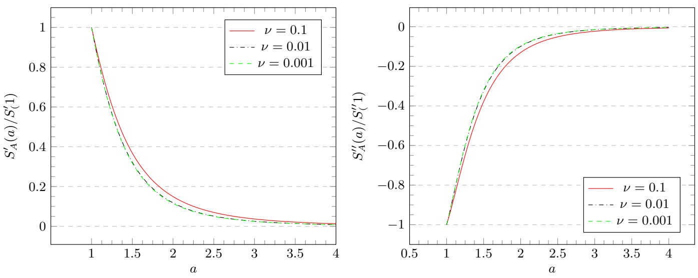

FIG. 2: Evolution of the first (left) and second (right) derivatives of the horizon entropy in the current universe with respect to a, normalized with respect to its value $S(a=1)$ and taken at different values for ν. Note that the first derivative is positive while the second is negative, as expected.

0. This doesn't contradict the Generalized Second Law which only says that S''_{total} must eventually be positive instead of always positive, and as we've seen it eventually becomes positive in the current universe. For a discussion of this see §7 of [1].

IV. CONCLUSIONS

- We obtained equations describing the Hubble parameter $H(a)$ (Eq. II.8), the matter energy density $\rho_m(a)$ (Eq. II.6) and the vacuum energy density $\rho_\Lambda(a)$ (Eq. II.7) within the running vacuum model $\rho_\Lambda(H) = \frac{3}{\kappa^2}\left(c_0 + \nu H^2 + \alpha H^4/H_I^2\right)$ and we showed that we get back the equations of ΛCDM by setting $\alpha = \nu = 0$.

- Moreover, we found expressions for $H(a)$ (Eq. II.14), $\rho_\Lambda(a)$ (Eq. II.16) and for the radiation energy density $\rho_r(a)$ (Eq. II.15) for the early universe for this model and we've seen that thanks to the term H^4 on $\rho_\Lambda(H)$ we obtain an early inflation period (and a graceful exit from it) which solves the horizon problem of the ΛCDM model.

- The Generalized Second Law of thermodynamics says we need to take into account the entropy of horizon, and therefore we found an expression for entropy, both for the apparent horizon and for the volume inside it in the current and early universe (see equations III.1, III.2 III.4 and III.3) and we studied them (see figure 1).In both cases the total entropy was dominated by the horizon term.

- We showed that this inflation period we obtained causes a rapid increase in entropy for the early universe $\sim \hat{a}^4$ (see Eq. III.2) that explains the currently observed elevated value and solves the entropy horizon problem.

- Finally we discussed the behaviour of S'_{total} and S''_{total} (see figure 2). They were both positive in the early universe and we had $S'_{total} > 0$ and $S''_{total} < 0$ for the current universe. This doesn't contradict the second law of thermodynamics, since we don't ask that $S'' < 0$ at every point in the evolution of a system; we only need that S'' eventually becomes negative, as it does for the current universe. Since S''_{total} would not tend to a negative value as the scale factor a tends to infinity if there weren't a positive cosmological constant (that is, if $c_0 = 0$) we conclude that this is a necessity for the universe to comply with the laws of thermodynamics, both within the running vacuum model and the ΛCDM model since the former is just an extension of the latter.

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[1] J. Solà and H. Yu, "Particle and entropy production in the Running Vacuum Universe". Gen. Rel. and Grav. **180** (2020) 17.
[2] J. Solà and C. Moreno-Pulido. "Running vacuum in quantum field theory in curved spacetime: renormalizing ρ_{vac} without $\sim m^4$ terms". [arXiv:2005.03164v2].
[3] E. W. Kolb and M. S. Turner, *The Early Universe*, (Addison-Wesley Publishing Company, Redwood City, 1990).
[4] J. Solà, "Dark energy: a quantum fossil from the inflationary Universe?". J. Phys. **A 41** (2008) 164066.
[5] J. Solà, "Cosmological constant and vacuum energy: old and new ideas". J. Phys. Conf. Ser. **453** (2013) 012015.
[6] J. A. S. Lima, S. Basilakos and J. Solà. "Expansion history with decaying vacuum: a complete cosmological scenario". MNRAS **431** (2013) 923.
[7] J. Solà,"The $\bar{\Lambda}$CDM cosmology: from inflation to dark energy through running Λ". Int. J. Mod. Phys. **D 24**, (2015) 1544027.
[8] J. Solà, J. de Cruz Pérez and A. Gómez-Valent, "Possible signals of vacuum dynamics in the Universe". MNRAS **478** (2018) 4357.
[9] J.D. Bekenstein, "Black holes and entropy". Phys. Rev. **D7** (1973) 2333; "Generalized second law of thermodynamics in black-hole physics". Phys. Rev. **9** (1974) 3292.
[10] D. Bak and S. J. Rey, "Cosmic Holography". Class. Quant. Grav. **17** (2000) L83.